# Signal Processing Framework for Reflection

## Lectures #7 and #8

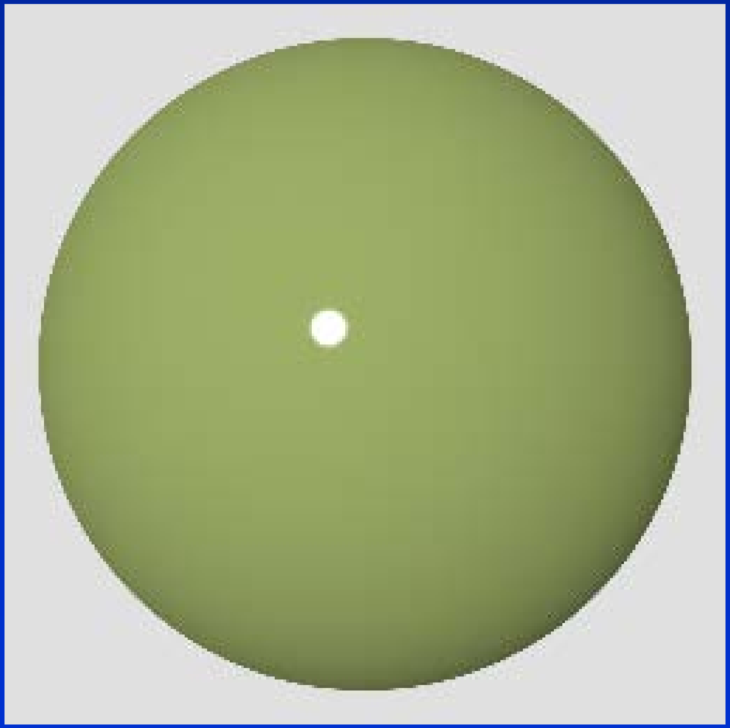
Thanks to Ravi Ramamoorthi, Pat Hanrahan, Ronen Basri, David Jacobs, Ron Dror, Ted Adelson.

Ravi Ramamoorthi's homepage is an excellent source for papers, videos, PPTs on this topic.
Many of the slides in these classes are obtained from his website.

http://www.cs.columbia.edu/~ravir/

# Illumination Illusion

People perceive materials more easily under natural illumination than simplified illumination.



Images courtesy Ron Dror and Ted Adelson

# Illumination Illusion

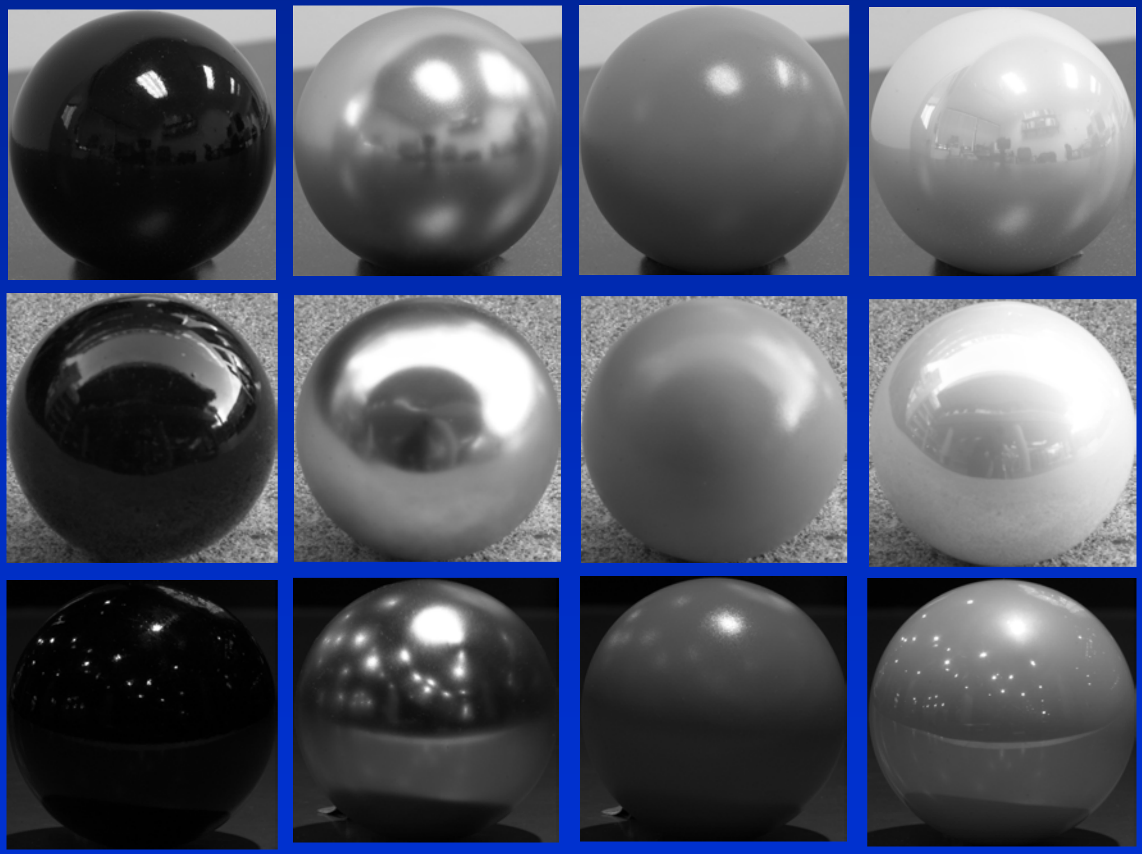
People perceive materials more easily under natural illumination than simplified illumination.

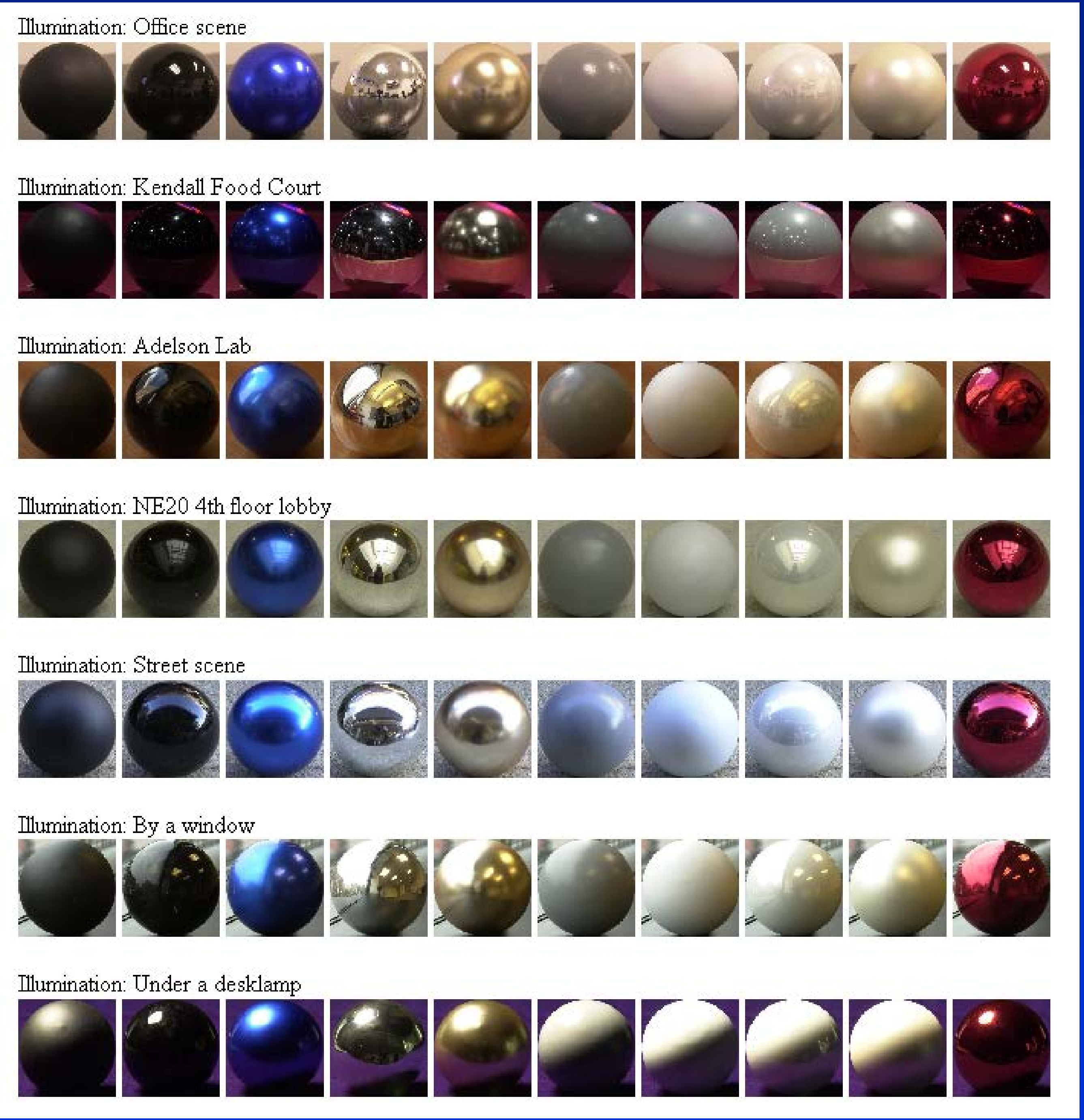
Images courtesy Ron Dror and Ted Adelson

# Material Recognition

Photographs of
4 spheres in 3 different
lighting conditions

courtesy
Dror and Adelson






Dror, Adelson, Wilsky

# Surface Appearance - RECAP

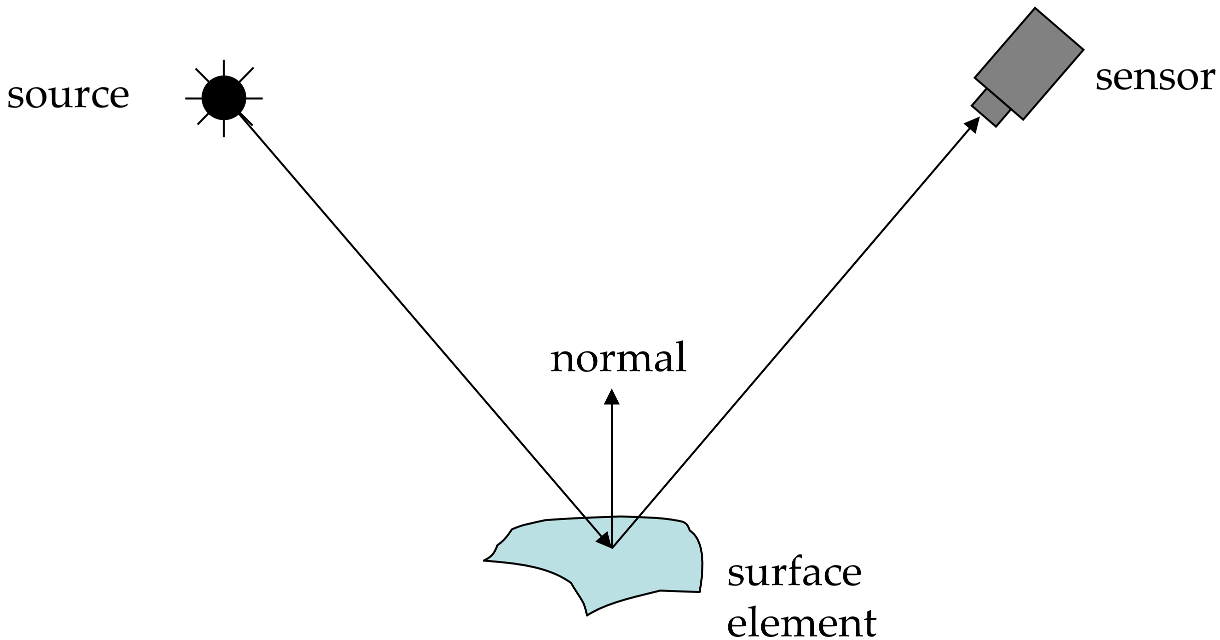
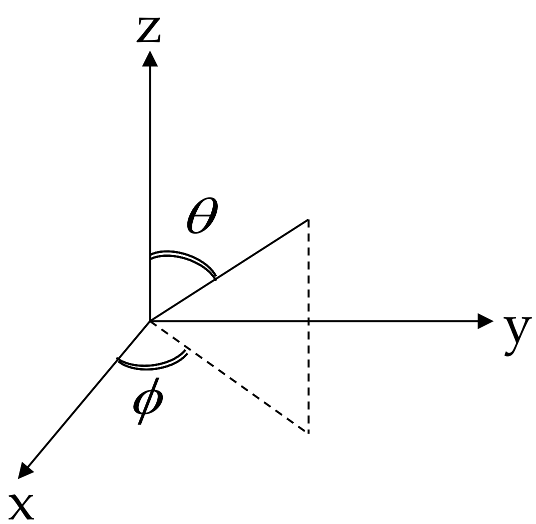
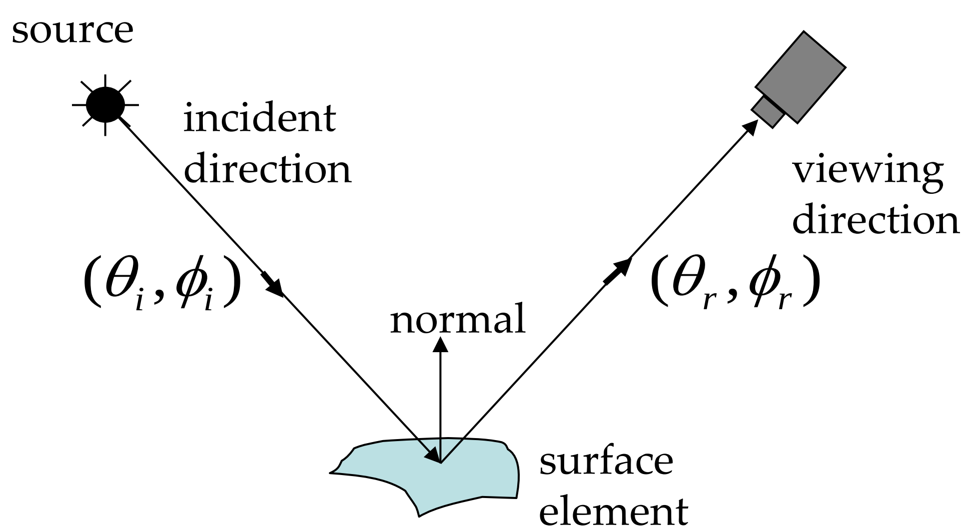



Image intensities = $f$ ( normal, surface reflectance, illumination )

Surface Reflection depends on both the viewing and illumination direction.

# BRDF: Bidirectional Reflectance Distribution Function



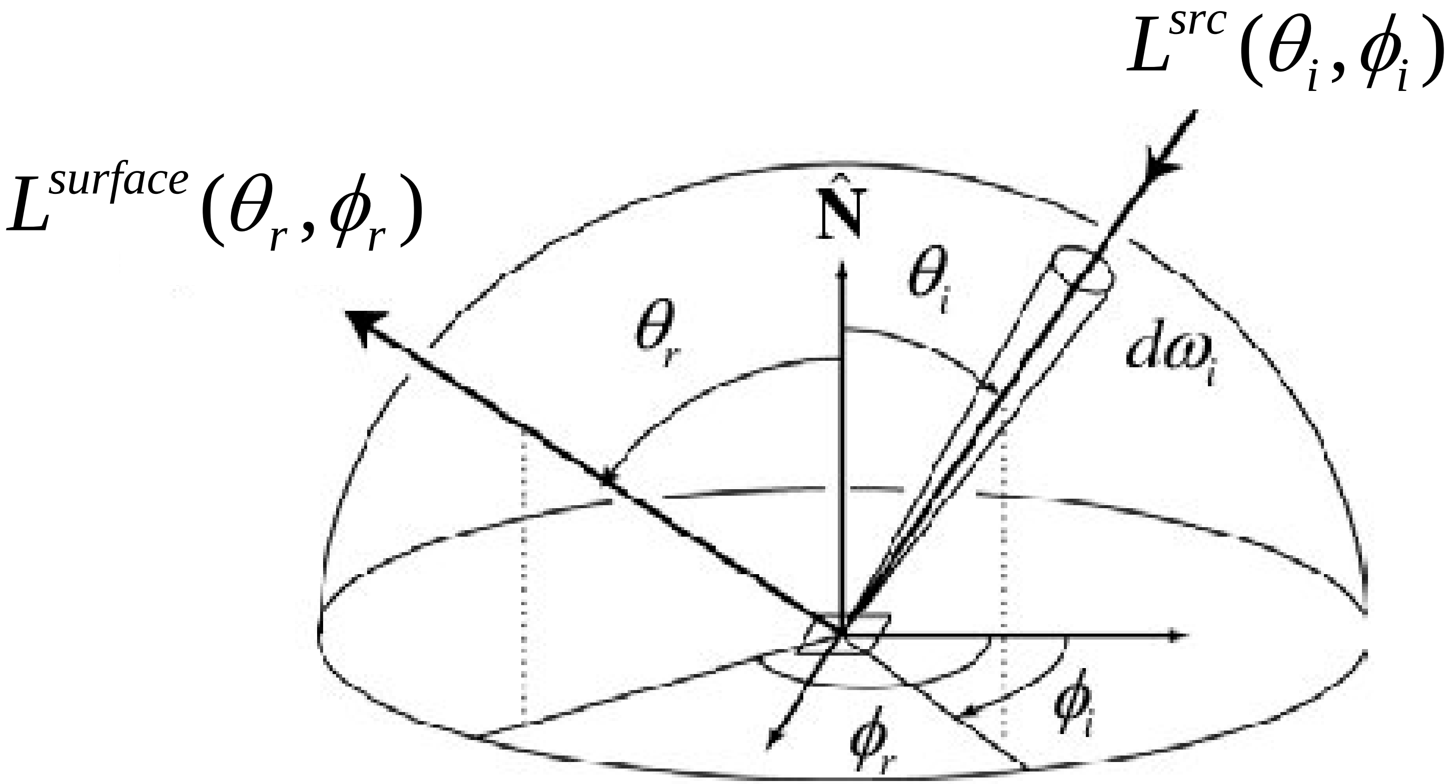




$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction $(\theta_i, \phi_i)$

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction $(\theta_r, \phi_r)$

$$\text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

# Derivation of the Scene Radiance Equation




From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) \quad = \quad E^{surface}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

# Derivation of the Scene Radiance Equation – Important!

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) = \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) = \underline{L^{src}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\,\underline{\cos\theta_i d\omega_i}$$

Integrate over entire hemisphere of possible source directions:

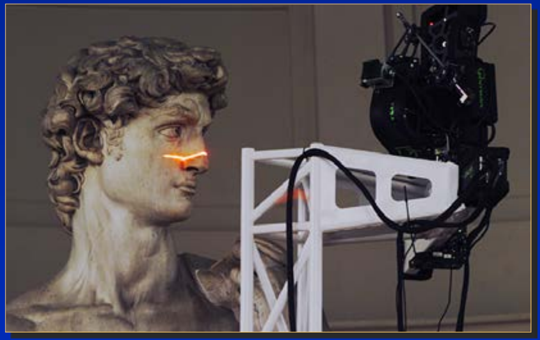
$$L^{surface}(\theta_r,\phi_r) = \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\, \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

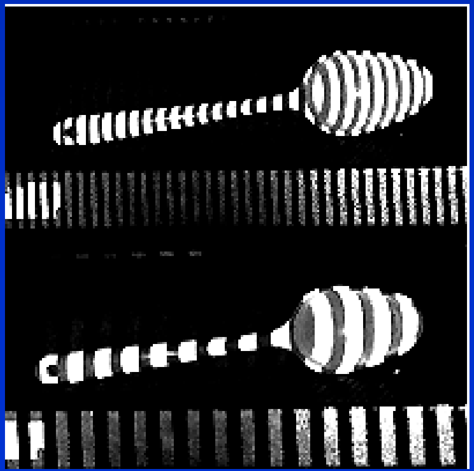
$$L^{surface}(\theta_r,\phi_r) = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\, \underline{\sin\theta_i d\theta_i d\phi_i}$$

# Assumptions

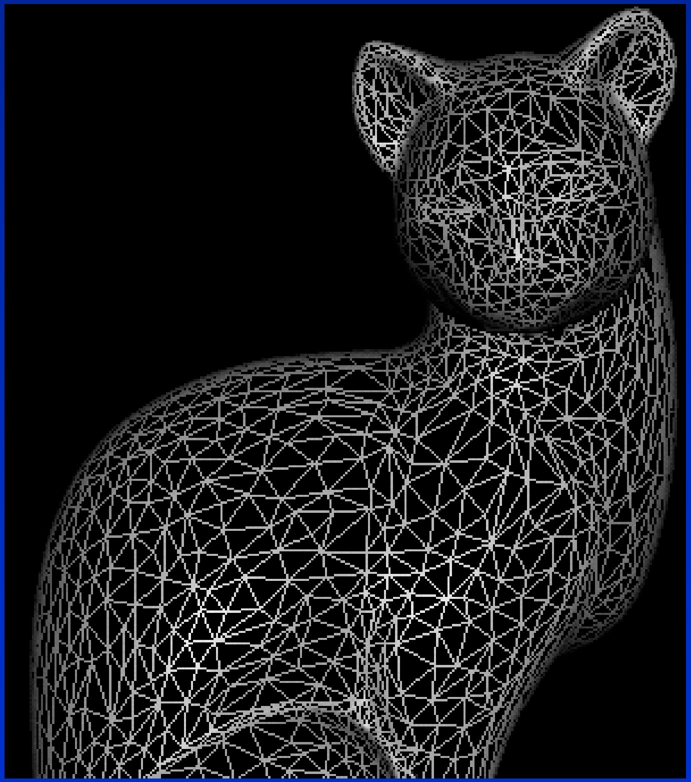
- Known geometry



Laser range scanner

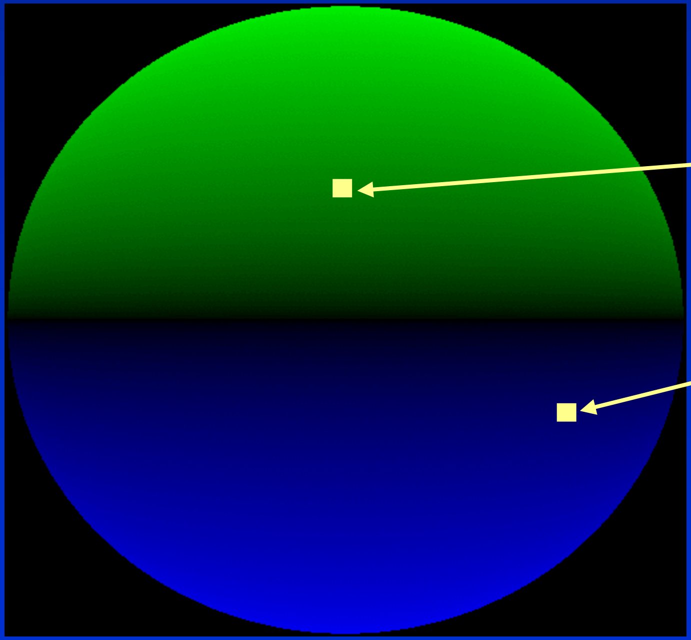


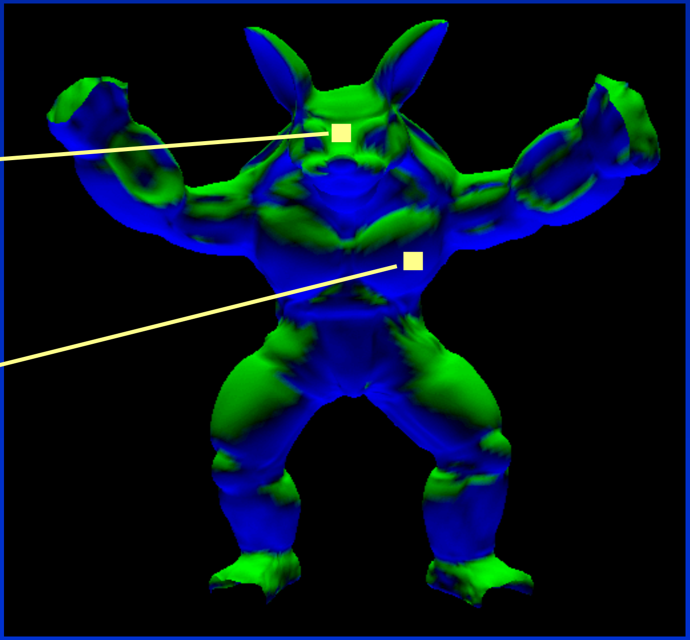
Structured light



# Assumptions

- Known geometry
- Convex curved surfaces: no shadows, interreflection





Complex geometry: use surface normal

# Assumptions

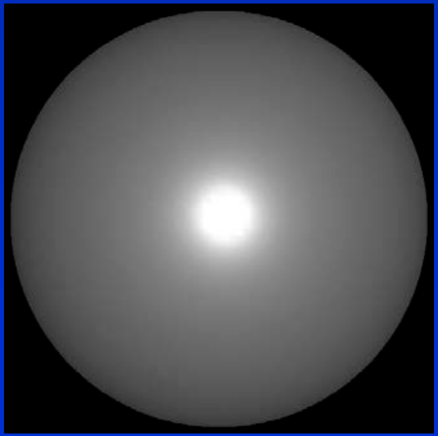
- Known geometry
- Convex curved surfaces: no shadows, interreflection
- Distant illumination

Photograph of mirror sphere

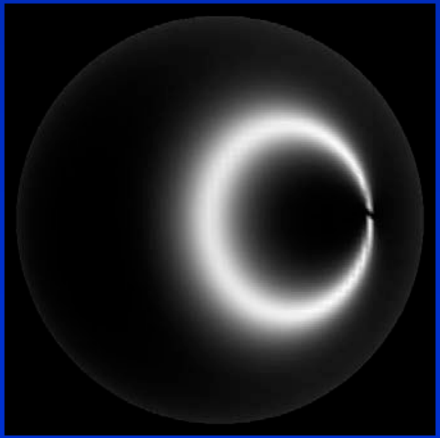
Illumination: Grace Cathedral
courtesy Paul Debevec

# Assumptions

- Known geometry
- Convex curved surfaces: no shadows, interreflection
- Distant illumination
- Homogeneous isotropic materials



Isotropic



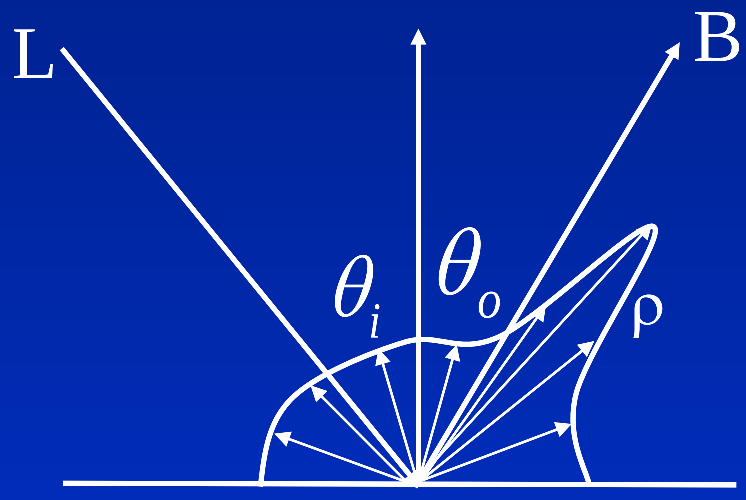
Anisotropic

# Assumptions

- Known geometry
- Convex curved surfaces: no shadows, interreflection
- Distant illumination
- Homogeneous isotropic materials

Later, practical algorithms: relax some assumptions

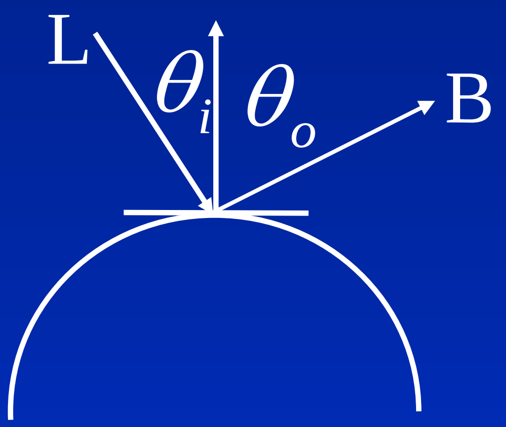
# Reflection

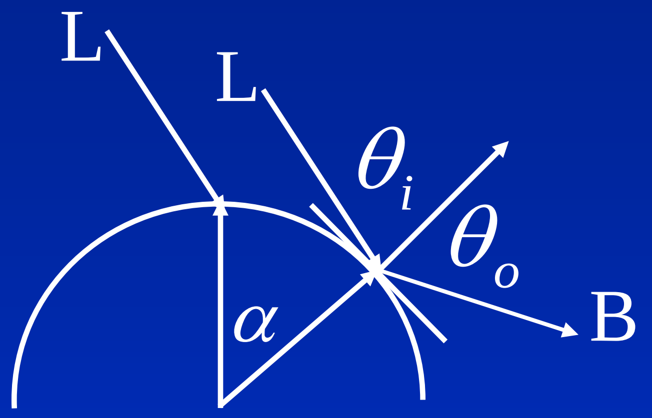



$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\, d\theta_i$$

**Reflected Light Field** **Lighting** **BRDF**

# Reflection as Convolution (2D)

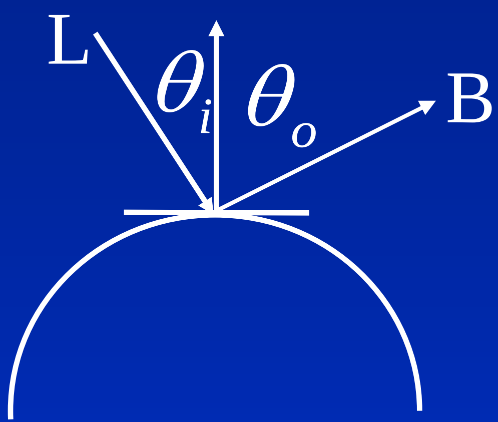


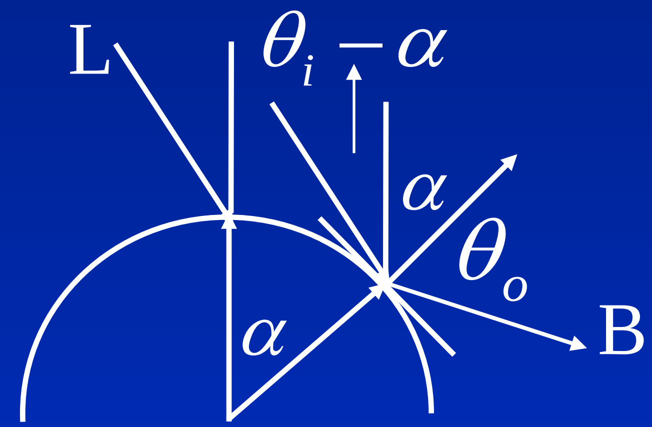




$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\ d\theta_i$$

**Reflected Light Field** **Lighting** **BRDF**

# Reflection as Convolution (2D)




$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\, d\theta_i$$

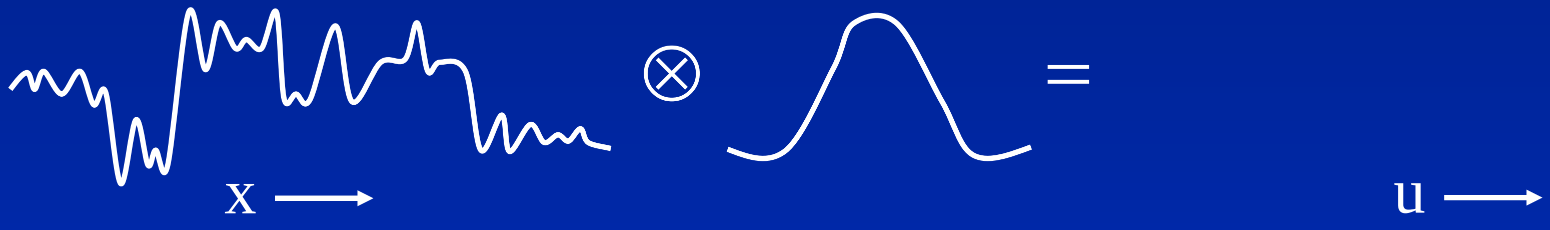
**Reflected Light Field** **Lighting** **BRDF**

$$B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \rho(\theta_i, \theta_o)\, d\theta_i$$

# Reflection as Convolution (2D)


$$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha)\rho(\theta_i,\theta_o)\, d\theta_i$$

# Convolution



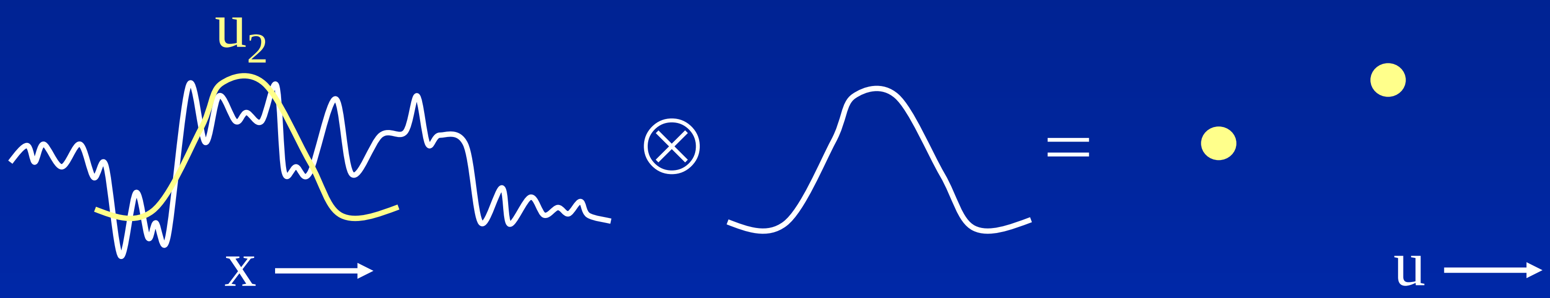

Signal f(x) Filter g(x) Output h(u)

# Convolution


Signal f(x)   Filter g(x)   Output h(u)

$$h(u_1) = \int g(x - u_1)\, f(x)\, dx$$

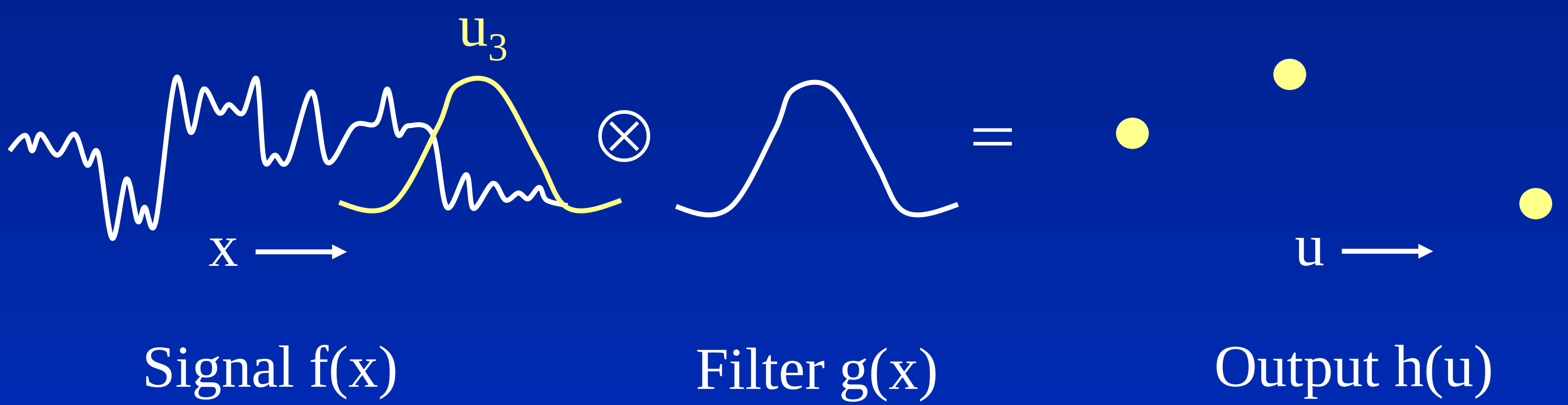
# Convolution




Signal f(x) Filter g(x) Output h(u)

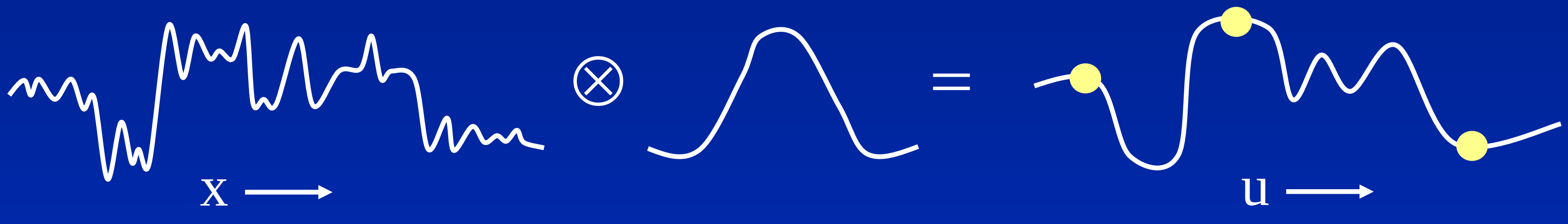
$$h(u_2) = \int g(x - u_2)\, f(x)\, dx$$

# Convolution




$$h(u_3) = \int g(x - u_3)\, f(x)\, dx$$

# Convolution




Signal f(x) Filter g(x) Output h(u)

$$h(u) = \int g(x-u)\, f(x)\, dx$$

$$h = f \otimes g = g \otimes f$$

**Fourier analysis**

$$h_\omega = f_\omega g_\omega$$

# Reflection as Convolution (2D)


$$B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha)\rho(\theta_i, \theta_o)\, d\theta_i$$

$$B = L \otimes \rho$$

**Fourier analysis**

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

**Spatial: integral**

**Frequency: product**

R. Ramamoorthi and P. Hanrahan “Analysis of Planar Light Fields from Homogeneous Convex Curved Surfaces under Distant Illumination” SPIE Photonics West 2001: Human Vision and Electronic Imaging VI pp 195-208

# Fourier Analysis

$$
\begin{aligned}
L(\theta_i) &= \sum_p L_p e^{Ip\theta_i} \\
\hat{\rho}(\theta_i', \theta_o') &= \sum_p \sum_q \hat{\rho}_{p,q} e^{Ip\theta_i'} e^{Iq\theta_o'} \\
B(\alpha, \theta_o') &= \sum_p \sum_q B_{p,q} e^{Ip\alpha} e^{Iq\theta_o'}
\end{aligned}
$$

# Fourier Analysis

$$L(\theta_i) = \sum_p L_p e^{Ip\theta_i}$$

$$\hat{\rho}(\theta_i', \theta_o') = \sum_p \sum_q \hat{\rho}_{p,q} e^{Ip\theta_i'} e^{Iq\theta_o'}$$

$$B(\alpha, \theta_o') = \sum_p \sum_q B_{p,q} e^{Ip\alpha} e^{Iq\theta_o'}$$

$$\boxed{B_{p,q} = 2\pi L_p \hat{\rho}_{-p,q}}$$

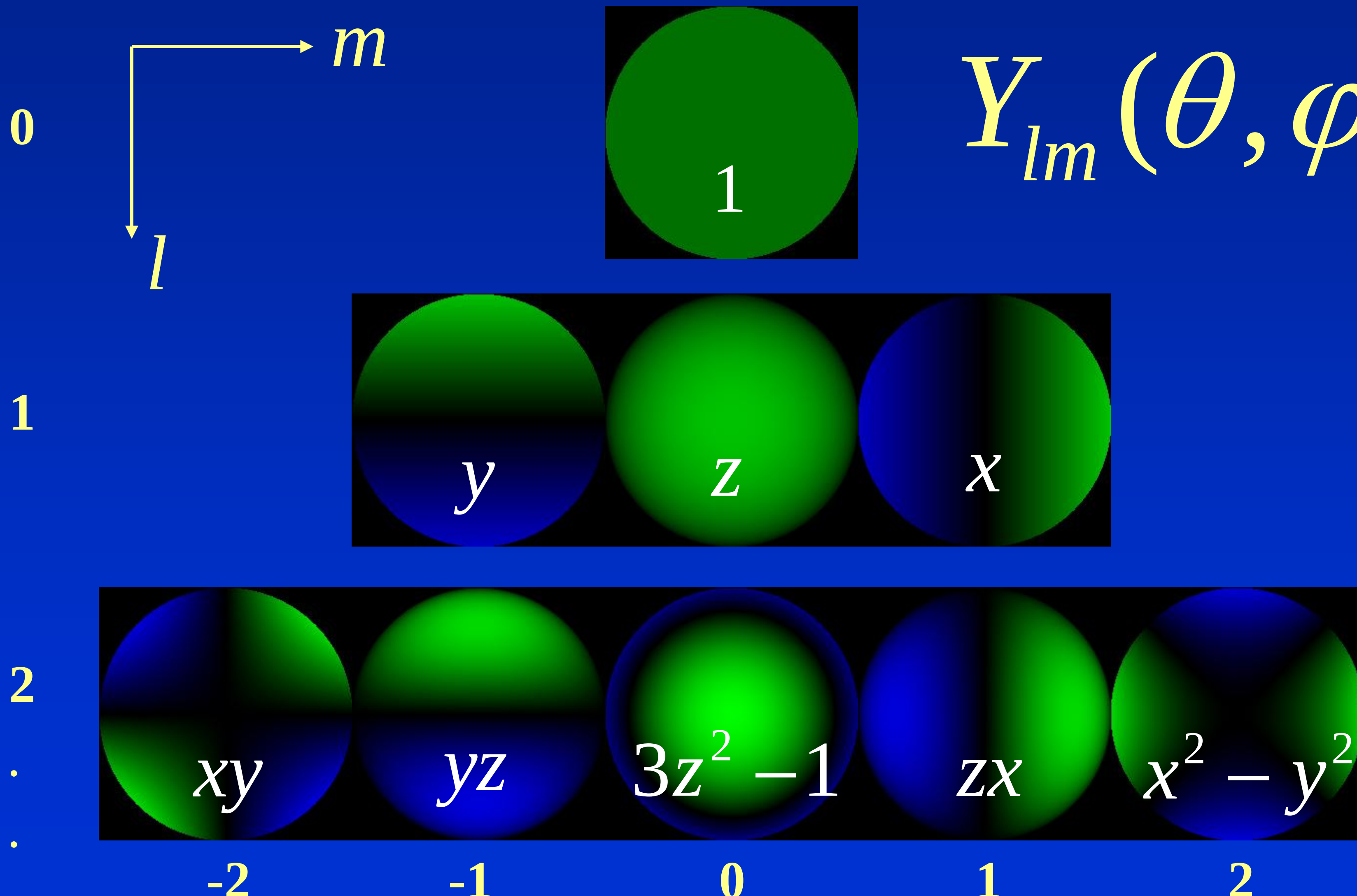
Note: Can fix output direction: $\boxed{B_p(\theta_o') = 2\pi L_p \hat{\rho}_{-p}(\theta_o')}$

# Spherical Harmonics (3D)

- Polynomials of polar and azimuth angles. $Y_{lm}(\theta, \varphi)$
- Represent all rotations on the sphere. $R_{\alpha,\beta}[\theta_i, \varphi_i]$
- Solutions to the angular part of Laplacian Equation in 3D
  - do not depend on radius of sphere.
  - very important in physics problems.
- They are Orthonormal basis on the sphere.
- Any function on the sphere can be expanded using a sum of spherical harmonics of different orders (like Fourier series in 2D)

# Spherical Harmonics

$$Y_{lm}(\theta, \varphi)$$



# Spherical Harmonic Analysis

**2D:**

$$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \quad \rho(\theta_i,\theta_o) \quad d\theta_i$$

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

**3D:**

$$B(\alpha,\beta,\theta_o,\varphi_o) = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} L(R_{\alpha,\beta}[\theta_i,\varphi_i]) \rho(\theta_i,\varphi_i,\theta_o,\varphi_o) d\theta_i d\varphi_i$$

$$B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq}$$

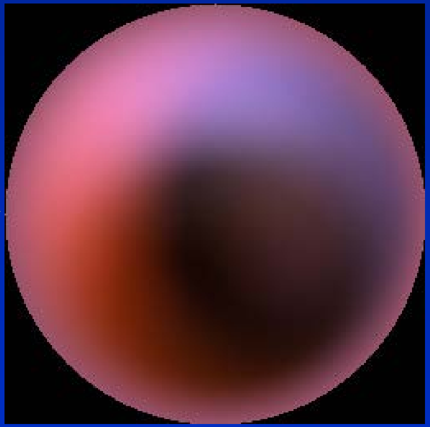
# Environment Maps

Miller and Hoffman, 1984

# Computing Irradiance

- Classically, hemispherical integral for each pixel

Incident Radiance



Irradiance

- Lambertian surface is like low pass filter
- Frequency-space analysis

# Assumptions

- Diffuse surfaces
- Distant illumination
- No shadowing, interreflection

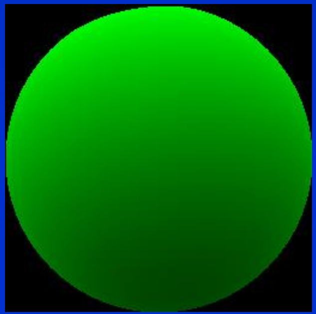
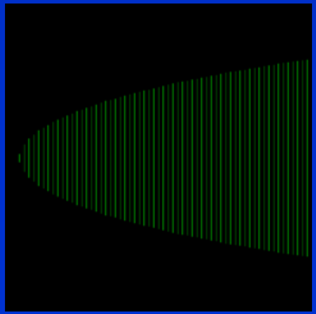
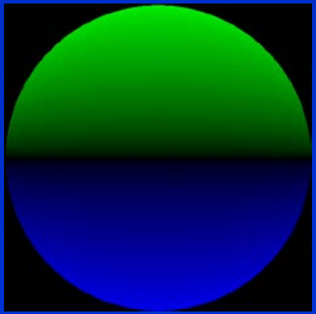
Hence, Irradiance is a function of surface normal

# Spherical Harmonic Expansion

Expand lighting (L), irradiance (E) in basis functions

$$L(\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{+l} L_{lm}Y_{lm}(\theta,\phi)$$

$$E(\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{+l} E_{lm}Y_{lm}(\theta,\phi)$$

 = .67  + .36  + …

# Computing Light Coefficients

Compute 9 lighting coefficients $L_{lm}$

- 9 numbers instead of integrals for every pixel
- Lighting coefficients are moments of lighting

$$L_{lm} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} L(\theta,\phi)\, Y_{lm}(\theta,\phi)\, \sin\theta \, d\theta d\phi$$

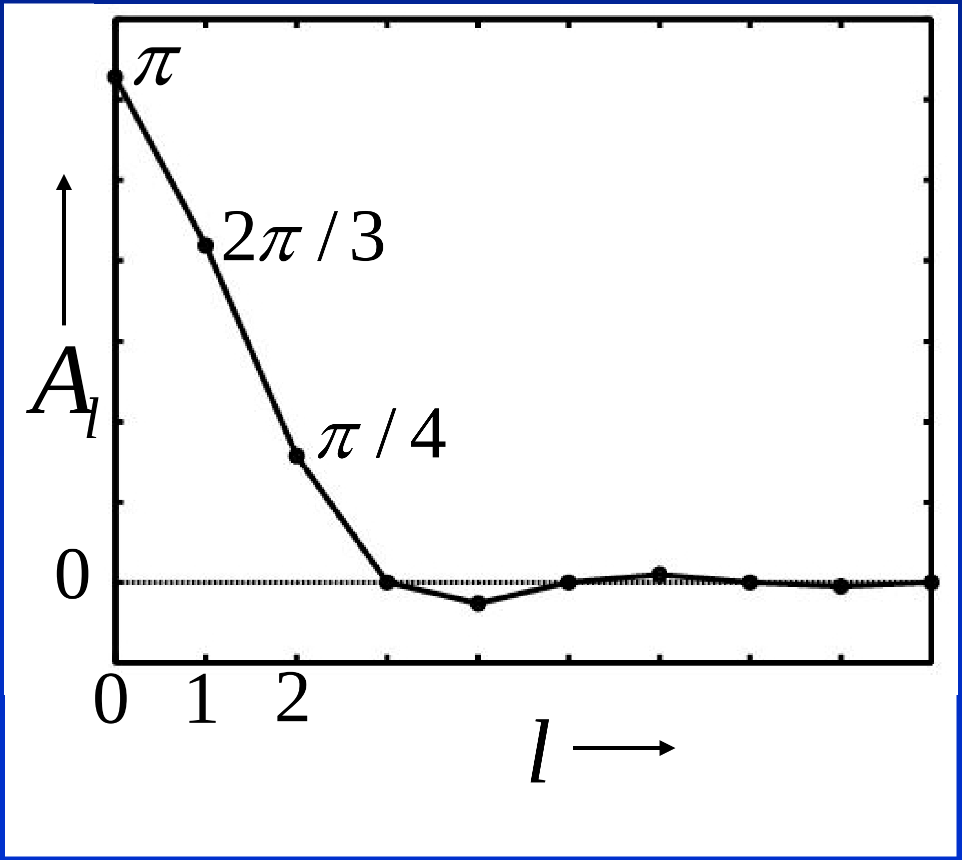
- Weighted sum of pixels in the environment map

$$L_{lm} = \sum_{pixels(\theta,\phi)} envmap[pixel] \times basisfunc_{lm}[pixel]$$

# Analytic Irradiance Formula

Lambertian surface acts like low-pass filter

$$E_{lm} = A_l L_{lm}$$

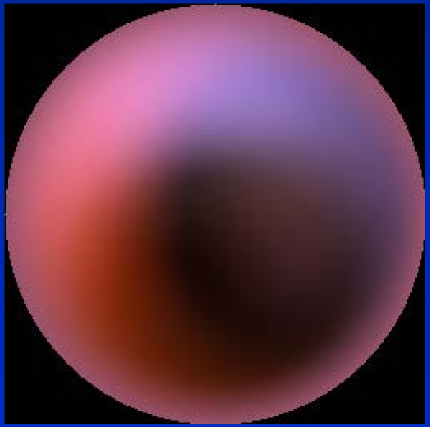



$$A_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[ \frac{l!}{2^l \left(\frac{l}{2}!\right)^2} \right] \quad l \text{ even}$$

# Computing Irradiance

- Classically, hemispherical integral for each pixel

Incident Radiance

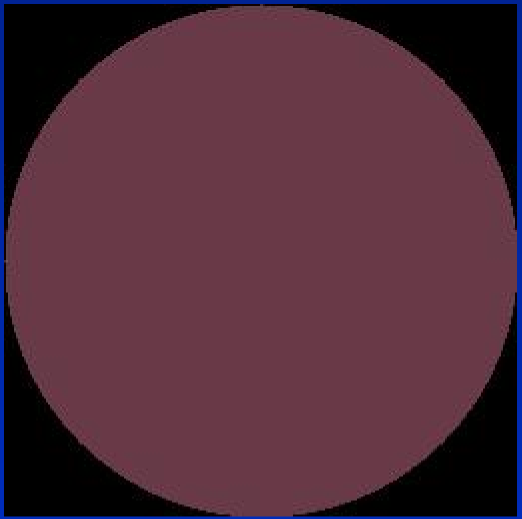


Irradiance

- Lambertian surface is like low pass filter
- Frequency-space analysis

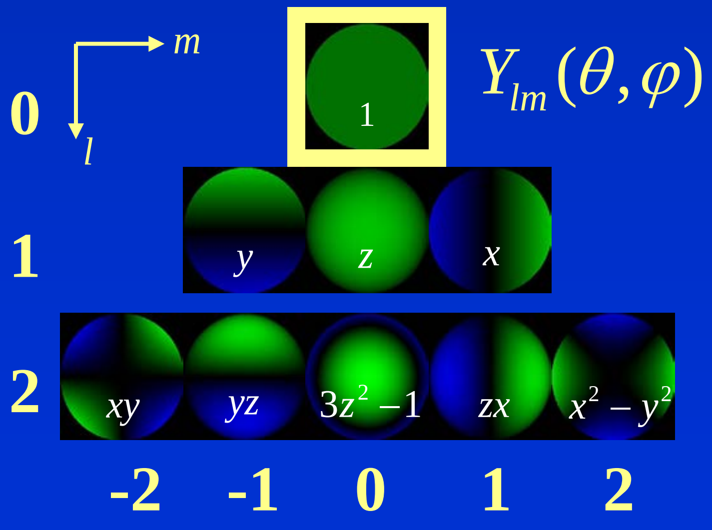
# 9 Parameter Approximation

Exact image



Order 0
1 term
(constant)

**RMS error = 25 %**

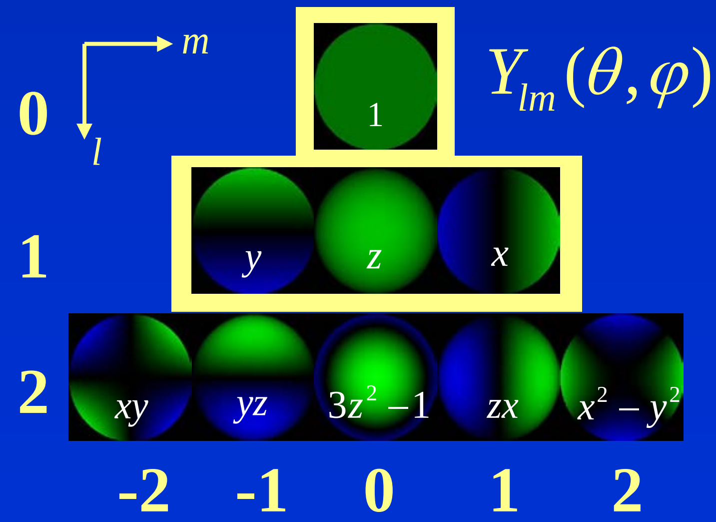


# 9 Parameter Approximation

Exact image

Order 1
4 terms
(linear)

**RMS Error = 8%**



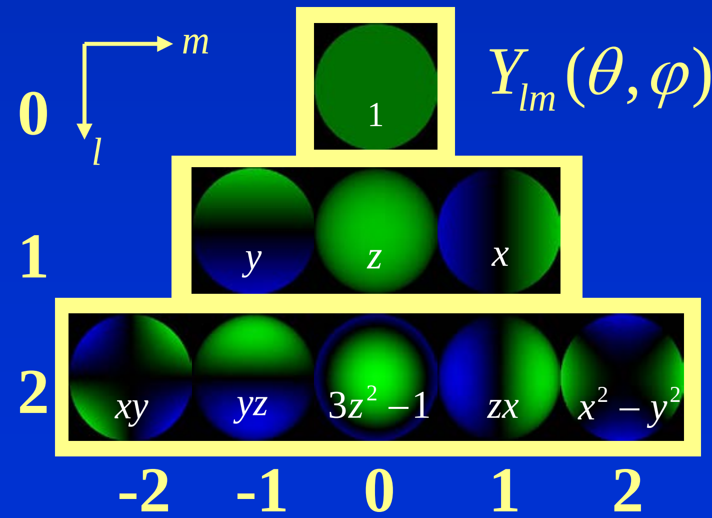
# 9 Parameter Approximation

Exact image

Order 2
9 terms
(quadratic)

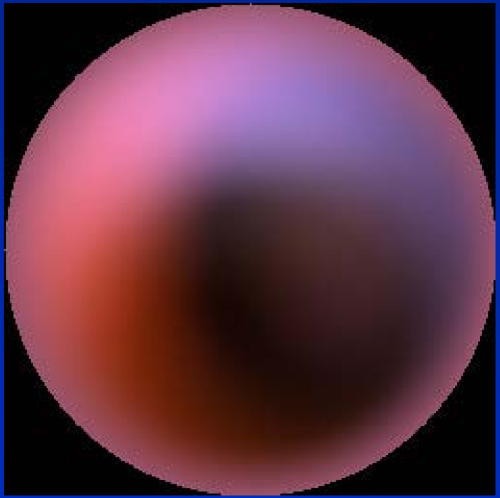
**RMS Error = 1%**

For any illumination, average error < 2% [Basri Jacobs 01]

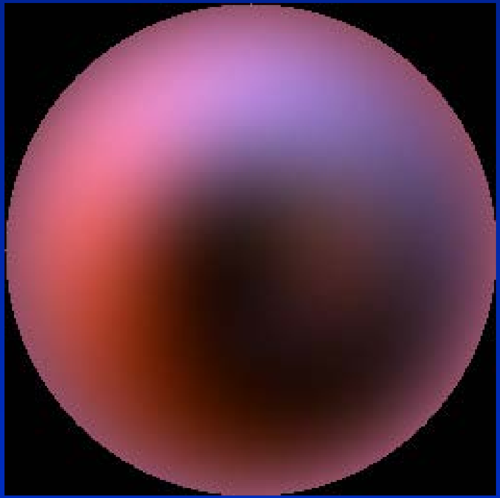


# Comparison

Incident illumination 300x300



Irradiance map Texture: 256x256 Hemispherical Integration 2Hrs



Irradiance map Texture: 256x256 Spherical Harmonic Coefficients 1sec

Time $\propto$ $300 \times 300 \times 256 \times 256$

Time $\propto$ $9 \times 256 \times 256$

# Dual Representation

Diffuse BRDF: Filter width small in frequency domain

Specular: Filter width small in spatial (angular) domain

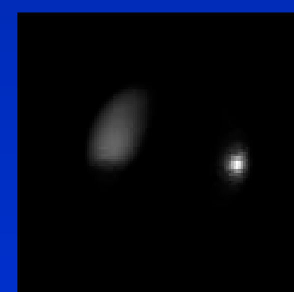
Practical Representation: Dual angular, frequency-space

B

=

$B_d$ diffuse
Frequency

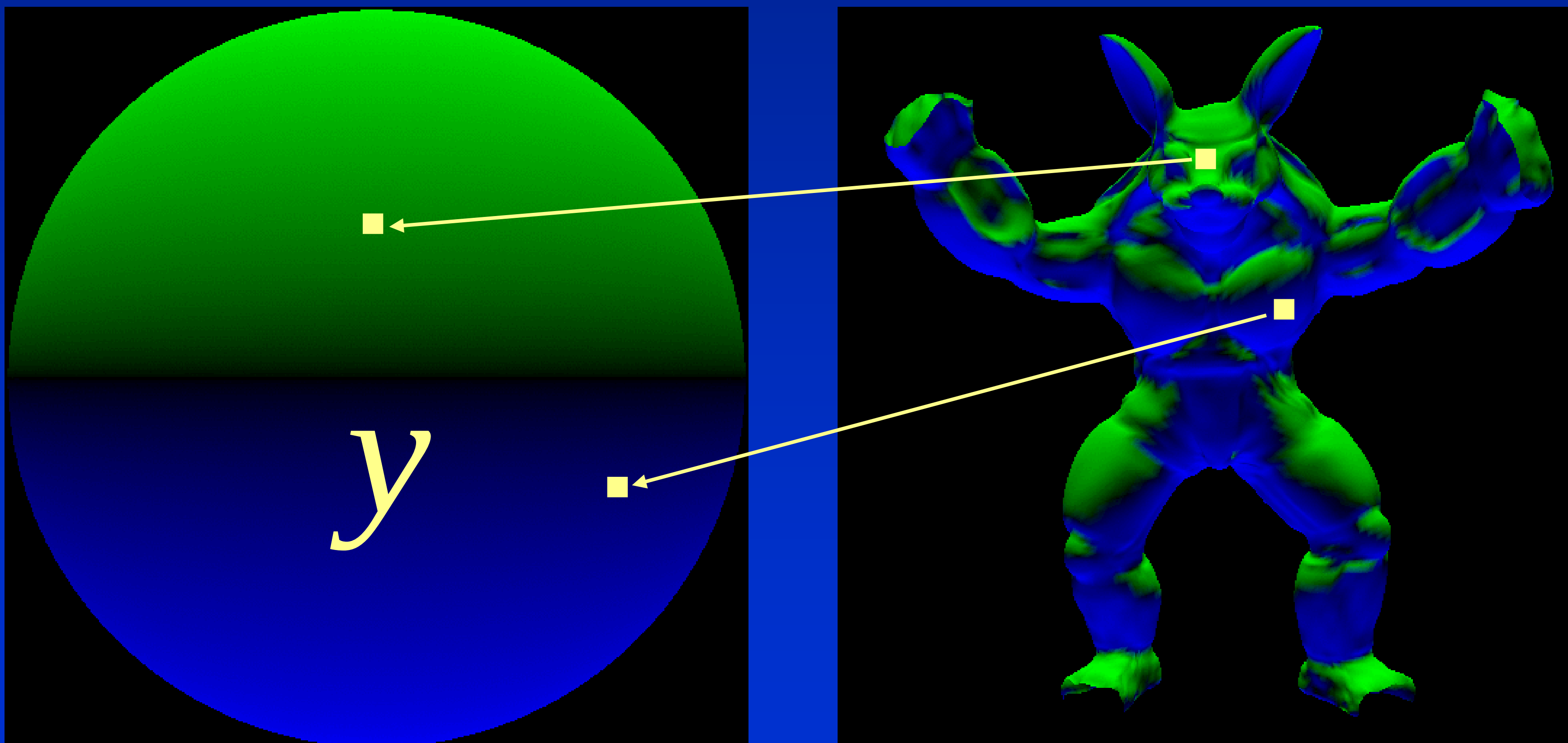
+



$B_s$ specular
Angular

# Complex Geometry

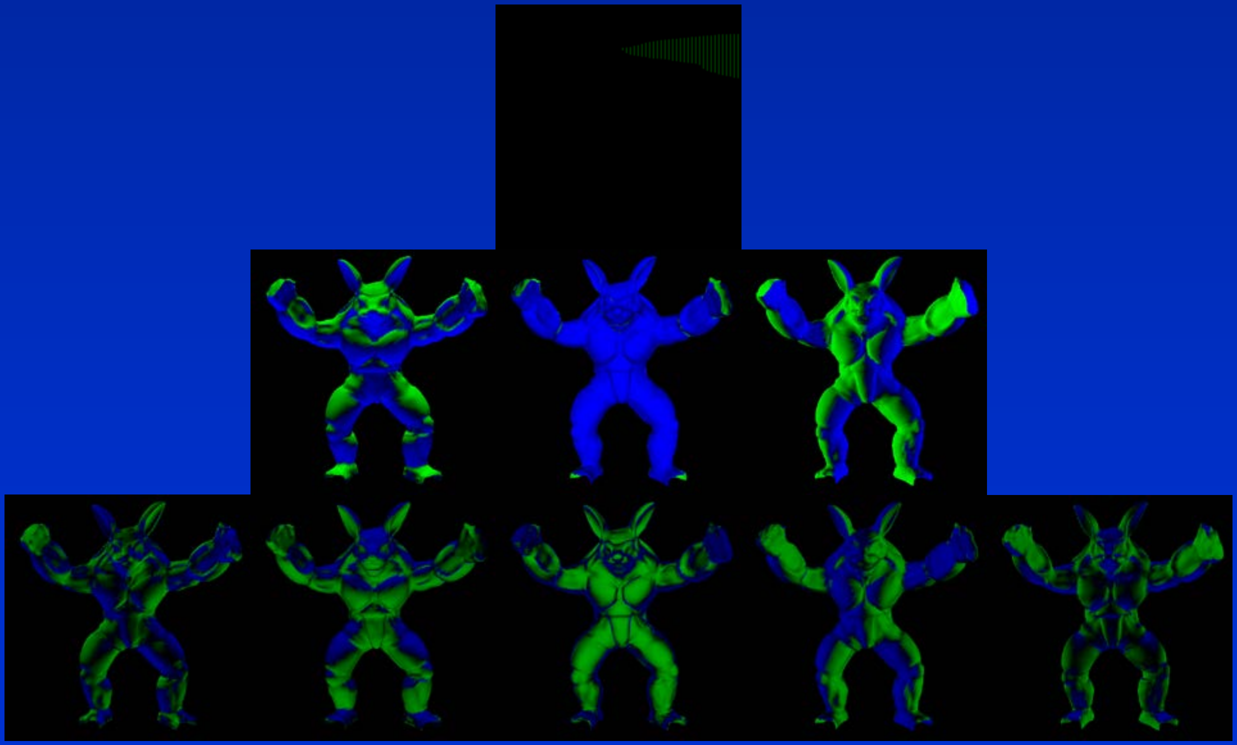
Assume no shadowing: Simply use surface normal



# Lighting Design

Final image sum of 3D basis functions scaled by $L_{lm}$

Alter appearance by changing weights of basis functions



# Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

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Signal processing framework for reflection

- Light is the signal
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Filter is Delta function : Output = Signal

Mirror BRDF : Image = Lighting [Miller and Hoffman 84]

Image courtesy Paul Debevec

# Example: Mirror BRDF

$$\hat{\rho}(\theta_i', \theta_o') \;=\; \delta(\theta_i' + \theta_o') \qquad \hat{\rho}_{p,q} = \frac{\delta_{p,q}}{2\pi}$$

$$\boxed{B_{p,q} = \delta_{p,q} L_{-p}}$$

Reflected Light Field corresponds directly to Lighting

Gazing Sphere

# Insights: Signal Processing

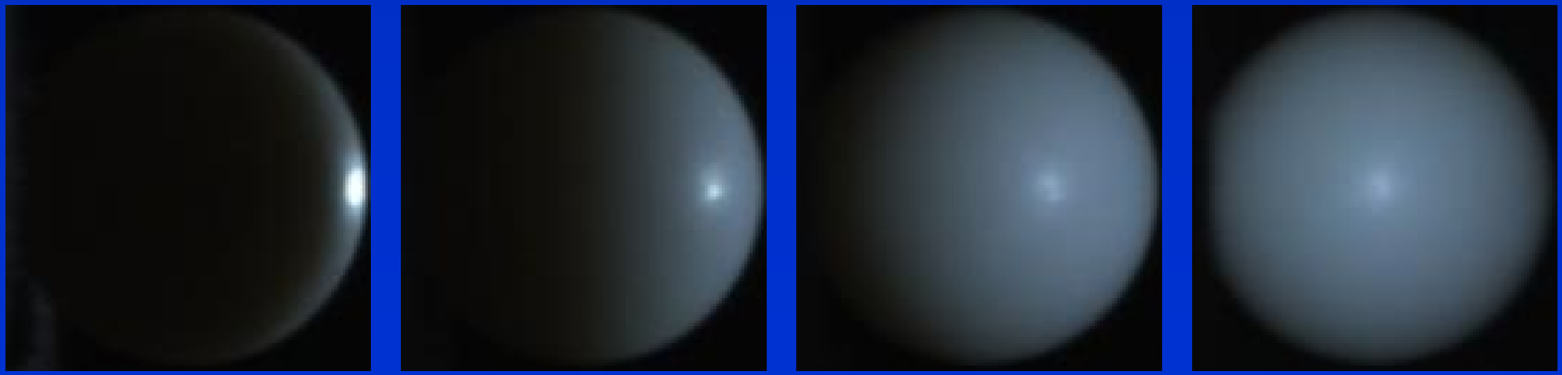
Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Signal is Delta function : Output = Filter

Point Light Source : Images = BRDF

[Marschner et al. 00]



# Example: Directional Source at $\theta_i = 0$

$$L(\theta_i) \;=\; \delta(\theta_i) \qquad L_p = \frac{1}{2\pi}$$

$$\boxed{B_{p,q} = \hat{\rho}_{-p,q}}$$

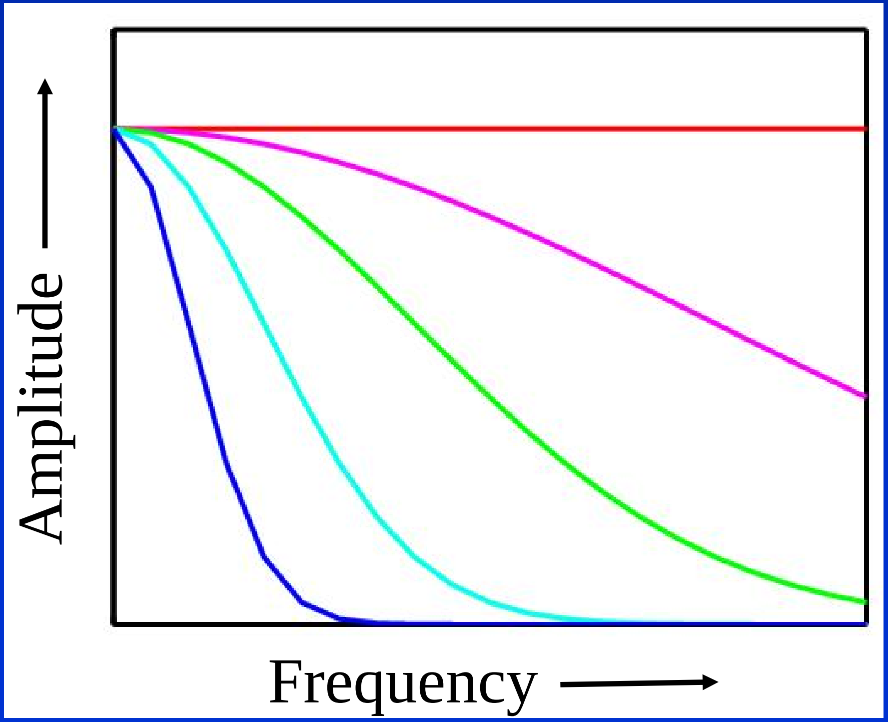
Reflected Light Field corresponds directly to BRDF

- *Impulse Response* of BRDF filter

# Phong, Microfacet Models

Mirror → Roughness

Illumination estimation
ill-posed for rough surfaces

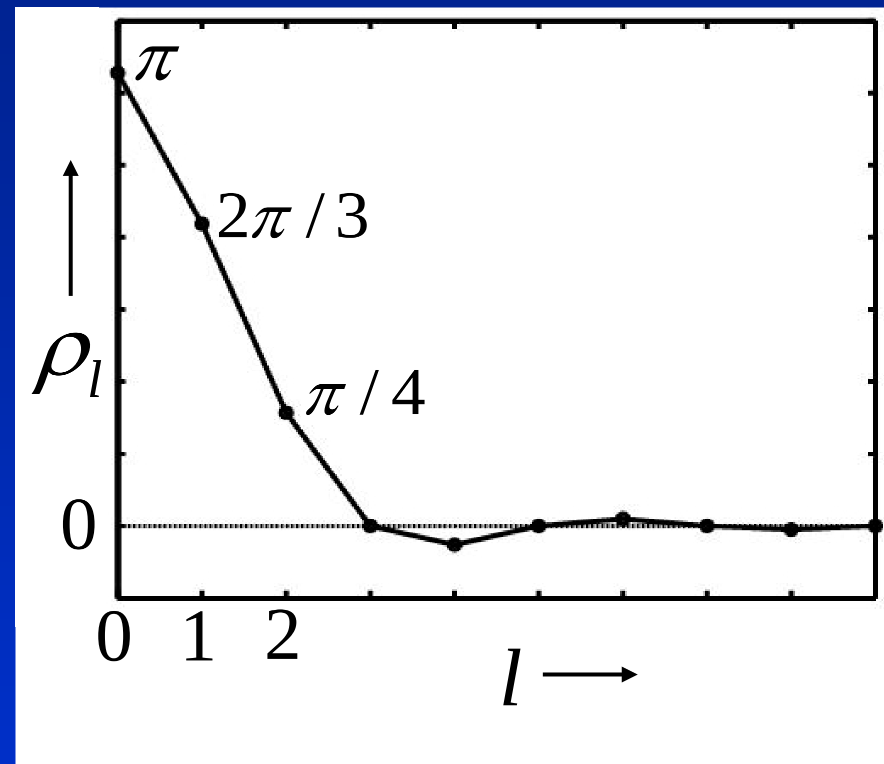



Analytic formulae in R. Ramamoorthi and P. Hanrahan
"A Signal-Processing Framework for Inverse Rendering"
SIGGRAPH 2001 pp 117-128

# Lambertian

Incident radiance (mirror sphere)


Irradiance (Lambertian)




$$\rho_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[ \frac{l!}{2^l \left(\frac{l}{2}!\right)^2} \right] \quad l \text{ even}$$

R. Ramamoorthi and P. Hanrahan "On the Relationship between Radiance and Irradiance: Determining the Illumination from Images of a Convex Lambertian Object" Journal of the Optical Society of America A 18(10) Oct 2001 pp 2448-2459

R. Basri and D. Jacobs "Lambertian Reflectance and Linear Subspaces" ICCV 2001 pp 383-390

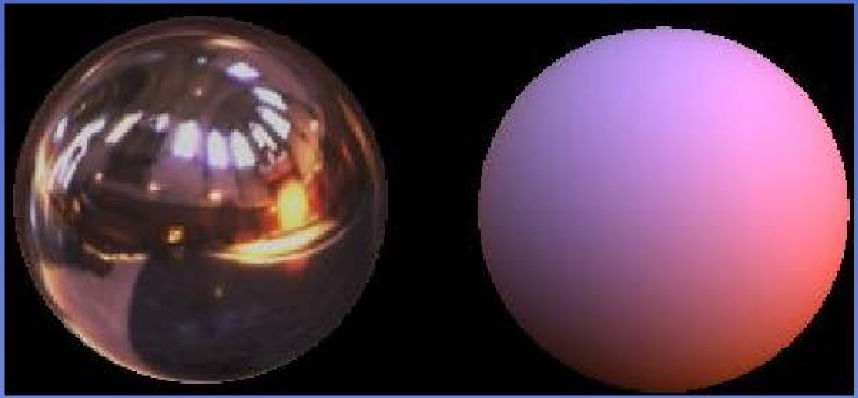
# Example: Lambertian BRDF

Transfer function is *Clamped Cosine*

No output dependence, drop index $q$

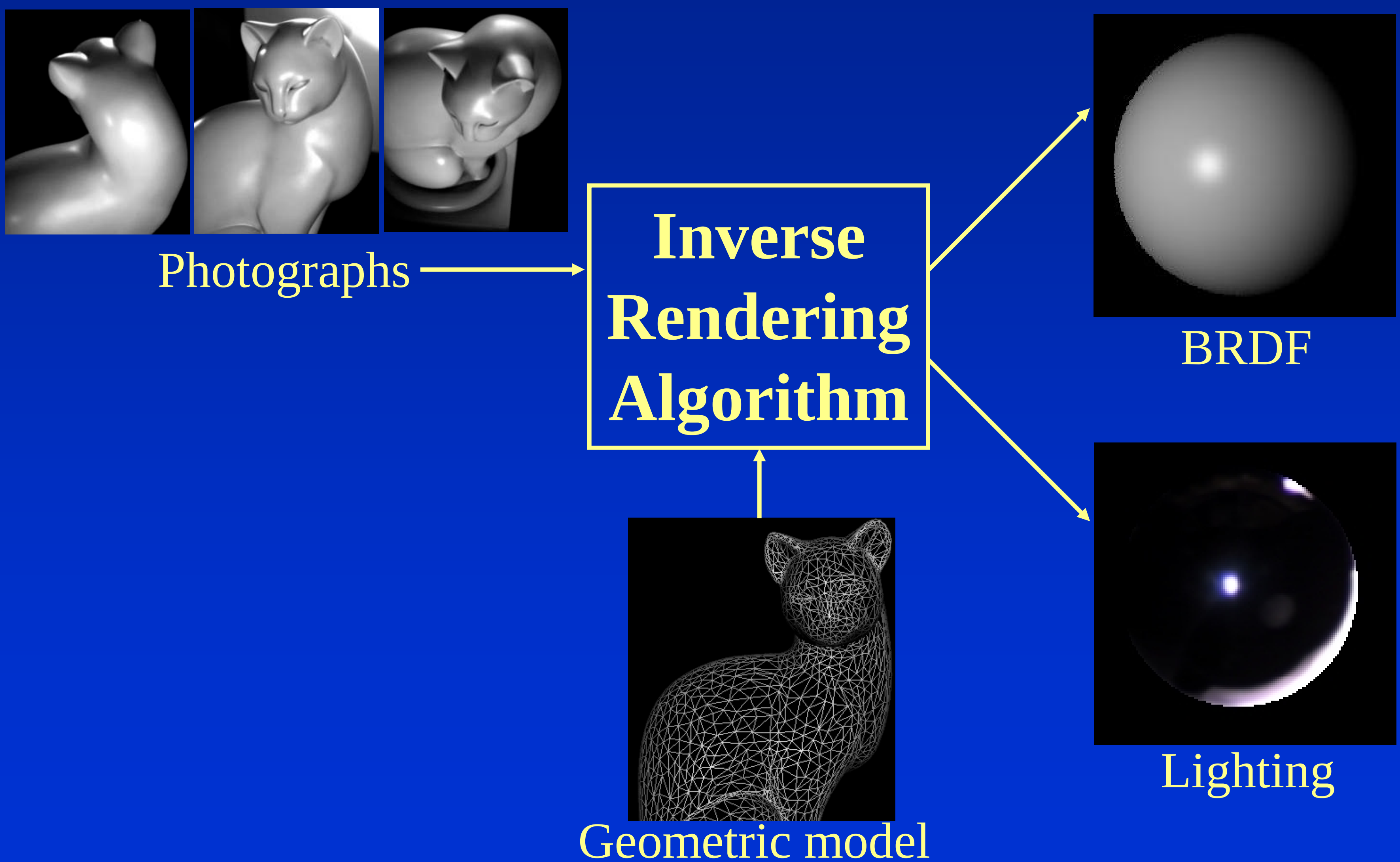
$$B_p = 2\pi L_p \hat{\rho}_{-p}$$

Lambertian BRDF is *Low-Pass* filter

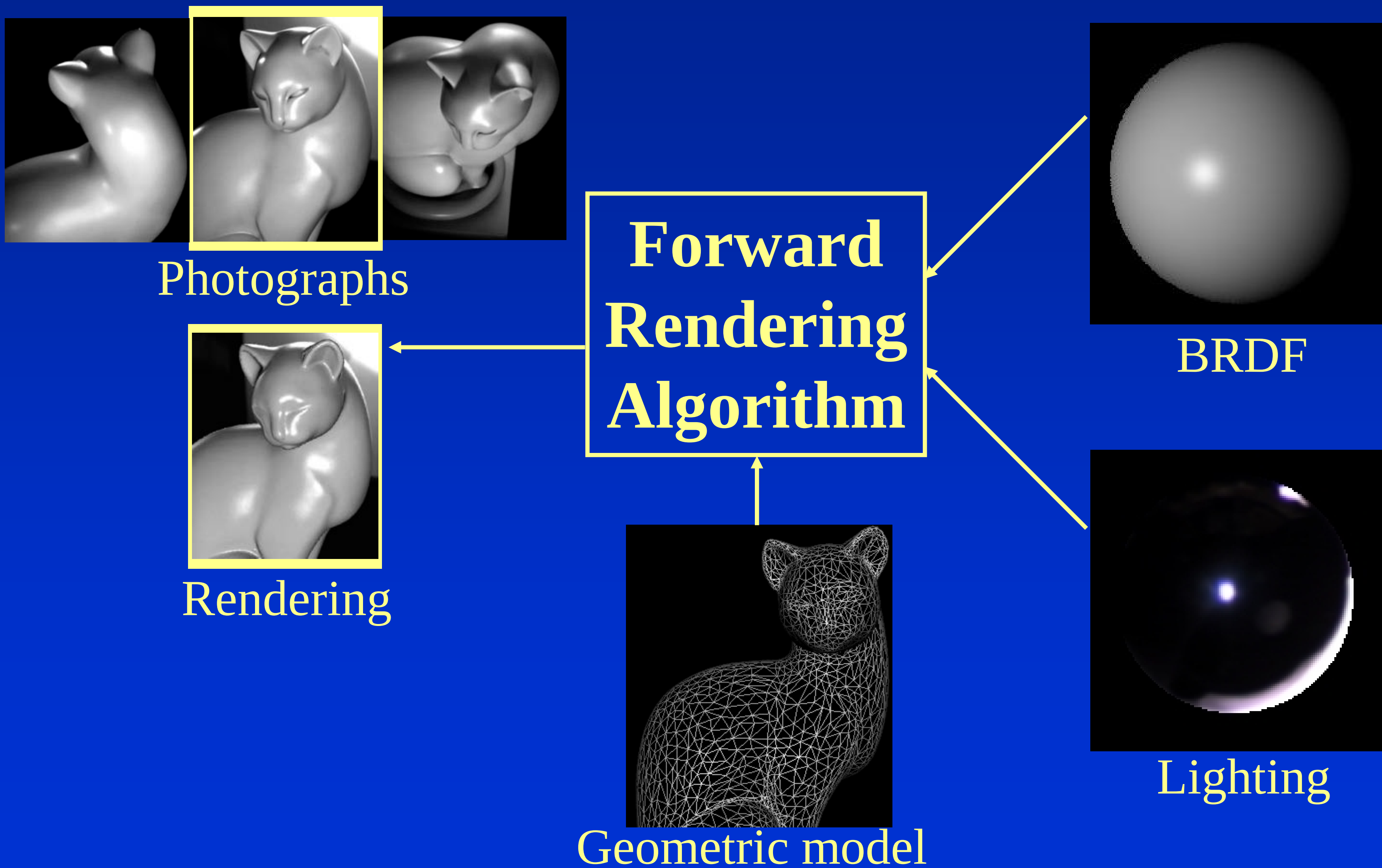


Incident Reflected

# Estimating BRDF and Lighting

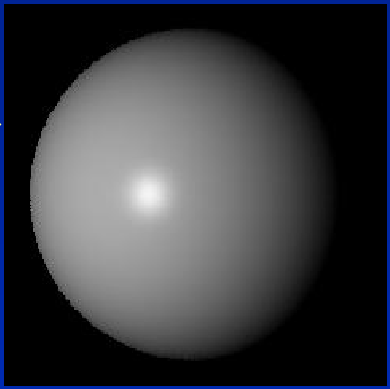


# Estimating BRDF and Lighting



# Estimating BRDF and Lighting

Photographs

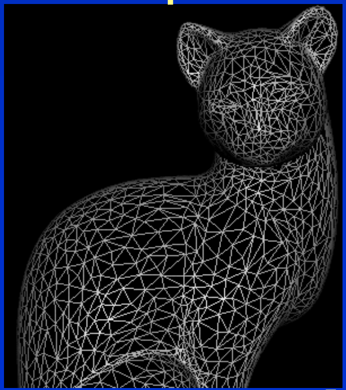


BRDF

**Forward Rendering Algorithm**

Novel lighting

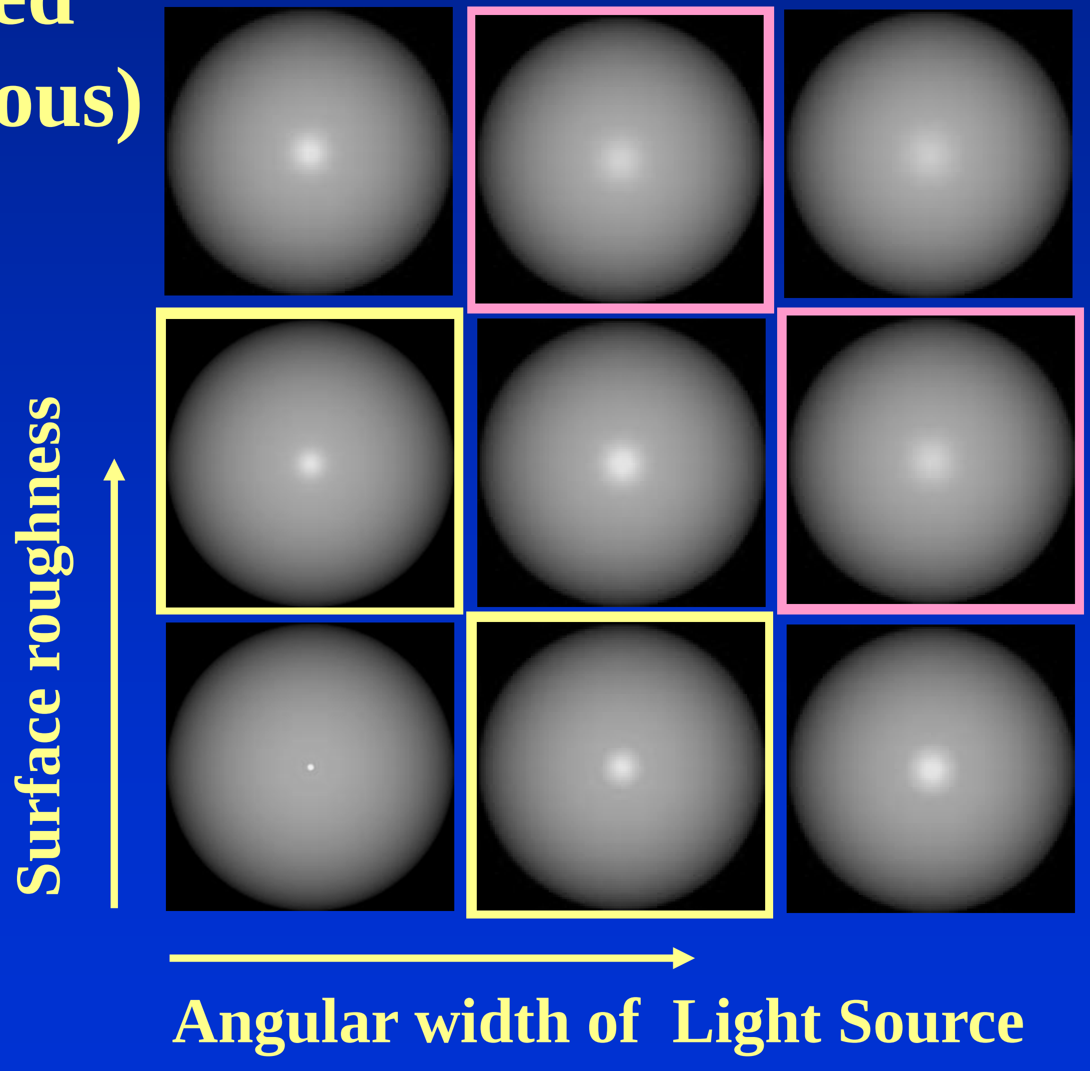
Rendering



Geometric model

# Inverse Problems: Difficulties

**Ill-posed (ambiguous)**



# Motivation

Understand nature of reflection and illumination

Applications in computer graphics

- Real-time forward rendering
- Inverse rendering

# Inverse Lighting

Given: B,ρ  find L

$$B = L \otimes \rho$$

$$B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq}$$

$$L_{lm} = \frac{1}{\Lambda_l} \frac{B_{lm,pq}}{\rho_{lq,pq}}$$

Well-posed unless denominator vanishes

- BRDF should contain high frequencies : Sharp highlights
- Diffuse reflectors low pass filters: Inverse lighting ill-posed

# Inverse BRDF

Given: B,L find ρ

$$\rho_{lq,pq} = \frac{1}{\Lambda_l} \frac{B_{lm,pq}}{L_{lm}}$$

Well-posed unless $L_{lm}$ vanishes

- Lighting should have sharp features (point sources, edges)
- BRDF estimation ill-conditioned for soft lighting

**Directional Source**

**Area source Same BRDF**

# Factoring the Light Field

Given: B  find L and ρ

$$B = L \otimes \rho$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{4D} & \text{2D} & \text{3D} \end{array}$$

More knowns (4D)
than unknowns (2D/3D)

Light Field can be factored

- Up to global scale factor
- Assumes reciprocity of BRDF
- Can be ill-conditioned
- Analytic formula derived

# Factoring the Light Field

$$\tilde{B}_{lmpq} = \Lambda_l L_{lm} \tilde{\rho}_{lpq}$$

Lighting coefficients are independent of viewing directions

(indices L and M are independent of P and Q).

BRDF Reciprocity:

$$\tilde{\rho}_{lpq} = \tilde{\rho}_{lqp}.$$

# Factoring the Light Field

$$\tilde{B}_{lmpq} = \Lambda_l L_{lm} \tilde{\rho}_{lpq}$$

$$\tilde{\rho}_{lpq} = \tilde{\rho}_{plq}.$$

Bootstrapping Method for Factorization: (Start by assuming DC component of Lighting)

$$\begin{aligned}
L_{00} &= \Lambda_0^{-1} \\
\tilde{\rho}_{0p0} &= \tilde{B}_{00p0} \\
L_{lm} &= \Lambda_l^{-1} \left( \frac{\tilde{B}_{lmpq}}{\tilde{\rho}_{lpq}} = \frac{\tilde{B}_{lm00}}{\tilde{\rho}_{l00}} = \frac{\tilde{B}_{lm00}}{\tilde{\rho}_{0l0}} \right) \\
&= \Lambda_l^{-1} \frac{\tilde{B}_{lm00}}{\tilde{B}_{00l0}} \\
\tilde{\rho}_{lpq} &= \Lambda_l^{-1} \frac{\tilde{B}_{lmpq}}{L_{lm}} \\
&= \frac{\tilde{B}_{lmpq} \tilde{B}_{00l0}}{\tilde{B}_{lm00}}
\end{aligned}$$

# Algorithm Validation

Photograph

"True" values

| | |
|---|---|
| $K_d$ | 0.91 |
| $K_s$ | 0.09 |
| $\mu$ | 1.85 |
| $\sigma$ | 0.13 |

# Algorithm Validation

Photograph

Renderings

Known lighting

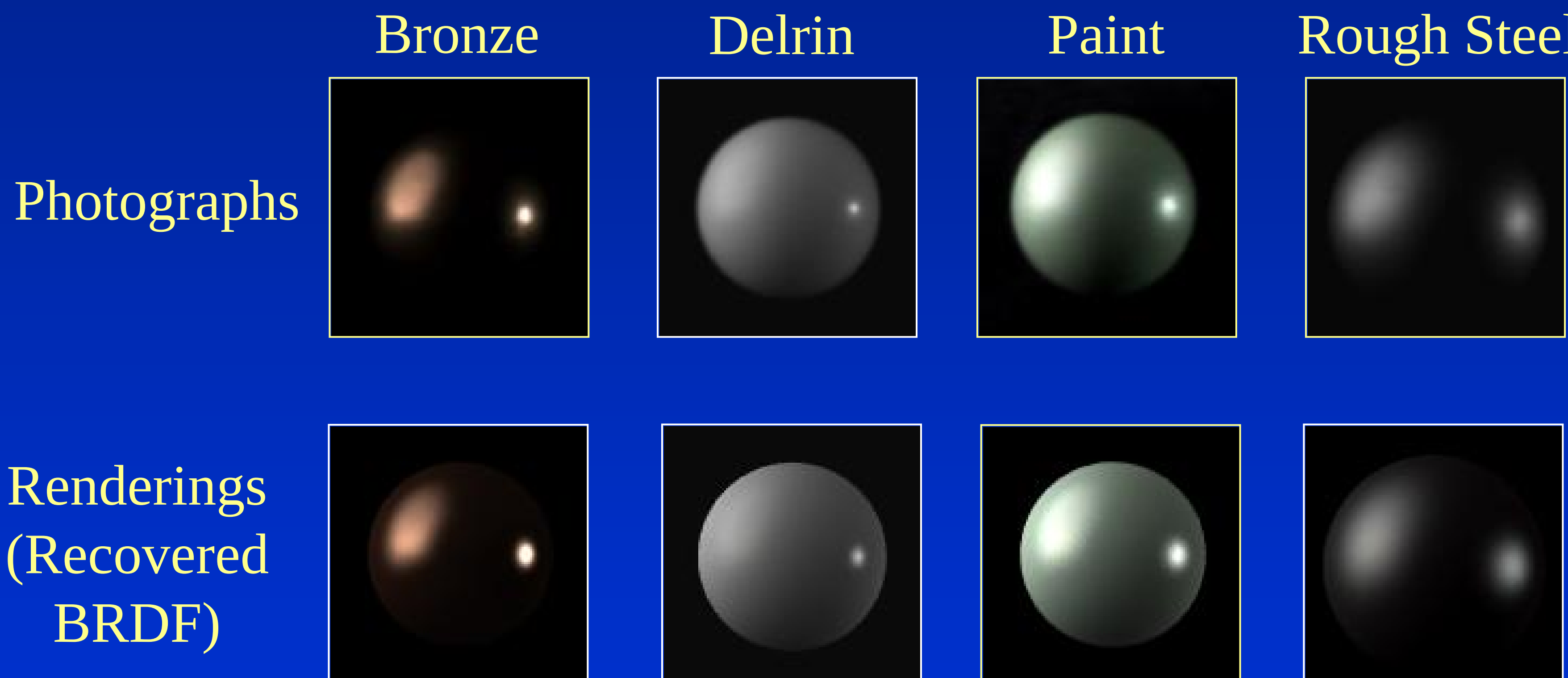
Unknown lighting

Image RMS error 5%

"True" values

| | | | |
|---|---|---|---|
| $K_d$ | 0.91 | 0.89 | 0.87 |
| $K_s$ | 0.09 | 0.11 | 0.13 |
| $\mu$ | 1.85 | 1.78 | 1.48 |
| $\sigma$ | 0.13 | 0.12 | 0.14 |

# Inverse BRDF: Spheres



# Complex Geometry

3 photographs of a sculpture

- Complex unknown illumination
- Geometry known
- Estimate microfacet BRDF *and* distant lighting

# Comparison

Photograph

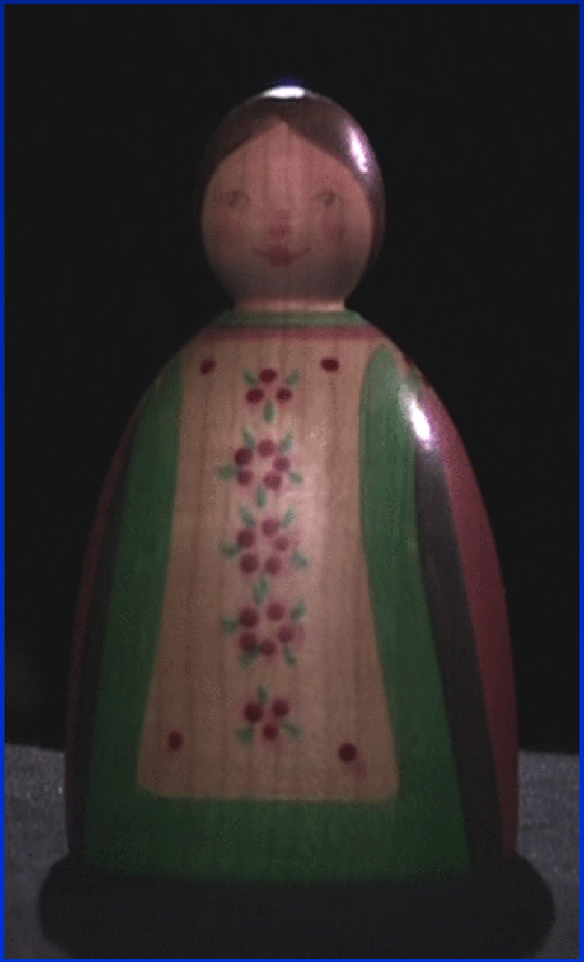
Rendering

# New View, Lighting

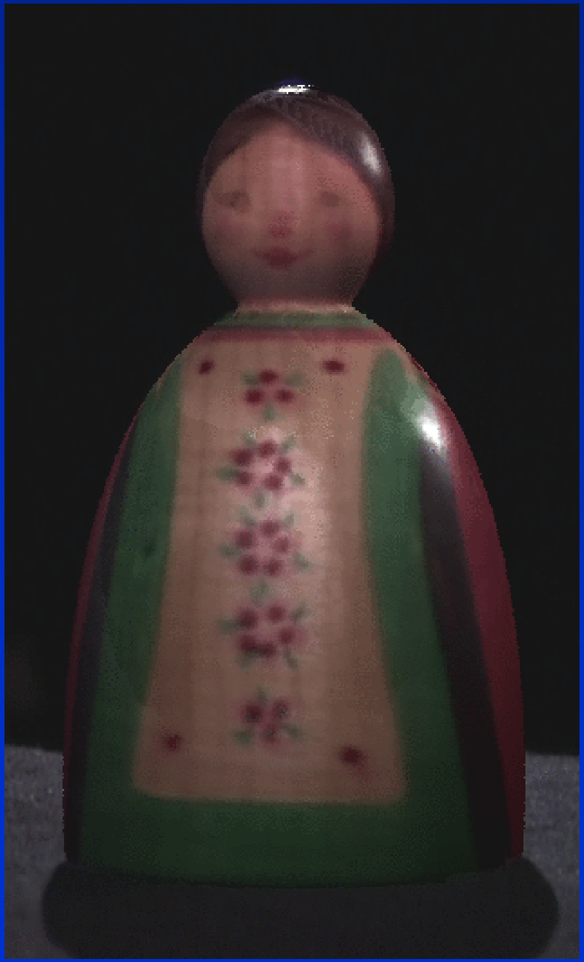
Photograph

Rendering

# Textured Objects



Photograph



Rendering