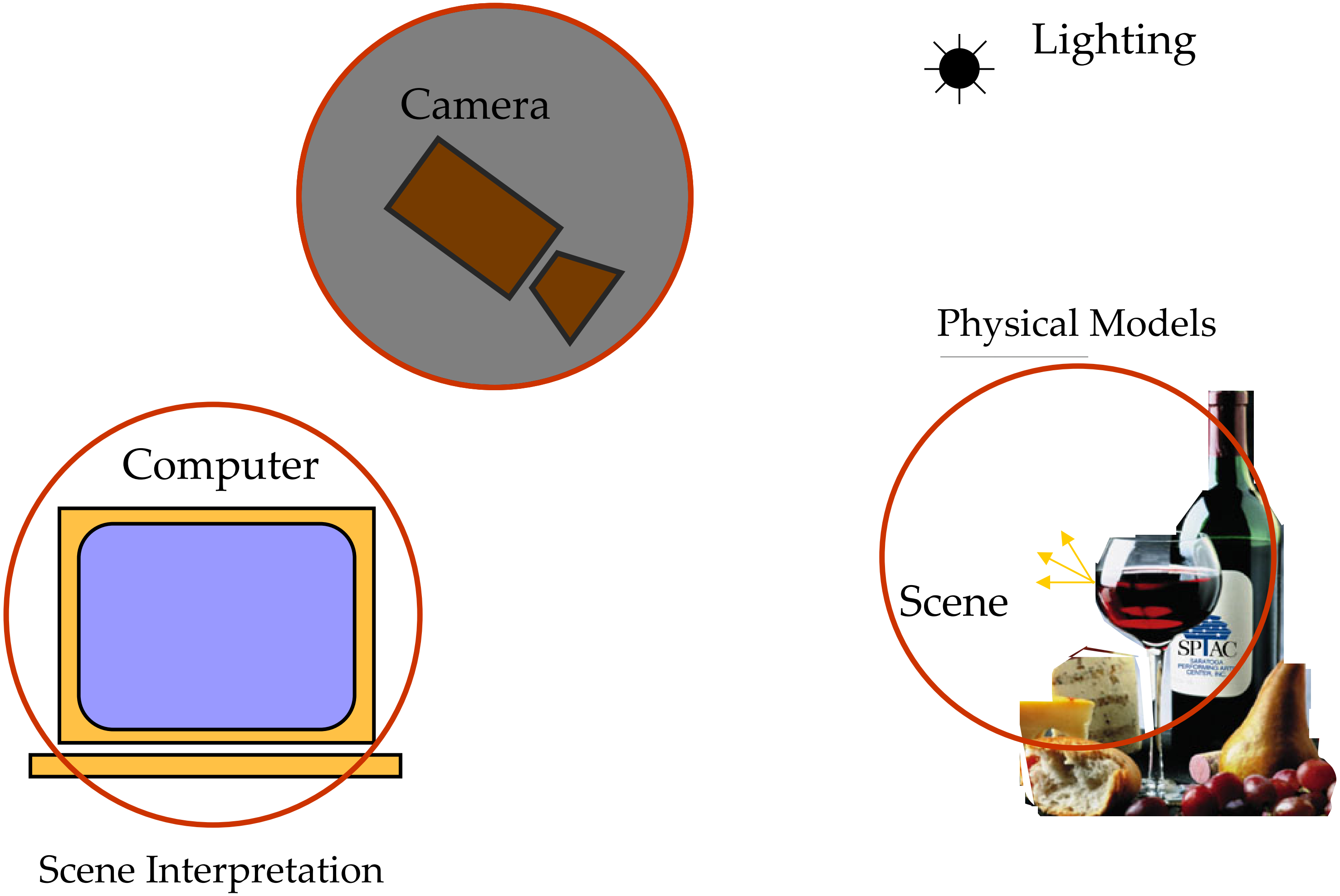
Basic Principles of Imaging and Photometry

Lecture #2

Thanks to Shree Nayar, Ravi Ramamoorthi, Pat Hanrahan

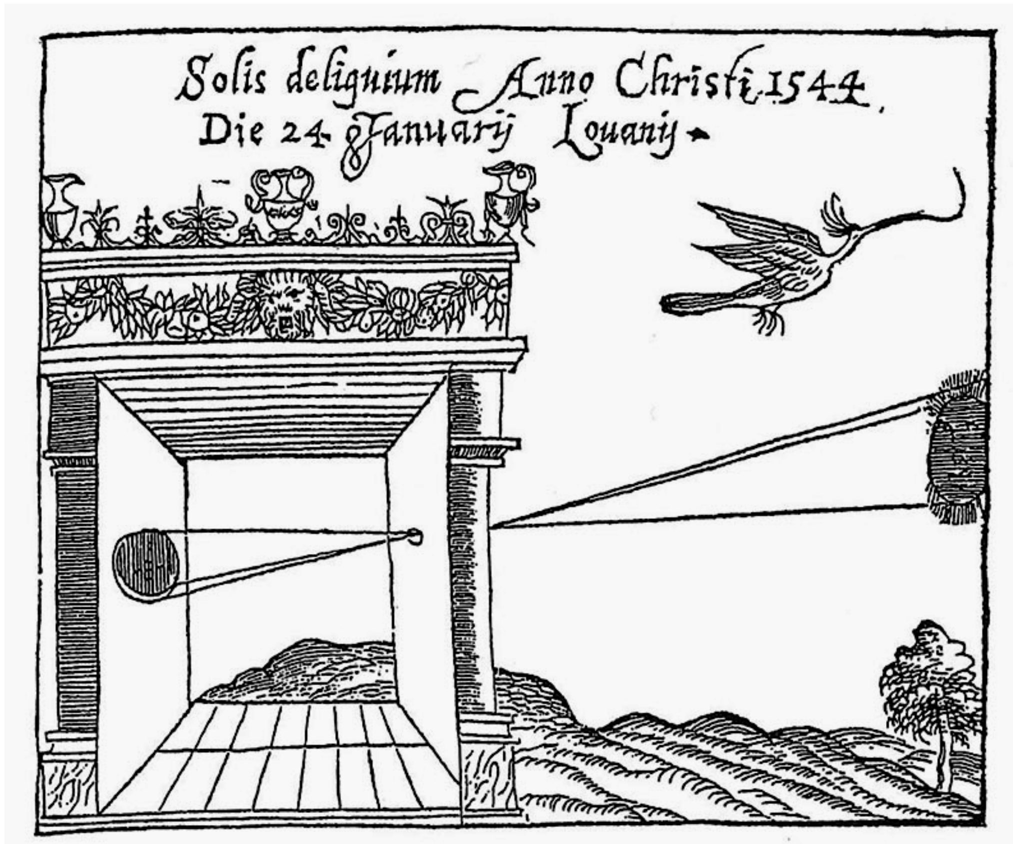
Computer Vision: Building Machines that See



We need to understand the Geometric and Radiometric relations between the scene and its image.

A Brief History of Images

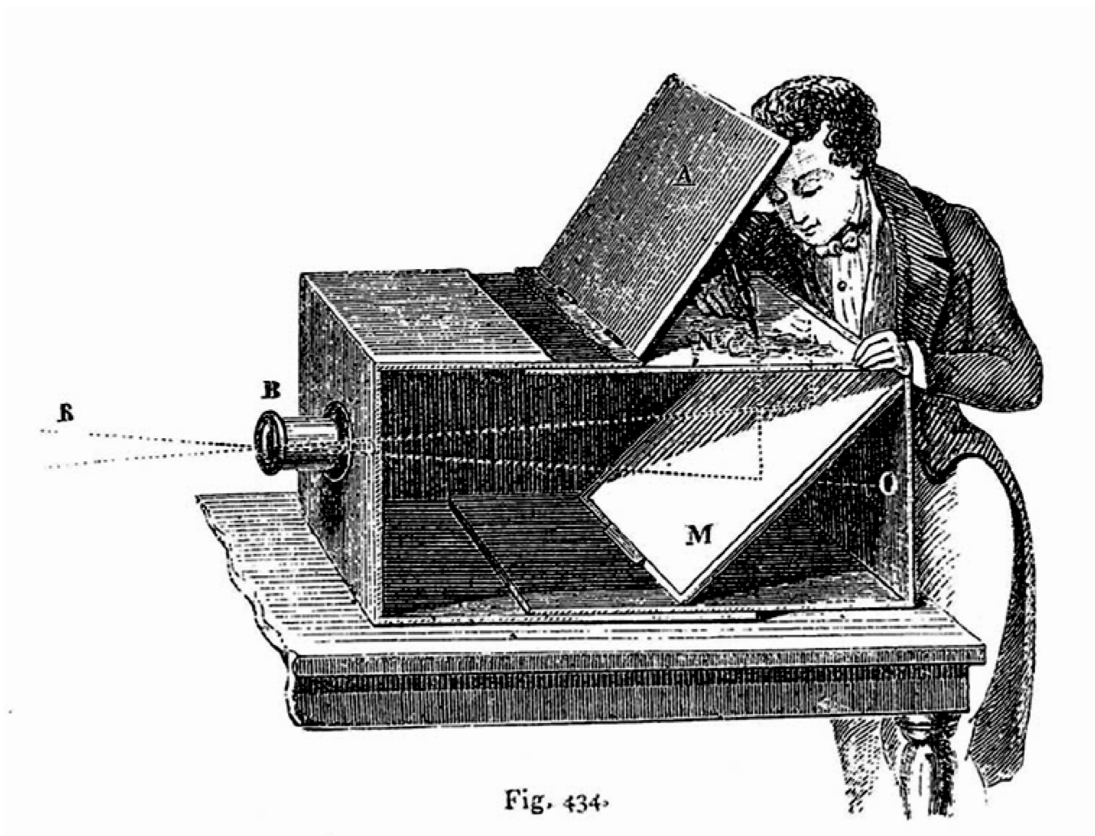
1558



Camera Obscura, Gemma Frisius, 1558

A Brief History of Images

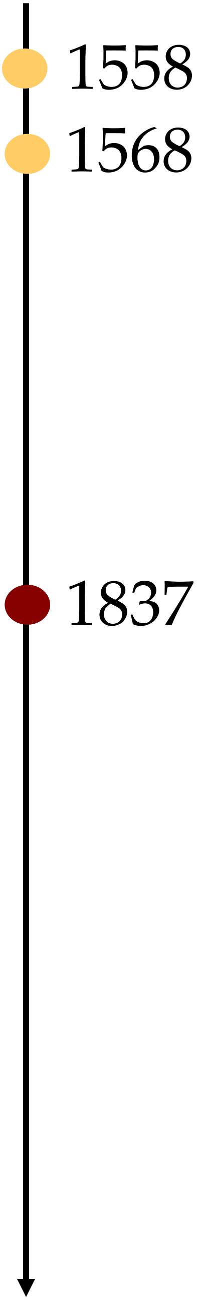
1558
1568



Lens Based Camera Obscura, 1568

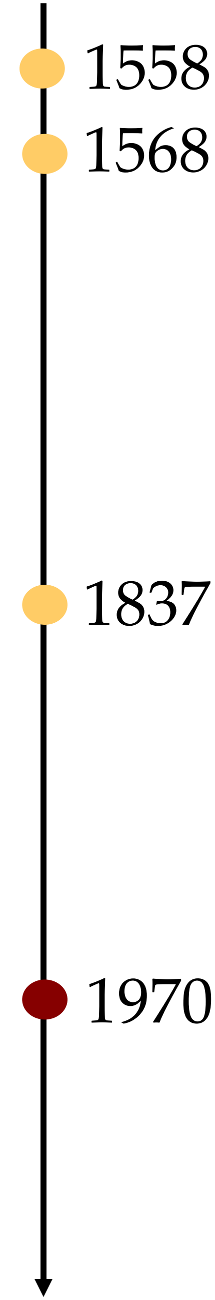
A Brief History of Images

Still Life, Louis Jaques Mande Daguerre, 1837



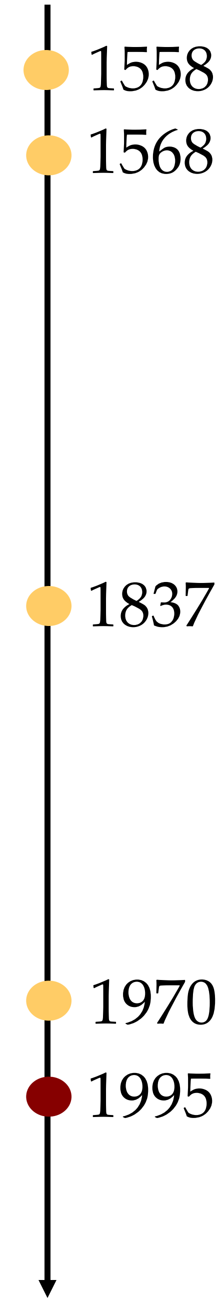
A Brief History of Images

Silicon Image Detector, 1970



A Brief History of Images

Digital Cameras



Geometric Optics and Image Formation

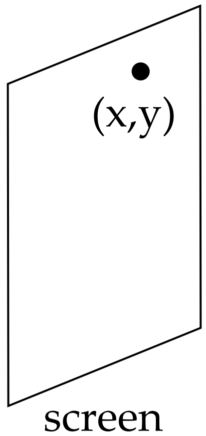
TOPICS TO BE COVERED :

1) Pinhole and Perspective Projection

2) Image Formation using Lenses

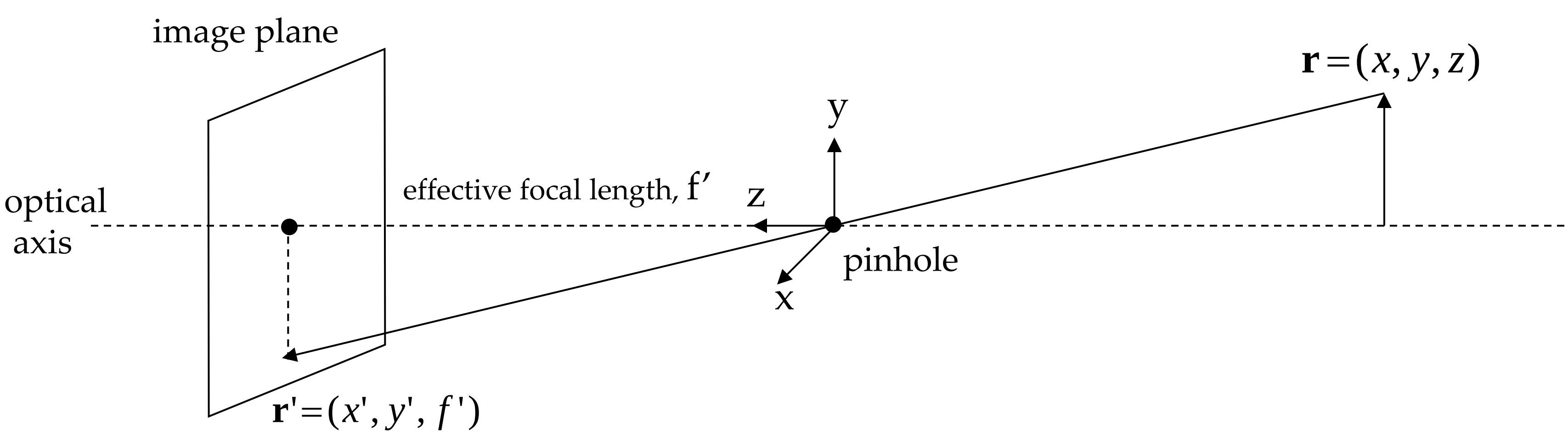
3) Lens related issues

Pinhole and the Perspective Projection



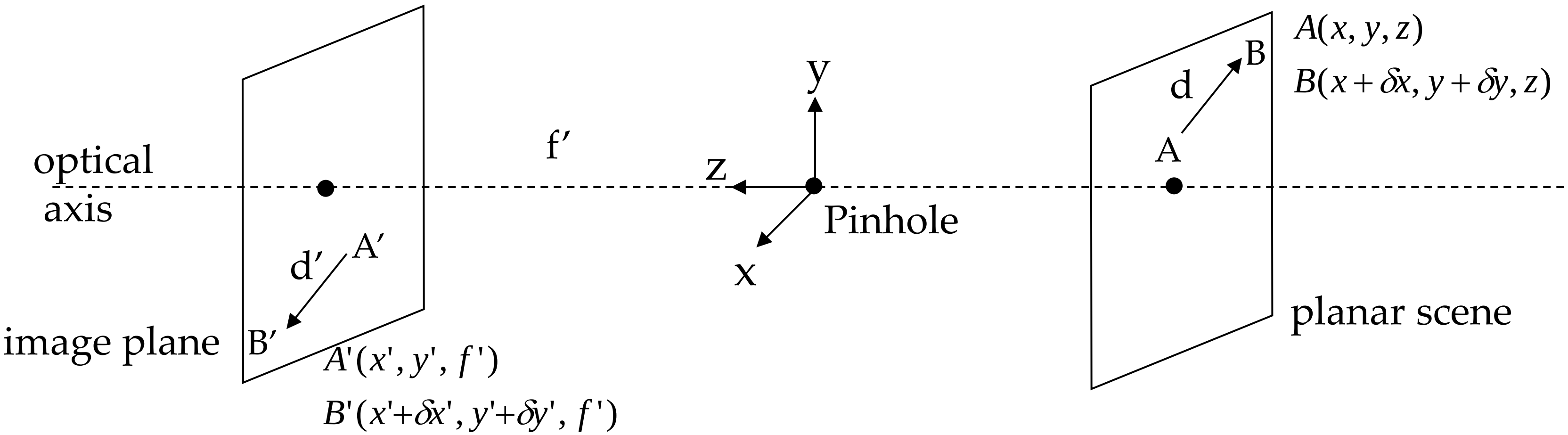
Is an image being formed on the screen?

YES! But, not a "clear" one.



$$\frac{\mathbf{r}'}{f'} = \frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'} = \frac{x}{z} \qquad \frac{y'}{f'} = \frac{y}{z}$$

Magnification



From perspective projection:

$$\frac{x'}{f'} = \frac{x}{z} \qquad \frac{y'}{f'} = \frac{y}{z}$$

$$\frac{x'+\delta x'}{f'} = \frac{x+\delta x}{z} \qquad \frac{y'+\delta y'}{f'} = \frac{y+\delta y}{z}$$

Magnification:

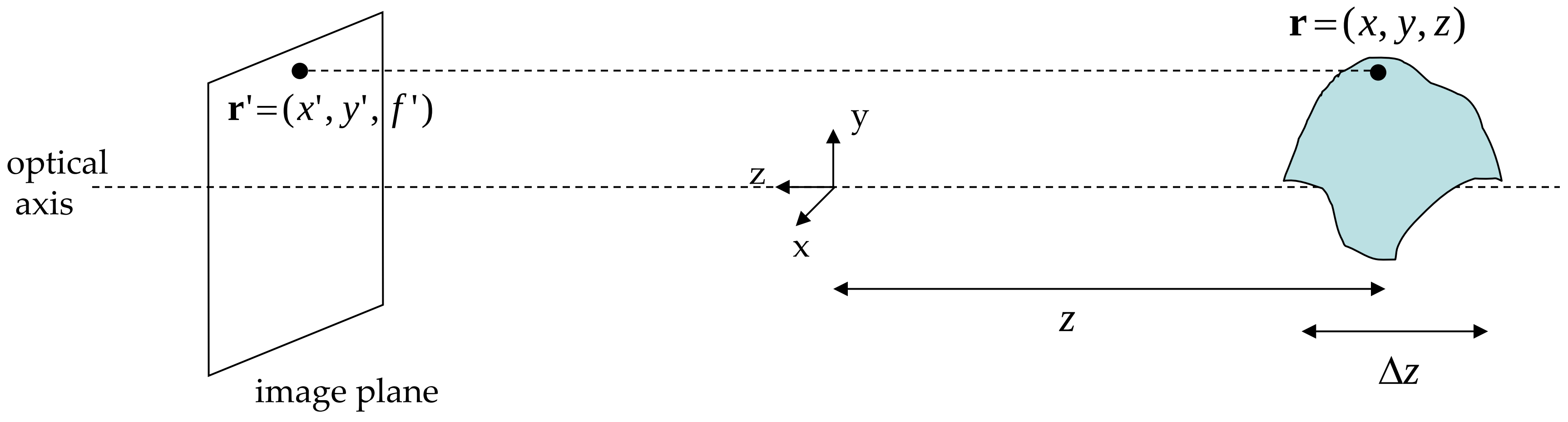
$$m = \frac{d'}{d} = \frac{\sqrt{(\delta x')^2 + (\delta y')^2}}{\sqrt{(\delta x)^2 + (\delta y)^2}} = \frac{f'}{z}$$

$$\frac{Area_{image}}{Area_{scene}} = m^2$$

Orthographic Projection

Magnification: $x' = m\ x \qquad y' = m\ y$

When m = 1, we have orthographic projection



This is possible only when $z >> \Delta z$
In other words, the range of scene depths is assumed to be much smaller than the average scene depth.

But, how do we produce non-inverted images?

Better Approximations to Perspective Projection

(NALWA

- Weak-Perspective

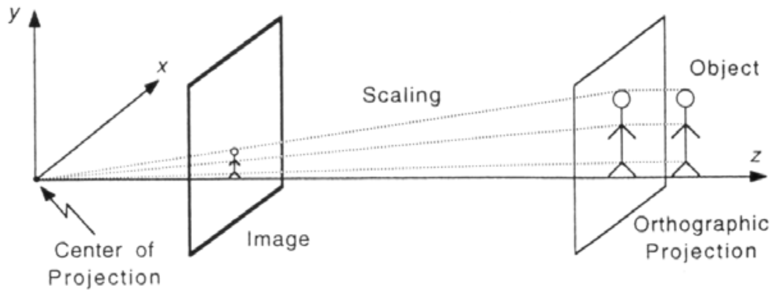


Figure 2.8 Approximation of perspective projection by orthographic projection. Perspective projection onto a plane can be approximated by orthographic projection, followed by scaling, when (1) the object dimensions are small compared to the distance of the object from the center of projection, and (2) compared to this distance, the object is close to the straight line that passes through the center of projection and is orthogonal to the image plane (this line is the z-axis here).

- Para-Perspective

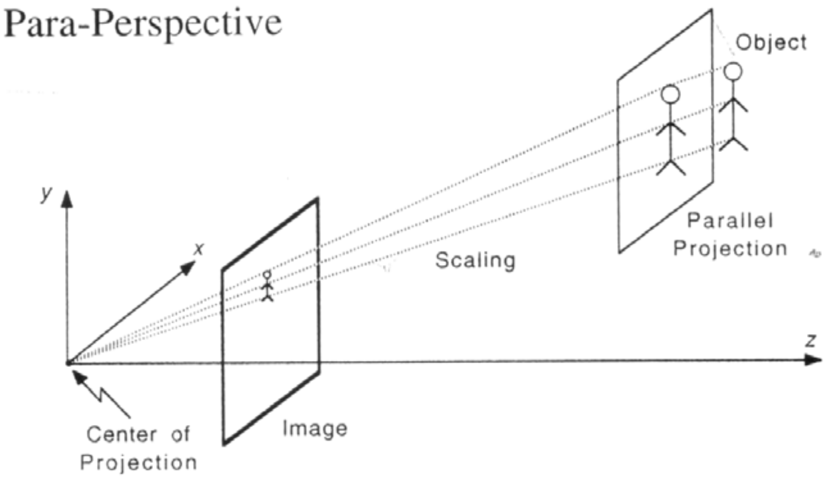


Figure 2.9 Approximation of perspective projection by parallel projection. *Parallel projection* onto a plane is a generalization of orthographic projection in which all the object points are projected along a set of parallel straight lines that may or may not be orthogonal to the projection plane. Perspective projection onto a plane can be approximated by parallel projection, followed by scaling, whenever the object dimensions are small compared to the distance of the object from the center of projection. The direction of parallel projection in such an approximation is along the "average direction" of perspective projection.

Problems with Pinholes

- Pinhole size (aperture) must be "very small" to obtain a clear image.
- However, as pinhole size is made smaller, less light is received by image plane.
- If pinhole is comparable to wavelength of incoming light, DIFFRACTION effects blur the image!
- Sharpest image is obtained when:

pinhole diameter $d = 2\sqrt{f'\lambda}$

Example: If $f' = 50mm$,

$\lambda = 600nm$ (red),

$d = 0.36mm$

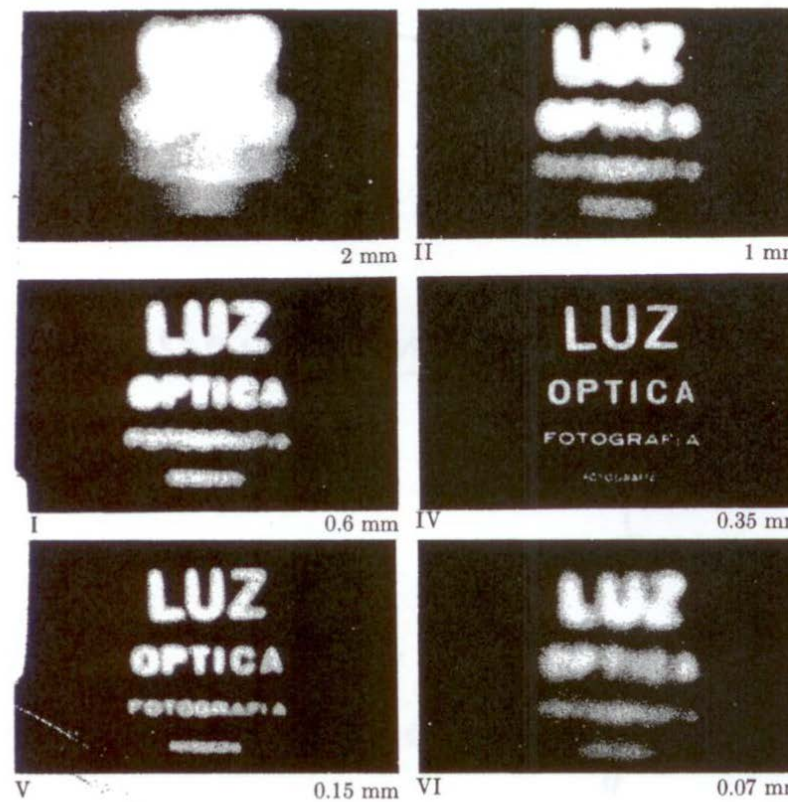
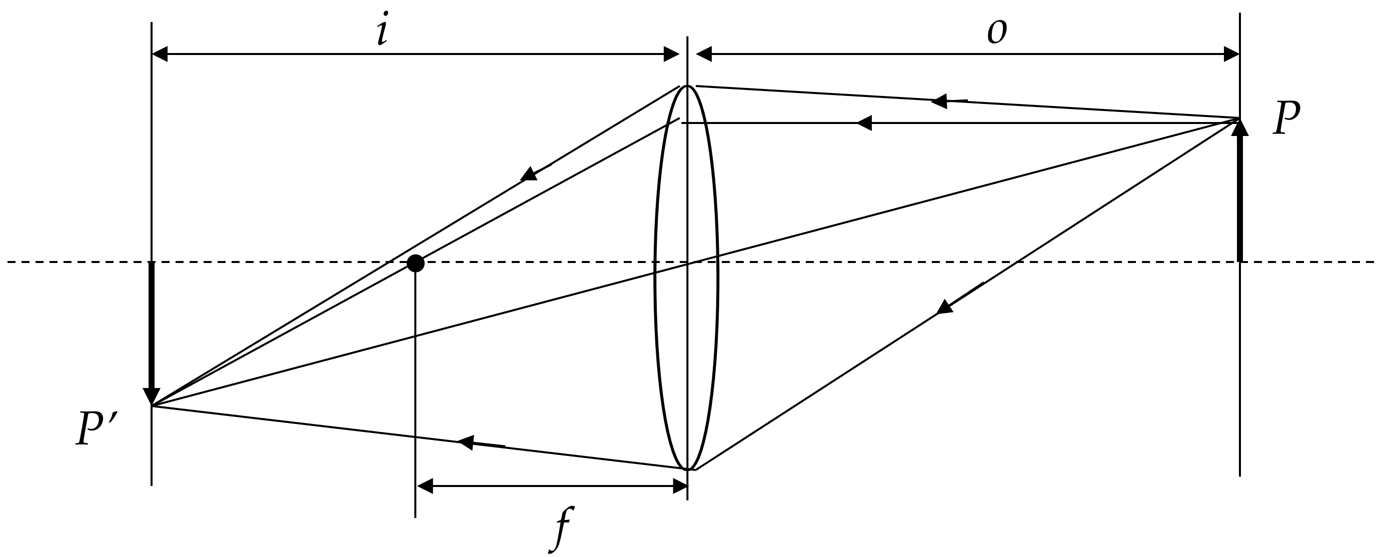


Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

Image Formation using Lenses

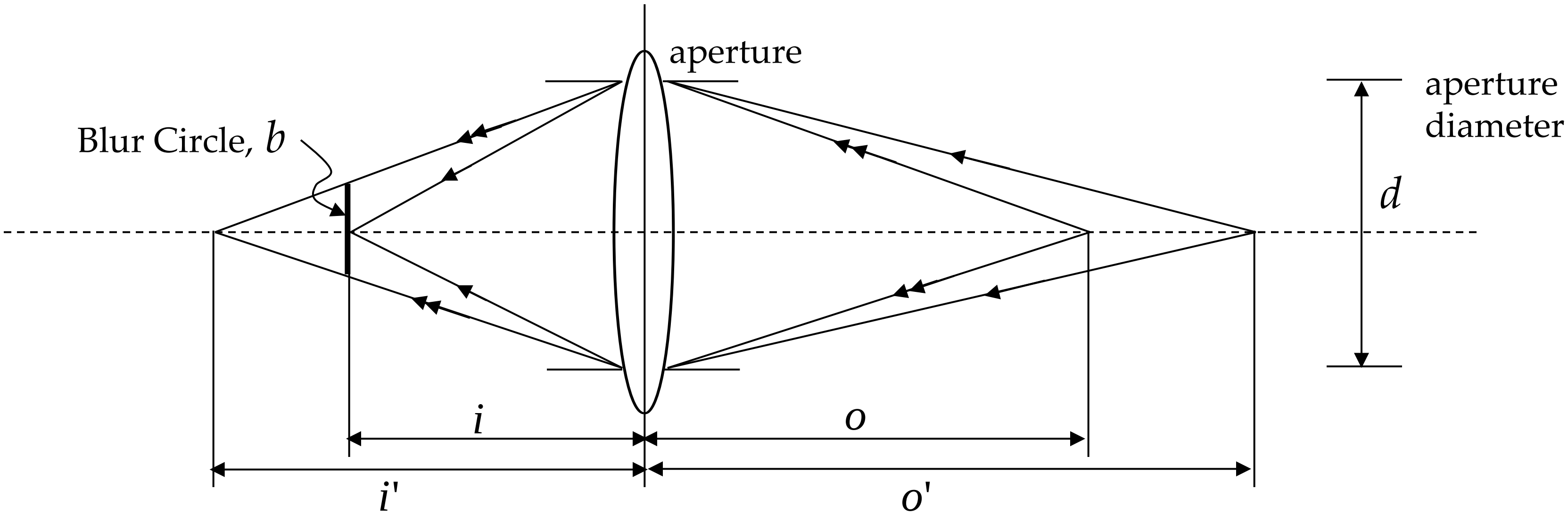
- Lenses are used to avoid problems with pinholes.
- Ideal Lens: Same projection as pinhole but gathers more light!



Gaussian Lens Formula: $$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

- f is the focal length of the lens – determines the lens's ability to bend (refract) light
- f different from the effective focal length f' discussed before!

Focus and Defocus

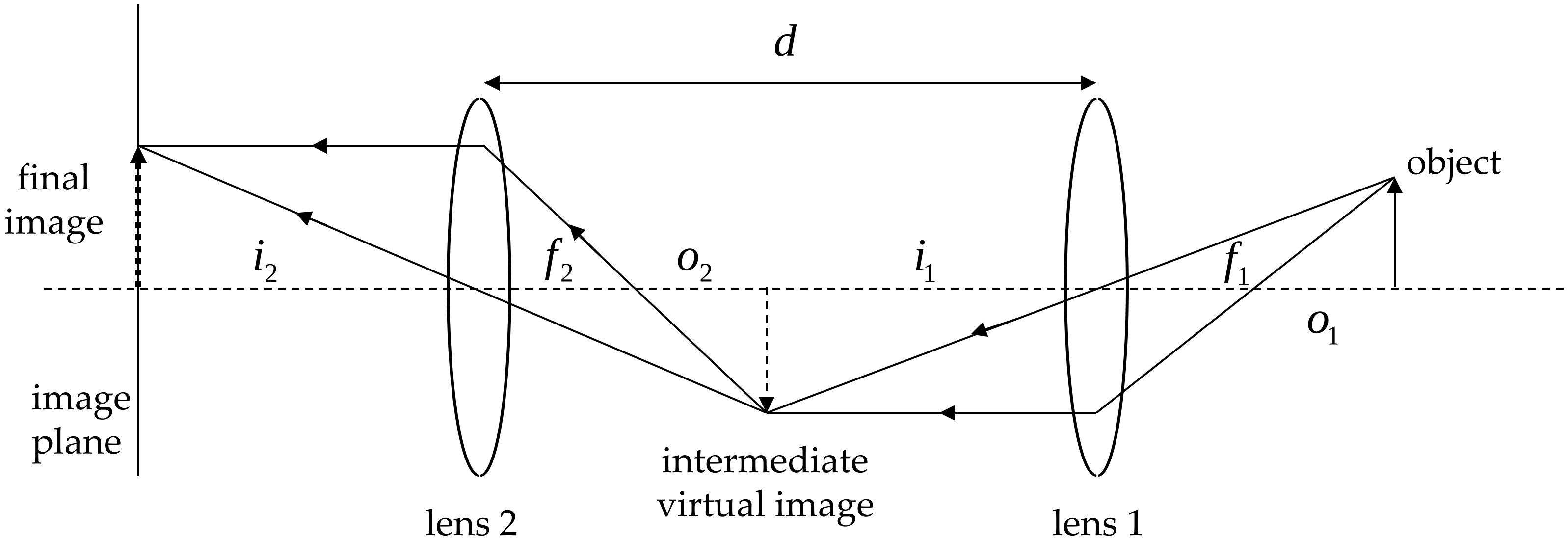


Gaussian Law: $\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$ $\quad \frac{1}{i'} + \frac{1}{o'} = \frac{1}{f}$ $\Rightarrow$ $(i' - i) = \frac{f}{(o' - f)} \frac{f}{(o - f)} (o - o')$

Blur Circle Diameter : $b = \frac{d}{i'} (i' - i)$

Depth of Field: Range of object distances over which image is sufficiently well focused.
i.e. Range for which *blur circle* is less than the resolution of the imaging sensor.

Two Lens System



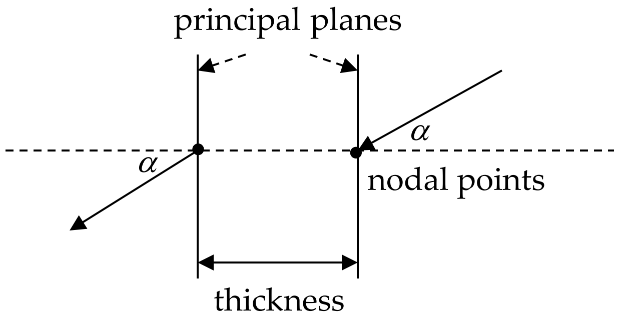
- Rule : Image formed by first lens is the object for the second lens.
- Main Rays : Ray passing through focus emerges parallel to optical axis.
  Ray through optical center passes un-deviated.

- Magnification: $m = \frac{i_2}{o_2}\frac{i_1}{o_1}$

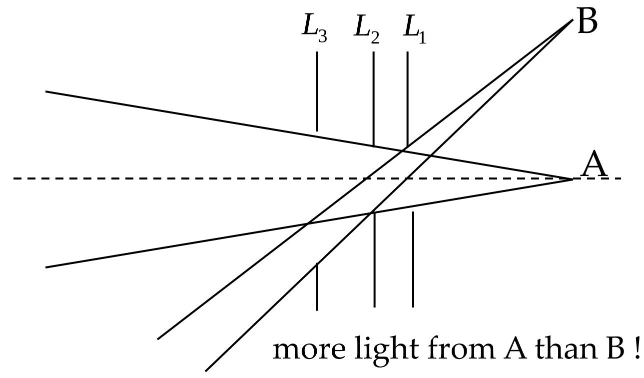
Exercises: What is the combined focal length of the system?
What is the combined focal length if d = 0?

Lens related issues

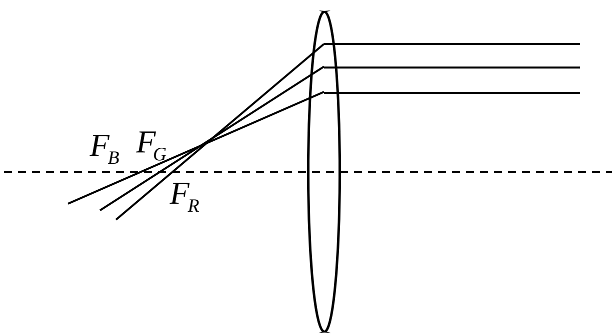
Compound (Thick) Lens



Vignetting

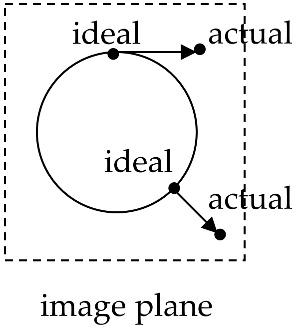


Chromatic Abberation



Lens has different refractive indices
for different wavelengths.

Radial and Tangential Distortion



Radiometry and Image Formation

- To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties.

- TOPICS TO BE COVERED:

  1) Image Intensities: Overview

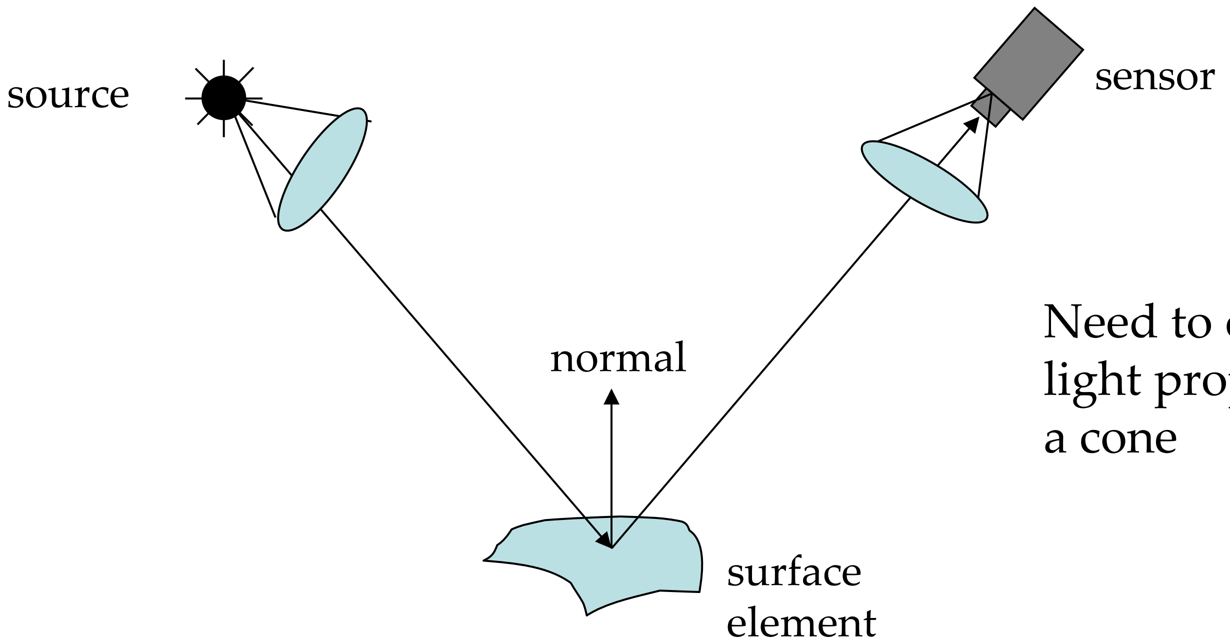
  2) Radiometric Concepts:

     Radiant Intensity
     Irradiance
     Radiance
     BRDF

  3) Image Formation using a Lens

  4) Radiometric Camera Calibration

Image Intensities

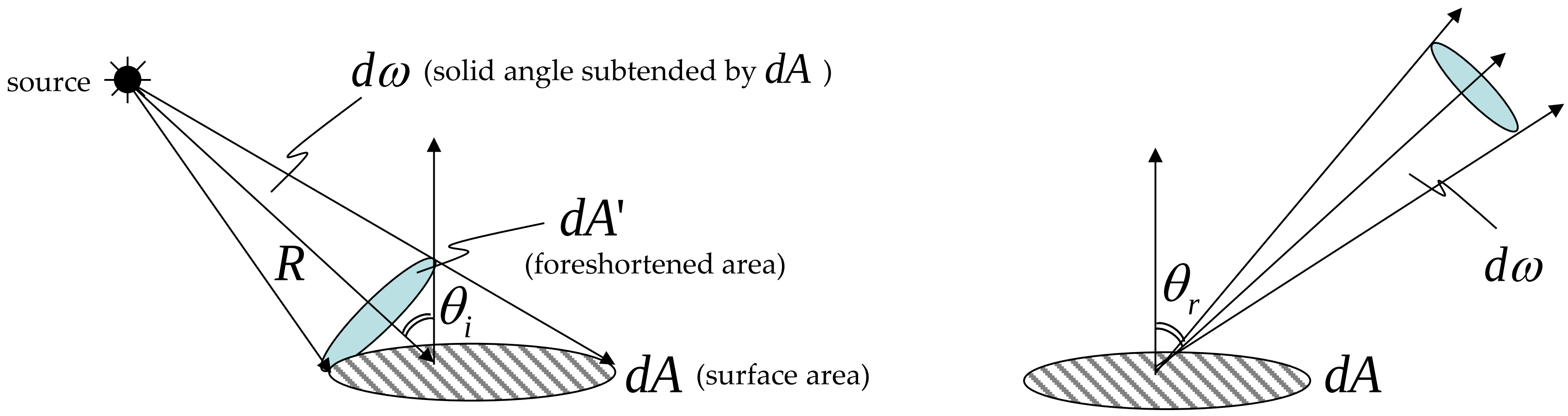


Need to consider light propagation in a cone

Image intensities = f (normal, surface reflectance, illumination)

Note: Image intensity understanding is an under-constrained problem!

Radiometric concepts – boring…but, important!



(1) Solid Angle : $d\omega = \frac{dA'}{R^2} = \frac{dA \cos\theta_i}{R^2}$ (steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source : $J = \frac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \frac{d\Phi}{dA}$ (watts / m)

Light Flux (power) incident per unit surface area.

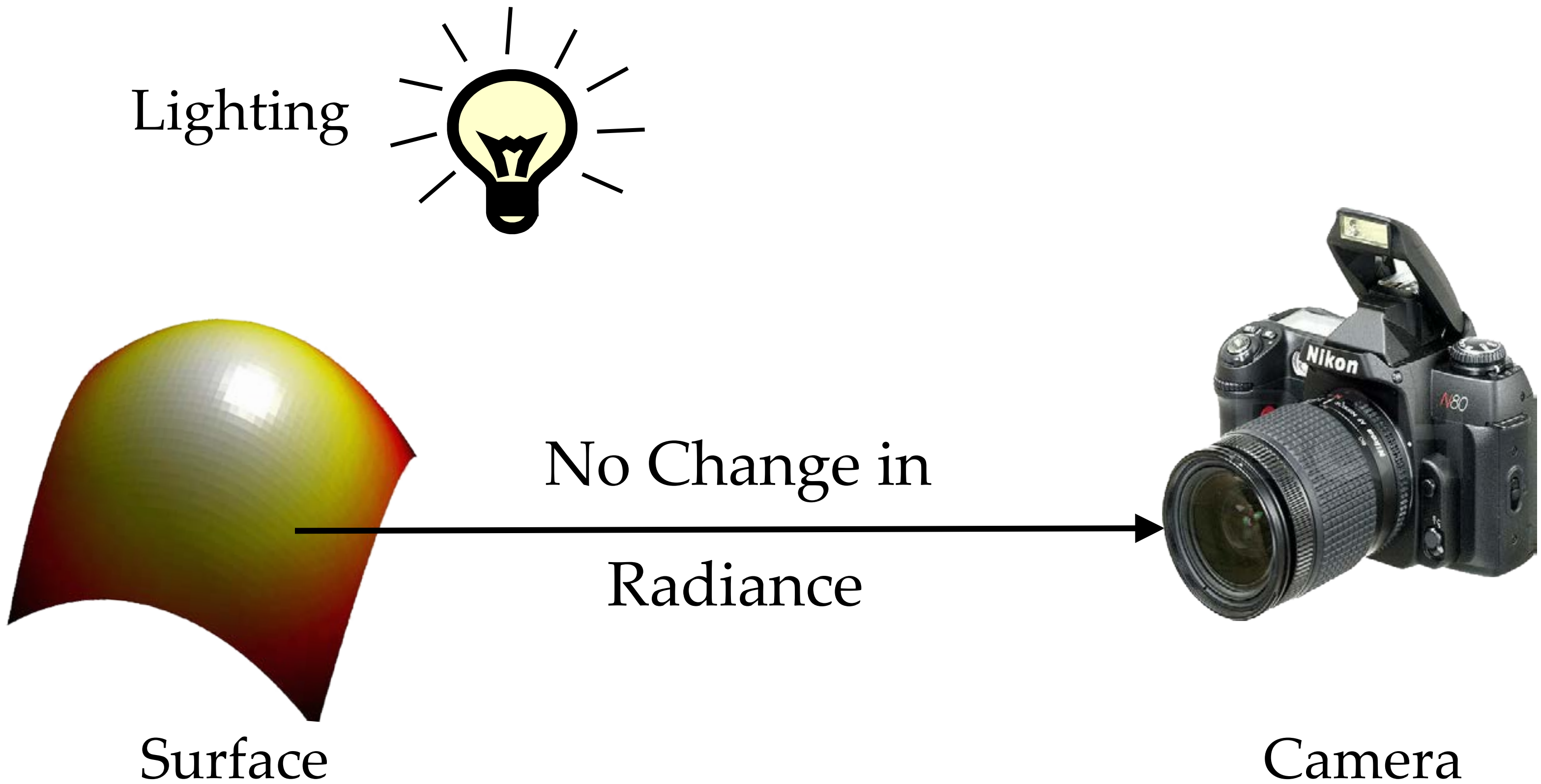
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

$$L = \frac{d^2\Phi}{(dA \cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

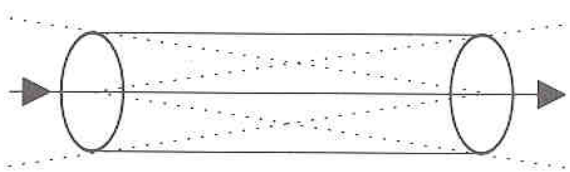
- Flux emitted per unit foreshortened area per unit solid angle.
- L depends on direction θ_r
- Surface can radiate into whole hemisphere.
- L depends on reflectance properties of surface.

The Fundamental Assumption in Vision

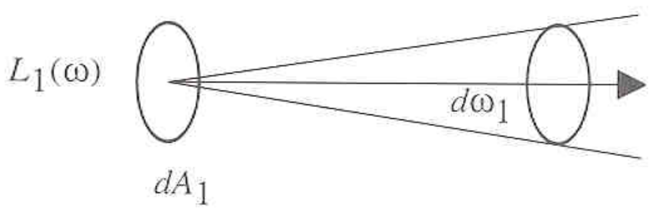


Radiance properties

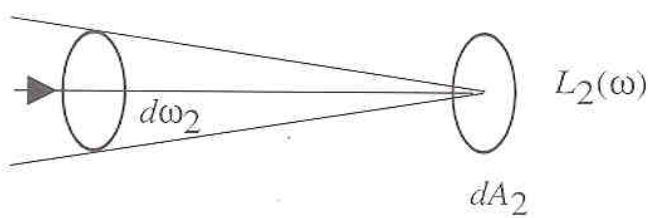
- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$



$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

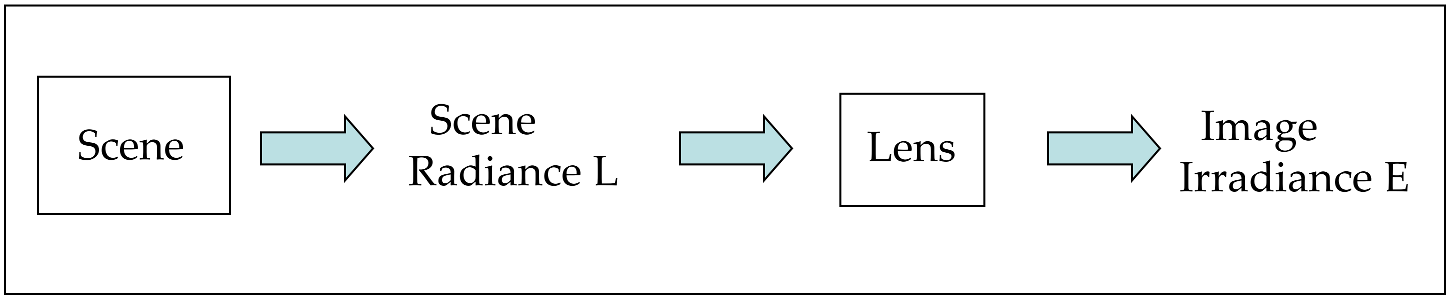


$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

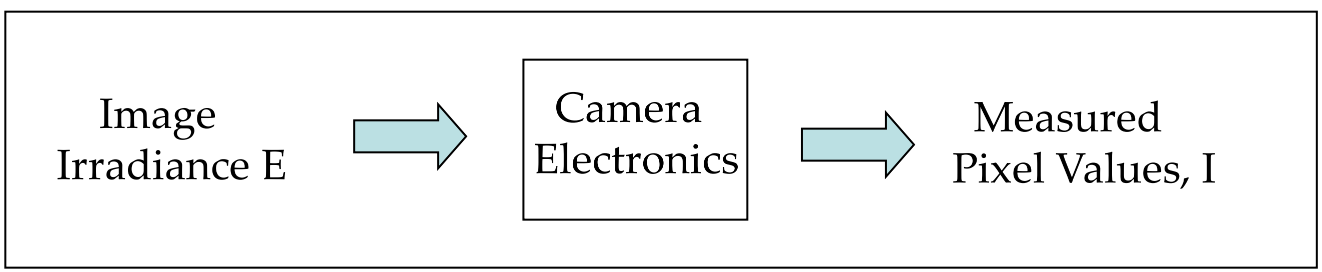
Relationship between Scene and Image Brightness

- Before light hits the image plane:



Linear Mapping!

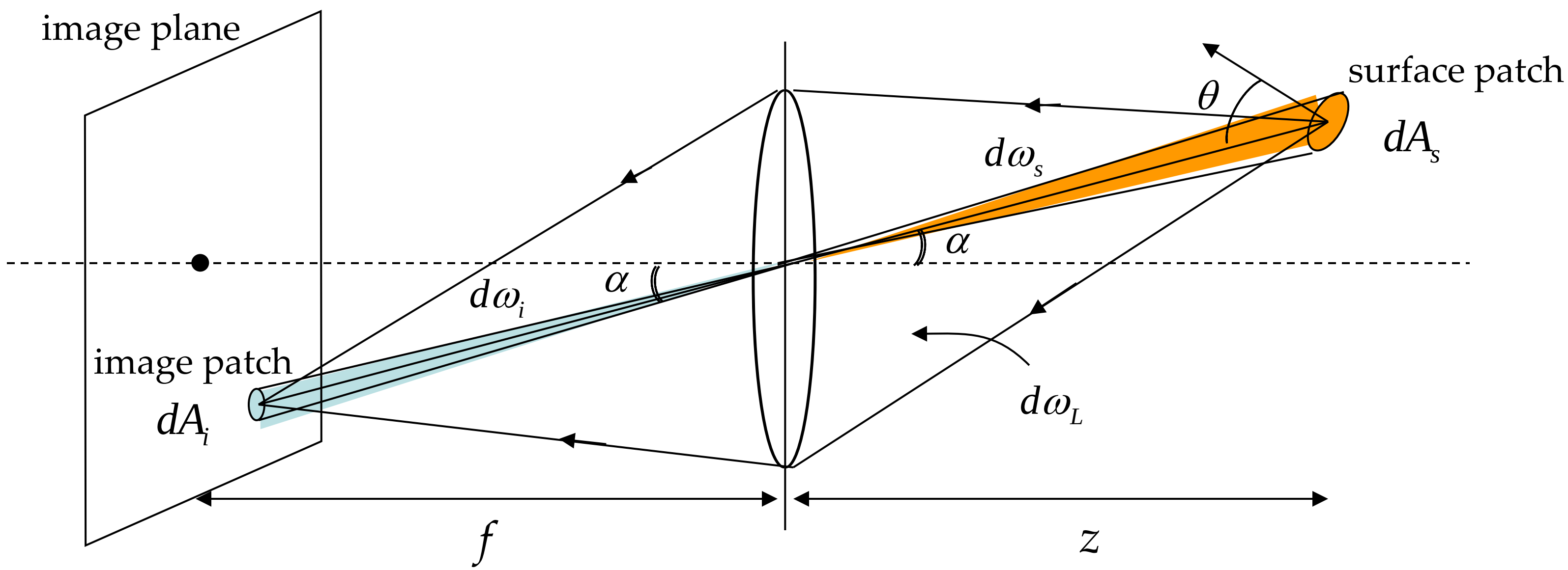
- After light hits the image plane:



Non-linear Mapping!

Can we go from measured pixel value, I, to scene radiance, L?

Relation between Image Irradiance E and Scene Radiance L



- Solid angles of the double cone (orange and green):

$$d\omega_i = d\omega_s \qquad \frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2}$$

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2$$

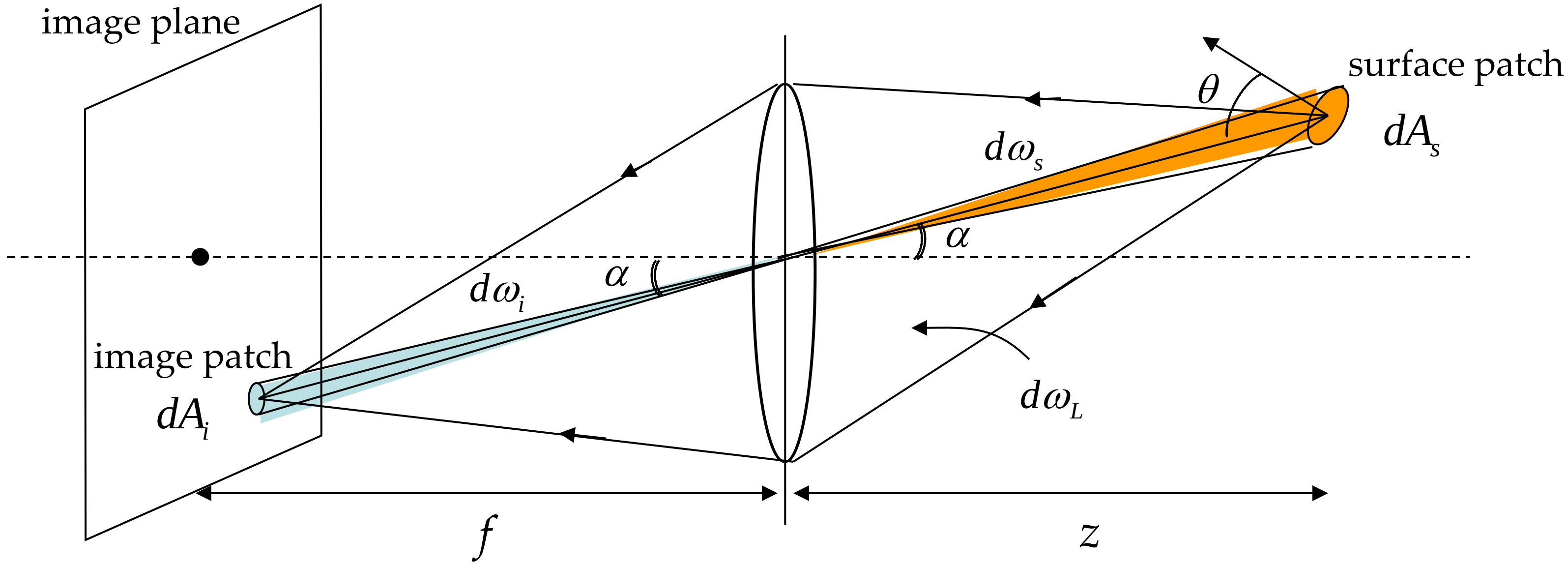
(1)

- Solid angle subtended by lens:

$$d\omega_L = \frac{\pi d^2}{4} \frac{\cos\alpha}{(z/\cos\alpha)^2}$$

(2)

Relation between Image Irradiance E and Scene Radiance L



- Flux received by lens from dA_s = Flux projected onto image dA_i

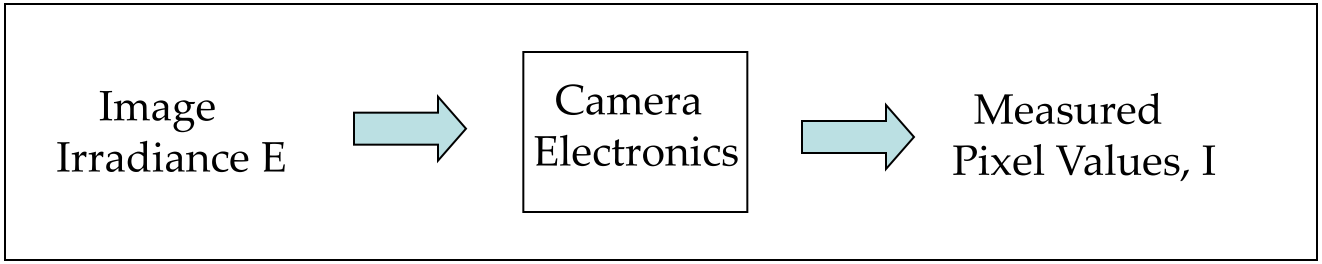
$$L\,(dA_s \cos\theta)\,d\omega_L = E\,dA_i \quad \longrightarrow \quad (3)$$

- From (1), (2), and (3):

$$E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos\alpha^4$$

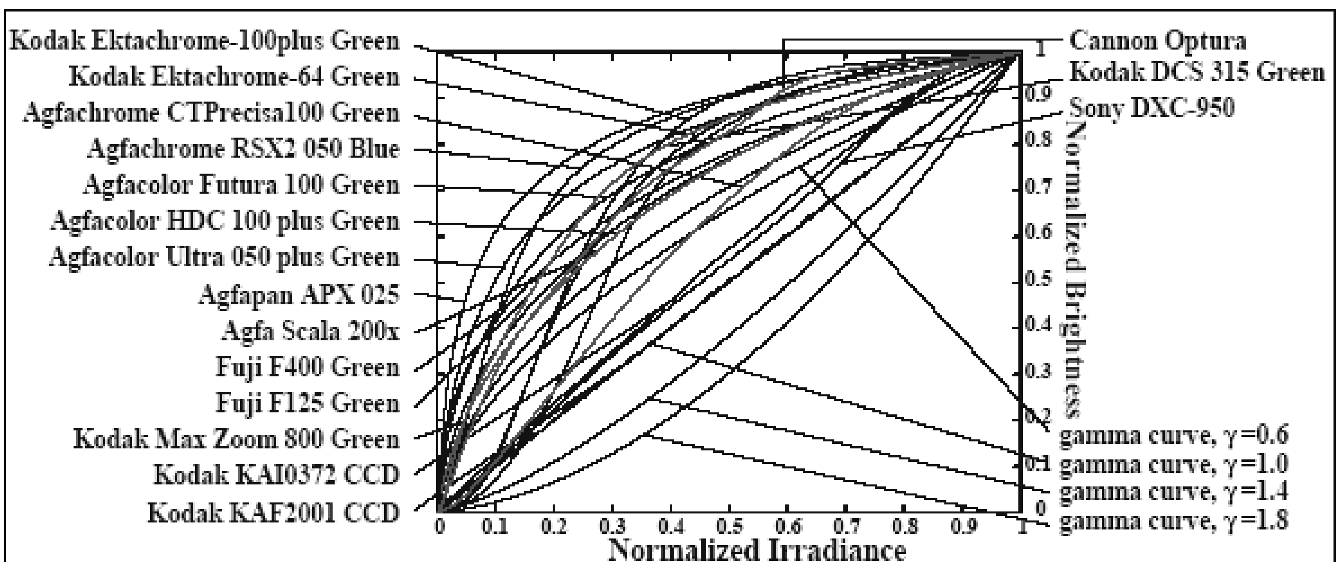
  - Image irradiance is proportional to Scene Radiance!
  - Small field of view → Effects of 4th power of cosine are small.

Relation between Pixel Values I and Image Irradiance E



- The camera response function relates image irradiance at the image plane to the measured pixel intensity values.

$$g : E \rightarrow I$$



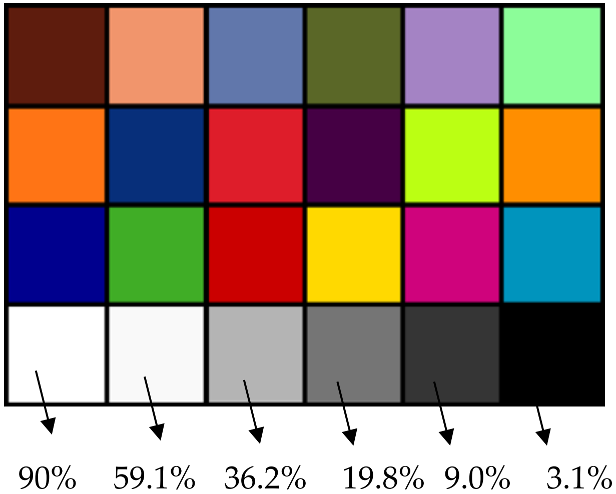
(Grossberg and Nayar)

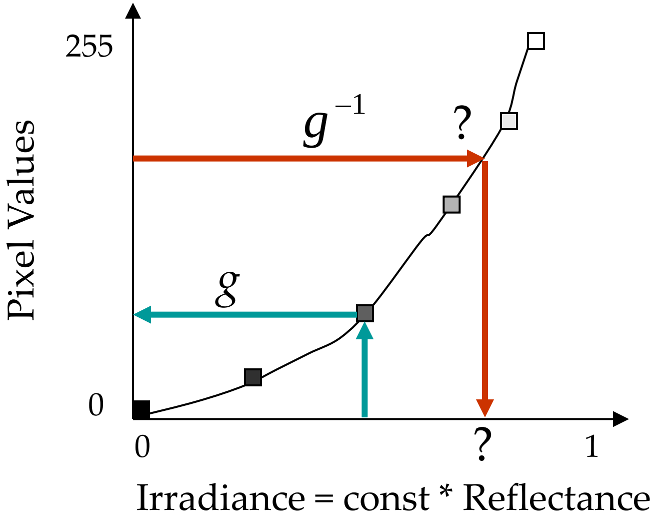
Radiometric Calibration

- Important preprocessing step for many vision and graphics algorithms such as photometric stereo, invariants, de-weathering, inverse rendering, image based rendering, etc.

$$g^{-1} : I \rightarrow E$$

- Use a color chart with precisely known reflectances.



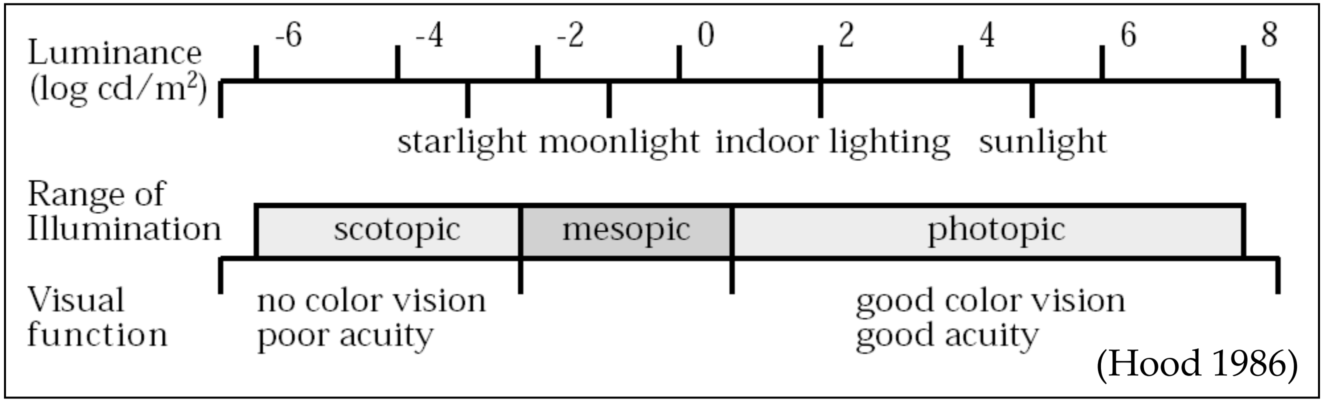


- Use more camera exposures to fill up the curve.
- Method assumes constant lighting on all patches and works best when source is far away (example sunlight).

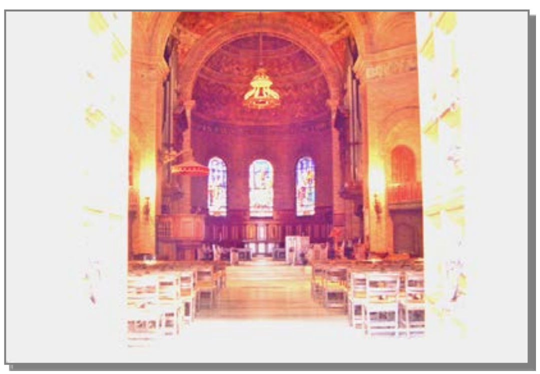
- Unique inverse exists because g is monotonic and smooth for all cameras.

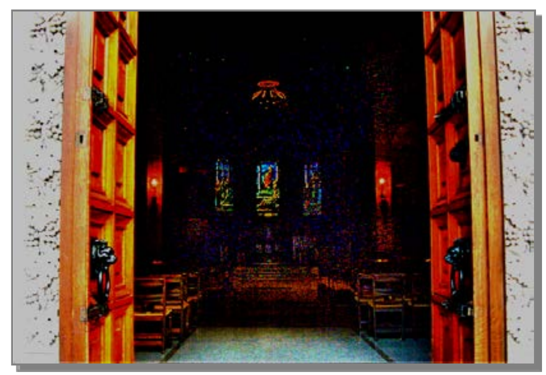
The Problem of Dynamic Range

- Dynamic Range: Range of brightness values measurable with a camera



- Today's Cameras: Limited Dynamic Range


High Exposure Image


Low Exposure Image

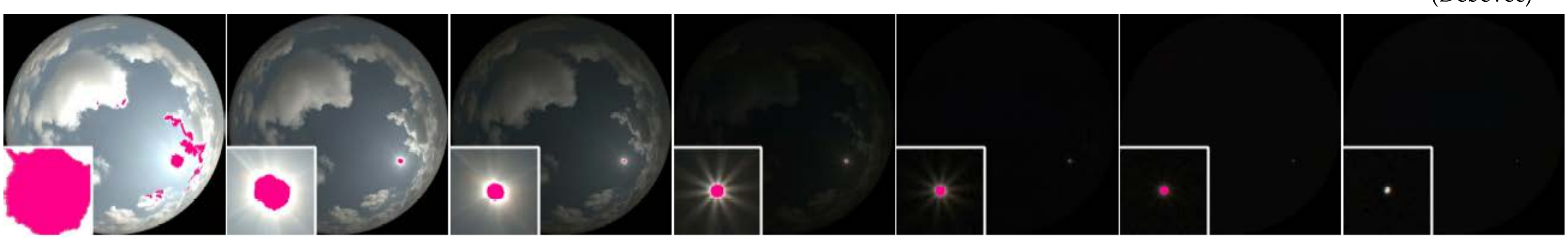
- We need 5-10 million values to store all brightnesses around us.
- But, typical 8-bit cameras provide only 256 values!!

High Dynamic Range Imaging

- Capture a lot of images with different exposure settings.
- Apply radiometric calibration to each camera.
- Combine the calibrated images (for example, using averaging weighted by exposures).

(Mitsunaga)

(Debevec)



f/4, 1s | f/4, $\frac{1}{4}$s | f/4, $\frac{1}{32}$s | f/16, $\frac{1}{16}$s | f/16, $\frac{1}{125}$s | f/16, $\frac{1}{1000}$s | f/16, $\frac{1}{8000}$s

Images taken with a fish-eye lens of the sky show the wide range of brightnesses.