Signal Processing Framework for Reflection

Lectures #7 and #8

Thanks to Ravi Ramamoorthi, Pat Hanrahan, Ronen Basri, David Jacobs, Ron Dror, Ted Adelson.

Ravi Ramamoorthi’s homepage is an excellent source for papers, videos, PPTs on this topic. Many of the slides in these classes are obtained from his website.

http://www.cs.columbia.edu/~ravir/
Illumination Illusion

People perceive materials more easily under natural illumination than simplified illumination.

Images courtesy Ron Dror and Ted Adelson
Illumination Illusion

People perceive materials more easily under natural illumination than simplified illumination.

Images courtesy Ron Dror and Ted Adelson
Material Recognition

Photographs of 4 spheres in 3 different lighting conditions
courtesy Dror and Adelson
Surface Appearance - RECAP

Image intensities = $f(\text{normal, surface reflectance, illumination})$

Surface Reflection depends on both the viewing and illumination direction.
BRDF: Bidirectional Reflectance Distribution Function

\[ E_{\text{surface}}(\theta_i, \phi_i) \quad \text{Irradiance at Surface in direction } (\theta_i, \phi_i) \]
\[ L_{\text{surface}}(\theta_r, \phi_r) \quad \text{Radiance of Surface in direction } (\theta_r, \phi_r) \]

\[ \text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L_{\text{surface}}(\theta_r, \phi_r)}{E_{\text{surface}}(\theta_i, \phi_i)} \]
Derivation of the Scene Radiance Equation

From the definition of BRDF:

\[ L_{\text{surface}}(\theta_r, \phi_r) = E_{\text{surface}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \]
Derivation of the Scene Radiance Equation – Important!

From the definition of BRDF:

\[
L^{\text{surface}}(\theta_r, \phi_r) = \frac{E^{\text{surface}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r)}{\dot{\omega}}
\]

Write Surface Irradiance in terms of Source Radiance:

\[
L^{\text{surface}}(\theta_r, \phi_r) = L^{\text{src}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i
\]

Integrate over entire hemisphere of possible source directions:

\[
L^{\text{surface}}(\theta_r, \phi_r) = \int \int L^{\text{src}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \, d\omega_i
\]

Convert from solid angle to theta-phi representation:

\[
L^{\text{surface}}(\theta_r, \phi_r) = \int \int L^{\text{src}}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i \, d\theta_i \, d\phi_i
\]
Assumptions

- Known geometry

Laser range scanner

Structured light
Assumptions

- Known geometry
- Convex curved surfaces: no shadows, interreflection

Complex geometry: use surface normal
Assumptions

- Known geometry
- Convex curved surfaces: no shadows, interreflection
- Distant illumination

Photograph of mirror sphere

Illumination: Grace Cathedral courtesy Paul Debevec
Assumptions

• Known geometry
• Convex curved surfaces: no shadows, interreflection
• Distant illumination
• Homogeneous isotropic materials

Isotropic

Anisotropic
Assumptions

• Known geometry
• Convex curved surfaces: no shadows, interreflection
• Distant illumination
• Homogeneous isotropic materials

Later, practical algorithms: relax some assumptions
Reflection

\[ B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i) \rho(\theta_i, \theta_o) \, d\theta_i \]

Reflected Light Field | Lighting | BRDF
Reflection as Convolution (2D)

\[ B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i) \rho(\theta_i, \theta_o) \, d\theta_i \]

Reflected Light Field

Lighting

BRDF
Reflection as Convolution (2D)

\[ B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i) \rho(\theta_i, \theta_o) \, d\theta_i \]

Reflected Light Field

\[ B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \rho(\theta_i, \theta_o) \, d\theta_i \]

Lighting BRDF
Reflection as Convolution (2D)

\[ B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \rho(\theta_i, \theta_o) \, d\theta_i \]
Convolution

\[ \text{Signal } f(x) \otimes g(x) = h(u) \]

- Signal \( f(x) \)
- Filter \( g(x) \)
- Output \( h(u) \)
Convolution

Signal $f(x)$ \hspace{1cm} Filter $g(x)$ \hspace{1cm} Output $h(u)$

$$h(u_1) = \int g(x - u_1) f(x) \, dx$$
Convolution

Signal $f(x)$  Filter $g(x)$  Output $h(u)$

$$h(u_2) = \int g(x - u_2) f(x) \, dx$$
Convolution

\[ h(u_3) = \int g(x - u_3) f(x) \, dx \]
Convolution

\[ h(u) = \int g(x - u) f(x) \, dx \]

\[ h = f \otimes g = g \otimes f \]

Fourier analysis

\[ h_\omega = f_\omega g_\omega \]
Reflection as Convolution (2D)

\[ B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha)\rho(\theta_i, \theta_o) \, d\theta_i \]

\[ B = L \otimes \rho \]

\[ B_{l,p} = 2\pi L_l \rho_{l,p} \]

Spatial: integral

Frequency: product

Fourier Analysis

\[ L(\theta_i) = \sum_p L_p e^{i p \theta_i} \]

\[ \hat{\rho}(\theta'_i, \theta'_o) = \sum_p \sum_q \hat{\rho}_{p,q} e^{i p \theta_i} e^{i q \theta_o} \]

\[ B(\alpha, \theta'_o) = \sum_p \sum_q B_{p,q} e^{i p \alpha} e^{i q \theta'_o} \]
Fourier Analysis

\[ L(\theta_i) = \sum_p L_p e^{Ip\theta_i} \]

\[ \hat{\rho}(\theta_i', \theta_o') = \sum_p \sum_q \hat{\rho}_{p,q} e^{Ip\theta_i'} e^{Iq\theta_o'} \]

\[ B(\alpha, \theta_o') = \sum_p \sum_q B_{p,q} e^{Ip\alpha} e^{Iq\theta_o'} \]

\[ B_{p,q} = 2\pi L_p \hat{\rho}_{-p,q} \]

Note: Can fix output direction:

\[ B_p(\theta_o') = 2\pi L_p \hat{\rho}_{-p}(\theta_o') \]
Spherical Harmonics (3D)

- Polynomials of polar and azimuth angles. \( Y_{lm}(\theta, \varphi) \)
- Represent all rotations on the sphere. \( R_{\alpha, \beta}[\theta_i, \varphi_i] \)
- Solutions to the angular part of Laplacian Equation in 3D
  - do not depend on radius of sphere.
  - very important in physics problems.
- They are Orthonormal basis on the sphere.
- Any function on the sphere can be expanded using a sum of spherical harmonics of different orders (like Fourier series in 2D)
Spherical Harmonics

\[ Y_{lm}(\theta, \varphi) \]

\[ l \quad m \]

\[
\begin{align*}
0 & \quad 0 & 1 & \quad 1 \\
1 & \quad y & z & \quad x \\
2 & \quad xy & yz & 3z^2 - 1 & \quad zx & x^2 - y^2 \\
& \quad -2 & -1 & 0 & 1 & 2
\end{align*}
\]
Spherical Harmonic Analysis

2D:

\[ B(\alpha, \theta_\circ) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \rho(\theta_i, \theta_\circ) \, d\theta_i \]

\[ B_{l,p} = 2\pi L_l \rho_{l,p} \]

3D:

\[ B(\alpha, \beta, \theta_\circ, \varphi_\circ) = \int_0^\pi \int_0^{2\pi} L(R_{\alpha, \beta}[\theta_i, \varphi_i]) \rho(\theta_i, \varphi_i, \theta_\circ, \varphi_\circ) \, d\theta_i \, d\varphi_i \]

\[ B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq} \]
Environment Maps

Miller and Hoffman, 1984
Computing Irradiance

- Classically, hemispherical integral for each pixel
- Lambertian surface is like low pass filter
- Frequency-space analysis
Assumptions

- Diffuse surfaces
- Distant illumination
- No shadowing, interreflection

Hence, Irradiance is a function of surface normal
Spherical Harmonic Expansion

Expand lighting ($L$), irradiance ($E$) in basis functions

$$L(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} L_{lm} Y_{lm}(\theta, \phi)$$

$$E(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} E_{lm} Y_{lm}(\theta, \phi)$$

$$= .67 + .36 + \ldots$$
Computing Light Coefficients

Compute 9 lighting coefficients $L_{lm}$

- 9 numbers instead of integrals for every pixel
- Lighting coefficients are moments of lighting

\[
L_{lm} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} L(\theta, \phi) Y_{lm}(\theta, \phi) \sin \theta \, d\theta \, d\phi
\]

- Weighted sum of pixels in the environment map

\[
L_{lm} = \sum_{\text{pixels}(\theta, \phi)} \text{envmap}[\text{pixel}] \times \text{basisfunc}_{lm}[\text{pixel}]
\]
Lambertian surface acts like low-pass filter

\[ E_{lm} = A_l L_{lm} \]

\[ A_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[ \frac{l!}{2^l \left( \frac{l}{2} \right)!} \right] \] for even \( l \)
Computing Irradiance

• Classically, hemispherical integral for each pixel

• Lambertian surface is like low pass filter

• Frequency-space analysis
9 Parameter Approximation

RMS error = 25 %

Exact image

Order 0
1 term
(constant)

$Y_{lm}(\theta, \varphi)$
9 Parameter Approximation

Exact image

Order 1
4 terms (linear)

RMS Error = 8%

$Y_{lm}(\theta, \varphi)$
9 Parameter Approximation

Exact image

RMS Error = 1%

For any illumination, average error < 2% [Basri Jacobs 01]
Comparison

Incident illumination
300x300

Irradiance map
Texture: 256x256
Hemispherical Integration 2Hrs
Time $\propto 300 \times 300 \times 256 \times 256$

Irradiance map
Texture: 256x256
Spherical Harmonic Coefficients 1sec
Time $\propto 9 \times 256 \times 256$
Dual Representation

Diffuse BRDF: Filter width small in frequency domain

Specular: Filter width small in spatial (angular) domain

Practical Representation: Dual angular, frequency-space

\[ B = B_d \text{ diffuse} + B_s \text{ specular} \]
Complex Geometry

Assume no shadowing: Simply use surface normal
Lighting Design

Final image sum of 3D basis functions scaled by $L_{lm}$
Alter appearance by changing weights of basis functions
Insights: Signal Processing

Signal processing framework for reflection
• Light is the signal
• BRDF is the filter
• Reflection on a curved surface is convolution
Insights: Signal Processing

Signal processing framework for reflection
- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Filter is Delta function : Output = Signal

Mirror BRDF : Image = Lighting

[Miller and Hoffman 84]

Image courtesy Paul Debevec
Example: Mirror BRDF

\[ \hat{\rho}(\theta_i', \theta_o') = \delta(\theta_i' + \theta_o') \]

\[ \hat{\rho}_{p,q} = \frac{\delta_{p,q}}{2\pi} \]

\[ B_{p,q} = \delta_{p,q} L_{-p} \]

Reflected Light Field corresponds directly to Lighting

Gazing Sphere
Insights: Signal Processing

Signal processing framework for reflection
- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Signal is Delta function : Output = Filter
Point Light Source : Images = BRDF

[Marschner et al. 00]
Example: Directional Source at $\theta_i = 0$

\[ L(\theta_i) = \delta(\theta_i) \]
\[ L_p = \frac{1}{2\pi} \]

\[ B_{p,q} = \hat{\rho}_{-p,q} \]

Reflected Light Field corresponds directly to BRDF

- Impulse Response of BRDF filter
Phong, Microfacet Models

Mirrors

Roughness

Illumination estimation
ill-posed for rough surfaces

Analytic formulae in R. Ramamoorthi and P. Hanrahan
“A Signal-Processing Framework for Inverse Rendering”
SIGGRAPH 2001 pp 117-128
Lambertian

Incident radiance (mirror sphere)

Irradiance (Lambertian)

\[
\rho_l = \frac{3\pi (1+\rho)(1-\rho)}{(1+\rho)(1-\rho)_{\frac{\pi}{2}}} \left[ \frac{3\pi (\frac{\pi}{2})_{\frac{\pi}{2}}}{\lambda} \right]
\]


Example: Lambertian BRDF

Transfer function is *Clamped Cosine*

No output dependence, drop index $q$

$$B_p = 2\pi L_p \hat{\rho}_{-p}$$

Lambertian BRDF is *Low-Pass filter*
Estimating BRDF and Lighting

Photographs

Forward Rendering Algorithm

BRDF

Lighting

Geometric model

Rendering
Estimating BRDF and Lighting

Photographs

Forward Rendering Algorithm

Geometric model

Novel lighting

Rendering

BRDF
Inverse Problems: Difficulties

Ill-posed (ambiguous)

Angular width of Light Source

Surface roughness
Motivation

Understand nature of reflection and illumination

Applications in computer graphics
  • Real-time forward rendering
  • Inverse rendering
Inverse Lighting

Given: $B, \rho$ find $L$

\[ B = L \otimes \rho \]

\[ B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq} \]

\[ L_{lm} = \frac{1}{\Lambda_l \rho_{lq,pq}} B_{lm,pq} \]

Well-posed unless denominator vanishes

- BRDF should contain high frequencies: Sharp highlights
- Diffuse reflectors low pass filters: Inverse lighting ill-posed
Inverse BRDF

Given: \( B, L \) find \( \rho \)

\[
\rho_{lq,pq} = \frac{1}{\Lambda_l} \frac{B_{lm,pq}}{L_{lm}}
\]

Well-posed unless \( L_{lm} \) vanishes

- Lighting should have sharp features (point sources, edges)
- BRDF estimation ill-conditioned for soft lighting

Directional Source

Area source

Same BRDF
Given: $B$ find $L$ and $\rho$

$$B = L \otimes \rho$$

Light Field can be factored
- Up to global scale factor
- Assumes reciprocity of BRDF
- Can be ill-conditioned
- Analytic formula derived

More knowns (4D) than unknowns (2D/3D)
Factoring the Light Field

\[ \tilde{B}_{lm,pq} = \Lambda_l L_{lm} \tilde{\rho}_{lpq} \]

Lighting coefficients are independent of viewing directions

(indices L and M are independent of P and Q).

BRDF Reciprocity:
Factoring the Light Field

\[ \tilde{B}_{lm\rho q} = \Lambda_l L_{lm} \tilde{\rho}_{lpq} \]

\[ \tilde{\rho}_{pq} = \tilde{\rho}_{plq} \]

Bootstrapping Method for Factorization: (Start by assuming DC component of Lighting)

\[ L_{00} = \Lambda_0^{-1} \]
\[ \tilde{\rho}_{0p0} = \tilde{B}_{00p0} \]
\[ L_{lm} = \Lambda_l^{-1} \left( \frac{\tilde{B}_{lm\rho q}}{\tilde{\rho}_{lpq}} \right) = \frac{\tilde{B}_{lm00}}{\tilde{\rho}_{l00}} = \frac{\tilde{B}_{lm00}}{\tilde{\rho}_{0l0}} \]
\[ \tilde{\rho}_{lpq} = \Lambda_l^{-1} \frac{\tilde{B}_{lm\rho q}}{L_{lm}} = \frac{\tilde{B}_{lm\rho q}}{\tilde{B}_{00l0}} = \frac{\tilde{B}_{lm\rho q}}{\tilde{B}_{lm00}} \]
Algorithm Validation

Photograph

“True” values

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<td>$\sigma$</td>
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Algorithm Validation

Photograph  Renderings

Known lighting  Unknown lighting

Image RMS error 5%

“True” values

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Inverse BRDF: Spheres

Photographs
- Bronze
- Delrin
- Paint
- Rough Steel

Renderings (Recovered BRDF)
3 photographs of a sculpture
• Complex unknown illumination
• Geometry known
• Estimate microfacet BRDF and distant lighting
Comparison

Photograph

Rendering
New View, Lighting

Photograph

Rendering
Textured Objects

Photograph

Rendering