Basic Principles of Imaging and Photometry

Lecture #2
We need to understand the **Geometric** and **Radiometric** relations between the scene and its image.
A Brief History of Images

Camera Obscura, Gemma Frisius, 1558
A Brief History of Images

Lens Based Camera Obscura, 1568
A Brief History of Images

Still Life, Louis Jaques Mande Daguerre, 1837
A Brief History of Images

Silicon Image Detector, 1970
A Brief History of Images

Digital Cameras
Geometric Optics and Image Formation

TOPICS TO BE COVERED:

1) Pinhole and Perspective Projection
2) Image Formation using Lenses
3) Lens related issues
Pinhole and the Perspective Projection

Is an image being formed on the screen?

YES! But, not a “clear” one.

\[ \frac{\mathbf{r}'}{f'} = \frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'} = \frac{x}{z} \quad \frac{y'}{f'} = \frac{y}{z} \]
Magnification

From perspective projection:

\[ \frac{x'}{f'} = \frac{x}{z} \quad \frac{y'}{f'} = \frac{y}{z} \]

\[ \frac{x' + \delta x'}{f'} = \frac{x + \delta x}{z} \quad \frac{y' + \delta y'}{f'} = \frac{y + \delta y}{z} \]

Magnification:

\[ m = \frac{d'}{d} = \frac{\sqrt{(\delta x')^2 + (\delta y')^2}}{\sqrt{\delta x^2 + \delta y^2}} = \frac{f'}{f} \]

\[ \frac{\text{Area}_{\text{image}}}{\text{Area}_{\text{scene}}} = m^2 \]
Magnification: \( x' = m \ x \quad y' = m \ y \)

When \( m = 1 \), we have orthographic projection

This is possible only when \( z >> \Delta z \)

In other words, the range of scene depths is assumed to be much smaller than the average scene depth.

But, how do we produce non-inverted images?
Better Approximations to Perspective Projection

- Weak-Perspective

![Diagram of Weak-Perspective Projection]

**Figure 2.8** Approximation of perspective projection by orthographic projection. Perspective projection onto a plane can be approximated by orthographic projection, followed by scaling, when (1) the object dimensions are small compared to the distance of the object from the center of projection, and (2) compared to this distance, the object is close to the straight line that passes through the center of projection and is orthogonal to the image plane (this line is the z-axis here).

- Para-Perspective

![Diagram of Para-Perspective Projection]

**Figure 2.9** Approximation of perspective projection by parallel projection. Parallel projection onto a plane is a generalization of orthographic projection in which all the object points are projected along a set of parallel straight lines that may or may not be orthogonal to the projection plane. Perspective projection onto a plane can be approximated by parallel projection, followed by scaling, whenever the object dimensions are small compared to the distance of the object from the center of projection. The direction of parallel projection in such an approximation is along the "average direction" of perspective projection.
Problems with Pinholes

- Pinhole size (aperture) must be “very small” to obtain a clear image.

- However, as pinhole size is made smaller, less light is received by image plane.

- If pinhole is comparable to wavelength of incoming light, DIFFRACTION effects blur the image!

- Sharpest image is obtained when:
  
  \[
  \text{pinhole diameter } d = 2 \sqrt{f' \lambda}
  \]

Example: If \( f' = 50\text{mm}, \)

\[ \lambda = 600\text{nm (red)}, \]

\[ d = 0.36\text{mm} \]
Image Formation using Lenses

- Lenses are used to avoid problems with pinholes.
- Ideal Lens: Same projection as pinhole but gathers more light!

Gaussian Lens Formula: \( \frac{1}{i} + \frac{1}{o} = \frac{1}{f} \)

- \( f \) is the focal length of the lens – determines the lens’s ability to bend (refract) light
- \( f \) different from the effective focal length \( f' \) discussed before!
Focus and Defocus

Depth of Field: Range of object distances over which image is sufficiently well focused. i.e. Range for which blur circle is less than the resolution of the imaging sensor.

Gaussian Law:

\[ \frac{1}{i} + \frac{1}{o} = \frac{1}{f} \quad \frac{1}{i'} + \frac{1}{o'} = \frac{1}{f} \quad \implies \quad (i' - i) = \frac{f}{(o' - f)} \frac{f}{(o - f)} (o - o') \]

Blur Circle Diameter:

\[ b = \frac{d}{i'} (i' - i) \]
Two Lens System

- **Rule**: Image formed by first lens is the object for the second lens.
- **Main Rays**: Ray passing through focus emerges parallel to optical axis. Ray through optical center passes un-deviated.

- **Magnification**: \( m = \frac{i_2 \cdot i_1}{o_2 \cdot o_1} \)

**Exercises**: What is the combined focal length of the system? What is the combined focal length if \( d = 0 \)?
Lens related issues

Compound (Thick) Lens

Vignetting

Chromatic Abberation

Radial and Tangential Distortion

Lens has different refractive indices for different wavelengths.

more light from A than B!
Radiometry and Image Formation

• To interpret image intensities, we need to understand **Radiometric Concepts** and **Reflectance Properties**.

• **TOPICS TO BE COVERED:**

  1) Image Intensities: Overview
  
  2) Radiometric Concepts:
      Radiant Intensity
      Irradiance
      Radiance
      BRDF
  
  3) Image Formation using a Lens
  
  4) Radiometric Camera Calibration
Image intensities = $f(\text{normal, surface reflectance, illumination})$

Note: Image intensity understanding is an **under-constrained** problem!
Radiometric concepts – boring… but, important!

(1) Solid Angle: 
\[ d\omega = \frac{dA'}{R^2} = \frac{dA \cos \theta_i}{R^2} \]  
(steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source: 
\[ J = \frac{d\Phi}{d\omega} \]  
(watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance: 
\[ E = \frac{d\Phi}{dA} \]  
(watts / m)

Light Flux (power) incident per unit surface area.

Does not depend on where the light is coming from!

(4) Surface Radiance (tricky): 
\[ L = \frac{d^2\Phi}{(dA \cos \theta_r) \ d\omega} \]  
(watts / m² steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- \( L \) depends on direction \( \theta_r \).
- Surface can radiate into whole hemisphere.
- \( L \) depends on reflectance properties of surface.
The Fundamental Assumption in Vision

Surface Camera

No Change in Radiance

Lighting
Radiance properties

- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[ d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2 \]

\[ d\omega_1 = dA_2 / r^2 \quad d\omega_2 = dA_1 / r^2 \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]
Relationship between Scene and Image Brightness

• Before light hits the image plane:

- Scene → Scene Radiance L → Lens → Image Irradiance E

  Linear Mapping!

• After light hits the image plane:

- Image Irradiance E → Camera Electronics → Measured Pixel Values, I

  Non-linear Mapping!

Can we go from measured pixel value, I, to scene radiance, L?
Relation between Image Irradiance $E$ and Scene Radiance $L$

- Solid angles of the double cone (orange and green):

$$d\omega_i = d\omega_s = \frac{dA_i \cos \alpha}{(f / \cos \alpha)^2} = \frac{dA_s \cos \theta}{(z / \cos \alpha)^2}$$

- Solid angle subtended by lens:

$$d\omega_L = \frac{\pi d^2}{4} \frac{\cos \alpha}{(z / \cos \alpha)^2}$$
Relation between Image Irradiance $E$ and Scene Radiance $L$

- Flux received by lens from $dA_s$ = Flux projected onto image $dA_i$

$$L \ (dA_s \ \cos \ \theta) \ d\omega_L = E \ dA_i \quad (3)$$

- From (1), (2), and (3):

$$E = L \ \frac{\pi}{4} \ \left( \frac{d}{f} \right)^2 \ \cos \ \alpha^4$$

- Image irradiance is proportional to Scene Radiance!
- Small field of view $\Rightarrow$ Effects of 4th power of cosine are small.
The camera response function relates image irradiance at the image plane to the measured pixel intensity values.

\[ g : E \rightarrow I \]
Radiometric Calibration

• Important preprocessing step for many vision and graphics algorithms such as photometric stereo, invariants, de-weathering, inverse rendering, image based rendering, etc.

\[ g^{-1} : I \rightarrow E \]

• Use a color chart with precisely known reflectances.

- Use more camera exposures to fill up the curve.
- Method assumes constant lighting on all patches and works best when source is far away (example sunlight).
- Unique inverse exists because \( g \) is monotonic and smooth for all cameras.
The Problem of Dynamic Range

- Dynamic Range: Range of brightness values measurable with a camera

Today’s Cameras: Limited Dynamic Range

- We need 5-10 million values to store all brightnesses around us.
- But, typical 8-bit cameras provide only 256 values!!
High Dynamic Range Imaging

- Capture a lot of images with different exposure settings.
- Apply radiometric calibration to each camera.
- Combine the calibrated images (for example, using averaging weighted by exposures).

Images taken with a fish-eye lens of the sky show the wide range of brightnesses.