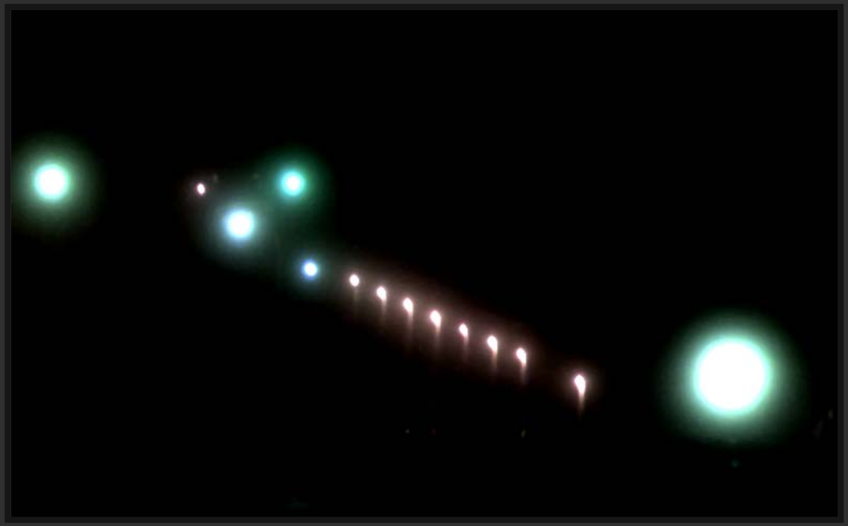
Multiple Scattering in Vision and Graphics

Lecture #21

Thanks to Henrik Wann Jensen

Glows of Light Sources



Mist

Fog

Properties of Scattering Media

Scattering Coefficient: Fractional loss in intensity due to scattering per unit cross section β

Absorption Coefficient: Fractional loss in intensity due to absorption per unit cross section κ

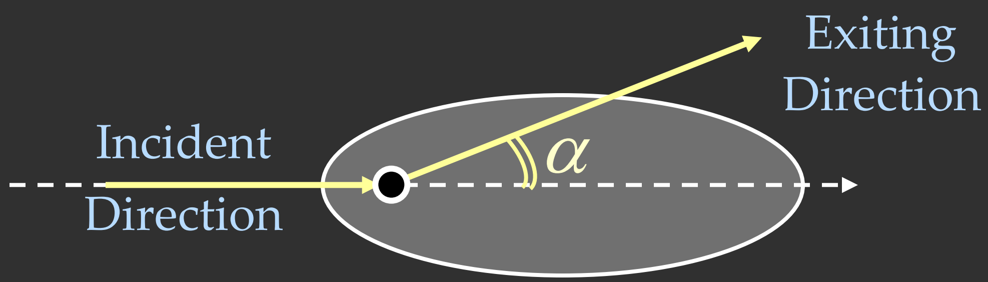
Extinction Coefficient: Scattering Coefficient + Absorption Coefficient σ

Scattering Albedo: Scat. Coeff. / Ext. Coeff. ϖ_0

Phase Function

- Probability of light getting scattered in a single direction

$$P(\alpha)$$



- Phase function integrates to 1
- Light Scattered in any direction :

$$\frac{\beta}{4\pi} P(\alpha)$$

Recap

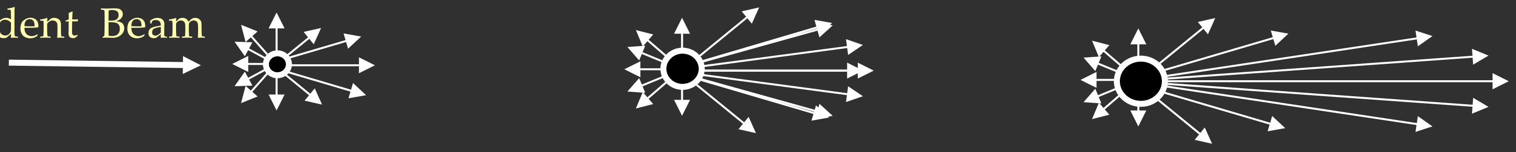
Different Orders of Scattering

Particle Scattering Mechanisms

(Mie 1908)

Single Scattering:

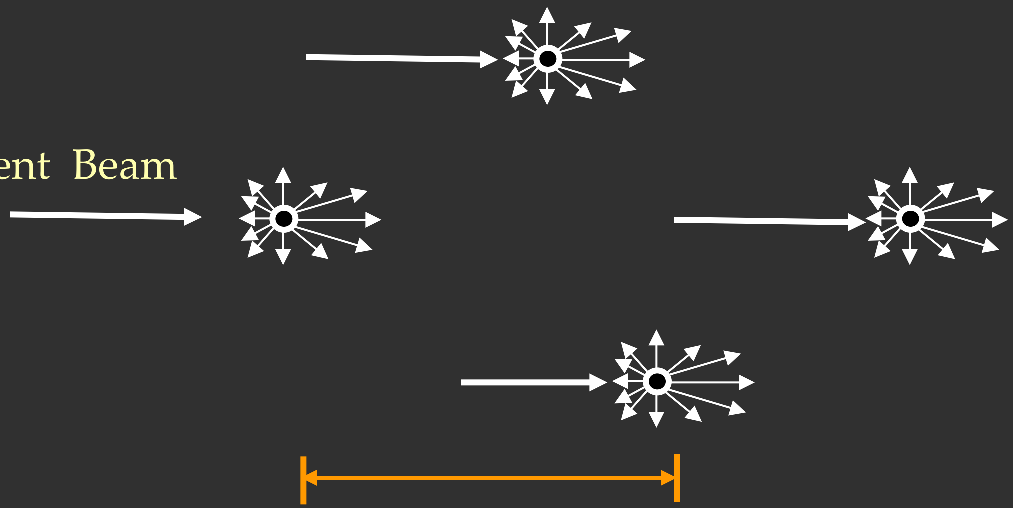
Incident Beam



Size: 0.01 μm Size: 0.1 μm Size: 1 μm

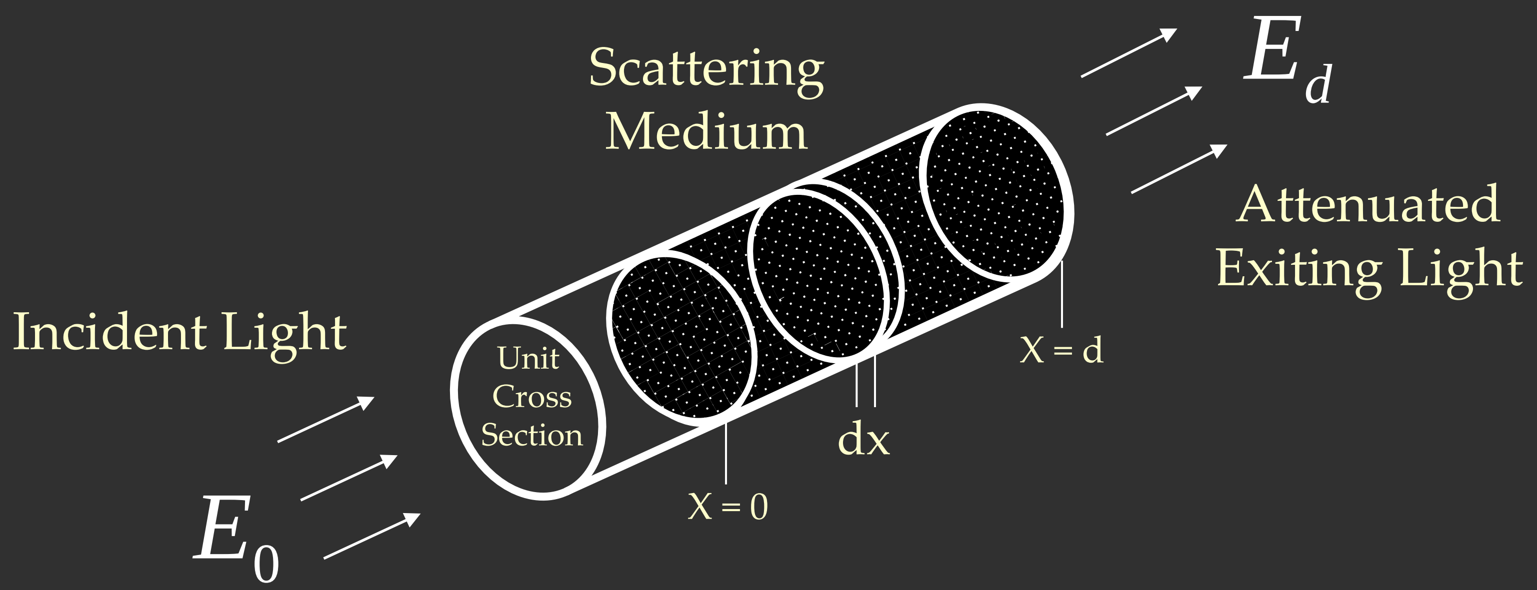
Independent Scattering:

Incident Beam



Distance of Separation >> Size of Particles

Attenuation Model – Zeroth Order Scattering



Brightness at Distance *d* :

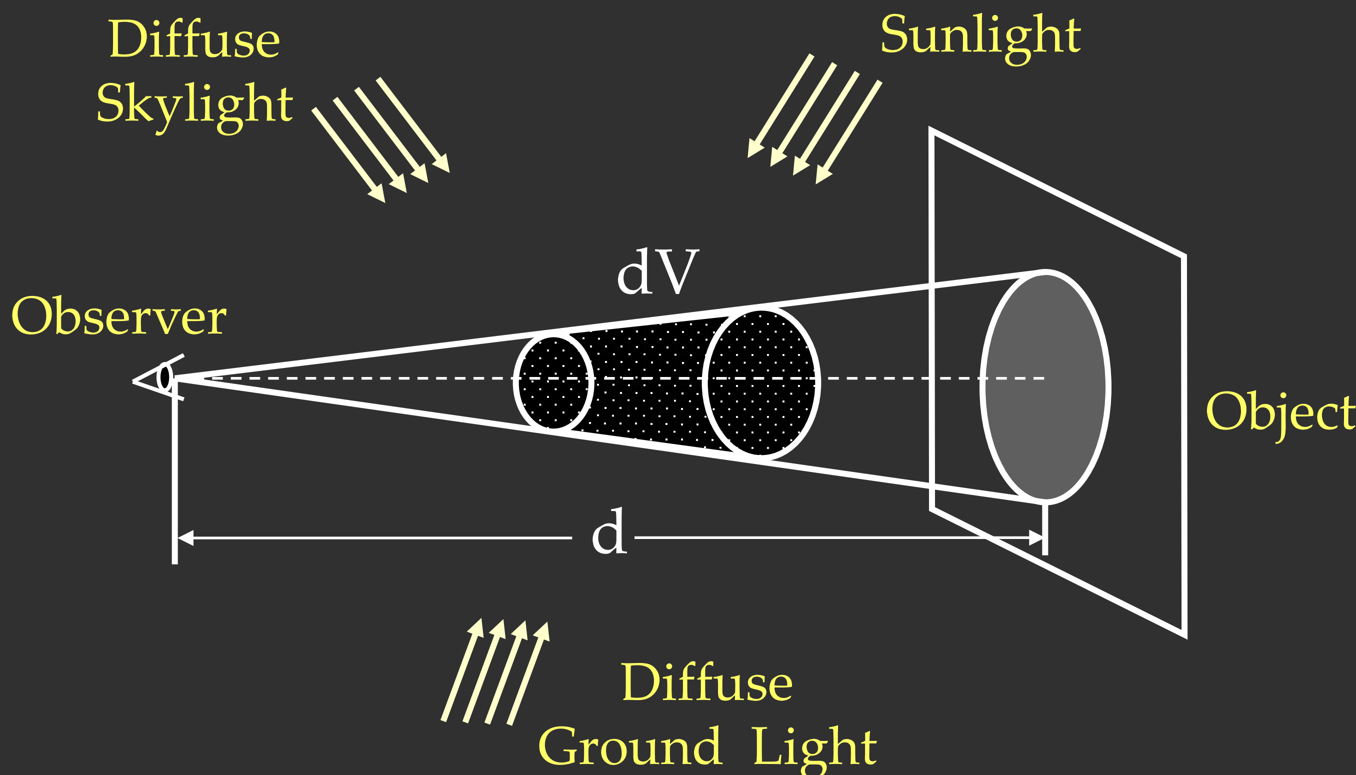
$$E(d) = E_0 \; e^{-\beta d}$$

(Bouguer's Law, 1729)

Scattering Coefficient

Airlight Model – First Order (Single) Scattering

(Koschmeider, 1924)



Brightness due to a Path of Length d :

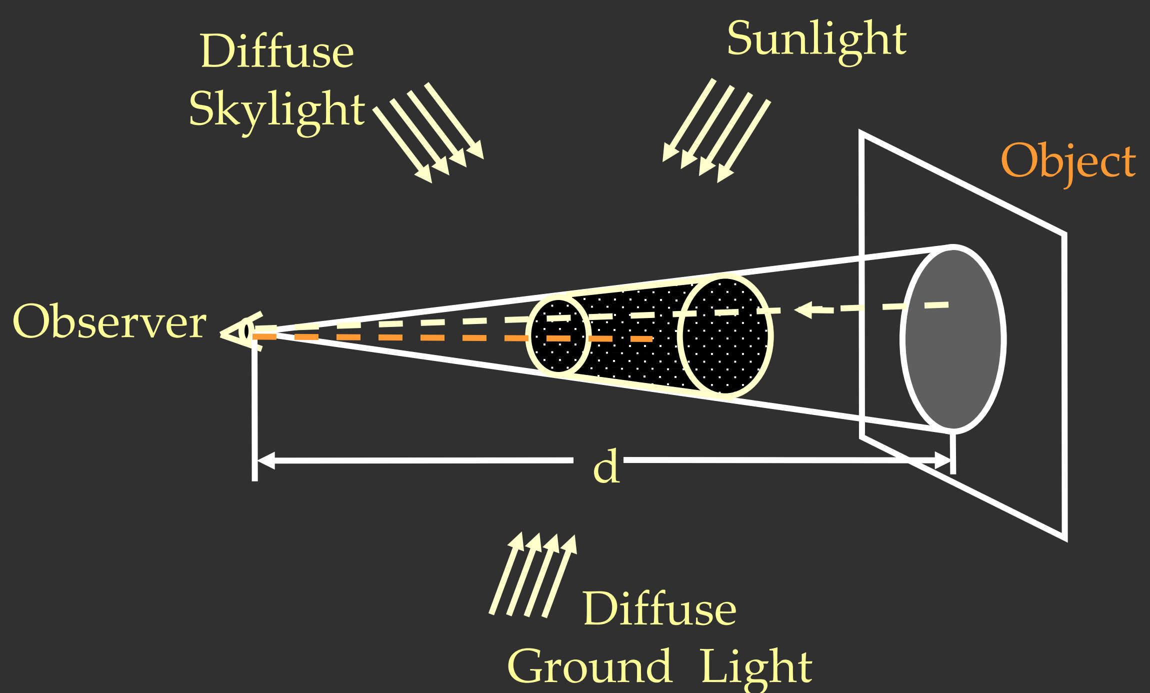
Horizon Brightness

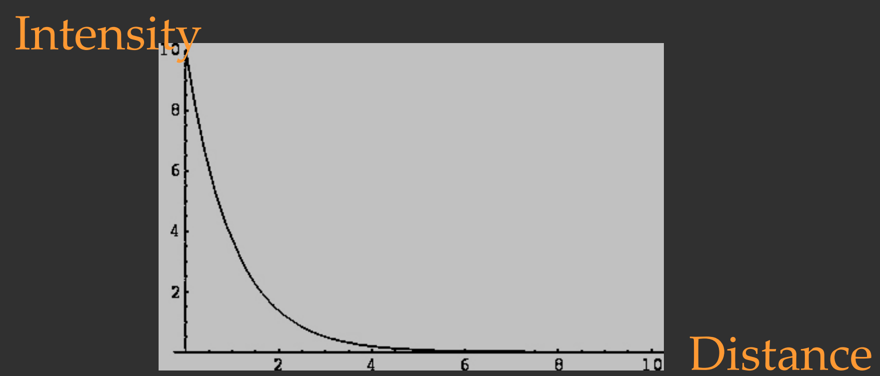
$$E(d) = E_{\infty}\,(1 - e^{-\beta d})$$

Distant objects appear Bright !

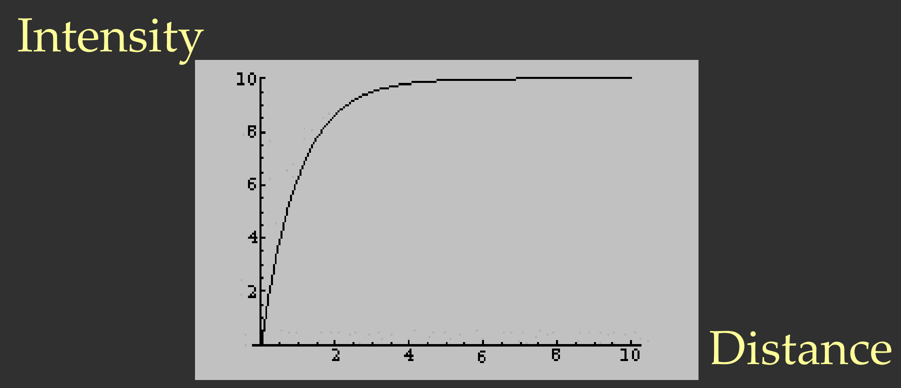
Mountains

Combining 0th and 1st orders: Useful for Vision



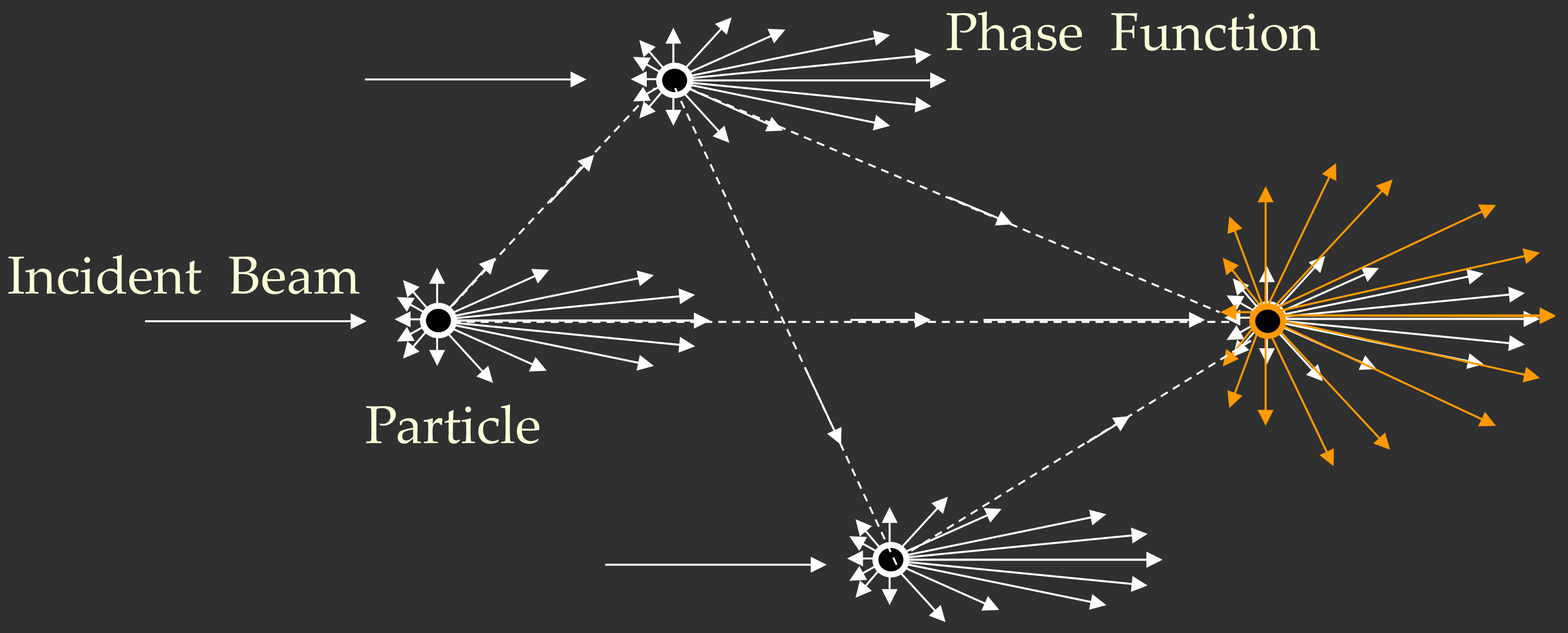


Attenuation



Airlight

Multiple Scattering : Higher orders of scattering



Radiative Transfer

Mathematical study of transport of radiation (in particular light).

Finite Difference method used to model the rate of change of radiation along any direction in an infinitesimal volume.

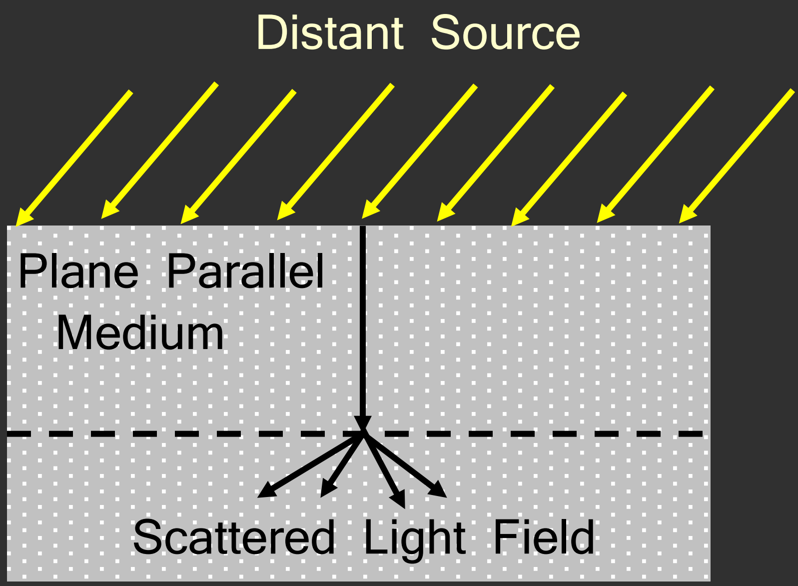
Can model multiple scattering elegantly.

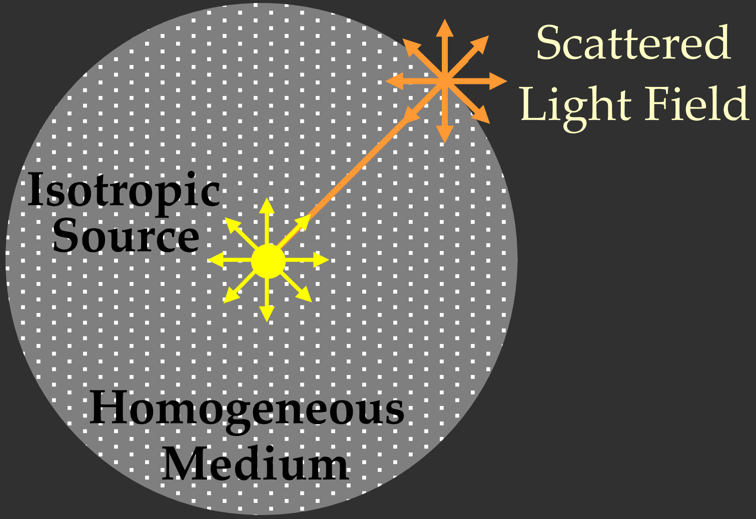
Solution to light transport gives the **Light Field** in the medium.

But, hard to solve analytically. Why?
Depends on medium geometry and location of sources.

Only few special cases are known to have analytic solutions.
(Plane Parallel, Spherical)

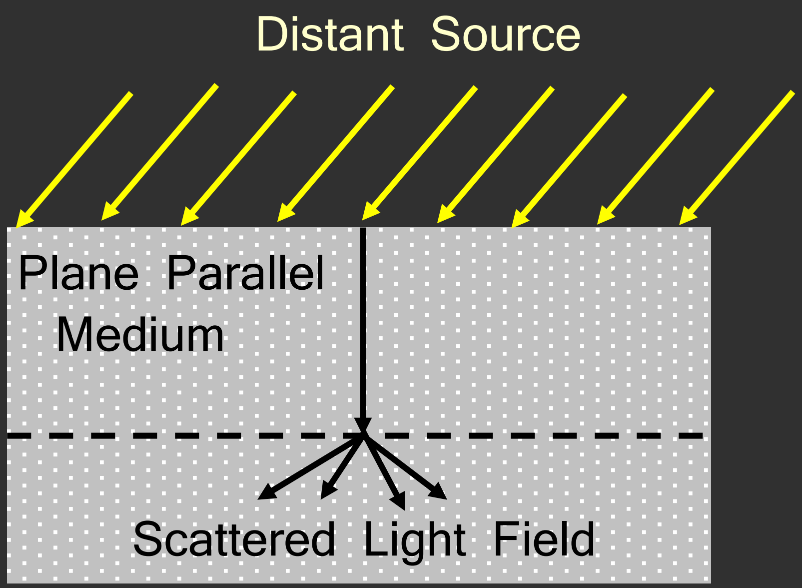
Plane Parallel and Spherical Radiative Transfer





Radiative Transfer in Plane Parallel Media

[Chandrasekhar 1960 , Ishimaru 1997]



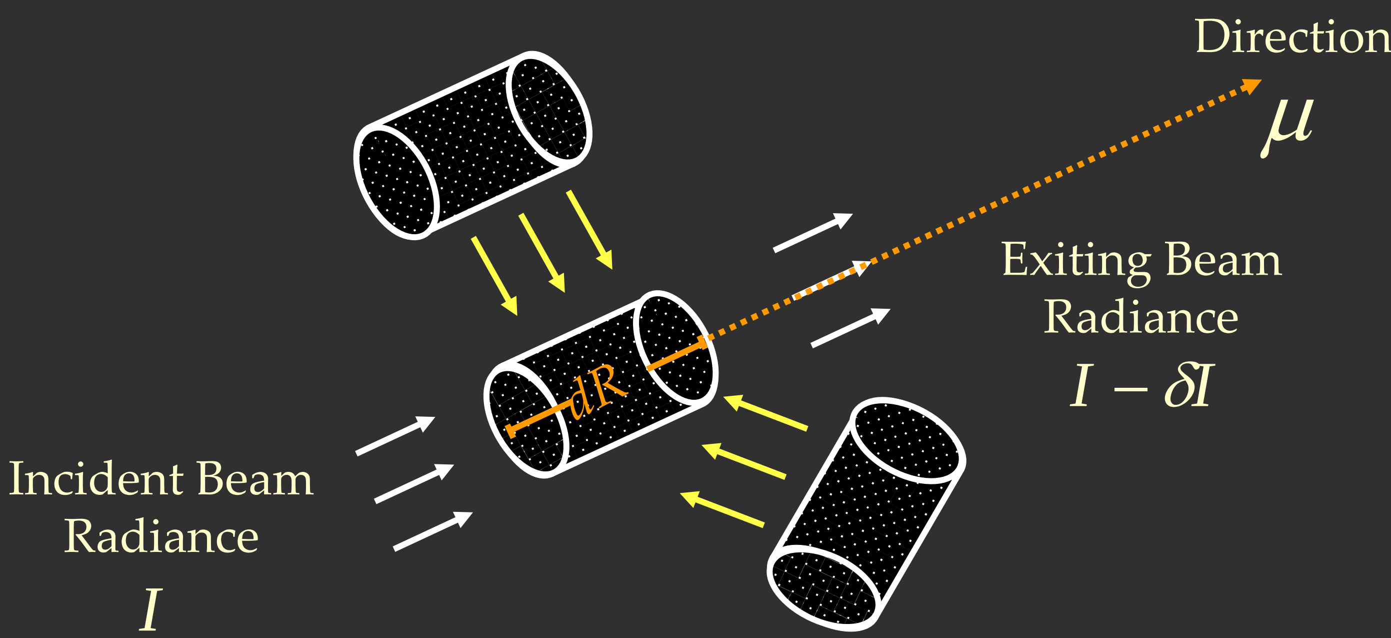
Collimated Source Outside Medium

Widely used in Atmospheric Optics, Remote Sensing

Popular configuration for Subsurface Scattering in Graphics

Radiative Transfer in Plane Parallel Medium

Infinitesimal Scattering Volume : $dT = \sigma \; dR$



Radiative Transfer Equation :

Phase Function   Optical Thickness

$$\frac{\partial I}{\partial T} = -I(T,\mu) + \frac{1}{4\pi}\int P(\mu,\mu')\; I(T,\mu')\, dw$$

Radiance Rate of Change   Extinction   Source Function

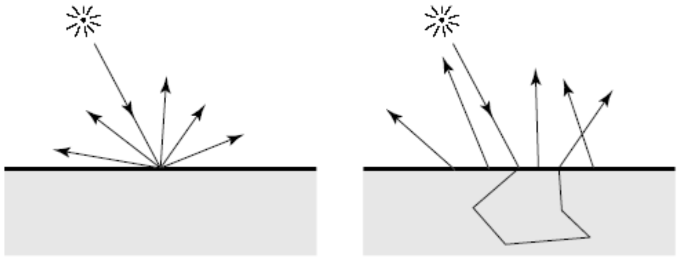
BSSRDFs

- Bidirectional Surface Scattering Reflectance Distribution Function
- The BSSRDF relates the outgoing radiance to the incident flux

$$dL_o(x_o, \vec{\omega}_o) = S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, d\Phi_i(x_i, \vec{\omega}_i)$$

- The BRDF is an approximation of the BSSRDF for which it is assumes that light enters and leaves at the same point
- The outgoing radiance is computed by integrating the incident radiance over incoming directions *and area, A*

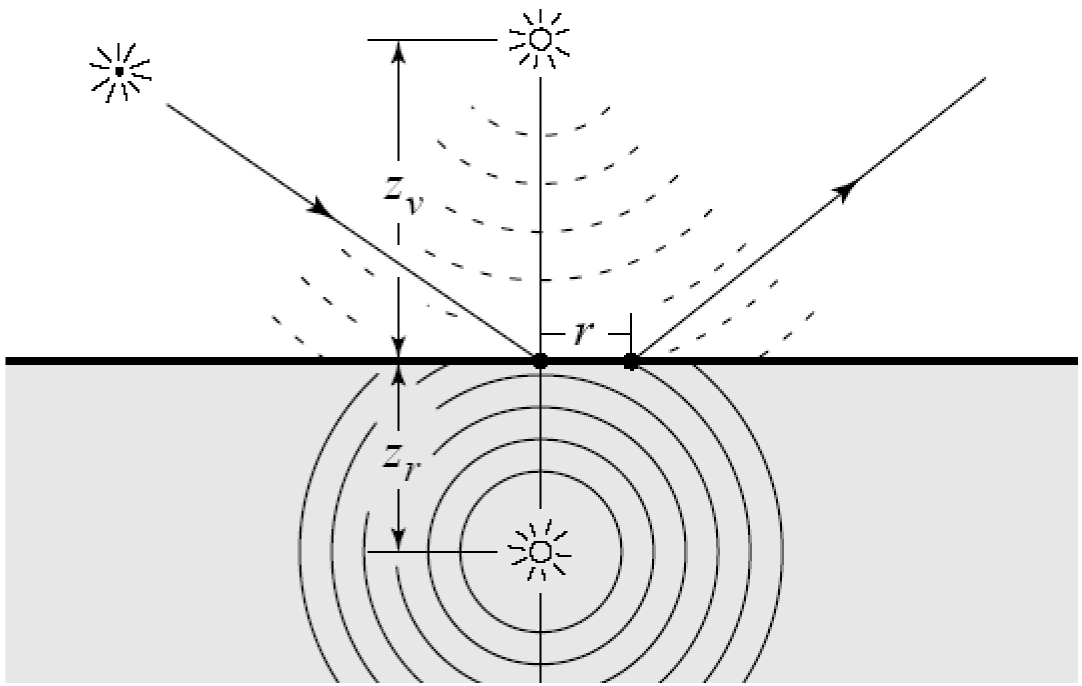
$$L_o(x_o, \vec{\omega}_o) = \int_A \int_{2\pi} S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, L_i(x_i, \vec{\omega}_i)\, (\vec{n} \cdot \vec{\omega}_i)\, d\omega_i dA(x_i)$$



Symbol Reference

S	BSSRDF
R_d	Diffuse BSSRDF
F_r	Fresnel reflectance
F_t	Fresnel transmittance
F_{dr}	Diffuse Fresnel reflectance
$\vec{E}$	Vector irradiance
ϕ	Radiant fluence
σ_a	Absorption coefficient
σ_s	Scattering coefficient
σ_t	Extinction coefficient
σ_t'	Reduced extinction coefficient
σ_{tr}	Effective extinction coefficient
D	Diffusion constant
α	Albedo
p	Phase function
η	Relative index of refraction
g	Mean cosine of the scattering angle
Q	Volume source distribution
Q_0	0th-order source distribution
$\vec{Q}_1$	1st-order source distribution

Diffusion Approximation for Multiple Scattering



An incoming ray is transformed into a dipole source for the diffusion approximation

The Diffusion Approximation

- The diffusion approximation is based on the observation that the light distribution in highly scattering media tends to become isotropic
- The volumetric source distribution can be approximated using the dipole method
- The dipole method consists of positioning two point sources near the surface in such a way as to satisfy the required boundary condition
- The diffuse reflectance due to the dipole source can be computed as

$$\begin{aligned} R_d(r) &= -D\frac{(\vec{n}\cdot\vec{\nabla}\phi(x_s))}{d\Phi_i} \\ &= \frac{\alpha'}{4\pi}\left[(\sigma_{tr}d_r+1)\frac{e^{-\sigma_{tr}d_r}}{\sigma_t' d_r^3} + z_v(\sigma_{tr}d_v+1)\frac{e^{-\sigma_{tr}d_v}}{\sigma_t' d_v^3}\right] \end{aligned}$$

- Taking into account the Fresnel reflection at the boundary for both the incoming light and the outgoing radiance

$$S_d(x_i,\vec{\omega}_i;x_o,\vec{\omega}_o) = \frac{1}{\pi}F_t(\eta,\vec{\omega}_i)R_d(||x_i-x_o||)F_t(\eta,\vec{\omega}_o)$$

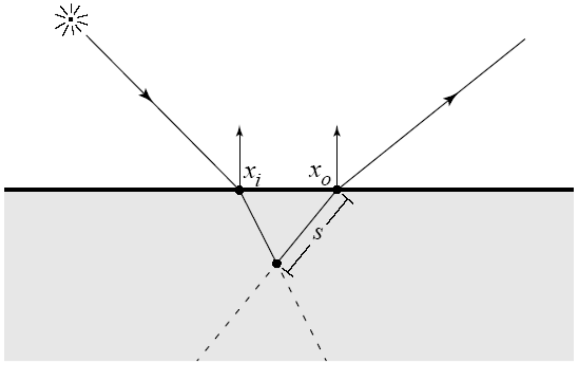
- Where S_d is the diffusion term of the BSSRDF, which represents multiple scattering

Single Scattering Term

- The total outgoing radiance, due to single scattering is computed by integrating the incident radiance along the refracted outgoing ray

$$L_o^{(1)}(x_o, \vec{\omega}_o) = \sigma_s(x_o) \int F\, p(\vec{\omega}_i' \cdot \vec{\omega}_o') \int_0^{\infty} e^{-\sigma_{tc} s} L_i(x_i, \vec{\omega}_i)\, ds\, d\vec{\omega}_i$$
$$= \int_A \int_{2\pi} S^{(1)}(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, L_i(x_i, \vec{\omega}_i)\, (\vec{n} \cdot \vec{\omega}_i)\, d\omega_i dA(x_i)$$

- The single scattering BSSRDF is defined implicitly by the second line of this equation



Single scattering occurs only when the refracted incoming and outgoing rays intersect, and is computed as an integral over path length *s* along the refracted outgoing ray

The BSSRDF Model

- The complete BSSRDF model is a sum of the diffusion approximation and the single scattering term

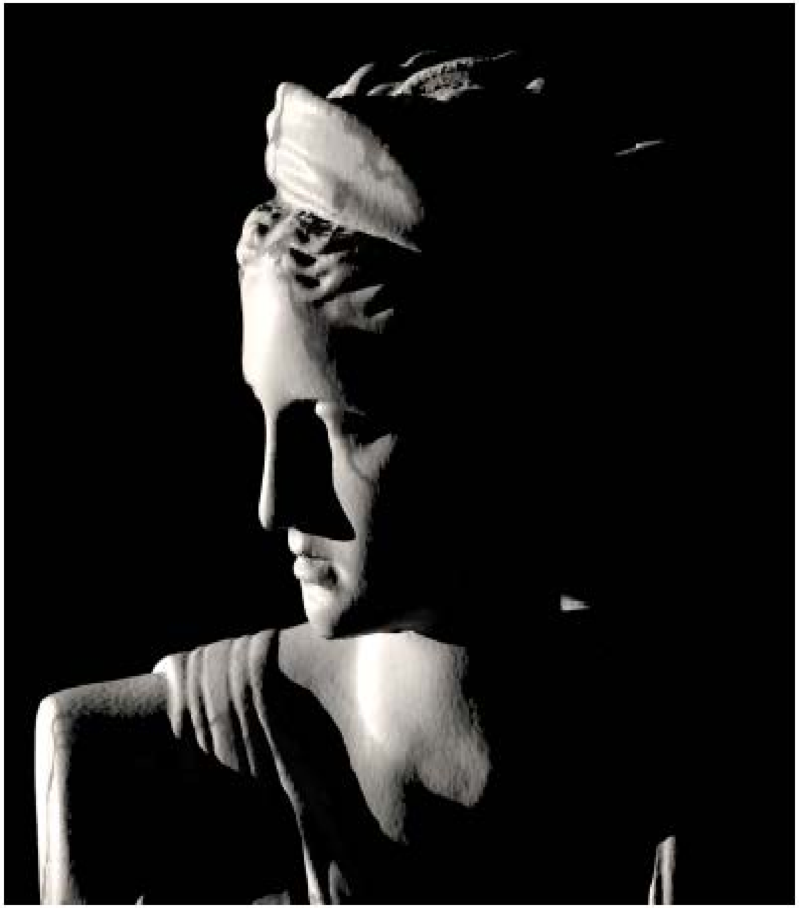
$$S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) = S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) + S^{(1)}(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)$$

- This model accounts for light transport between different locations on the surface, and it simulates both the directional component (due to single scattering) as well as the diffuse component (due to multiple scattering)

Rendering Using the BSSRDF

- The BSSRDF model derived only applies to semi-infinite homogeneous media, for a practical model we must consider
  - Efficient integration of the BSSRDF (importance sampling)
  - Single scattering evaluation for arbitrary geometry
  - Diffusion approximation for arbitrary geometry
  - Texture (spatial variation on the object surface)

BRDF vs BSSRDF



BRDF vs BSSRDF

BRDF vs BSSRDF

BRDF

BSSRDF

Diffusion Approximation for Multiple Layers

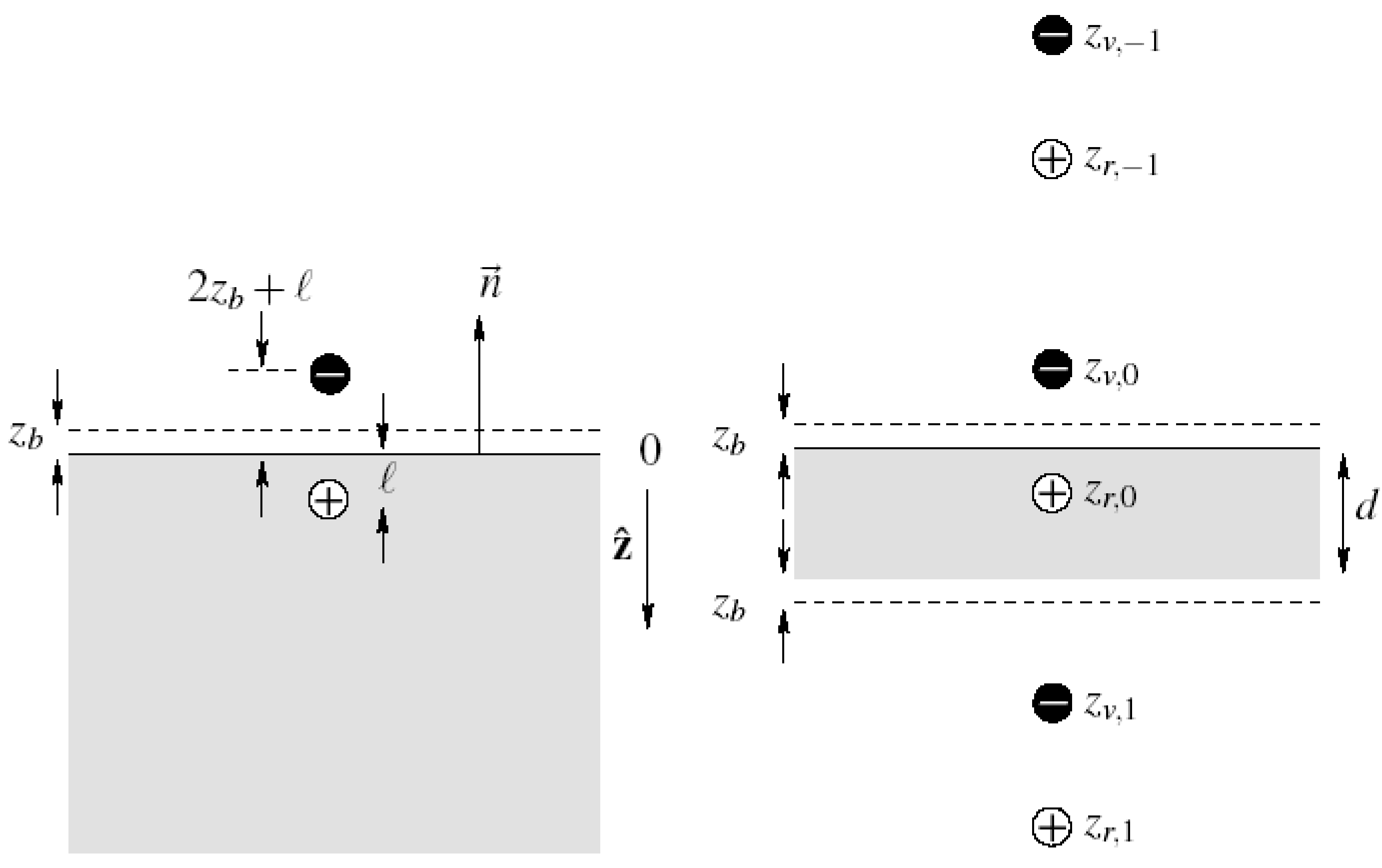
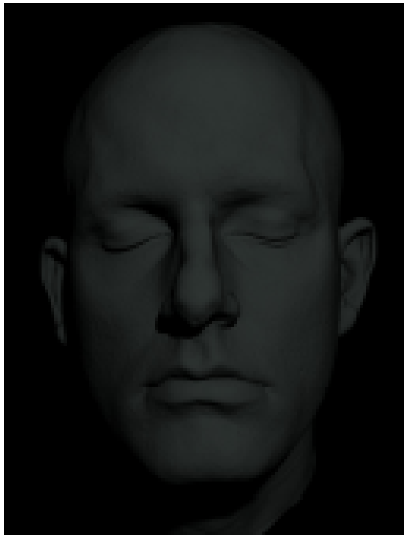
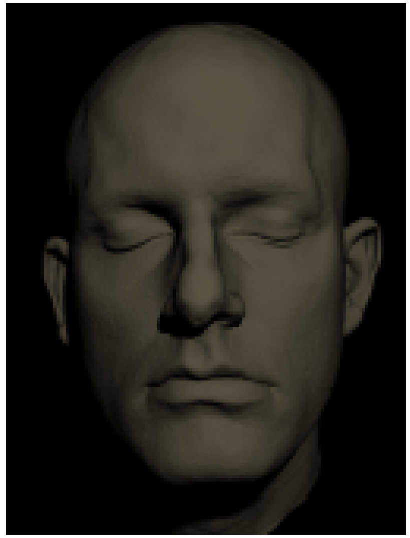


Figure 1: *Dipole configuration for semi-infinite geometry (left), and the multipole configuration for thin slabs (right).*

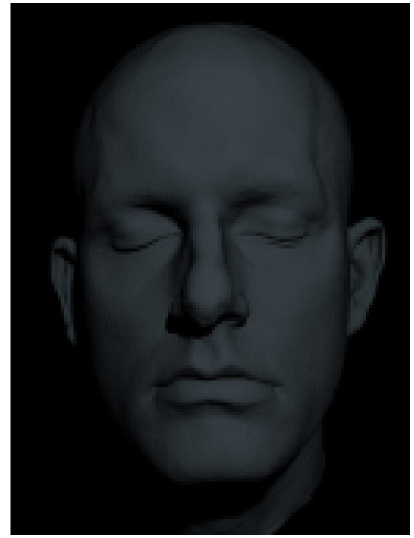
Donner, Jensen, Siggraph 05



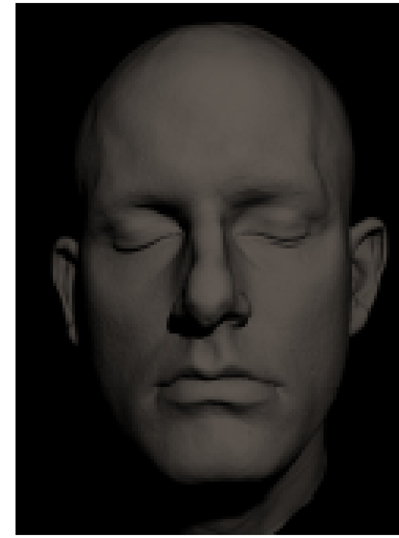
Epidermis Reflectance



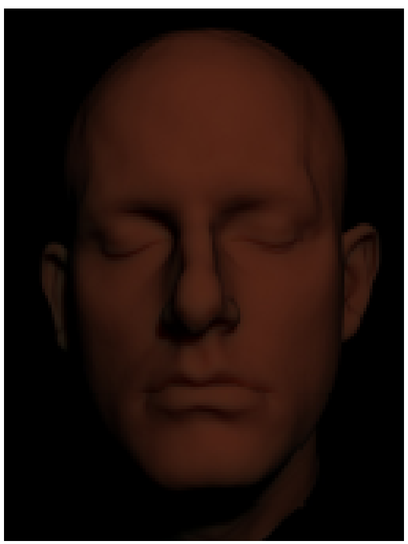
Epidermis Transmittance



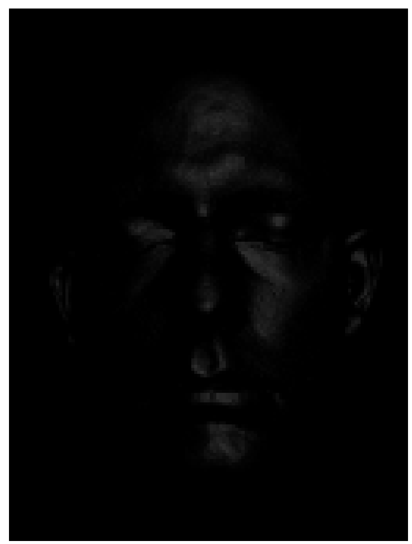
Upper Dermis Reflectance



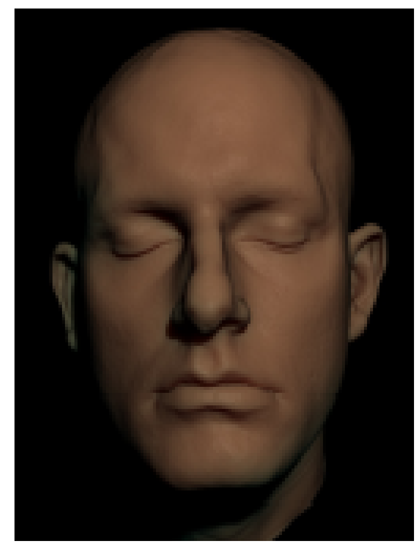
Upper Dermis Transmittance



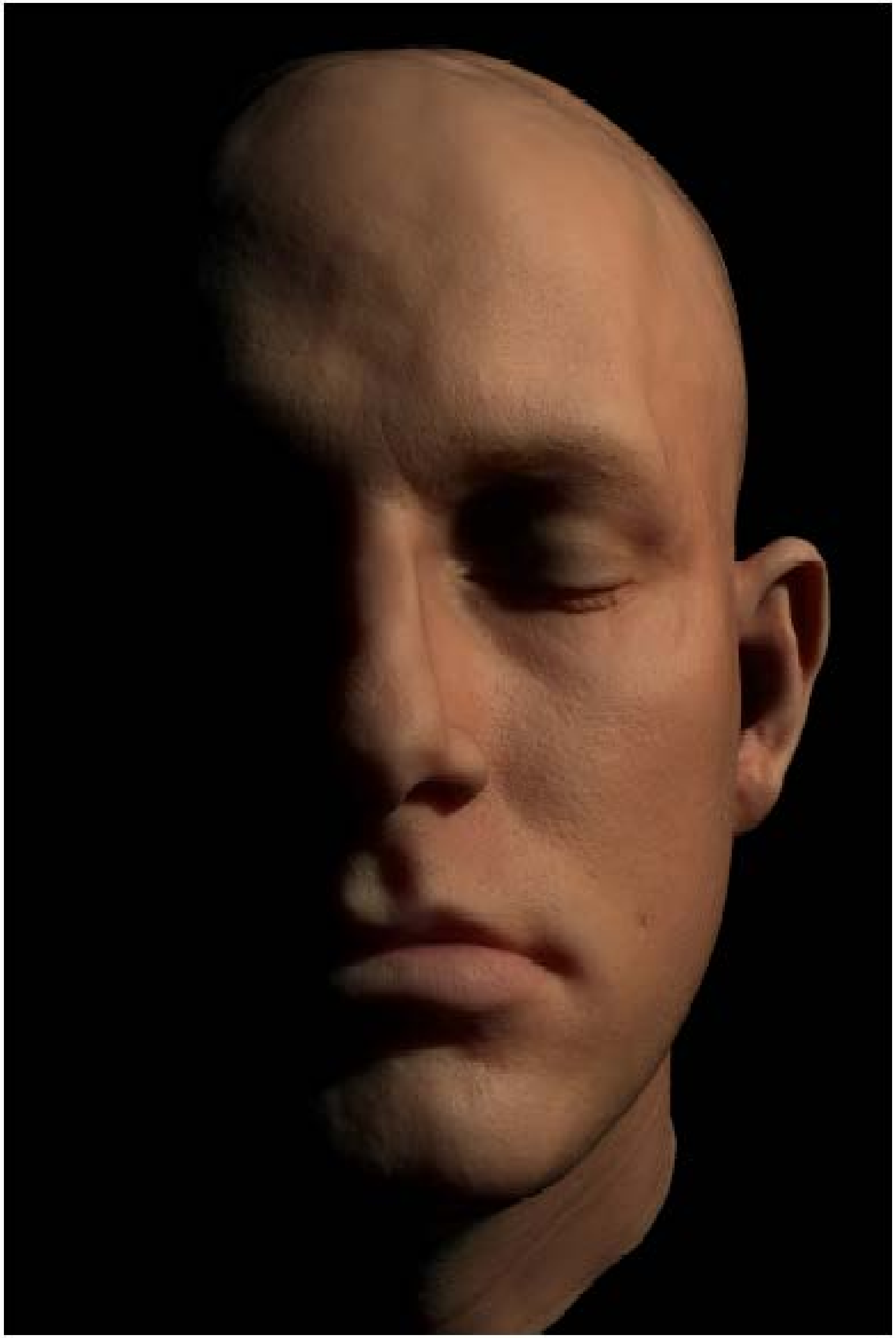
Bloody Dermis Reflectance

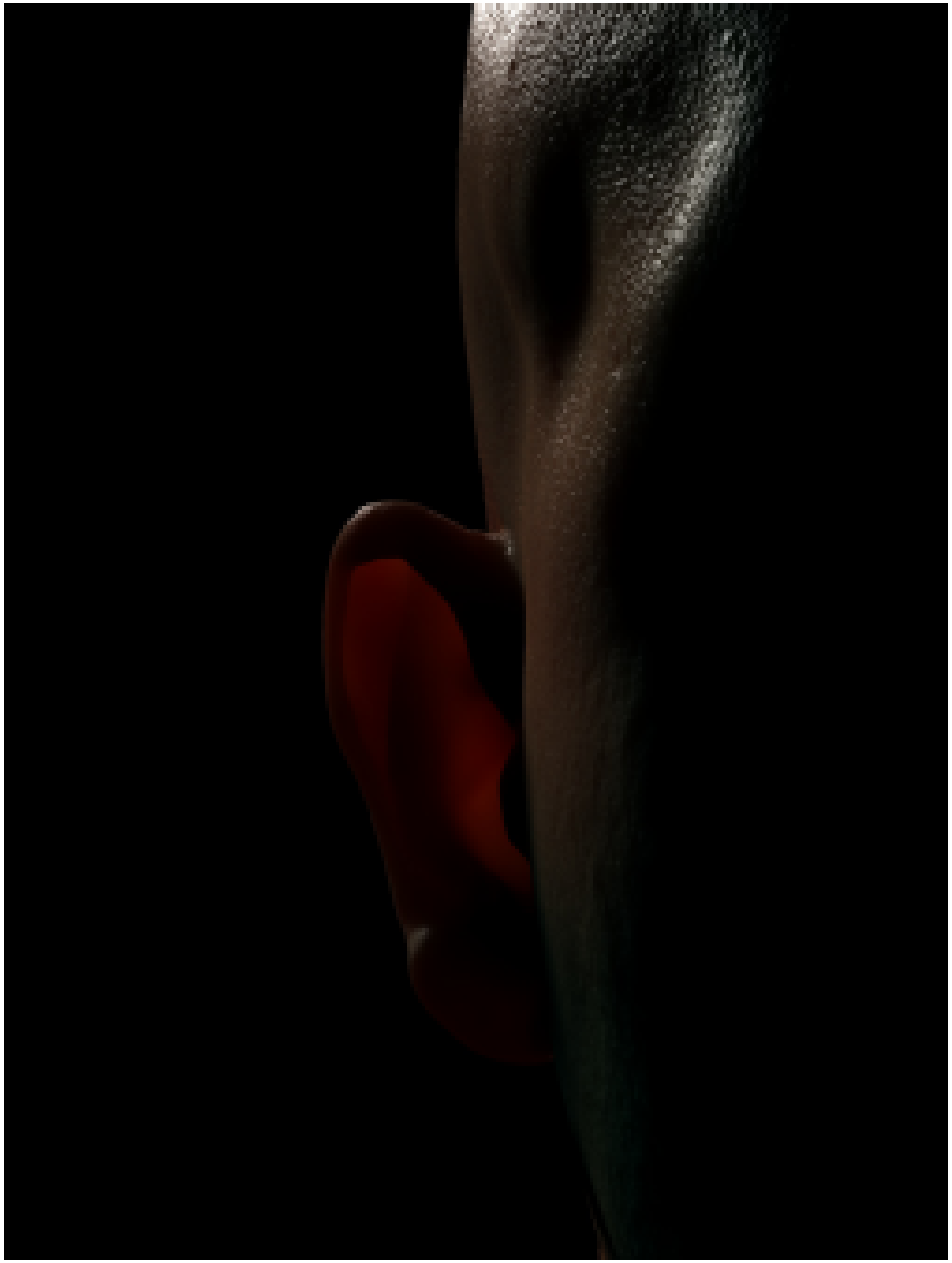


Surface Roughness



All Layers

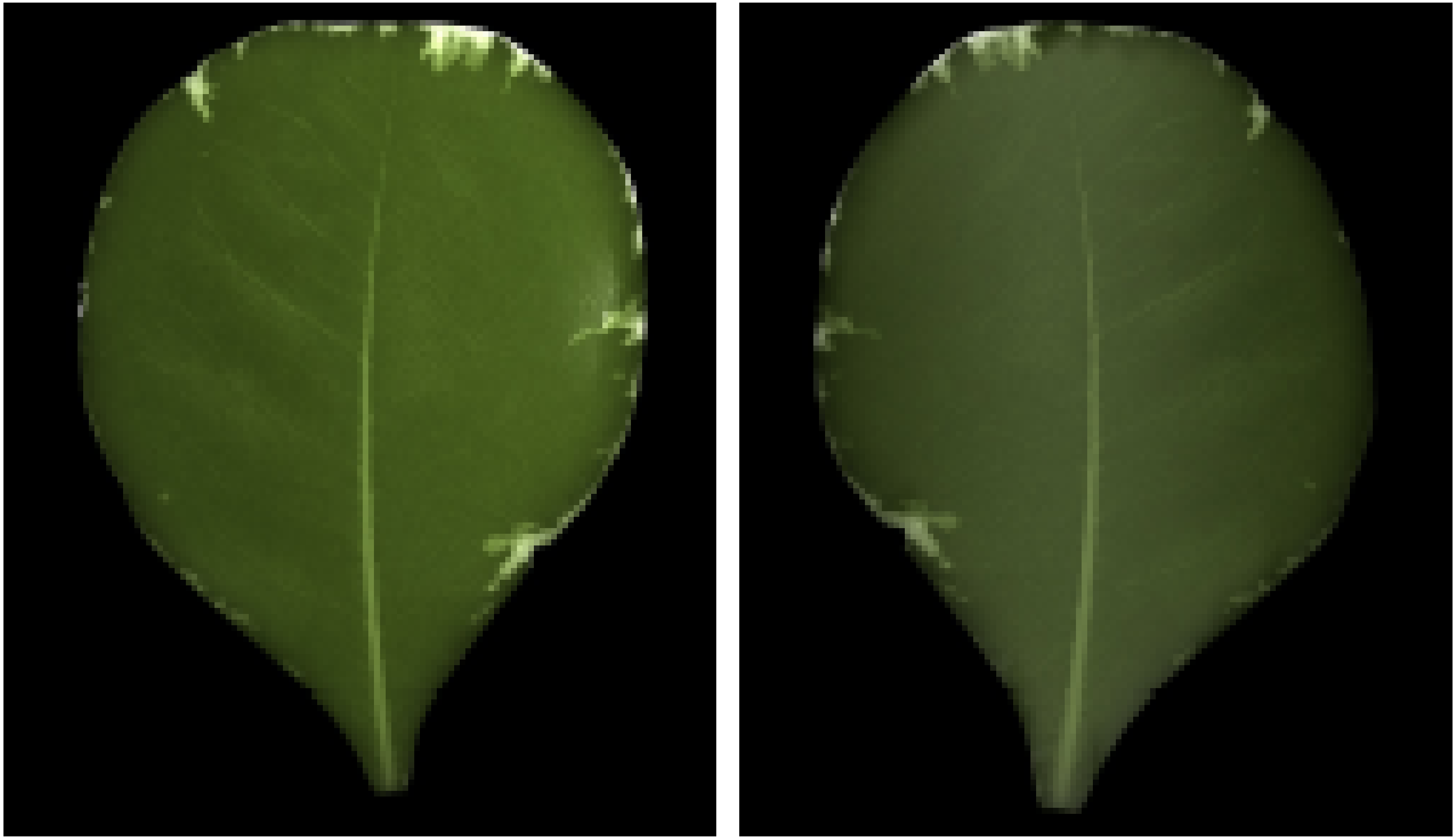




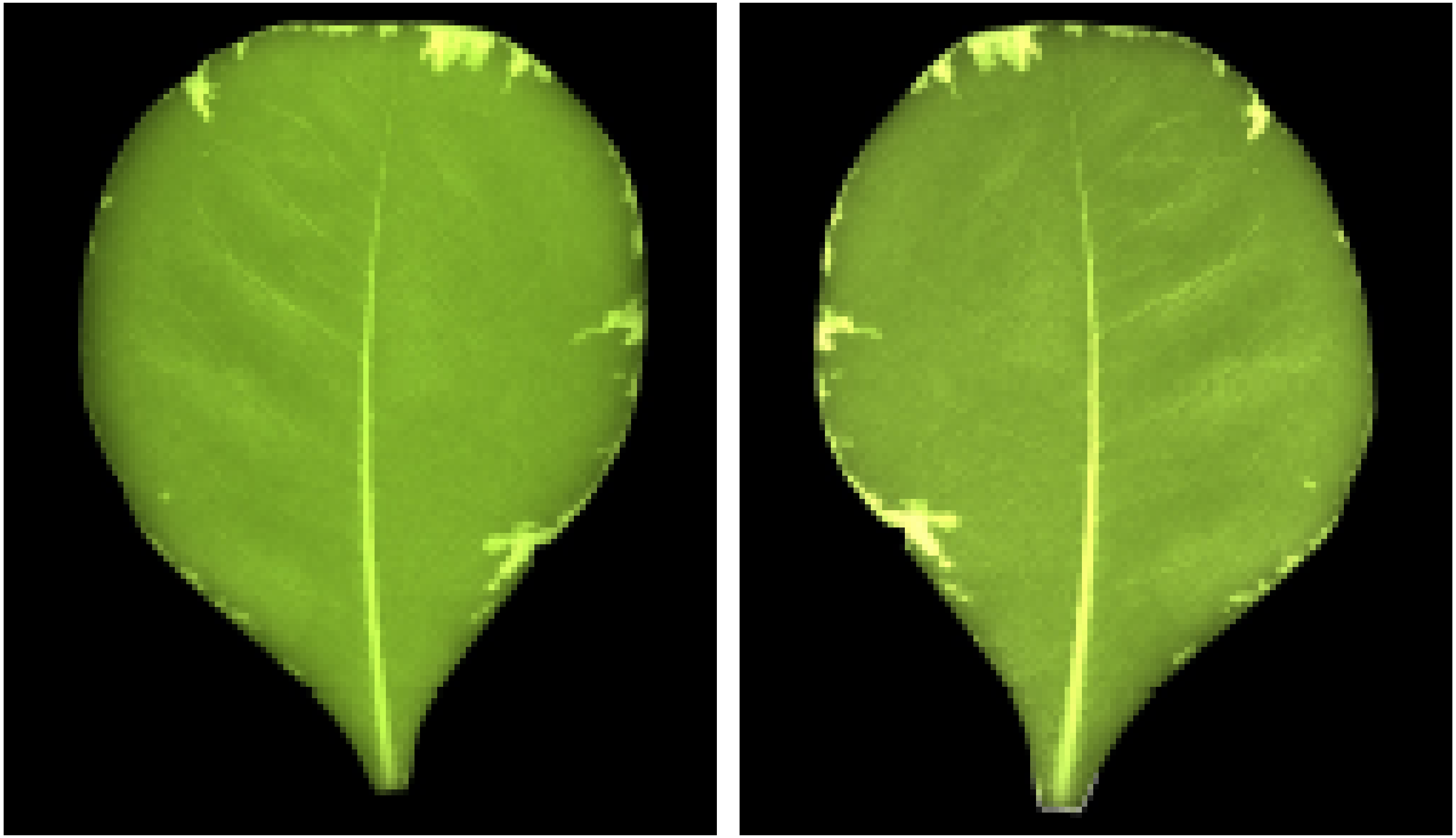
Backlit close-up of the left ear

Dipole close-up using parameters from [Jensen et al. 2001]

Multi-layer close-up using parameters from [Tuchin 2000]

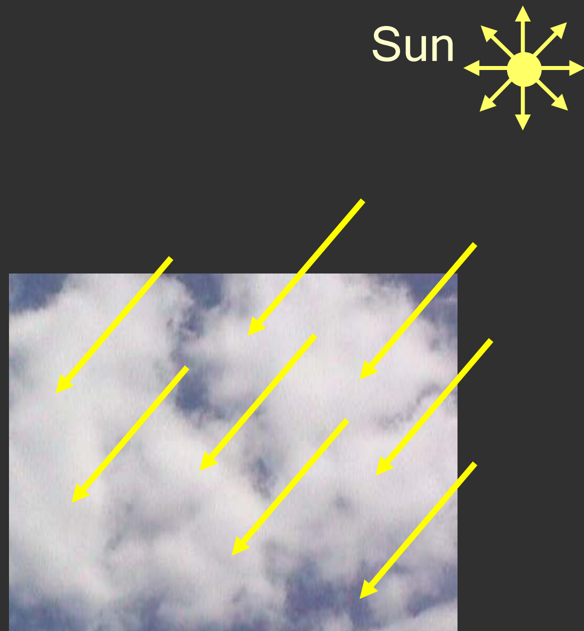


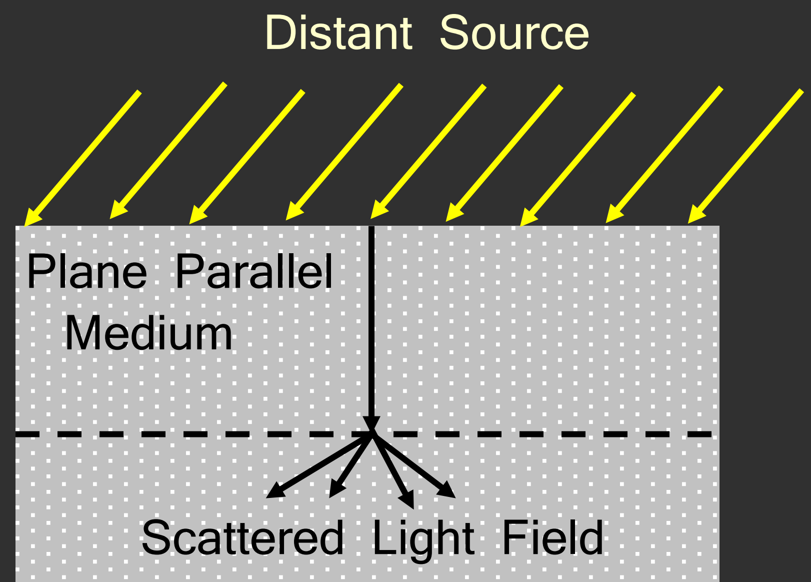
Front and back, frontlit.

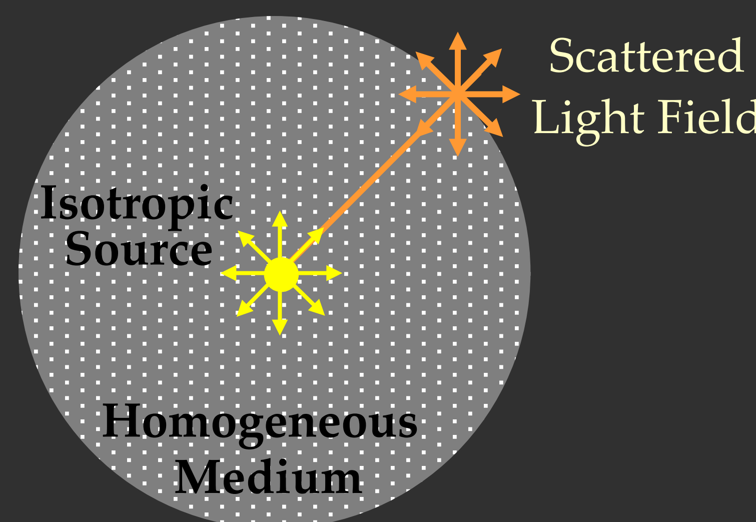


Front and back, backlit.

Plane Parallel and Spherical Radiative Transfer

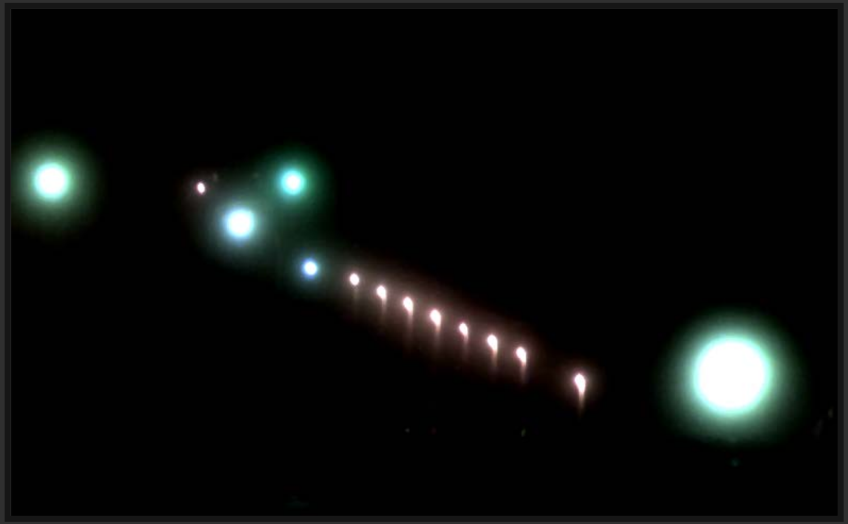






Glows of Light Sources

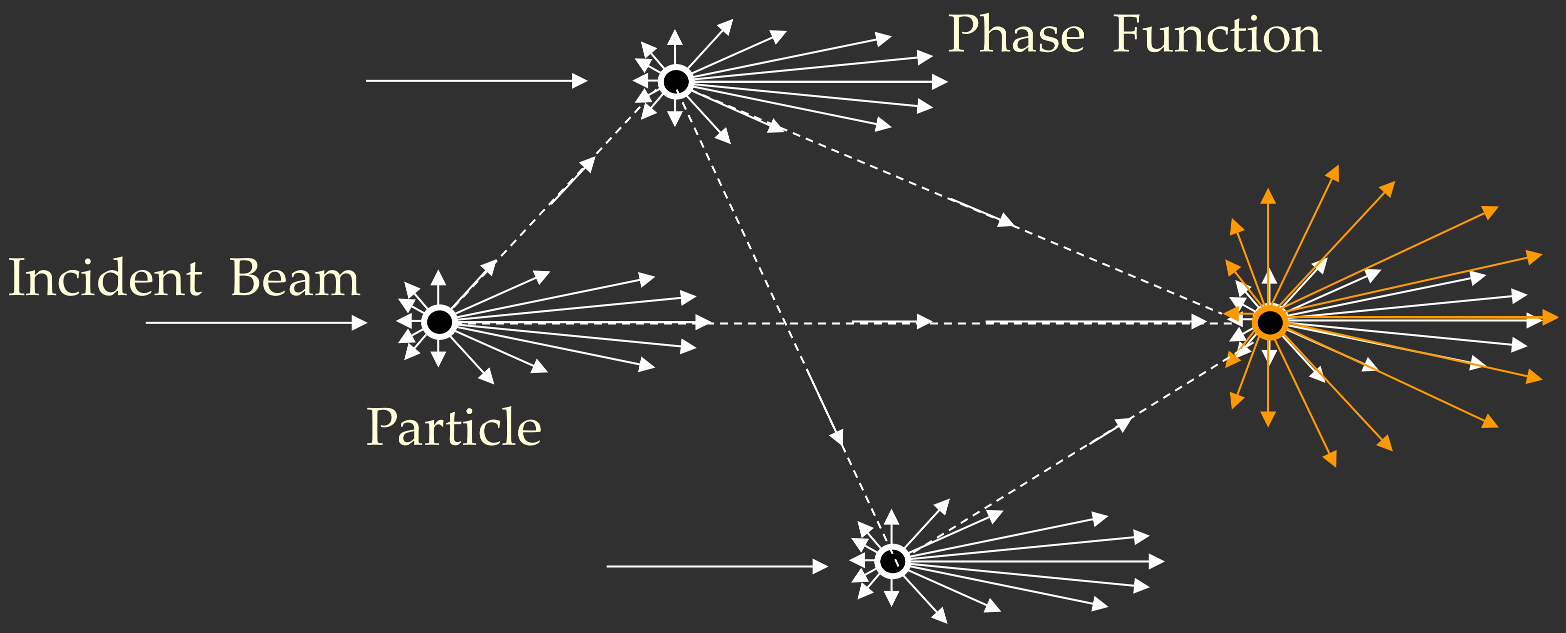
Mist

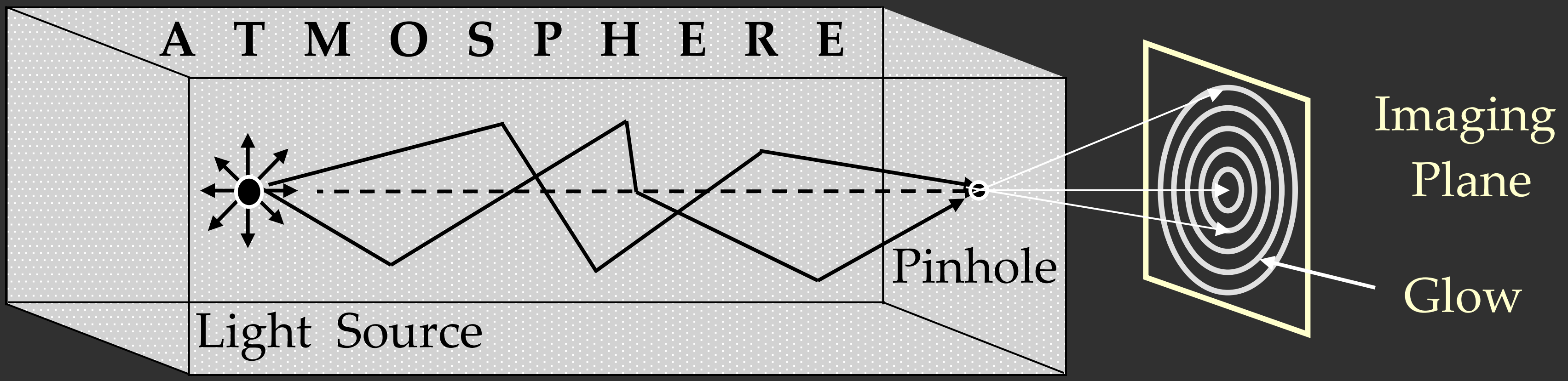


Fog

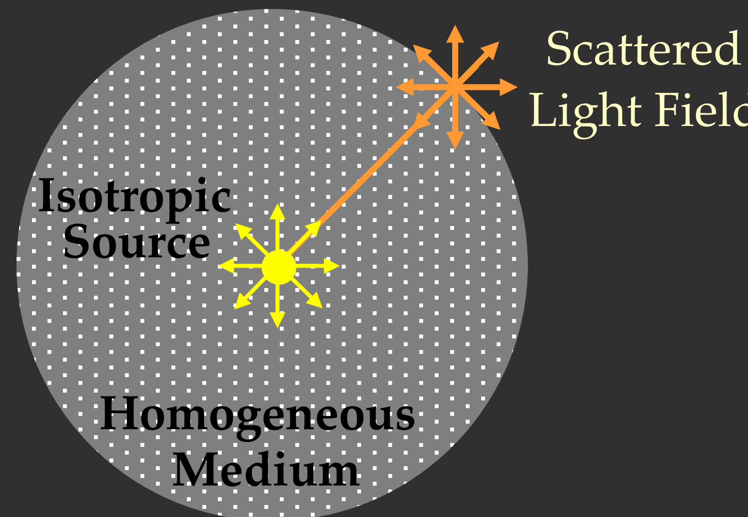
(Narasimhan & Nayar, CVPR 2003)

Multiple Scattering in the Atmosphere





Light Source in a Spherical Medium



Spherical Radiative Transfer Equation:

[Chandrasekhar 1960]

$$\mu \frac{\partial I}{\partial T} + \frac{1-\mu^2}{T} \frac{\partial I}{\partial \mu} = -I(T,\mu) + \frac{1}{4\pi} \int_0^{2\pi} \int_{-1}^{+1} P(\mu, \mu') I(T,\mu') \, d\mu' \, d\phi'$$

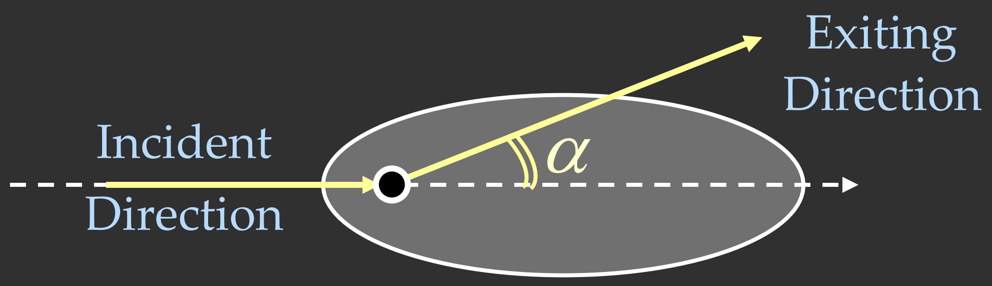
Cosine of Angle

Optical Thickness

Light Field

Phase Function

Axially Symmetric Phase Functions



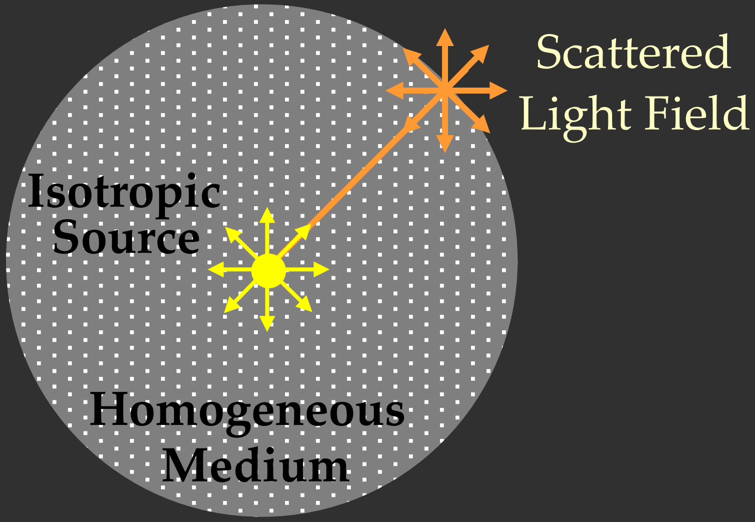
Legendre Polynomial Expansion: [Ishimaru 1997] [Henyey et al., 1941]

Forward Scattering Parameter

$$P(\cos\alpha) = \sum_{m=0}^{\infty} [(2m+1)\, q^m]\; L_m(\cos\alpha)$$

Legendre Polynomial

Light Source in a Spherical Medium



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Cosine of Angle — μ

Optical Thickness — T

Light Field — $I(T,\mu)$

Phase Function — $P(\mu,\ \mu')$

Analytic Multiple Scattering Solution

Scattered Light Field :

Legendre Polynomial

$$I(T,\mu) = \sum_{m=0}^{\infty} \left(g_m(T) + g_{m+1}(T)\right) L_m(\mu)$$

Exponential Coefficients :

Optical Thickness

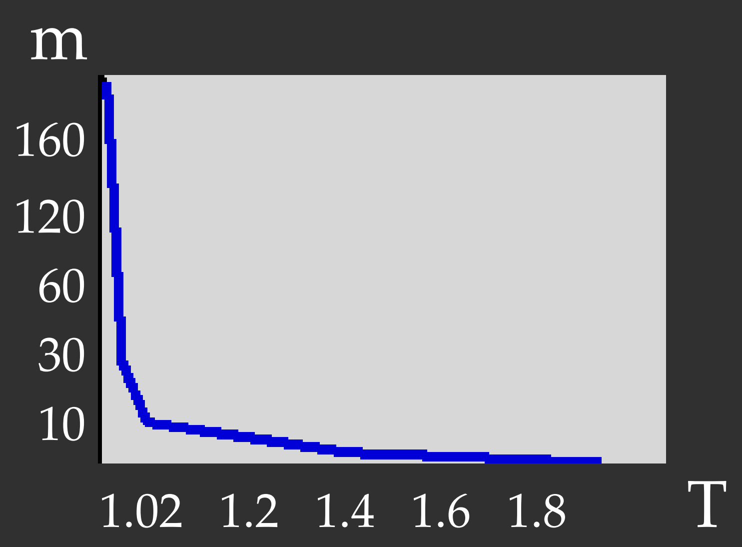
Phase Function Parameter

$$g_m(T) = I_0 \exp\left[-(m+1)T - \frac{2m+1}{m}(1-q^{m-1})\,T \right]$$

Radiant Intensity of Source

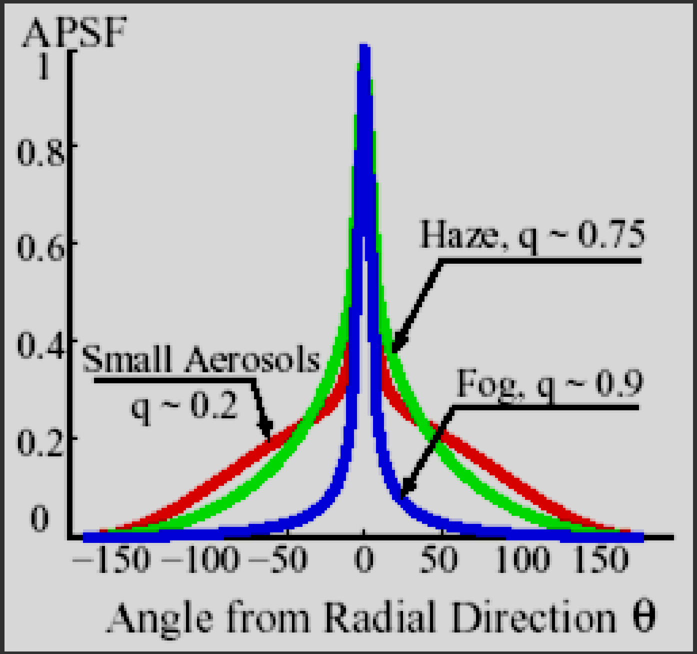
Highlights of the Model

- Single and Multiple Scattering
- Absorbing and Purely Scattering Media
- Isotropic and Anisotropic Phase Functions
- Small Number of Coefficients (m) :

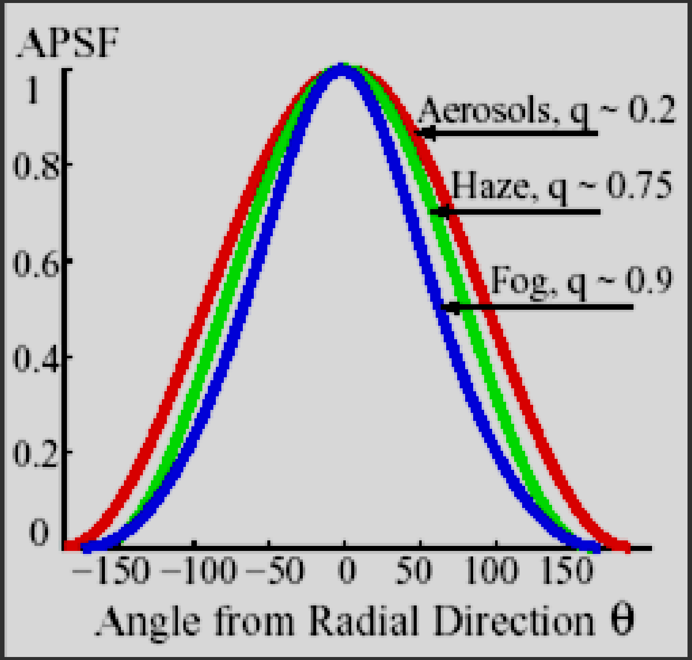


Scattered Light Field vs. Weather Condition

Angular PSF : Scattered Light Field at a Point



Mild Weather (T = 1.2)



Dense Weather (T = 4)

Validation: Multiple Scattering in Milk

Image acquired With No Milk

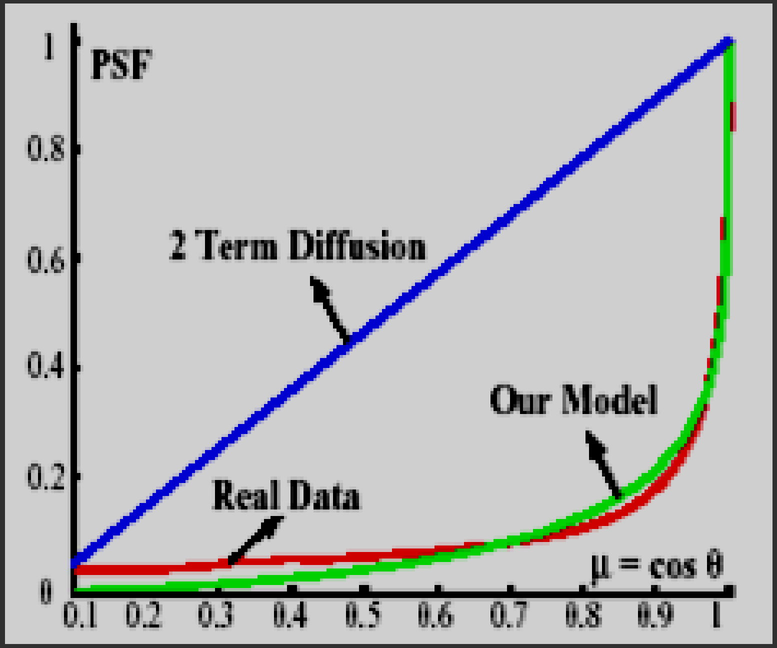
Increasing Milk Concentrations

Original Milk Images

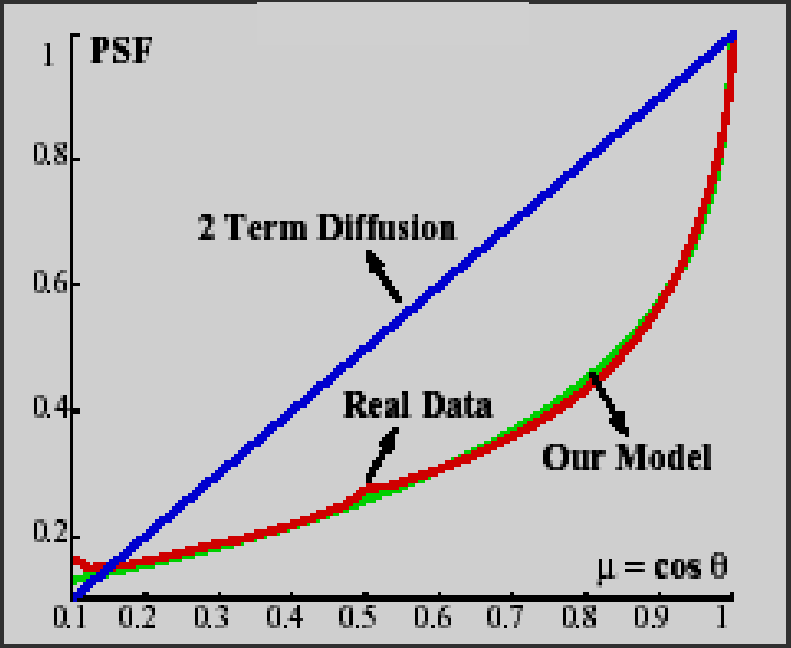
Rendered Milk Images

Model Fit Accuracy

Low Milk Concentration



High Milk Concentration

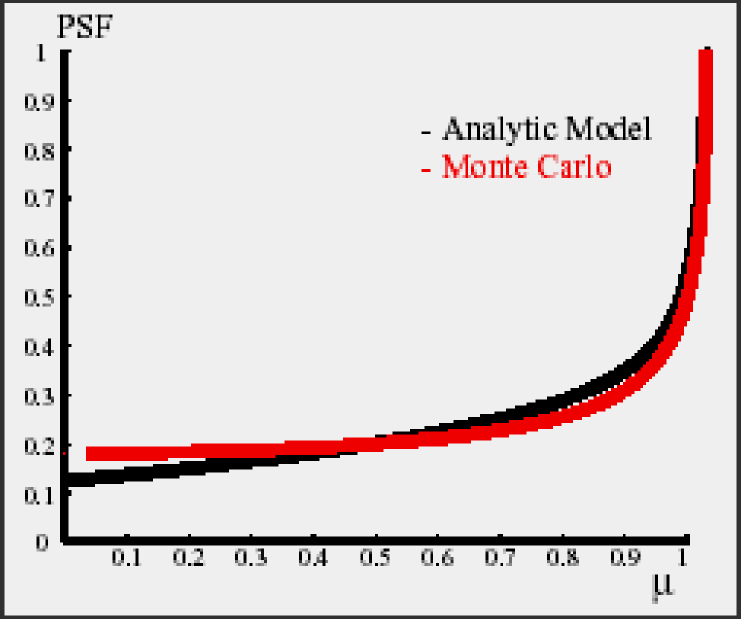


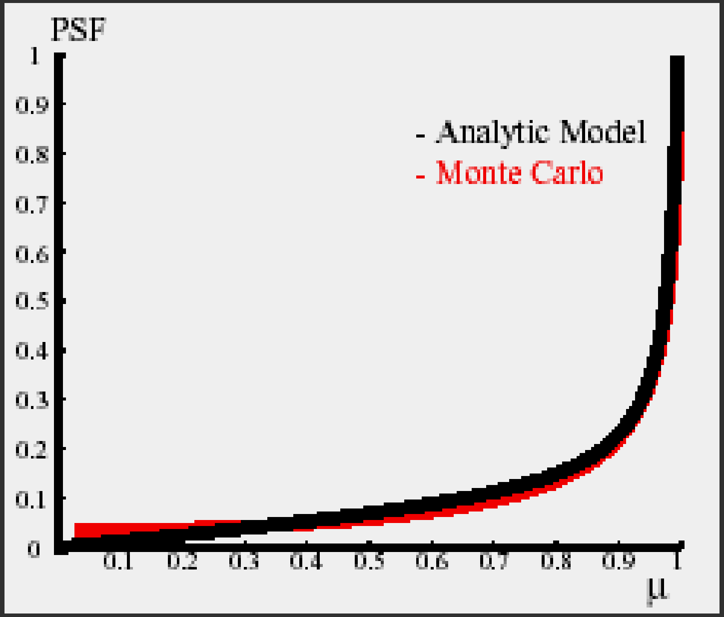
Number of Milk Concentrations : 15

Model Fitting Error : [1 % to 3 %]

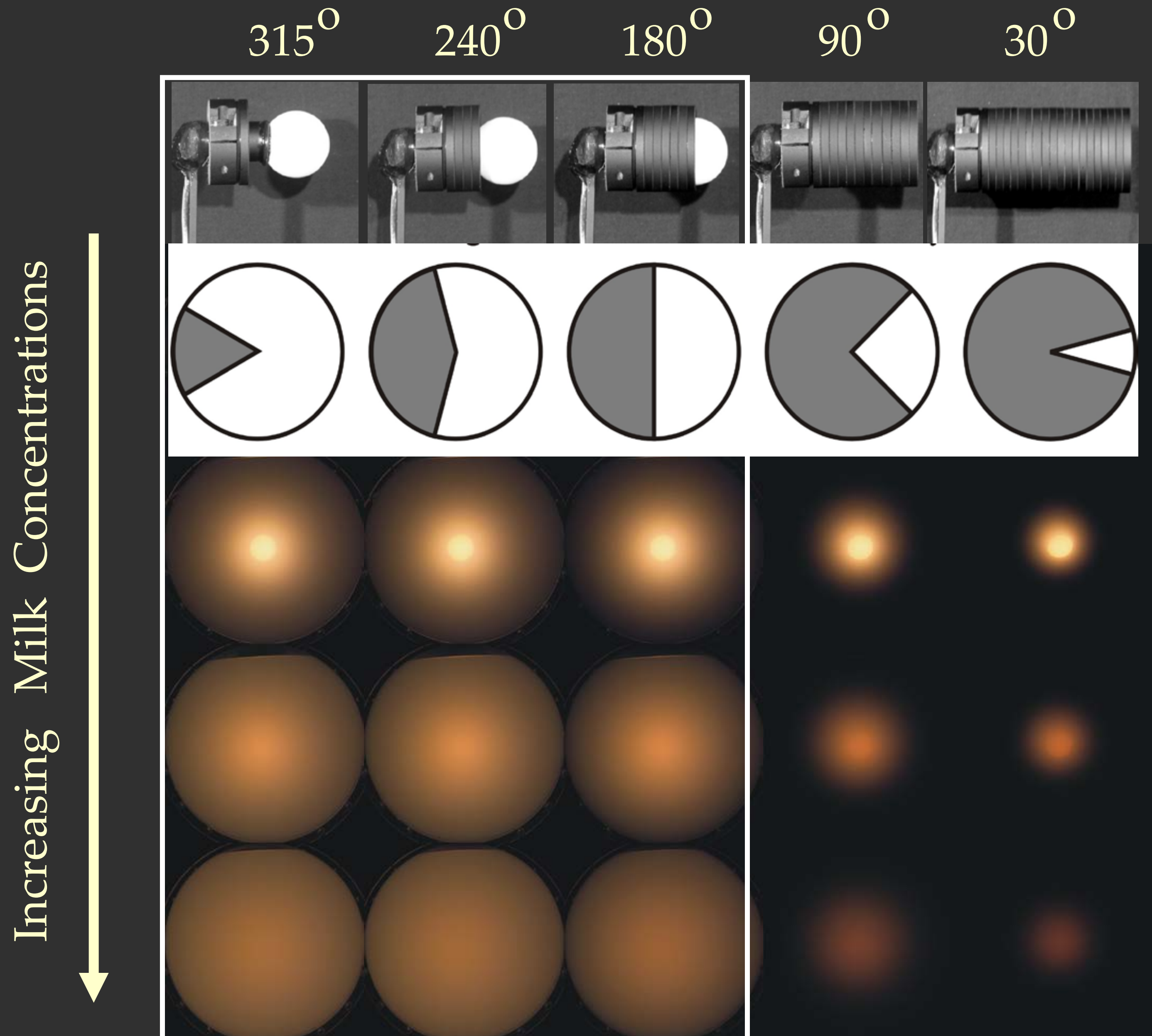
Diffusion Fitting Error : [20 % to 50 %]

Model Fit Accuracy: Monte Carlo Simulations





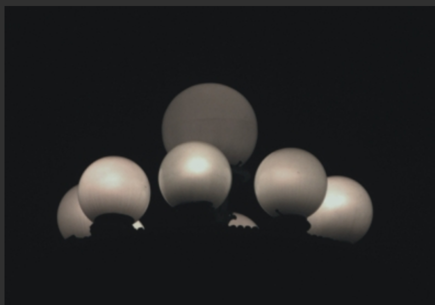
Effect of Source Visibility



Observed Milk Images

Rendering Glows using Convolution

Original Image



Rendered Images

Increasing Fog →

Joint work with Ramamoorthi

Single versus Multiple Scattering

Original Image

Single Scattering

Multiple Scattering (Mild Condition)

Multiple Scattering (Dense Condition)

Joint work with Ramamoorthi

Inverse RTE : Weather from APSF

Measured APSF : $I(T,\mu)$

Objective Function :

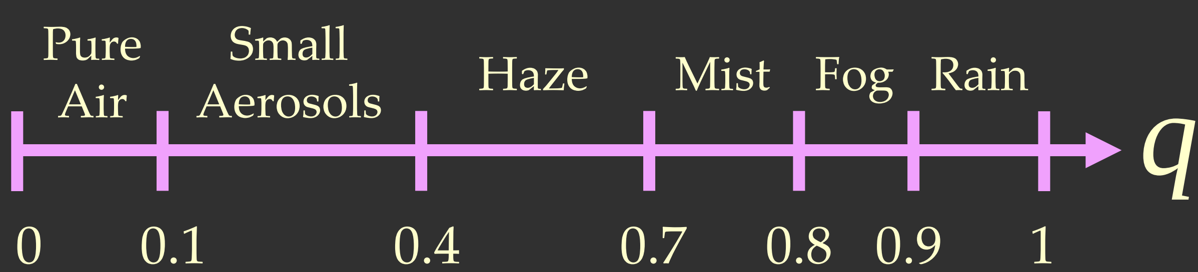
$$\underset{T,q}{\arg\min} \left\| I(T,\mu) - \sum_{m=0}^{\infty} \left(g_m(T,q) + g_{m+1}(T,q)\right) L_m(\mu) \right\|$$

Meteorological Visibility: [Middleton 1952]

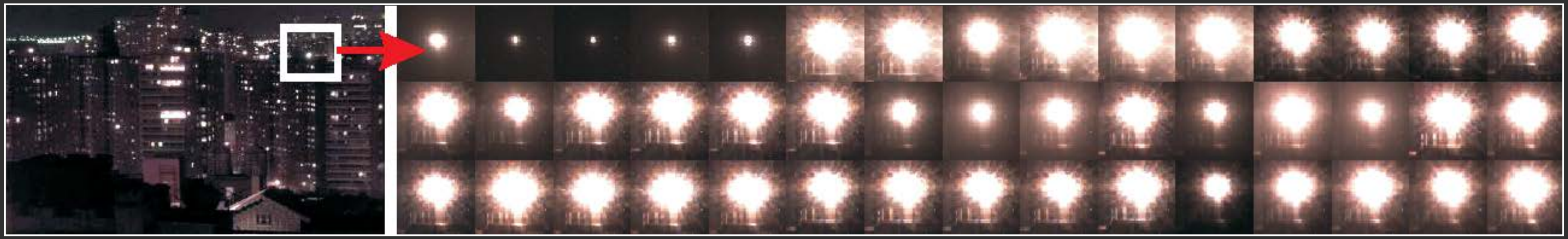
$$V \approx \frac{3.912}{T} R$$

Weather Condition:

[Van de Hulst 1957]

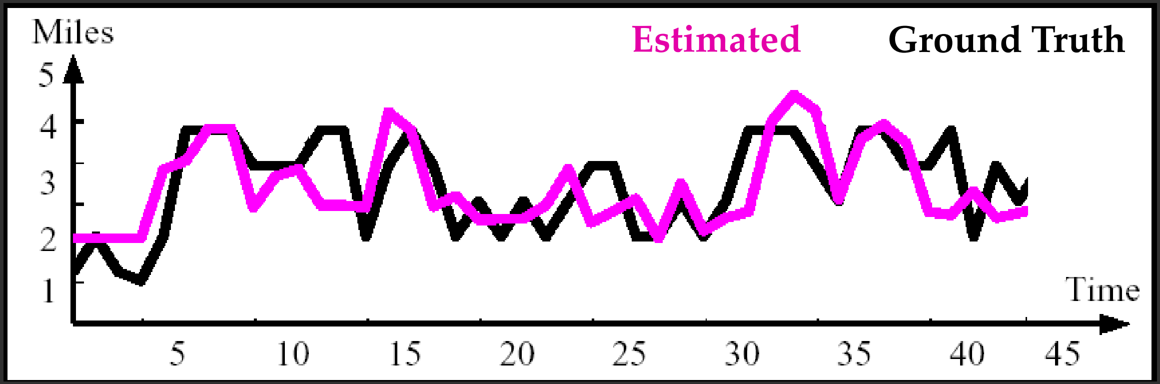


A Camera-based Weather Station

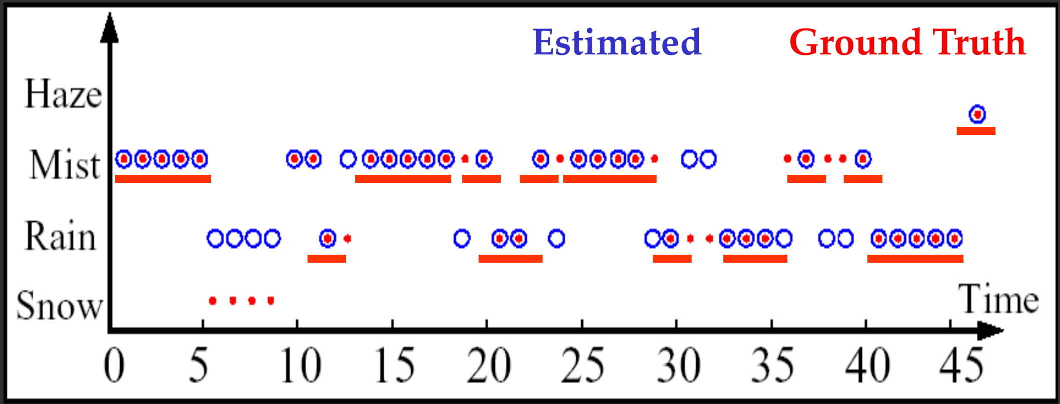


45 images of a light source (WILD Database ECCV 02)

Computed
Atmospheric Visibilities



Computed
Weather Conditions

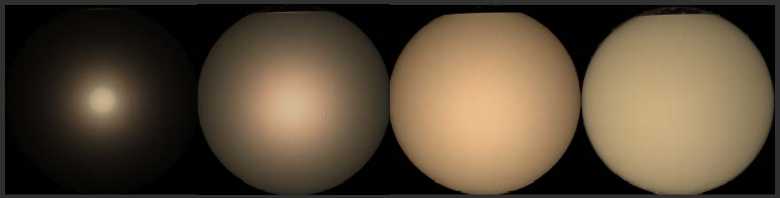


Summary

Analytic Multiple Scattering

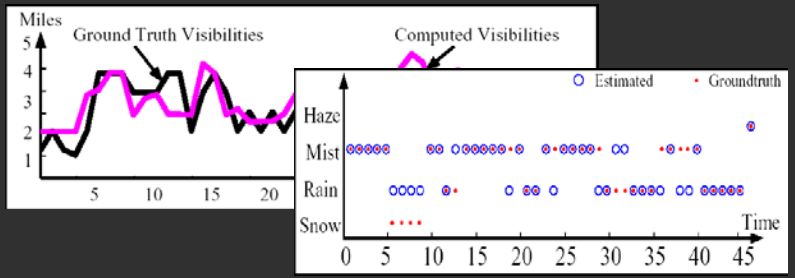
$$I(T,\mu) = \sum_{m=0}^{\infty} \left(g_m(T) + g_{m+1}(T)\right) L_m(\mu)$$

Model Validation using Milk



Volume Rendering as Convolution

Shedding Light on the Weather



Next Class: Fluids

Lectures #22