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# Code Optimization

18-213/18-613: Introduction to Computer Systems
26th Lecture, April 22, 2025

# Performance Realities

- ***There's more to performance than asymptotic complexity***
- **Constant factors matter too!**
  - Easily see 10:1 performance range depending on how code is written
  - Must optimize at multiple levels:
    - algorithm, data representations, procedures, and loops
- **Must understand system to optimize performance**
  - How programs are compiled and executed
  - How modern processors + memory systems operate
  - How to measure program performance and identify bottlenecks
  - How to improve performance without destroying code modularity and generality

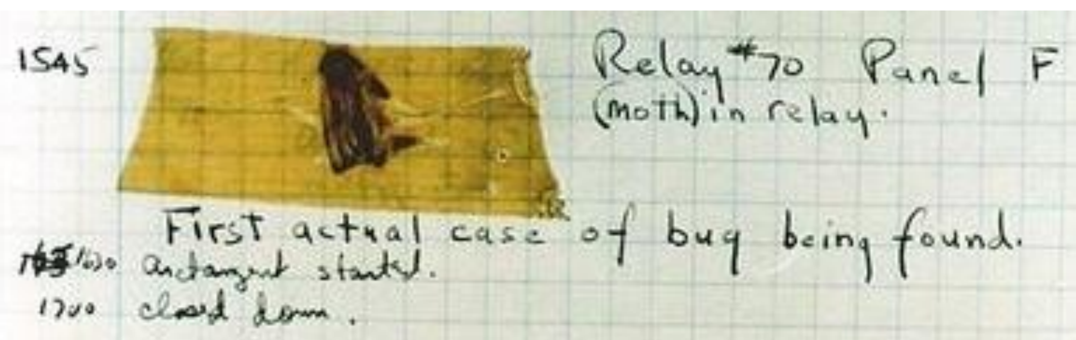
# Optimizing Compilers

- **Provide efficient mapping of program to machine**
  - register allocation
  - code selection and ordering (scheduling)
  - dead code elimination
  - eliminating minor inefficiencies
- **Don’t (usually) improve asymptotic efficiency**
  - up to programmer to select best overall algorithm
  - big-O savings are (often) more important than constant factors
    - but constant factors also matter
- **Have difficulty overcoming “optimization blockers”**
  - potential procedure side-effects
  - potential memory aliasing

# Today

- **Generally Useful Optimizations** CSAPP 5.1
  - Code motion/precomputation
  - Strength reduction
  - Sharing of common subexpressions
  - Example: Bubblesort
- **Optimization Blockers** CSAPP 5.1
  - Procedure calls
  - Memory aliasing
- **Exploiting Instruction-Level Parallelism** CSAPP 5.2-5.10
- **Dealing with Conditionals** CSAPP 5.11

- **Rear Admiral Grace Hopper (1906-1992)**
  - **Invented first compiler in 1951 (technically it was a linker)**
  - **Coined "compiler" (and "bug")**
  - **Compiled for Harvard Mark I**
  - **Eventually led to COBOL (which ran the world for years)**
  - **"I decided data processors ought to be able to write their programs in English, and the computers would translate them into machine code"**




# Generally Useful Optimizations

- **Optimizations that you or the compiler should do regardless of processor / compiler**

- **Code Motion**
  - Reduce frequency with which computation performed
    - If it will always produce same result
    - Especially moving code out of loop

```
void set_row(double *a, double *b,
   long i, long n)
{
    long j;
    for (j = 0; j < n; j++)
         a[n*i+j] = b[j];
}
```

→

```
    long j;
    int ni = n*i;
    for (j = 0; j < n; j++)
         a[ni+j] = b[j];
```

# Compiler-Generated Code Motion (-O1)

```
void set_row(double *a, double *b,
   long i, long n)
{
    long j;
    for (j = 0; j < n; j++)
         a[n*i+j] = b[j];
}
```

```
    long j;
    long ni = n*i;
    double *rowp = a+ni;
    for (j = 0; j < n; j++)
         *rowp++ = b[j];
```

```
set_row:
        testq    %rcx, %rcx                 # Test n
        jle      .L1                        # If <= 0, goto done
        imulq    %rcx, %rdx                 # ni = n*i
        leaq     (%rdi,%rdx,8), %rdx        # rowp = A + ni*8
        movl     $0, %eax                   # j = 0
.L3:                                        # loop:
        movsd    (%rsi,%rax,8), %xmm0       # t = b[j]
        movsd    %xmm0, (%rdx,%rax,8)       # M[A+ni*8 + j*8] = t
        addq     $1, %rax                   # j++
        cmpq     %rcx, %rax                 # j:n
        jne      .L3                        # if !=, goto loop
.L1:                                        # done:
        rep ; ret
```

# Strength Reduction

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide

  `16*x` → `x << 4`

  - Utility is machine dependent
  - Depends on cost of multiply or divide instruction
    - Intel Nehalem: integer multiply takes 3 CPU cycles, add is 1 cycle[1]
- Recognize sequence of products

```
for (i = 0; i < n; i++) {
  int ni = n*i;
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
}
```

→

```
int ni = 0;
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
  ni += n;
}
```

[1]https://www.agner.org/optimize/instruction_tables.pdf

# Share Common Subexpressions

- Reuse portions of expressions
- GCC will do this with –O1

```
/* Sum neighbors of i,j */
up =    val[(i-1)*n + j  ];
down =  val[(i+1)*n + j  ];
left =  val[i*n     + j-1];
right = val[i*n     + j+1];
sum = up + down + left + right;
```

**3 multiplications: `i*n`, `(i–1)*n`, `(i+1)*n`**

```
leaq   1(%rsi), %rax   # i+1
leaq   -1(%rsi), %r8   # i-1
imulq  %rcx, %rsi      # i*n
imulq  %rcx, %rax      # (i+1)*n
imulq  %rcx, %r8       # (i-1)*n
addq   %rdx, %rsi      # i*n+j
addq   %rdx, %rax      # (i+1)*n+j
addq   %rdx, %r8       # (i-1)*n+j
...
```

```
long inj = i*n + j;
up =    val[inj - n];
down =  val[inj + n];
left =  val[inj - 1];
right = val[inj + 1];
sum = up + down + left + right;
```

**1 multiplication: `i*n`**

```
imulq   %rcx, %rsi   # i*n
addq    %rdx, %rsi   # i*n+j
movq    %rsi, %rax   # i*n+j
subq    %rcx, %rax   # i*n+j-n
leaq    (%rsi,%rcx), %rcx # i*n+j+n
...
```

# Optimization Example: Bubblesort

- **Bubblesort program that sorts an array `A` that is allocated in static storage:**
  - an element of `A` requires four bytes
  - elements of `A` are numbered 1 through `n` (`n` is a variable)
  - `A[j]` is in location `&A+4*(j-1)`

```
for (i = n-1; i >= 1; i--) {
  for (j = 1; j <= i; j++)
    if (A[j] > A[j+1]) {
      temp = A[j];
      A[j] = A[j+1];
      A[j+1] = temp;
    }
}
```

# Translated (Pseudo) Code

```
     i := n-1
L5:  if i<1 goto L1
     j := 1
L4:  if j>i goto L2
     t1 := j-1
     t2 := 4*t1
     t3 := A[t2]     // A[j]
     t4 := j+1
     t5 := t4-1
     t6 := 4*t5
     t7 := A[t6]     // A[j+1]
     if t3<=t7 goto L3
```

```
for (i = n-1; i >= 1; i--) {
  for (j = 1; j <= i; j++)
    if (A[j] > A[j+1]) {
      temp = A[j];
      A[j] = A[j+1];
      A[j+1] = temp;
    }
}
```

```
     t8 := j-1
     t9 := 4*t8
     temp := A[t9]    // temp:=A[j]
     t10 := j+1
     t11:= t10-1
     t12 := 4*t11
     t13 := A[t12]    // A[j+1]
     t14 := j-1
     t15 := 4*t14
     A[t15] := t13    // A[j]:=A[j+1]
     t16 := j+1
     t17 := t16-1
     t18 := 4*t17
     A[t18]:=temp     // A[j+1]:=temp
L3:  j := j+1
     goto L4
L2:  i := i-1
     goto L5
L1:
```

**Instructions**

**29 in outer loop**

**25 in inner loop**

# Redundancy in Address Calculation

```
      i  := n-1
L5:   if i<1 goto L1
      j  := 1
L4:   if j>i goto L2
      t1  := j-1
      t2  := 4*t1
      t3  := A[t2]     // A[j]
      t4  := j+1
      t5  := t4-1
      t6  := 4*t5
      t7  := A[t6]     // A[j+1]
      if t3<=t7 goto L3
```

```
     t8  :=j-1
     t9  := 4*t8
     temp  := A[t9]    // temp:=A[j]
     t10  := j+1
     t11:= t10-1
     t12 := 4*t11
     t13  := A[t12]    // A[j+1]
     t14  := j-1
     t15  := 4*t14
     A[t15]  := t13    // A[j]:=A[j+1]
     t16  := j+1
     t17  := t16-1
     t18  := 4*t17
     A[t18]:=temp      // A[j+1]:=temp
L3:  j  := j+1
     goto L4
L2:  i  := i-1
     goto L5
L1:
```

# Redundancy Removed

```
     i := n-1
L5:  if i<1 goto L1
     j := 1
L4:  if j>i goto L2
     t1 := j-1
     t2 := 4*t1
     t3 := A[t2]      // A[j]
     t6 := 4*j
     t7 := A[t6]      // A[j+1]
     if t3<=t7 goto L3
```

```
     t8 :=j-1
     t9 := 4*t8
     temp := A[t9]   // temp:=A[j]
     t12 := 4*j
     t13 := A[t12]   // A[j+1]
     A[t9]:= t13     // A[j]:=A[j+1]
     A[t12]:=temp    // A[j+1]:=temp
L3:  j := j+1
     goto L4
L2:  i := i-1
     goto L5
L1:
```

**Instructions**

**20 in outer loop**

**16 in inner loop**

# More Redundancy

```
    i  := n-1
L5: if i<1 goto L1
    j  := 1
L4: if j>i goto L2
    t1  := j-1
    t2  := 4*t1
    t3  := A[t2]     // A[j]
    t6  := 4*j
    t7  := A[t6]     // A[j+1]
    if t3<=t7 goto L3
```

```
    t8  :=j-1
    t9  := 4*t8
    temp  := A[t9]  // temp:=A[j]
    t12  := 4*j
    t13  := A[t12]  // A[j+1]
    A[t9]:= t13     // A[j]:=A[j+1]
    A[t12]:=temp    // A[j+1]:=temp
L3: j  := j+1
    goto L4
L2: i  := i-1
    goto L5
L1:
```

# Redundancy Removed

```
     i  := n-1
L5:  if i<1 goto L1
     j  := 1
L4:  if j>i goto L2
     t1  := j-1
     t2  := 4*t1
     t3  := A[t2]      // old_A[j]
     t6  := 4*j
     t7  := A[t6]      // A[j+1]
     if t3<=t7 goto L3
```

```
     A[t2]  := t7     // A[j]:=A[j+1]
     A[t6]  := t3     // A[j+1]:=old_A[j]
L3:  j  := j+1
     goto L4
L2:  i  := i-1
     goto L5
L1:
```

**Instructions**
**15 in outer loop**
**11 in inner loop**

# Redundancy in Loops

```
     i := n-1
L5:  if i<1 goto L1
     j := 1
L4:  if j>i goto L2
     t1 := j-1
     t2 := 4*t1
     t3 := A[t2]     // A[j]
     t6 := 4*j
     t7 := A[t6]     // A[j+1]
     if t3<=t7 goto L3
     A[t2] := t7
     A[t6] := t3
L3:  j := j+1
     goto L4
L2:  i := i-1
     goto L5
L1:
```

# Redundancy Eliminated

```
     i  := n-1
L5:  if i<1 goto L1
     j  := 1
L4:  if j>i goto L2
     t1  := j-1
     t2  := 4*t1
     t3  := A[t2]      // A[j]
     t6  := 4*j
     t7  := A[t6]      // A[j+1]
     if t3<=t7 goto L3
     A[t2]  := t7
     A[t6]  := t3
L3:  j  := j+1
     goto L4
L2:  i  := i-1
     goto L5
L1:
```

```
     i  := n-1
L5:  if i<1 goto L1
     t2  := 0
     t6  := 4
     t19  := 4*i
L4:  if t6>t19 goto L2
     t3  := A[t2]
     t7  := A[t6]
     if t3<=t7 goto L3
     A[t2]  := t7
     A[t6]  := t3
L3:  t2  := t2+4
     t6  := t6+4
     goto L4
L2:  i  := i-1
     goto L5
L1:
```

# Final Pseudo Code (after strength reduction)

```
     i  := n-1
L5:  if i<1 goto L1
     t2  := 0
     t6  := 4
     t19  := i << 2
L4:  if t6>t19 goto L2
     t3  := A[t2]
     t7  := A[t6]
     if t3<=t7 goto L3
     A[t2]  := t7
     A[t6]  := t3
L3:  t2  := t2+4
     t6  := t6+4
     goto L4
L2:  i  := i-1
     goto L5
L1:
```

**Instructions**
**<u>Before Optimizations</u>**
**29 in outer loop**
**25 in inner loop**

**Instructions**
**<u>After Optimizations</u>**
**15 in outer loop**
**9 in inner loop**

- These were **Machine-Independent Optimizations**.
- Will be followed by **Machine-Dependent Optimizations**, including allocating temporaries to registers, converting to assembly code

# Today

- **Generally Useful Optimizations**
  - Code motion/precomputation
  - Strength reduction
  - Sharing of common subexpressions
  - Example: Bubblesort
- **Optimization Blockers**
  - Procedure calls
  - Memory aliasing
- **Exploiting Instruction-Level Parallelism**
- **Dealing with Conditionals**

- **John Backus** (1924-2007)
  - Led team at IBM invented the first commercially available compiler in 1957
  - Compiled FORTRAN code for the IBM 704 computer
  - FORTRAN still in use today for high performance code
  - "Much of my work has come from being lazy. I didn't like writing programs, and so, when I was working on the IBM 701, I started work on a programming system to make it easier to write programs"

# Limitations of Optimizing Compilers

- **Operate under fundamental constraint**
  - **Must not cause any change in program behavior**
  - Often prevents optimizations that affect only "edge case" behavior
- **Behavior obvious to the programmer is not obvious to compiler**
  - e.g., Data range may be more limited than types suggest (short vs. int)
- **Most analysis is only within a procedure**
  - Whole-program analysis is usually too expensive
  - Sometimes compiler does interprocedural analysis **within** a file (new GCC)
- **Most analysis is based only on *static* information**
  - Compiler has difficulty anticipating run-time inputs
- **When in doubt, the compiler must be conservative**

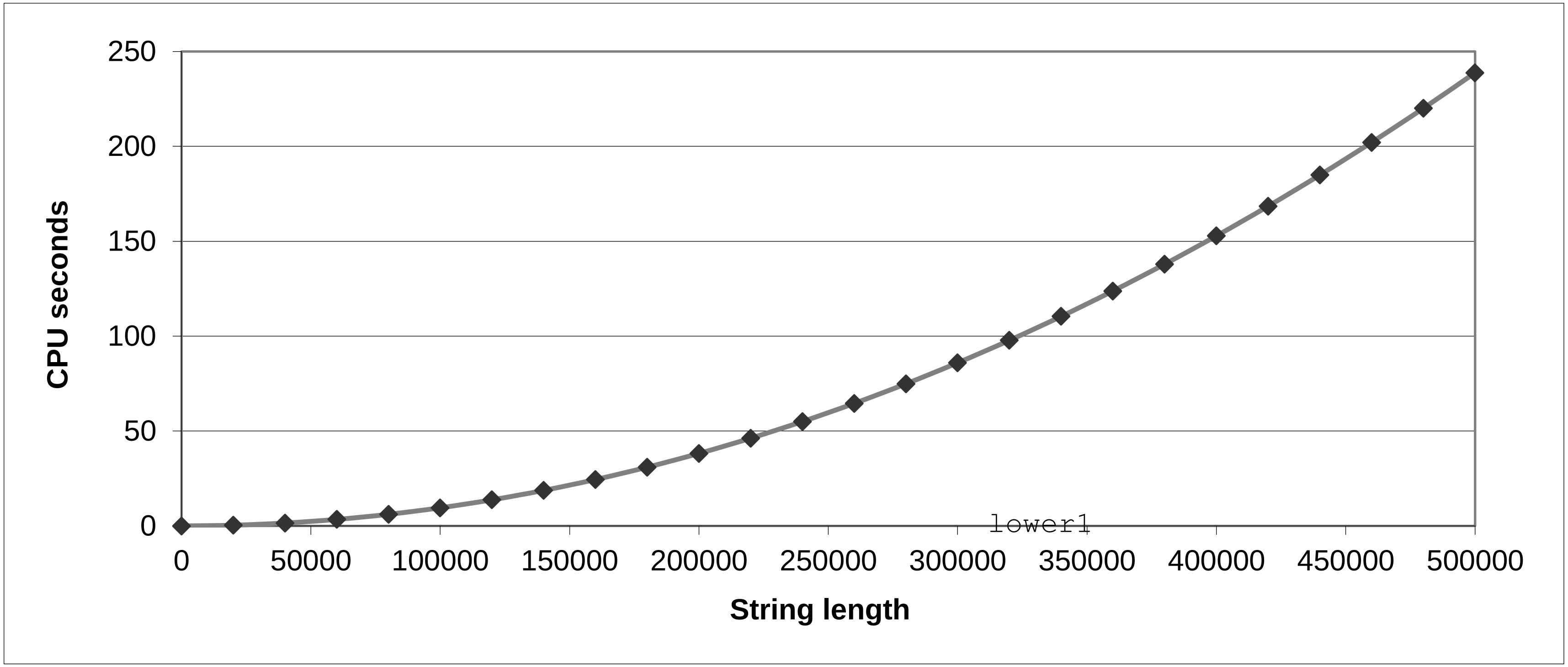
# Optimization Blocker #1: Procedure Calls

- **Procedure to Convert String to Lower Case**

```
void lower1(char *s)
{
  size_t i;
  for (i = 0; i < strlen(s); i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

  - Extracted from 213 lab submissions

# Lower Case Conversion Performance




- Time quadruples when double string length
- Quadratic performance

# Calling Strlen

```
void lower1(char *s)
{
  size_t i;
  for (i = 0; i < strlen(s); i++)
    if (s[i] >= 'A' &&
        s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

```
/* My version of strlen */
size_t strlen(const char *s)
{
    size_t length = 0;
    while (*s != '\0') {
       s++;
       length++;
    }
    return length;
}
```

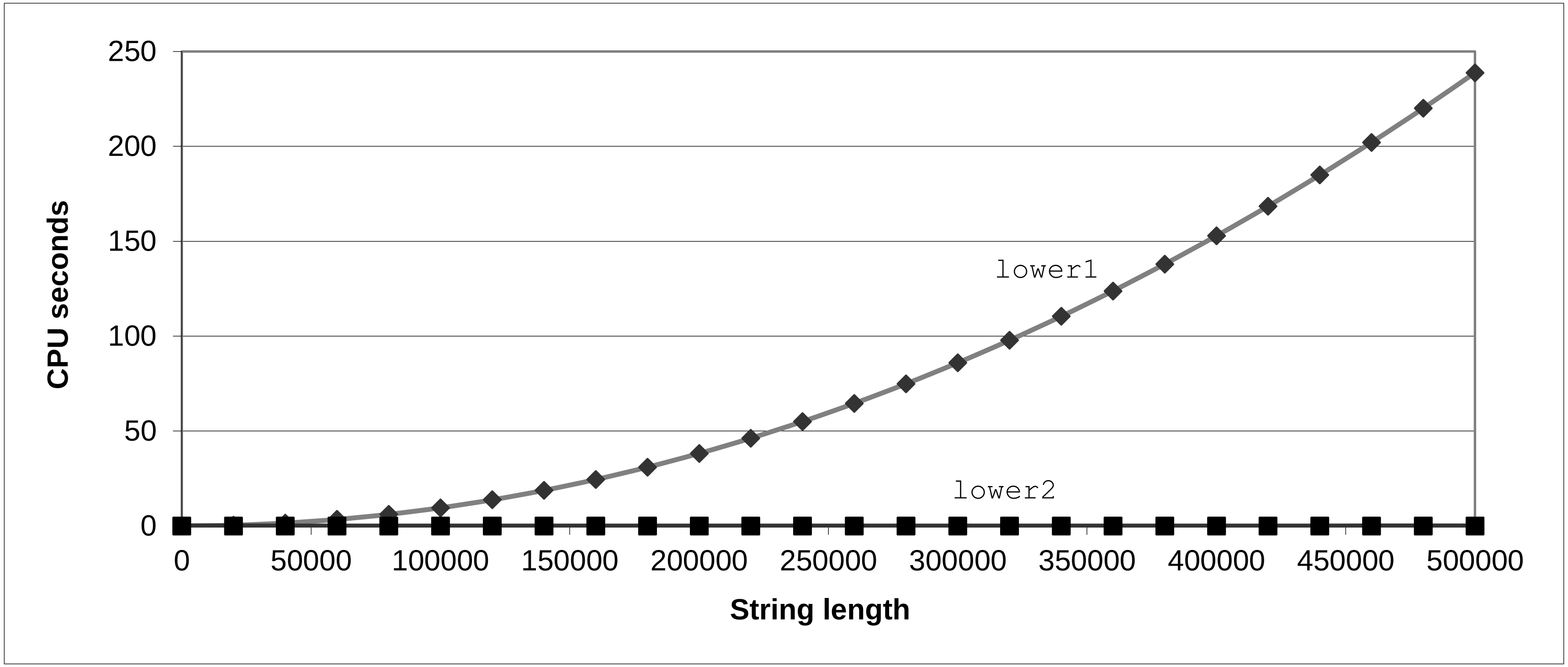
- **Strlen performance**
  - Only way to determine length of string is to scan its entire length, looking for null character.
- **Overall performance, string of length N**
  - N calls to strlen (called every time through the loop)
  - Require times N, N-1, N-2, …, 1
  - Overall O($N^2$) performance

# Improving Performance

```
void lower2(char *s)
{
  size_t i;
  size_t len = strlen(s);
  for (i = 0; i < len; i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

- Move call to **`strlen`** outside of loop
- Legal since result does not change from one iteration to another
- Form of code motion

# Lower Case Conversion Performance




- Time doubles when double string length
- Linear performance of lower2

# Optimization Blocker: Procedure Calls

- ***Why couldn't compiler move `strlen` out of inner loop?***
  - Procedure may have side effects
    - Alters global state each time called
  - Function may not return same value for given arguments
    - Depends on other parts of global state
    - Procedure **`lower1`** could interact with **`strlen`**
- **Warning:**
  - Compiler may treat procedure call as a black box
  - Weak optimizations near them
- **Remedies:**
  - Use of inline functions
    - GCC does this with –O1
      - Within single file
  - Do your own code motion

```
/* Alternative strlen */
size_t lencnt = 0;
size_t strlen(const char *s)
{
    size_t length = 0;
    while (*s != '\0') {
       s++; length++;
    }
    lencnt += length;
    return length;
}
```

# Optimization Blocker #2: Memory Aliasing

```
/* Sum rows of n X n matrix a
   and store in vector b  */
void sum_rows1(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

```
# sum_rows1 inner loop
.L4:
        movsd   (%rsi,%rax,8), %xmm0      # FP load
        addsd   (%rdi), %xmm0             # FP add
        movsd   %xmm0, (%rsi,%rax,8)      # FP store
        addq    $8, %rdi
        cmpq    %rcx, %rdi
        jne     .L4
```

- Code updates `b[i]` on every iteration
- Why couldn't compiler optimize this away?

# Memory Aliasing

```
/* Sum rows is of n X n matrix a
   and store in vector b  */
void sum_rows1(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

```
double A[9] =
  { 0,   1,   2,
    4,   8,   16,
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

```
double A[9] =
  { 0,   1,   2,
    3,  22, 224,
   32,  64, 128};
```

**Value of B:**

```
init:   [4, 8, 16]
```

```
i = 0:  [3, 8, 16]
```

```
i = 1:  [3, 22, 16]
```

```
i = 2:  [3, 22, 224]
```

- Code updates **b[i]** on every iteration
- Must consider possibility that these updates will affect program behavior

# Removing Aliasing

```
/* Sum rows is of n X n matrix a
   and store in vector b  */
void sum_rows2(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        double val = 0;
        for (j = 0; j < n; j++)
            val += a[i*n + j];
        b[i] = val;
    }
}
```

```
# sum_rows2 inner loop
.L10:
        addsd   (%rdi), %xmm0      # FP load + add
        addq    $8, %rdi
        cmpq    %rax, %rdi
        jne     .L10
```

- No need to store intermediate results

# Optimization Blocker: Memory Aliasing

- **Aliasing**
  - Two different memory references specify single location
  - Easy to have happen in C
    - Since allowed to do address arithmetic
    - Direct access to storage structures
  - Get in habit of introducing local variables
    - Accumulating within loops
    - Your way of telling compiler not to check for aliasing

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  - Sharing of common subexpressions
  - Example: Bubblesort
- **Optimization Blockers**
  - Procedure calls
  - Memory aliasing
- **Exploiting Instruction-Level Parallelism**
- **Dealing with Conditionals**

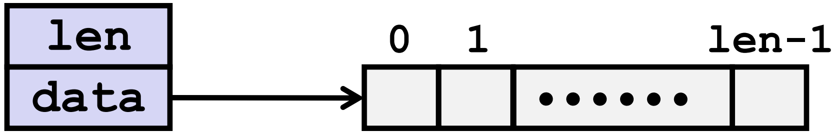
- **Fran Allen (1932-2020)**
  - **Pioneer of many optimizing compilation techniques**
  - **Wrote a paper simply called "Program Optimization" in 1966**
  - **"This paper introduced the use of graph-theoretic structures to encode program content in order to automatically and efficiently derive relationships and identify opportunities for optimization"**
  - **First woman to win the ACM Turing Award (the "Nobel Prize of Computer Science"), in 2006**

# Exploiting Instruction-Level Parallelism

- **Need general understanding of modern processor design**
  - Hardware can execute multiple instructions in parallel
- **Performance limited by data dependencies**
- **Simple transformations can cause big speedups**
  - Compilers often cannot make these transformations
  - Lack of associativity and distributivity in floating-point arithmetic

# Benchmark Example: Data Type for Vectors

```
/* data structure for vectors */
typedef struct{
    size_t len;
    data_t *data;
} vec;
```




- **Data Types**
  - Use different declarations for `data_t`
  - `int`
  - `long`
  - `float`
  - `double`

```
/* retrieve vector element
   and store at val */
int get_vec_element
  (*vec v, size_t idx, data_t *val)
{
    if (idx >= v->len)
        return 0;
    *val = v->data[idx];
    return 1;
}
```

# Benchmark Computation

```
void combine1(vec_ptr v, data_t *dest)
{
    long int i;
    *dest = IDENT;
    for (i = 0; i < vec_length(v); i++) {
       data_t val;
       get_vec_element(v, i, &val);
       *dest = *dest OP val;
    }
}
```

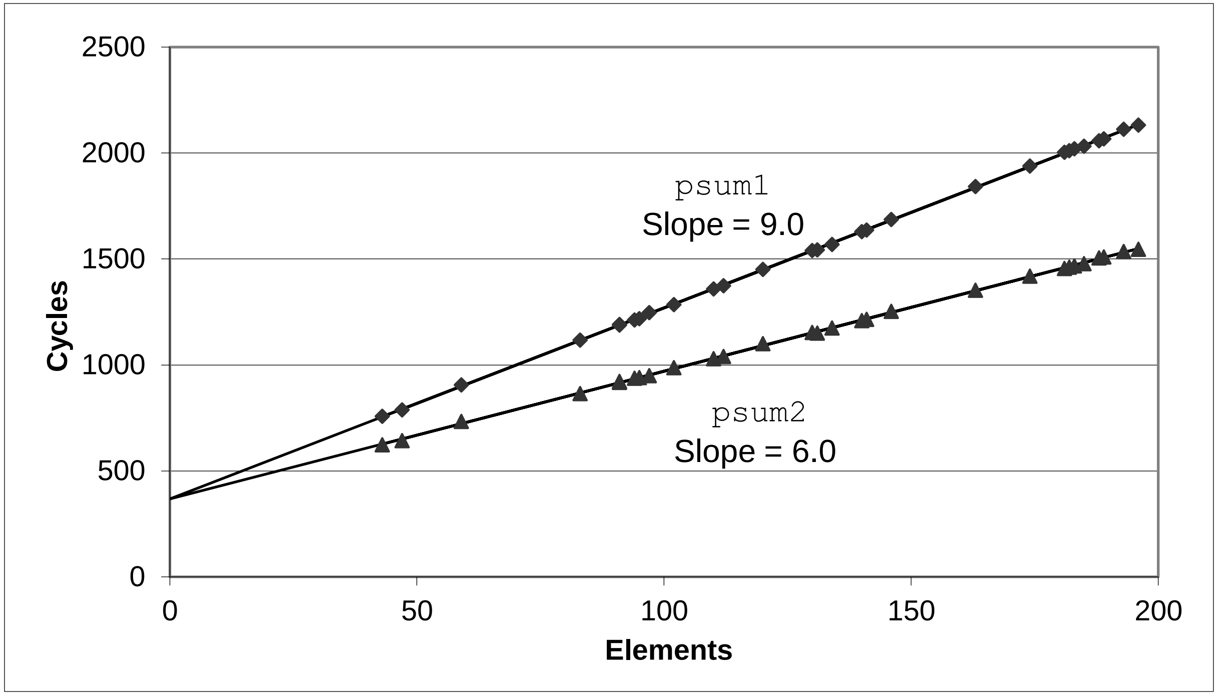
**Compute sum or product of vector elements**

- **Data Types**
  - Use different declarations for `data_t`
  - `int`
  - `long`
  - `float`
  - `double`

- **Operations**
  - Use different definitions of `OP` and `IDENT`
  - `+ / 0`
  - `* / 1`

# Cycles Per Element (CPE)

- Convenient way to express performance of program that operates on vectors or lists
- Length = n
- In our case: CPE = cycles per OP
- Cycles = CPE*n + Overhead
  - CPE is slope of line



# Benchmark Performance

```
void combine1(vec_ptr v, data_t *dest)
{
    long int i;
    *dest = IDENT;
    for (i = 0; i < vec_length(v); i++) {
       data_t val;
       get_vec_element(v, i, &val);
       *dest = *dest OP val;
    }
}
```

**Compute sum or product of vector elements**

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Combine1 unoptimized | 22.68 | 20.02 | 19.98 | 20.18 |
| Combine1 –O1 | 10.12 | 10.12 | 10.17 | 11.14 |
| Combine1 –O3 | 4.5 | 4.5 | 6 | 7.8 |

Results in CPE (cycles per element)

# Basic Optimizations

```
void combine4(vec_ptr v, data_t *dest)
{
  long i;
  long length = vec_length(v);
  data_t *d = get_vec_start(v);
  data_t t = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

- Move vec_length out of loop
- Avoid bounds check on each cycle
- Accumulate in temporary

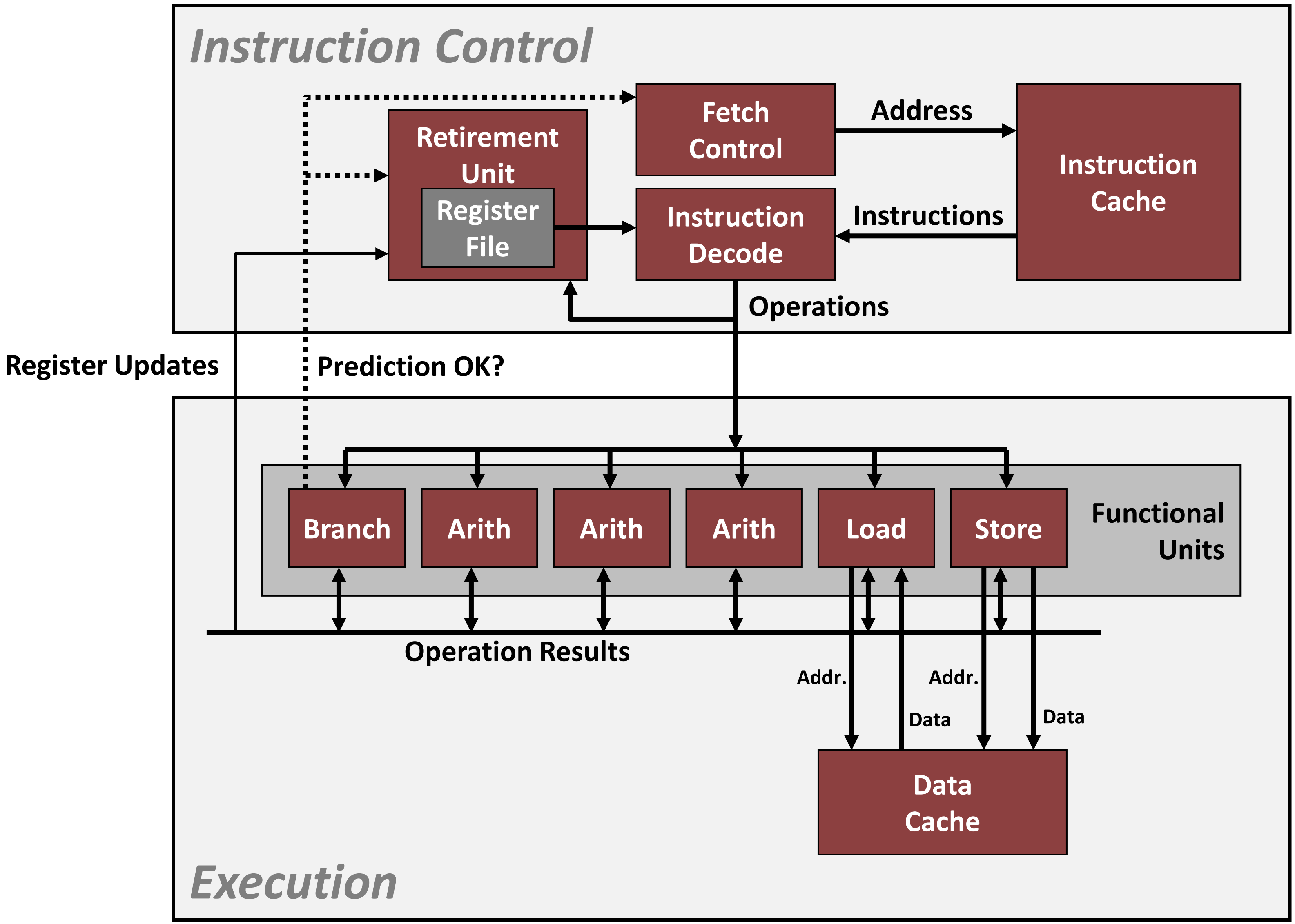
# Effect of Basic Optimizations

```
void combine4(vec_ptr v, data_t *dest)
{
  long i;
  long length = vec_length(v);
  data_t *d = get_vec_start(v);
  data_t t = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Combine1 –O1 | 10.12 | 10.12 | 10.17 | 11.14 |
| Combine4 | 1.27 | 3.01 | 3.01 | 5.01 |

- Eliminates sources of overhead in loop

# Modern CPU Design



# Superscalar Processor

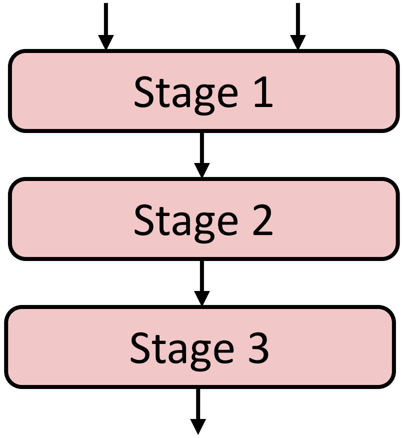
- **Definition:** **A superscalar processor can issue and execute *multiple instructions in one cycle*. The instructions are retrieved from a sequential instruction stream and are usually scheduled dynamically.**

- **Benefit: without programming effort, superscalar processor can take advantage of the *instruction level parallelism* that most programs have.**

- **Most CPUs are superscalar.**
  - Intel: since Pentium (1993)

# Pipelined Functional Units

```
long mult_eg(long a, long b, long c) {
    long p1 = a*b;
    long p2 = a*c;
    long p3 = p1 * p2;
    return p3;
}
```




| | Time | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Stage 1 | a*b | a*c | | | p1*p2 | | |
| Stage 2 | | a*b | a*c | | | p1*p2 | |
| Stage 3 | | | a*b | a*c | | | p1*p2 |

- Divide computation into stages
- Pass partial computations from stage to stage
- Stage i can start on new computation once values passed to i+1
- E.g., complete 3 multiplications in 7 cycles, even though each requires 3 cycles

# Haswell CPU

- **8 Total Functional Units**
- **Multiple instructions can execute in parallel**
  - 2 load, with address computation
  - 1 store, with address computation
  - 4 integer
  - 2 FP multiply
  - 1 FP add
  - 1 FP divide
- **Some instructions take > 1 cycle, but can be pipelined**

| *Instruction* | *Latency* | *Cycles/Issue* |
|---|---|---|
| Load / Store | 4 | 1 |
| Integer Multiply | 3 | 1 |
| **Integer/Long Divide** | **3-30** | **3-30** |
| Single/Double FP Multiply | 5 | 1 |
| Single/Double FP Add | 3 | 1 |
| **Single/Double FP Divide** | **3-15** | **3-15** |

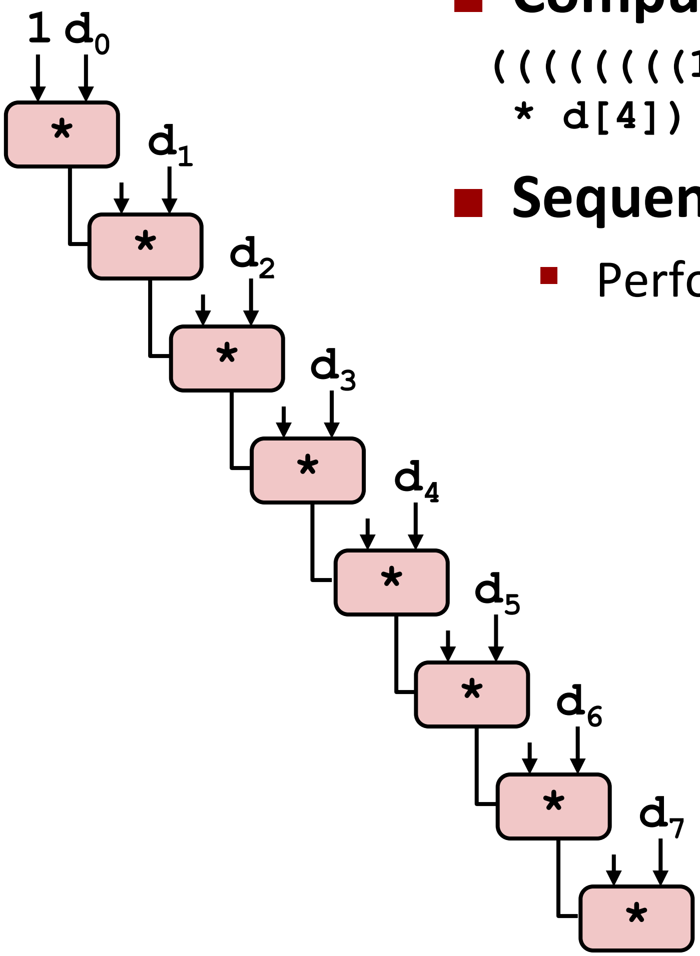
# x86-64 Compilation of Combine4

- Inner Loop (Case: Integer Multiply)

```
.L519:                          # Loop:
  imull  (%rax,%rdx,4), %ecx    # t = t * d[i]
  addq   $1, %rdx               # i++
  cmpq   %rdx, %rbp             # Compare length:i
  jg     .L519                  # If >, goto Loop
```

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Combine4 | 1.27 | 3.01 | 3.01 | 5.01 |
| *Latency Bound* | *1.00* | *3.00* | *3.00* | *5.00* |

# Combine4 = Serial Computation (OP = *)




- **Computation (length=8)**

```
((((((((1 * d[0]) * d[1]) * d[2]) * d[3])
 * d[4]) * d[5]) * d[6]) * d[7])
```

- **Sequential dependence**
  - Performance: determined by latency of OP

# Loop Unrolling (2x1)

```
void unroll2a_combine(vec_ptr v, data_t *dest)
{
    long length = vec_length(v);
    long limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x = IDENT;
    long i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
        x = (x OP d[i]) OP d[i+1];
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {
        x = x OP d[i];
    }
    *dest = x;
}
```

- Perform 2x more useful work per loop iteration

# Effect of Loop Unrolling

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Combine4 | 1.27 | 3.01 | 3.01 | 5.01 |
| Unroll 2x1 | 1.01 | 3.01 | 3.01 | 5.01 |
| *Latency Bound* | *1.00* | *3.00* | *3.00* | *5.00* |

- **Helps integer add**
  - Achieves latency bound
- **Others don't improve. *Why?***
  - Still sequential dependency

```
x = (x OP d[i]) OP d[i+1];
```

# Loop Unrolling with Separate Accumulators (2x2)

```
void unroll2a_combine(vec_ptr v, data_t *dest)
{
    long length = vec_length(v);
    long limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x0 = IDENT;
    data_t x1 = IDENT;
    long i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
       x0 = x0 OP d[i];
       x1 = x1 OP d[i+1];
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {
        x0 = x0 OP d[i];
    }
    *dest = x0 OP x1;
}
```

- **Can this change the result of the computation?**
- **Yes, for FP. *Why?***

# Effect of Separate Accumulators

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| **Operation** | **Add** | **Mult** | **Add** | **Mult** |
| Combine4 | 1.27 | 3.01 | 3.01 | 5.01 |
| Unroll 2x1 | 1.01 | 3.01 | 3.01 | 5.01 |
| Unroll 2x2 | 0.81 | 1.51 | 1.51 | 2.51 |
| *Latency Bound* | *1.00* | *3.00* | *3.00* | *5.00* |
| *Throughput Bound* | *0.50* | *1.00* | *1.00* | *0.50* |

4 func. units for int +,
2 func. units for load
*Why Not .25?*

1 func. unit for FP +
3-stage pipelined FP +

2 func. units for FP *,
2 func. units for load
5-stage pipelined FP *

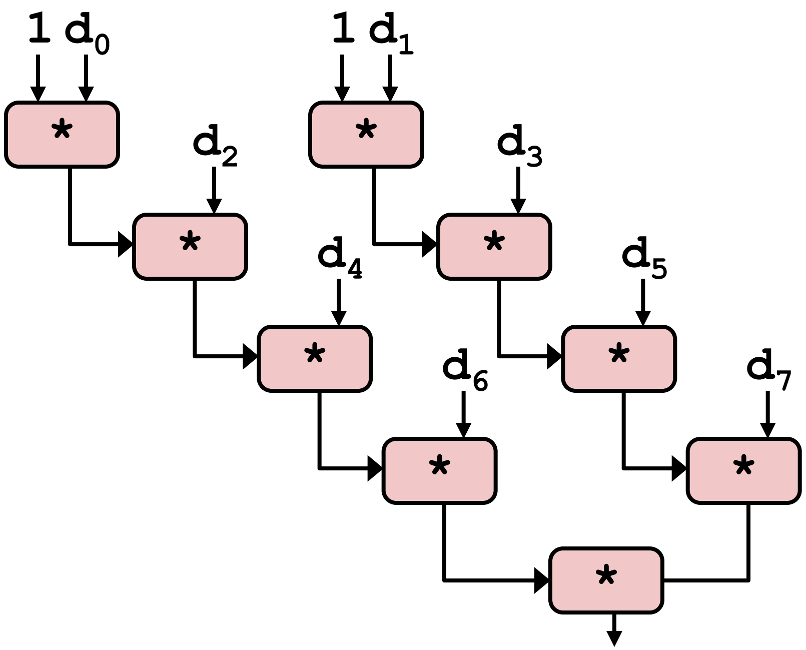
- **Nearly 2x speedup for Int *, FP +, FP ***
  - Reason: Breaks sequential dependency

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```

  - Why is that? (next slide)

# Separate Accumulators

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```




- **What changed:**
  - Two independent “streams” of operations
- **Overall Performance**
  - N elements, D cycles latency/op
  - Should be (N/2+1)*D cycles: **CPE = D/2**
  - CPE matches prediction!

***What Now?***

# Unrolling & Accumulating

- **Idea: Can choose L Unrolling Factor and K Accumulators**
  - L must be a multiple of K
- **Case: Double ***
  - Latency bound: 5.00.  Throughput bound: 0.50

*Accumulators*

| FP * | Unrolling Factor L | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **K** | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 |
| 1 | 5.01 | 5.01 | 5.01 | 5.01 | 5.01 | 5.01 | 5.01 | |
| 2 | | 2.51 | | 2.51 | | 2.51 | | |
| 3 | | | 1.67 | | | | | |
| 4 | | | | 1.25 | | 1.26 | | |
| 6 | | | | | 0.84 | | | 0.88 |
| 8 | | | | | | 0.63 | | |
| 10 | | | | | | | 0.51 | |
| 12 | | | | | | | | 0.52 |

# Achievable Performance

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Best | 0.54 | 1.01 | 1.01 | 0.52 |
| *Latency Bound* | *1.00* | *3.00* | *3.00* | *5.00* |
| *Throughput Bound* | *0.50* | *1.00* | *1.00* | *0.50* |

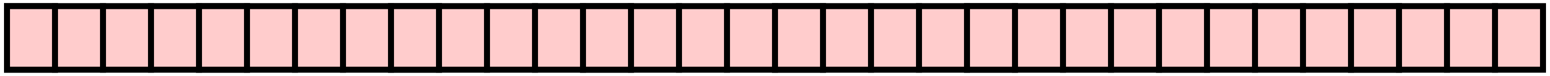
- **Limited only by throughput of functional units**
- **Up to 42X improvement over original, unoptimized code**

***Can we do even better?***

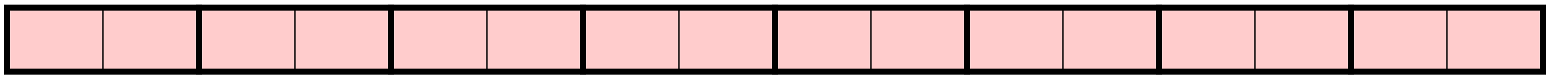
# Programming with AVX2

## YMM Registers

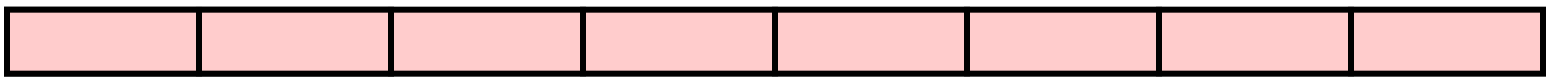
- 16 total, each 32 bytes
- 32 single-byte integers



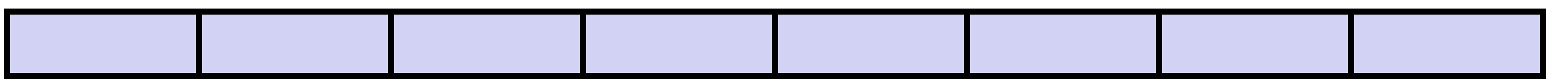
- 16 16-bit integers



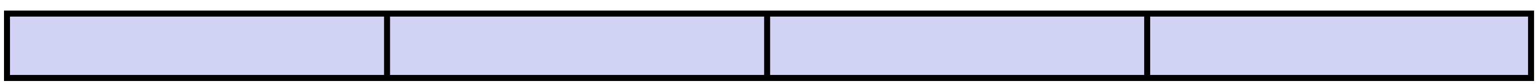
- 8 32-bit integers



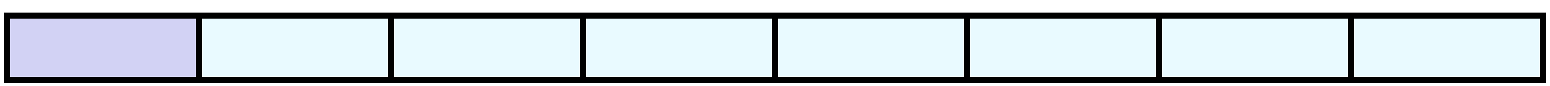
- 8 single-precision floats



- 4 double-precision floats



- 1 single-precision float

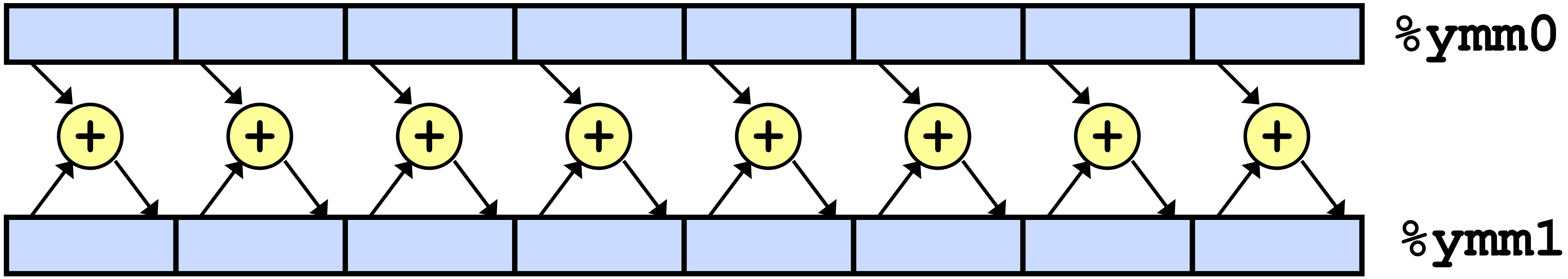


- 1 double-precision float

# SIMD Operations

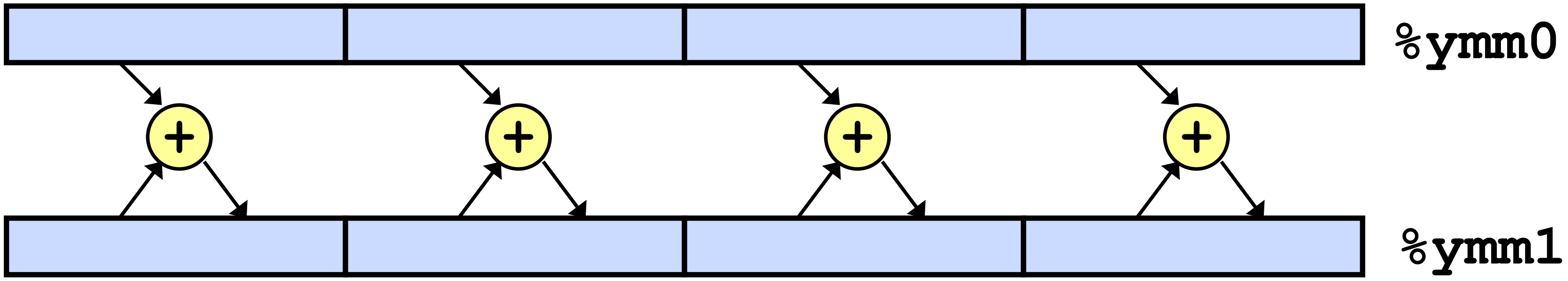
- SIMD Operations: Single Precision

```
vaddps %ymm0, %ymm1, %ymm1
```




- SIMD Operations: Double Precision

```
vaddpd %ymm0, %ymm1, %ymm1
```



# Using Vector Instructions

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Scalar Best | 0.54 | 1.01 | 1.01 | 0.52 |
| Vector Best | 0.06 | 0.24 | 0.25 | 0.16 |
| *Latency Bound* | *0.50* | *3.00* | *3.00* | *5.00* |
| *Throughput Bound* | *0.50* | *1.00* | *1.00* | *0.50* |
| *Vec Throughput Bound* | *0.06* | *0.12* | *0.25* | *0.12* |

- **Make use of AVX Instructions**
  - Parallel operations on multiple data elements
  - See Web Aside OPT:SIMD on CS:APP web page

# Today

- **Generally Useful Optimizations**
  - Code motion/precomputation
  - Strength reduction
  - Sharing of common subexpressions
  - Example: Bubblesort
- **Optimization Blockers**
  - Procedure calls
  - Memory aliasing
- **Exploiting Instruction-Level Parallelism**
- **Dealing with Conditionals**

# What About Branches?

- **Challenge**
  - Instruction Control Unit must work well ahead of Execution Unit to generate enough operations to keep EU busy

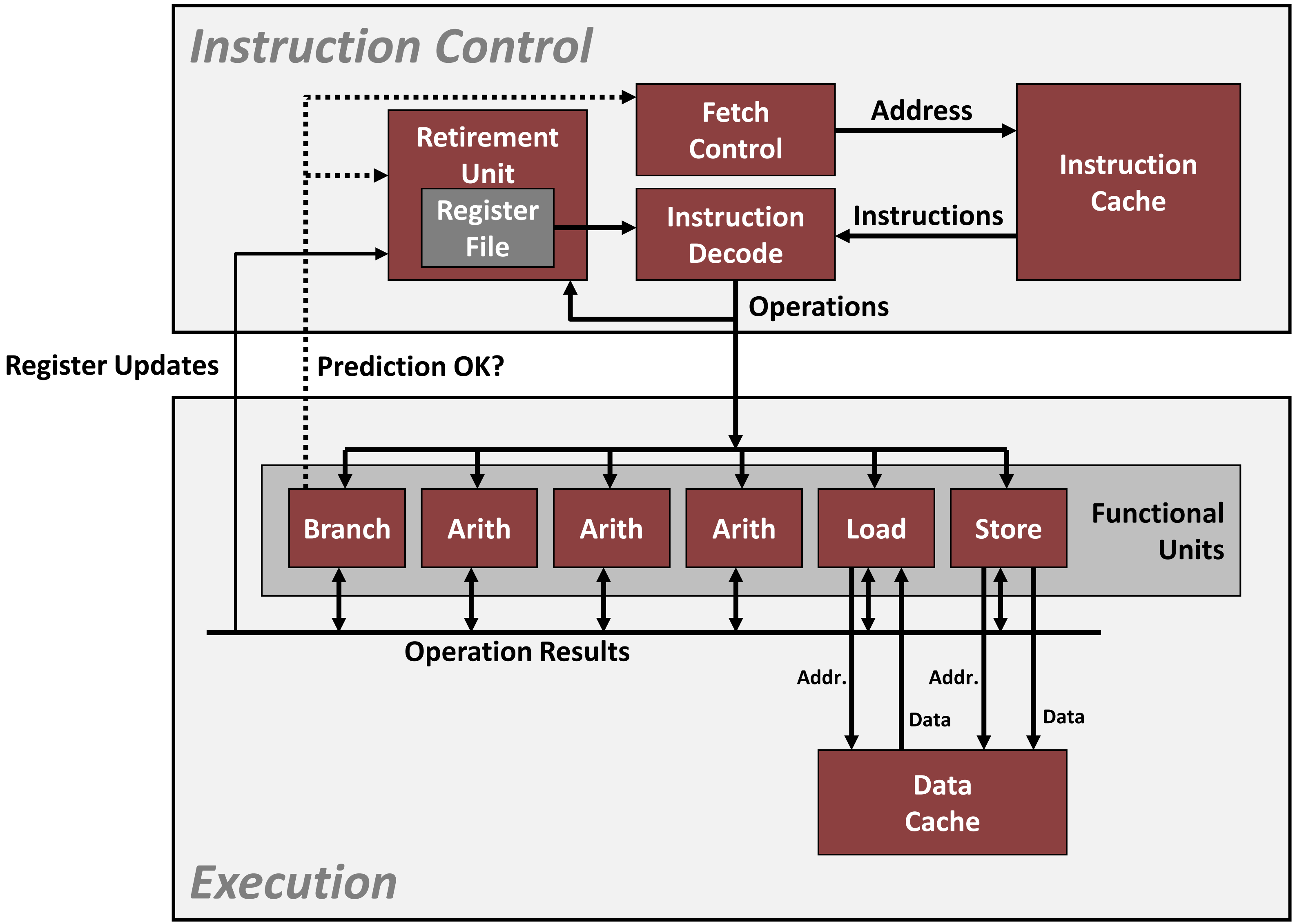
```
404663:  mov    $0x0,%eax
404668:  cmp    (%rdi),%rsi
40466b:  jge    404685
40466d:  mov    0x8(%rdi),%rax

 . . .

404685:  repz retq
```

**Executing** (404663, 404668)

**How to continue?** (40466b: jge 404685)

  - When encounters conditional branch, cannot reliably determine where to continue fetching

# Modern CPU Design



# Branch Outcomes

- **When encounter conditional branch, cannot determine where to continue fetching**
  - Branch Taken: Transfer control to branch target
  - Branch Not-Taken: Continue with next instruction in sequence
- **Cannot resolve until outcome determined by branch/integer unit**

```
404663:  mov    $0x0,%eax
404668:  cmp    (%rdi),%rsi
40466b:  jge    404685
40466d:  mov    0x8(%rdi),%rax

 . . .

404685:  repz retq
```

Branch Not-Taken

Branch Taken

# Branch Prediction

- **Idea**
  - Guess which way branch will go
  - Begin executing instructions at predicted position
    - But don’t actually modify register or memory data

```
404663:  mov     $0x0,%eax
404668:  cmp     (%rdi),%rsi
40466b:  jge     404685
40466d:  mov     0x8(%rdi),%rax

 . . .

404685:  repz retq
```

**Predict Taken**

**Begin Execution**

# Branch Prediction Through Loop

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 98*

Predict Taken (OK)

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 99*

Predict Taken (Oops)

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 100*

Read invalid location

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 101*

***Assume vector length = 100***

Executed

Fetched

# Branch Misprediction Invalidation

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 98*

*Assume*

*vector length = 100*

Predict Taken (OK)

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 99*

Predict Taken (Oops)

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 100*

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
```

*i = 101*

Invalidate

# Branch Misprediction Recovery

```
401029:   vmulsd  (%rdx),%xmm0,%xmm0
40102d:   add     $0x8,%rdx
401031:   cmp     %rax,%rdx
401034:   jne     401029
401036:   jmp     401040
  . . .
401040:   vmovsd  %xmm0,(%r12)
```

*i = 99*

**Definitely not taken**

**Reload Pipeline**

- **Performance Cost**
  - Multiple clock cycles on modern processor
  - Can be a major performance limiter

# Branch Prediction Numbers

- **Default behavior:**
  - Backwards branches are often loops so predict taken
  - Forwards branches are often if so predict not taken

- **Predictors average better than 95% accuracy**
  - Most branches are already predictable.

- **Annual branch predictor contests at top Computer Architecture conferences (2010-2016)**
  - Metrics: Size of branch predictor tables
    Mispredictions per kilo-instruction (MPKI)
  - 2016 Winners (https://www.jilp.org/cbp2016/)
    - Size 8KB: MPKI=4.1
    - Size 64KB: MPKI=3.3

# Summary: Getting High Performance

- **Good compiler and flags**
- **Don't do anything sub-optimal**
  - Watch out for hidden algorithmic inefficiencies
  - Write compiler-friendly code
    - Watch out for optimization blockers: procedure calls & memory references
  - Look carefully at innermost loops (where most work is done)

- **Tune code for machine**
  - Exploit instruction-level parallelism
  - Avoid unpredictable branches
  - Make code cache friendly

# ADDITIONAL SLIDES

# Loop Unrolling with Reassociation (2x1a)

```
void unroll2aa_combine(vec_ptr v, data_t *dest)
{
    long length = vec_length(v);
    long limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x = IDENT;
    long i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
        x = x OP (d[i] OP d[i+1]);
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {
        x = x OP d[i];
    }
    *dest = x;
}
```

Compare to before

```
x = (x OP d[i]) OP d[i+1];
```

- **Can this change the result of the computation?**
- **Yes, for FP.** ***Why?***

# Effect of Reassociation

| Method | Integer | | Double FP | |
|---|---|---|---|---|
| Operation | Add | Mult | Add | Mult |
| Combine4 | 1.27 | 3.01 | 3.01 | 5.01 |
| Unroll 2x1 | 1.01 | 3.01 | 3.01 | 5.01 |
| Unroll 2x1a | 1.01 | 1.51 | 1.51 | 2.51 |
| *Latency Bound* | *1.00* | *3.00* | *3.00* | *5.00* |
| *Throughput Bound* | *0.50* | *1.00* | *1.00* | *0.50* |

4 func. units for int +,
2 func. units for load
*Why Not .25?*

1 func. unit for FP +
3-stage pipelined FP +

2 func. units for FP *,
2 func. units for load
5-stage pipelined FP *

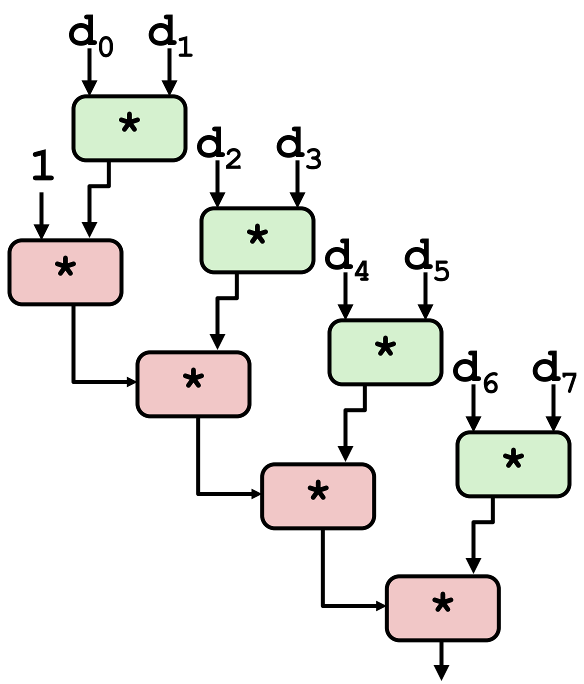
- **Nearly 2x speedup for Int *, FP +, FP ***
  - Reason: Breaks sequential dependency

```
x = x OP (d[i] OP d[i+1]);
```

  - Why is that? (next slide)

# Reassociated Computation

```
x = x OP (d[i] OP d[i+1]);
```




- **What changed:**
  - Ops in the next iteration can be started early (no dependency)
- **Overall Performance**
  - N elements, D cycles latency/op
  - (N/2+1)*D cycles:
    **CPE = D/2**

# Unrolling & Accumulating: Int +

- **Case**
  - Intel Haswell
  - Integer addition
  - Latency bound: 1.00. Throughput bound: 0.50

*Accumulators*

| FP * | Unrolling Factor L | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| K | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 |
| 1 | 1.27 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | |
| 2 | | 0.81 | | 0.69 | | 0.54 | | |
| 3 | | | 0.74 | | | | | |
| 4 | | | | 0.69 | | 1.24 | | |
| 6 | | | | | 0.56 | | | 0.56 |
| 8 | | | | | | 0.54 | | |
| 10 | | | | | | | 0.54 | |
| 12 | | | | | | | | 0.56 |