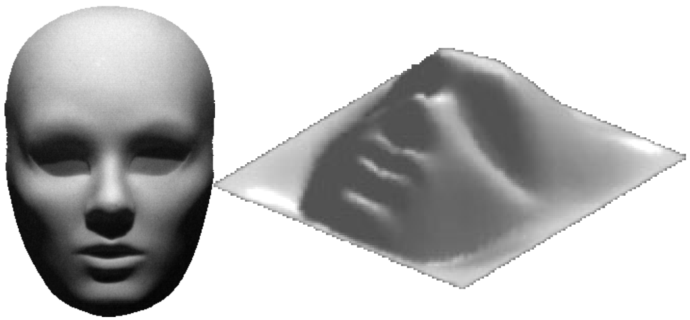
Basic Principles of Surface Reflectance

Lecture #3

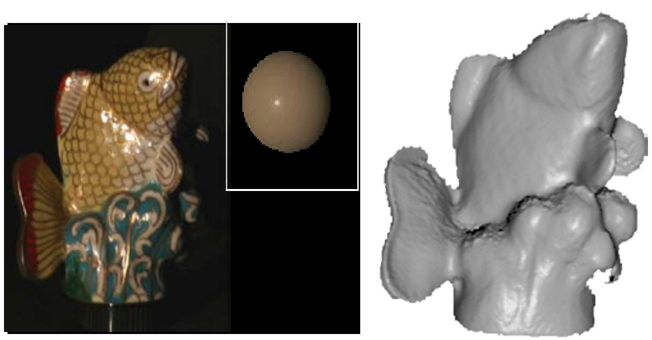
Thanks to Shree Nayar, Ravi Ramamoorthi, Pat Hanrahan

Methods Relying on Surface Reflectance



Shape from Shading

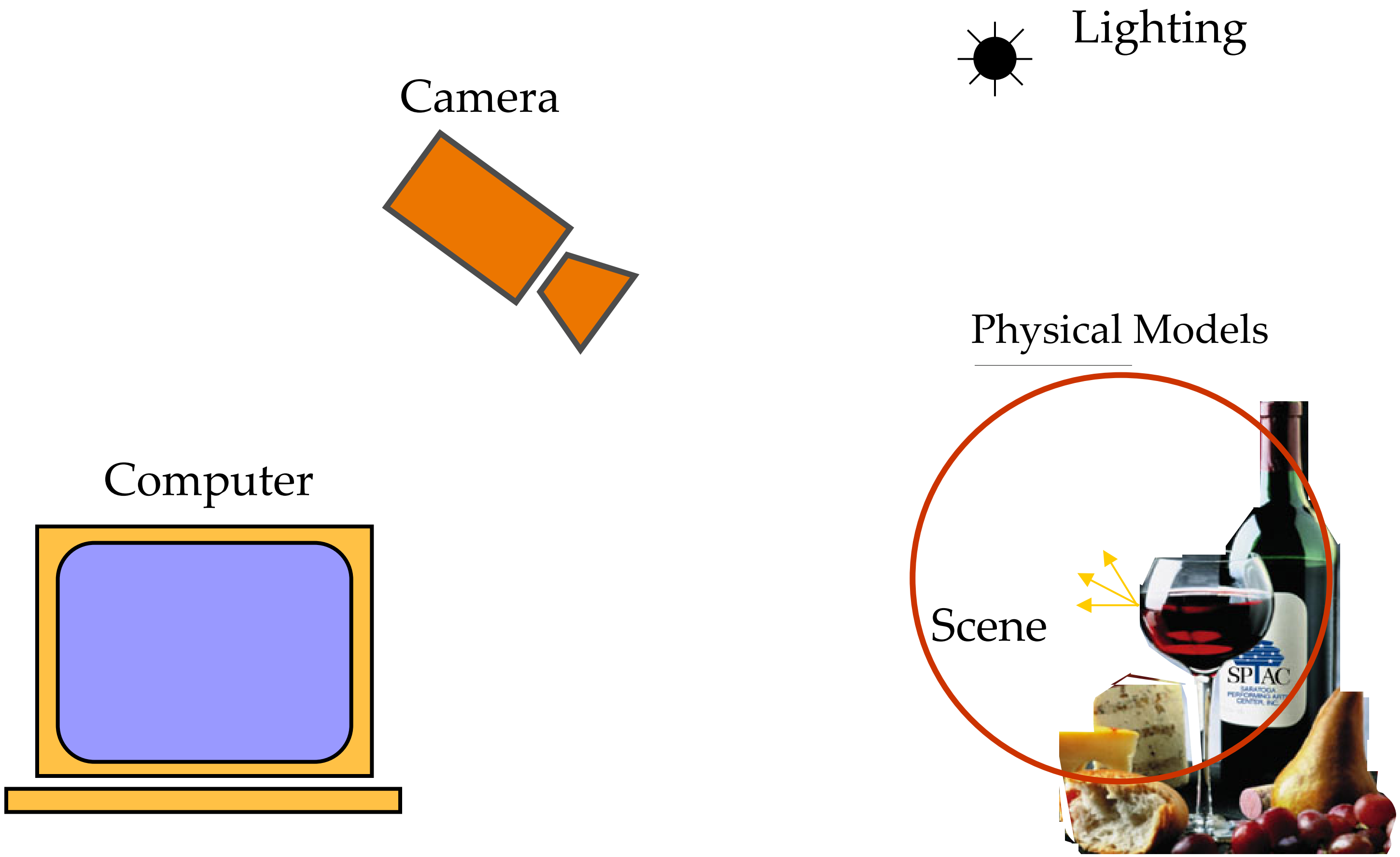
Texture Modeling



Photometric Stereo

Reflection Separation

Computer Vision: Building Machines that See



We need to understand the relation between
the lighting, surface reflectance and medium and the image of the scene.

Surface Appearance

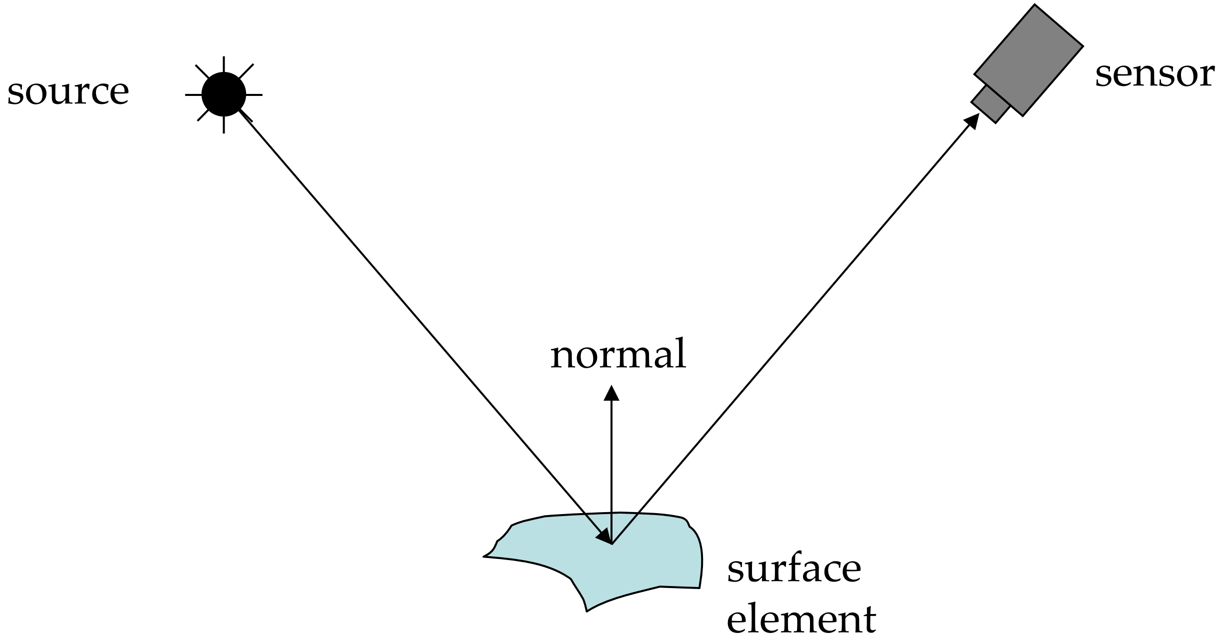
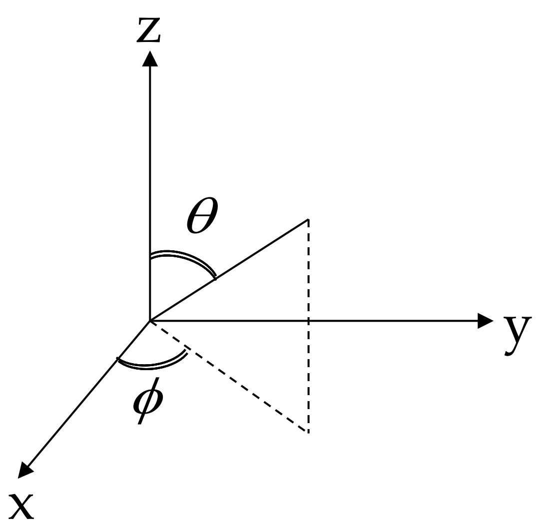
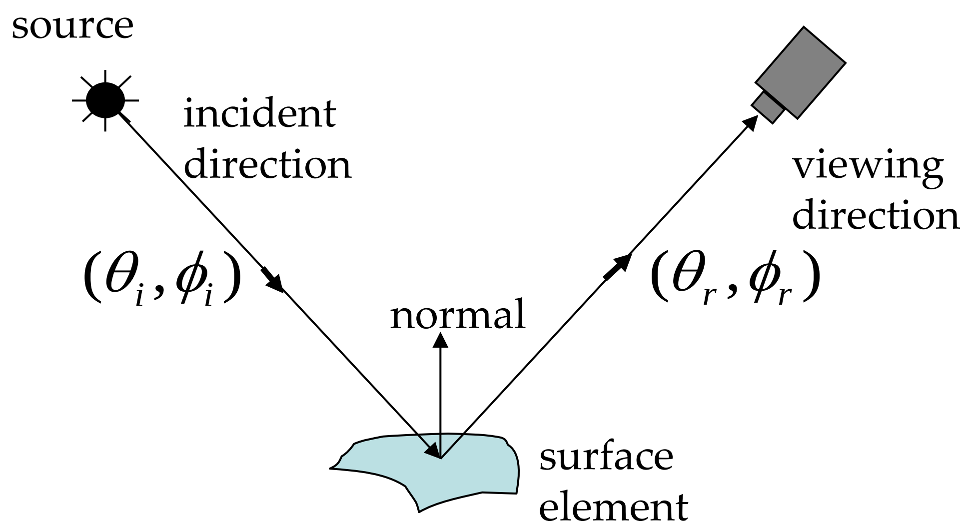


Image intensities = f (normal, surface reflectance, illumination)

Surface Reflection depends on both the viewing and illumination direction.

BRDF: Bidirectional Reflectance Distribution Function



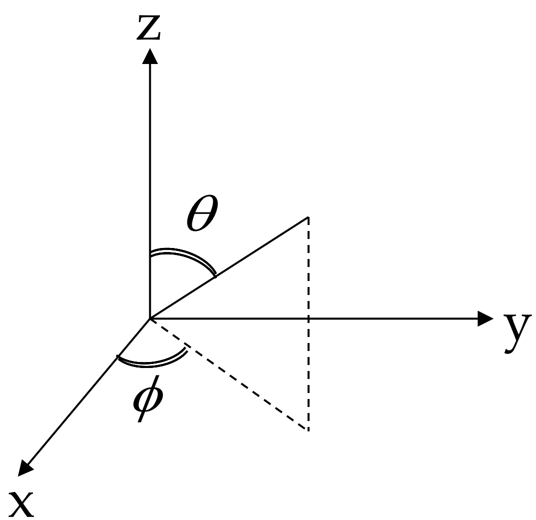


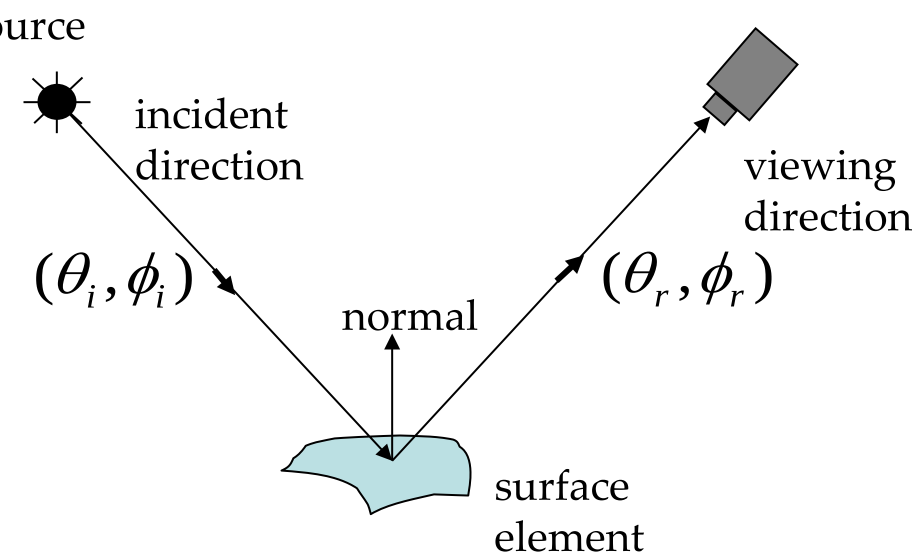
$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

Important Properties of BRDFs





- Rotational Symmetry:

BRDF does not change when surface is rotated about the normal.

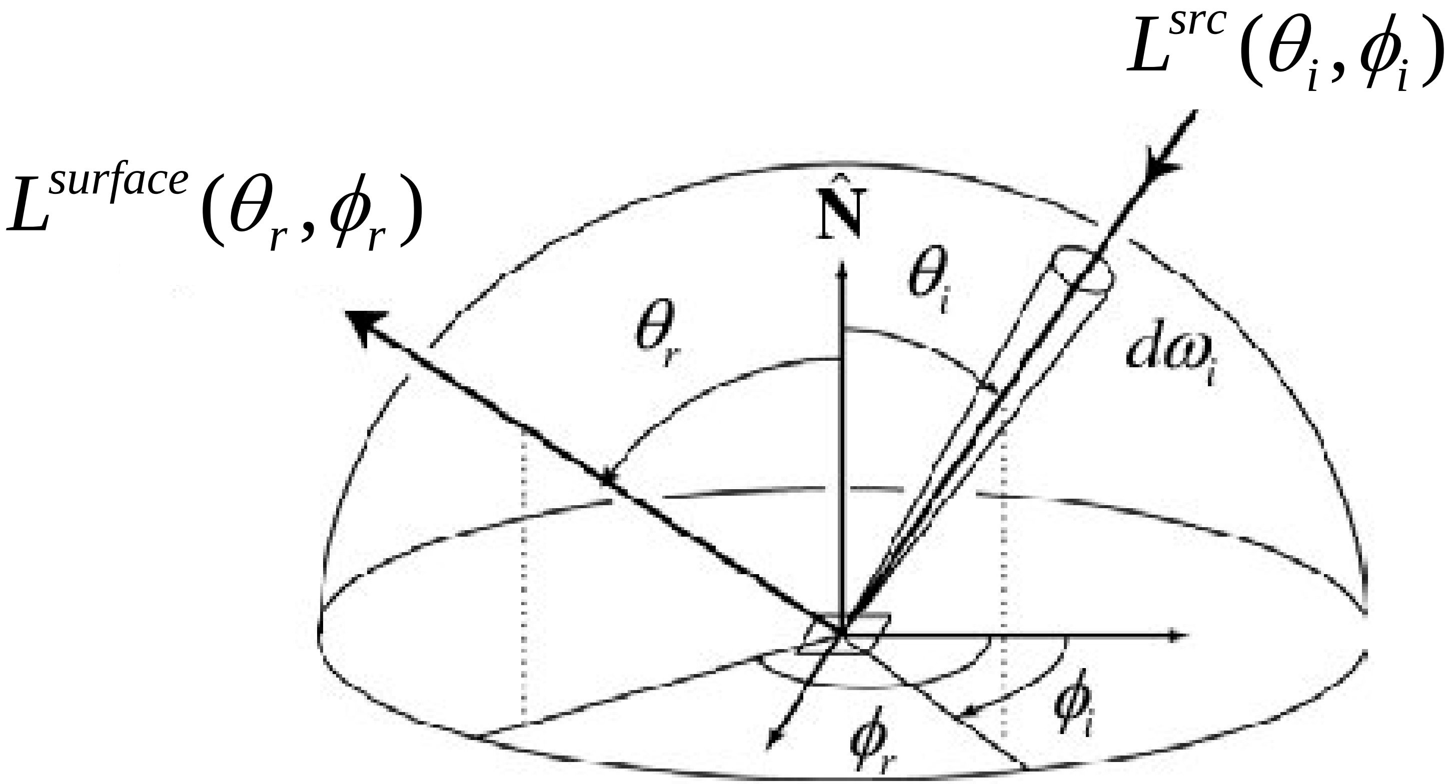
BRDF is only a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f(\theta_i, \phi_i; \theta_r, \phi_r) \quad = \quad f(\theta_r, \phi_r; \theta_i, \phi_i)$$

Derivation of the Scene Radiance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) \quad = \quad E^{surface}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation – Important!

From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) = \underline{E^{surface}(\theta_i, \phi_i)}\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r, \phi_r) = \underline{L^{src}(\theta_i, \phi_i)}\, f(\theta_i, \phi_i; \theta_r, \phi_r) \underline{\cos\theta_i d\omega_i}$$

Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r, \phi_r) = \int_{2\pi} L^{src}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \, \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{src}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \, \underline{\sin\theta_i d\theta_i d\phi_i}$$

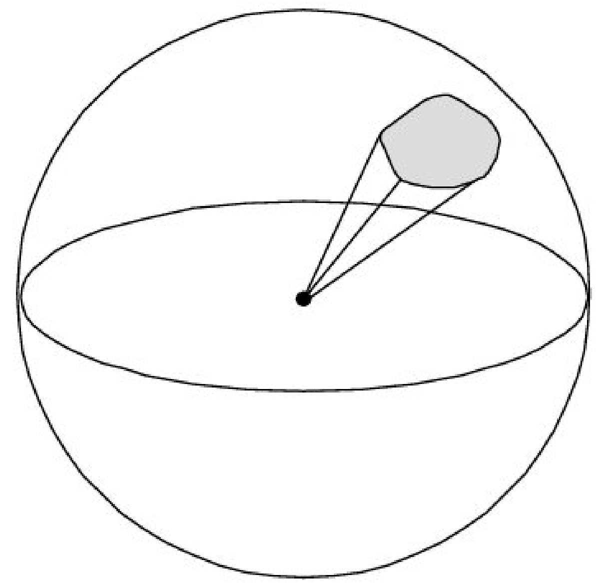
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

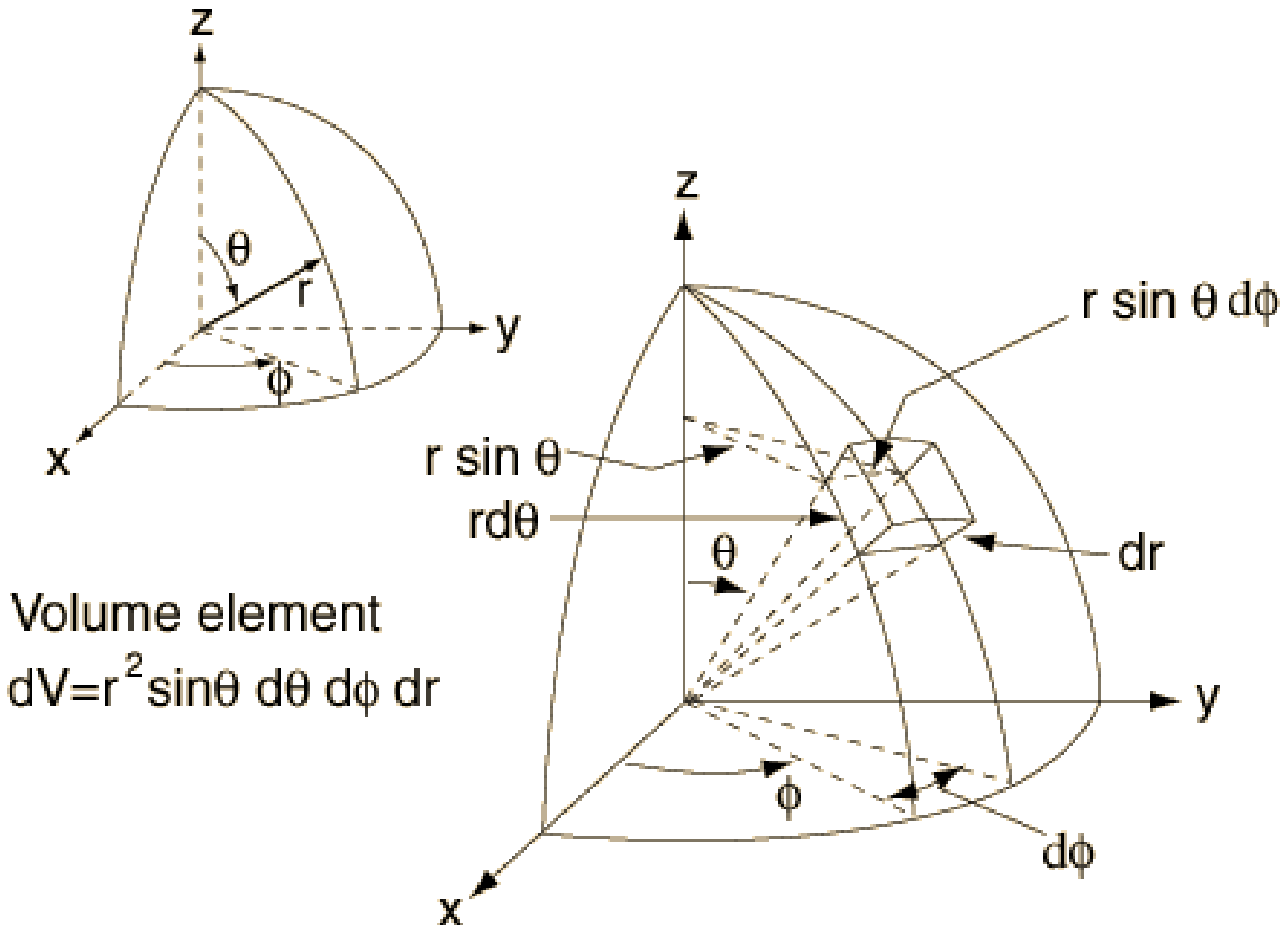
  $\Rightarrow$ **circle has 2π radians**

- **Solid angle** $\Omega = \frac{A}{R^2}$

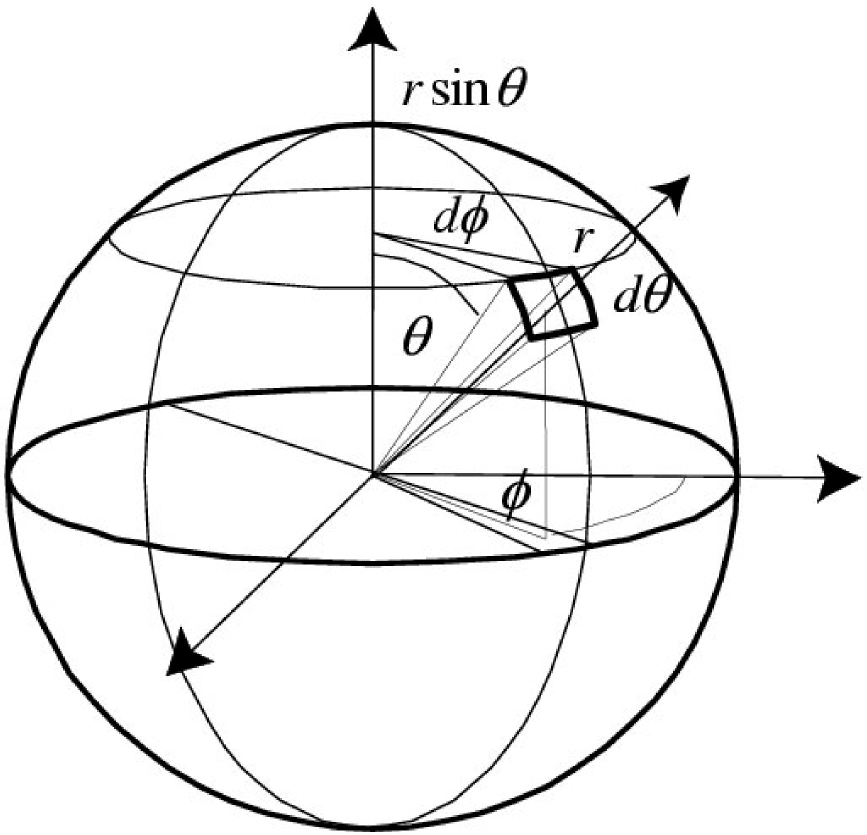
  $\Rightarrow$ **sphere has 4π steradians**



Differential Solid Angle and Spherical Polar Coordinates



Differential Solid Angles

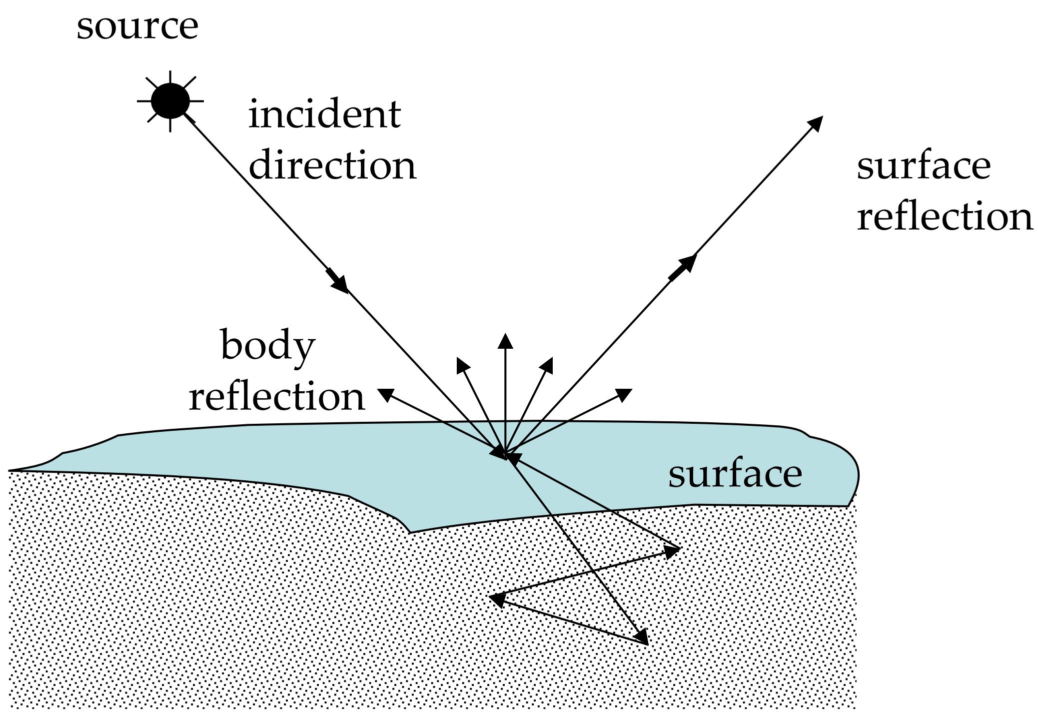


$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

Mechanisms of Reflection



Body Reflection:

- Diffuse Reflection
- Matte Appearance
- Non-Homogeneous Medium
- Clay, paper, etc

Surface Reflection:

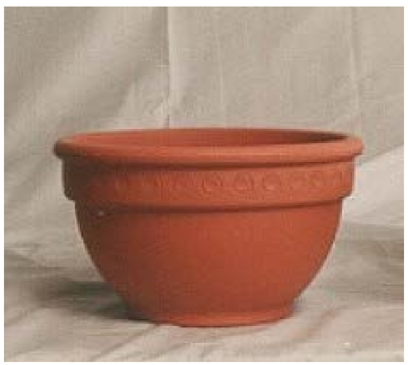
- Specular Reflection
- Glossy Appearance
- Highlights
- Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Mechanisms of Surface Reflection

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc

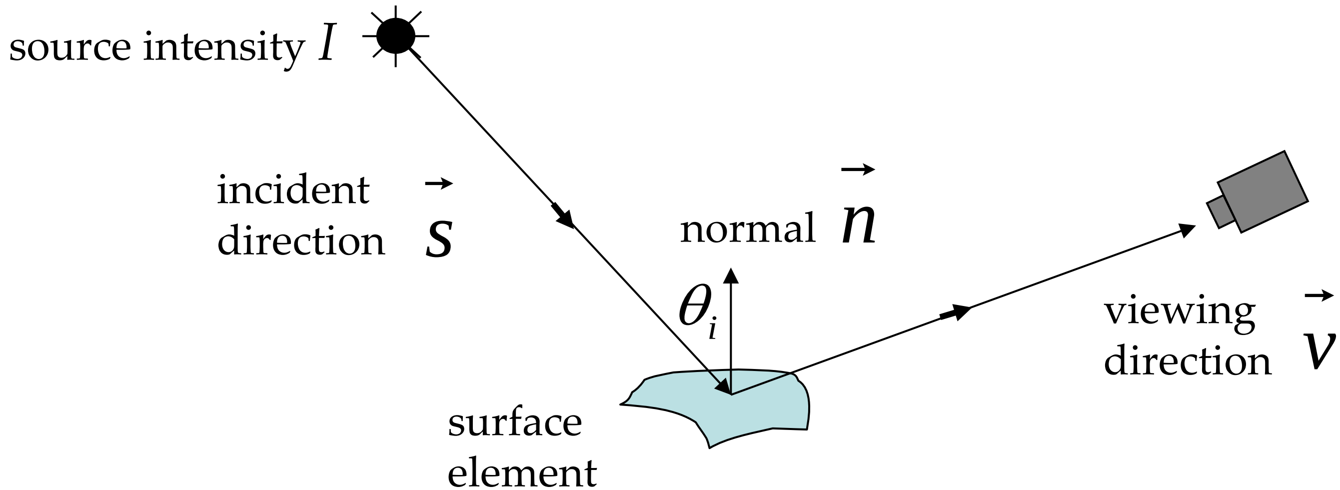


Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

Diffuse Reflection and Lambertian BRDF



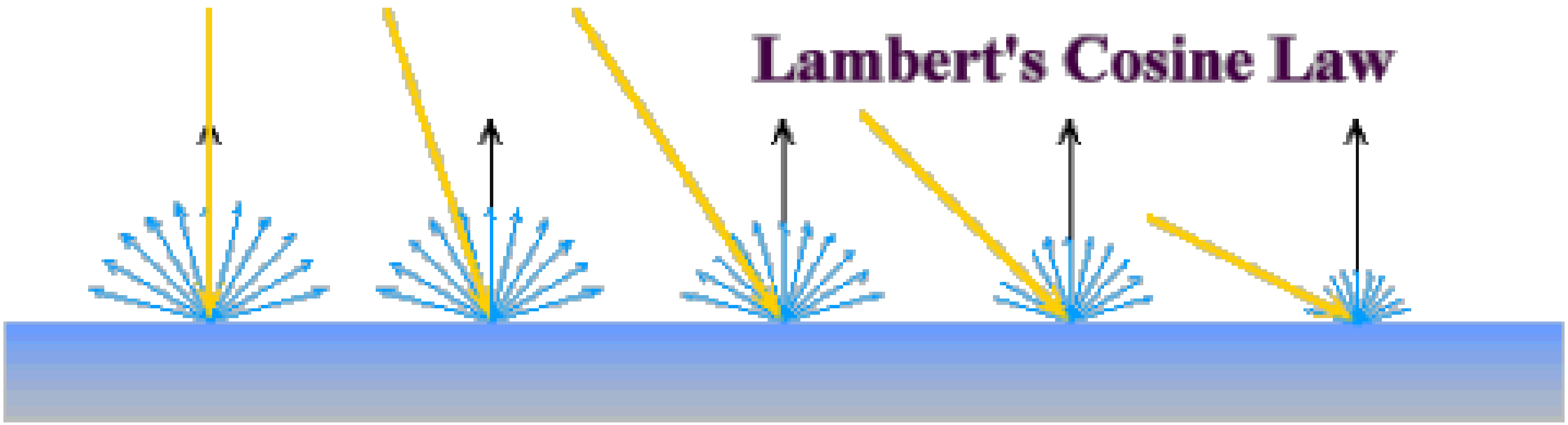
- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)

- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \dfrac{\rho_d}{\pi}$ → albedo

- Surface Radiance : $L = \dfrac{\rho_d}{\pi} I \cos\theta_i = \dfrac{\rho_d}{\pi} I \; \vec{n} \cdot \vec{s}$ → source intensity

- Commonly used in Vision and Graphics!

Diffuse Reflection and Lambertian BRDF



White-out Conditions from an Overcast Sky

CAN'T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?

Diffuse Reflection from Uniform Sky

$$L^{surface}(\theta_r, \phi_r) \quad = \quad \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{src}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \, \sin\theta_i d\theta_i d\phi_i$$

- Assume Lambertian Surface with Albedo = 1 (no absorption)

$$f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{1}{\pi}$$

- Assume Sky radiance is constant

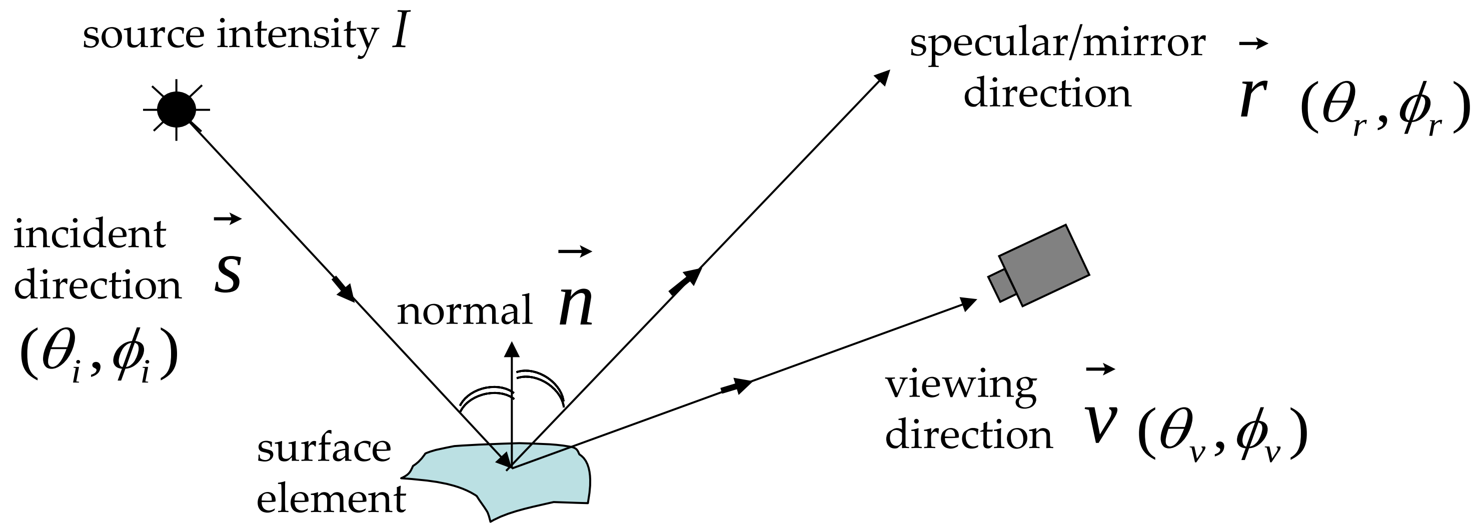
$$L^{src}(\theta_i, \phi_i) = L^{sky}$$

- Substituting in above Equation:

$$\boxed{L^{surface}(\theta_r, \phi_r) \quad = \quad L^{sky}}$$

Radiance of any patch is the same as Sky radiance !! (white-out condition)

Specular Reflection and Mirror BRDF



- Very smooth surface.
- All incident light energy reflected in a SINGLE direction. (only when $\vec{v} = \vec{r}$)
- Mirror BRDF is simply a double-delta function :

specular albedo

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s\ \delta(\theta_i - \theta_v)\ \delta(\phi_i + \pi - \phi_v)$$

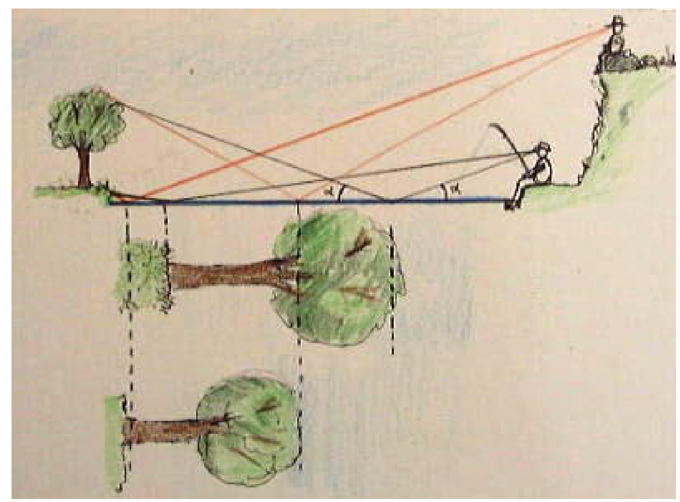
- Surface Radiance : $L = I\ \rho_s\ \delta(\theta_i - \theta_v)\ \delta(\phi_i + \pi - \phi_v)$

Specular Reflections in Nature

It's surprising how long the reflections are when viewed sitting on the river bank.

Compare sizes of objects and their reflections!

The reflections when seen from a lower view point are always longer than when viewed from a higher view point.



Specular Reflections in Nature

The reflections of bright objects have better perceived contrast.

Intensity of reflected light is a fraction of the direct light –
[Fresnel term (derivation in a later class)]

Papers to Read

Shape and Materials by Example: A Photometric Stereo Approach

http://grail.cs.washington.edu/projects/sam/

Helmholtz Stereopsis

http://www.eecs.harvard.edu/~zickler/helmholtz.html

Specularity Removal and Dichromatic Editing

http://www.eecs.harvard.edu/~zickler/dichromaticediting.html

Color Subspaces as Photometric Invariants

http://www.eecs.harvard.edu/~zickler/projects/colorsubspaces.html