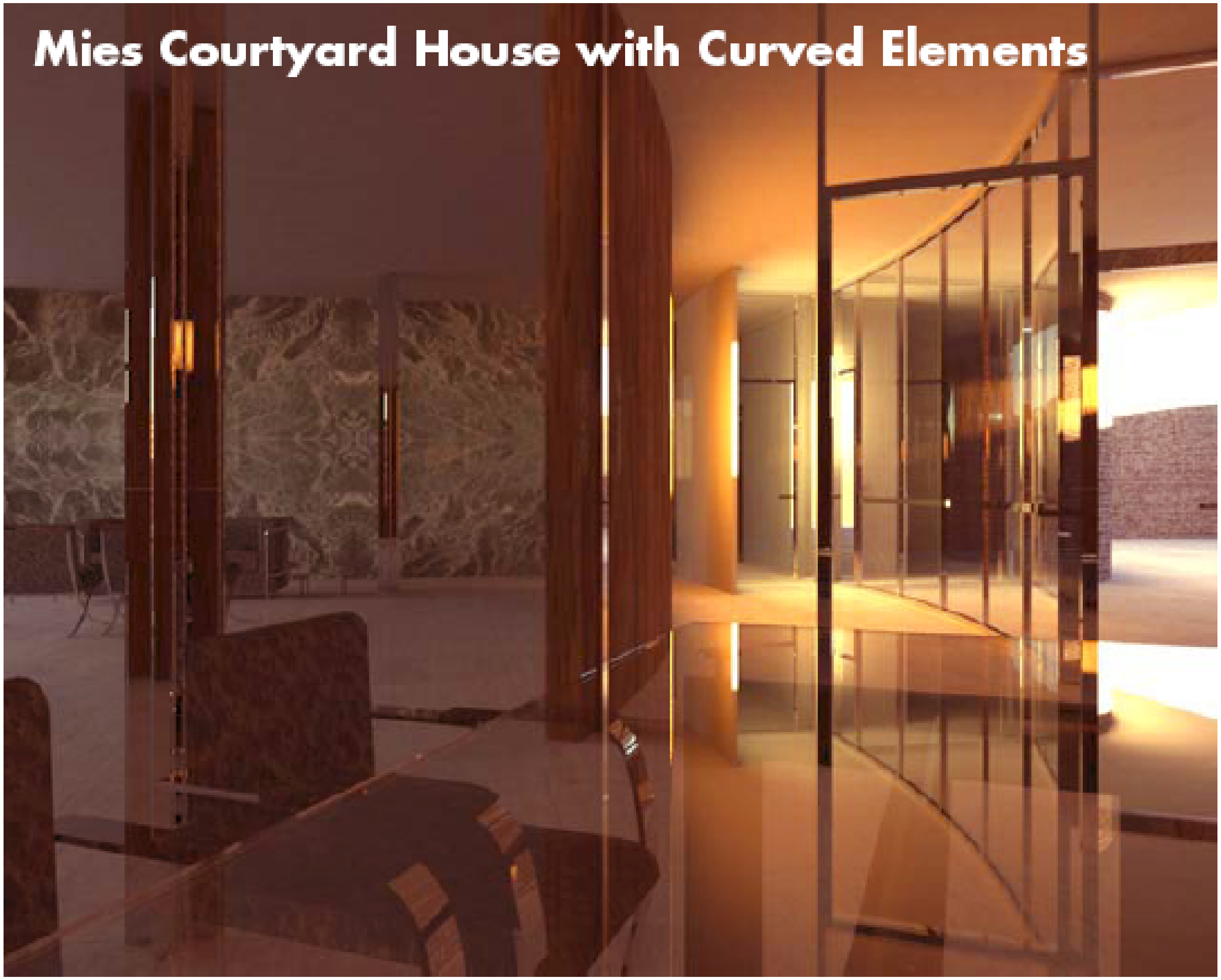
Interreflections :

The Inverse Problem

Lecture #12

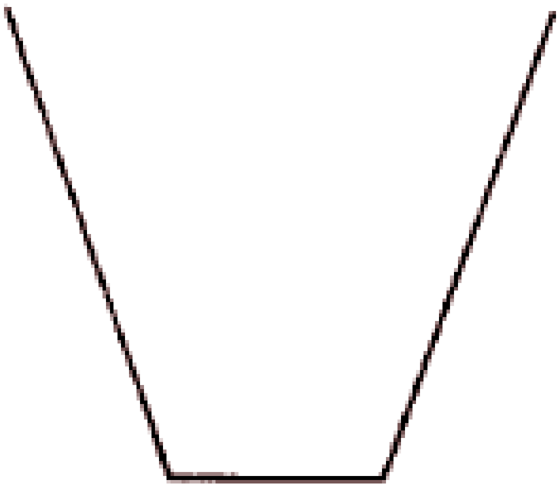
Thanks to Shree Nayar, Seitz et al, Levoy et al, David Kriegman

Graphics



Modeling: Stephen Duck; Rendering: Henrik Wann Jensen

Vision: Estimating Shape of Concave Surfaces



Actual Shape

Shape from
Photometric Stereo

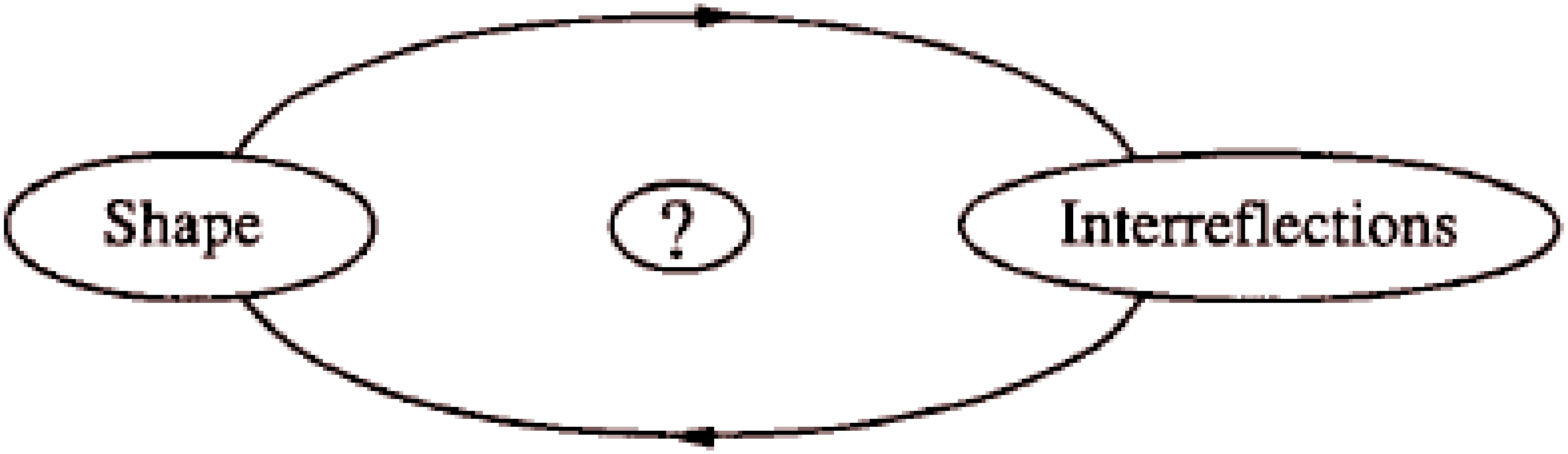
Need to account for Interreflections!!

Shape and Interreflections: Chicken and Egg

- If we remove the effects of interreflections, we know how to compute shape.

- But, interreflections depend on the shape!!

So, which comes first?



Linear System of Radiosity Equations - RECAP

$\forall$patches i: $$B_i = B_{ei} + \rho_i \sum_j F_{i \to j} B_j$$

$$\underbrace{\begin{bmatrix} 1-\rho_1 F_{1\to 1} & -\rho_1 F_{1\to 2} & \cdots & -\rho_1 F_{1\to n} \\ -\rho_2 F_{2\to 1} & 1-\rho_2 F_{2\to 2} & \cdots & -\rho_2 F_{2\to n} \\ \cdots & \cdots & \cdots & \cdots \\ -\rho_n F_{n\to 1} & -\rho_n F_{n\to 2} & \cdots & 1-\rho_n F_{n\to n} \end{bmatrix}}_{\text{Known}} \underbrace{\begin{bmatrix} B_1 \\ B_2 \\ \cdots \\ B_n \end{bmatrix}}_{\text{Unknown}} = \underbrace{\begin{bmatrix} B_{e1} \\ B_{e2} \\ \cdots \\ B_{en} \end{bmatrix}}_{\text{Known}}$$

- Matrix Inversion to Solve for Radiosities.

Use Radiance instead of Radiosity

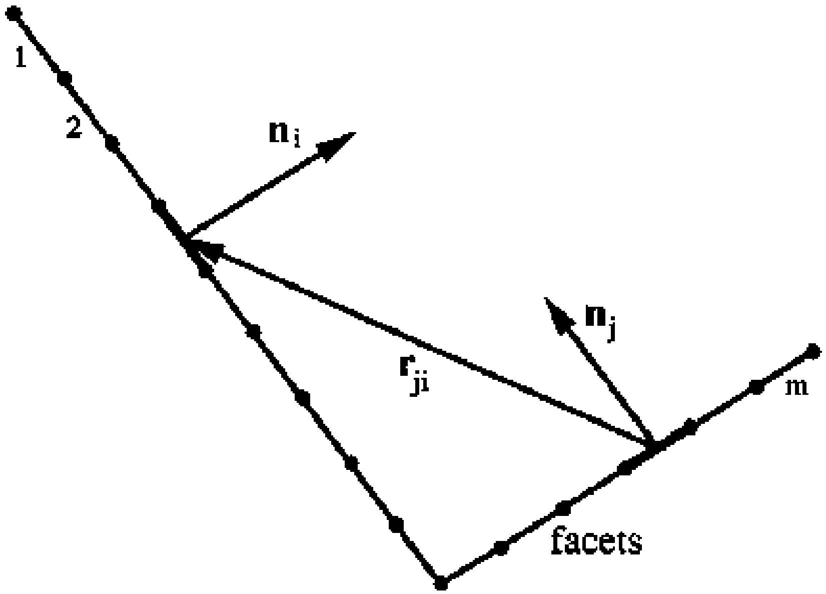
- Vision shape-from-intensity algorithms work on Radiance.
- Assume image pixel covers infinitesimal scene patch area .
- Form factor is called “Interreflection Kernel”, K (better name).

$$L_i = L_s + \rho_i \sum_j L_j K_{ij}$$

Radiance of a facet is given by the linear combination of radiances from other facets.

Loosely, we can say, this weighted averaging in the direction of concave curvature.

Why do concavities appear shallow?



$$L_i = L_s + \rho_i \sum_j L_j K_{ij}$$

Radiance of a facet is given by the linear combination of radiances from other facets.

Loosely, we can say this weighted averaging in the direction of concave curvature.

Matrix Form of Interreflection Equation

$$\mathbf{L} = \mathbf{L}_s + \mathbf{P}\,\mathbf{K}\,\mathbf{L}$$

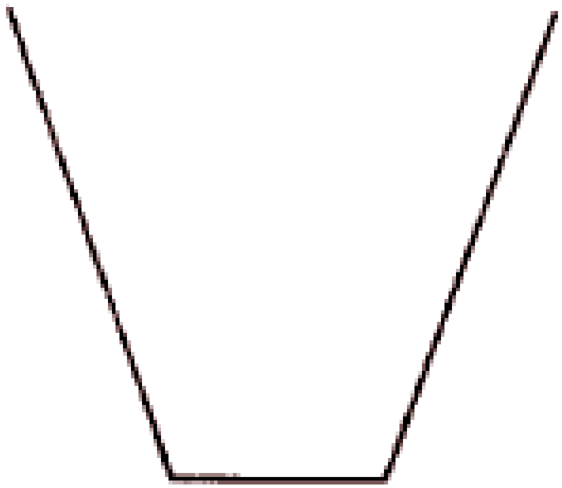
$$\mathbf{P} = \frac{1}{\pi}\begin{bmatrix} \rho_1 & 0 & . & . & . & 0 \\ 0 & \rho_2 & 0 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & \rho_m \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} 0 & K_{12} & . & . & . & . \\ K_{21} & 0 & . & . & . & . \\ . & . & 0 & . & . & . \\ . & . & . & 0 & . & . \\ . & . & . & . & . & 0 \end{bmatrix}$$

$$\downarrow$$

$$(\mathbf{I} - \mathbf{P}\,\mathbf{K})\,\mathbf{L} = \mathbf{L}_s$$

Pseudo Shape and Reflectance

- Apply any shape from intensity algorithm ignoring interreflections!

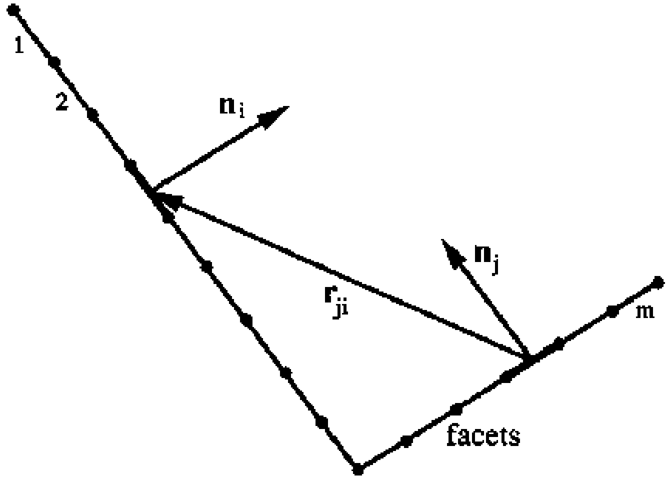


Actual Shape

Pseudo-shape from Photometric Stereo

- KEY IDEA: The pseudo shape and reflectance (albedo) is related to the actual shape and reflectance.

Pseudo Shape/Reflectance from Photometric Stereo



$$N_i = \rho_i \; n_i$$

Facet Matrix :

$$\mathbf{F} = [N_1, N_2, N_3, N_4 \ldots]$$

$$\mathbf{L} = (\mathbf{I} - \mathbf{P}\,\mathbf{K})^{-1}\,\mathbf{L_s}$$

$$\downarrow$$

$$\mathbf{L} = (\mathbf{I} - \mathbf{P}\,\mathbf{K})^{-1}\,\mathbf{F}\,.\,s$$

Source direction

Three Source Directions :

$$[\mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3] = (\mathbf{I} - \mathbf{P}\,\mathbf{K})^{-1}\,\mathbf{F}\,.[s_1, s_2, s_3]$$

Pseudo Shape/Reflectance from Photometric Stereo

Three Source Directions :

$$[\mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3] = (\mathbf{I} - \mathbf{P}\,\mathbf{K})^{-1}\,\mathbf{F}\,.[s_1, s_2, s_3]$$

Pseudo Shape :

$$\mathbf{F_p} = [\mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3]\ .[s_1, s_2, s_3]^{-1}$$

$$\mathbf{F_p} = (\mathbf{I} - \mathbf{P}\,\mathbf{K})^{-1}\,\mathbf{F}$$

Key Observations

$$\mathbf{F_p} = (\mathbf{I} - \mathbf{P}\,\mathbf{K})^{-1}\,\mathbf{F}$$

- Pseudo Shape and Albedos are independent of source direction! This allows us to reconstruct actual shape.

- Pseudo Facets: Lambertian!
  "Smoothed" versions of actual facets (shallow)
  Pseudo albedos may be greater than 1.

Iterative Refinement of Shape and Albedos

- Start with pseudo shape and albedos as initial guesses.
- Compute Interreflection Kernel K, and Albedo matrix P.
- Iterate until convergence.

$$\mathbf{F}^{i+1} = (\mathbf{I} - \mathbf{P}^{i}\ \mathbf{K}^{i})^{-1}\ \mathbf{F}_{p}$$

$$\mathbf{F}^{0} = \mathbf{F}_{p}$$

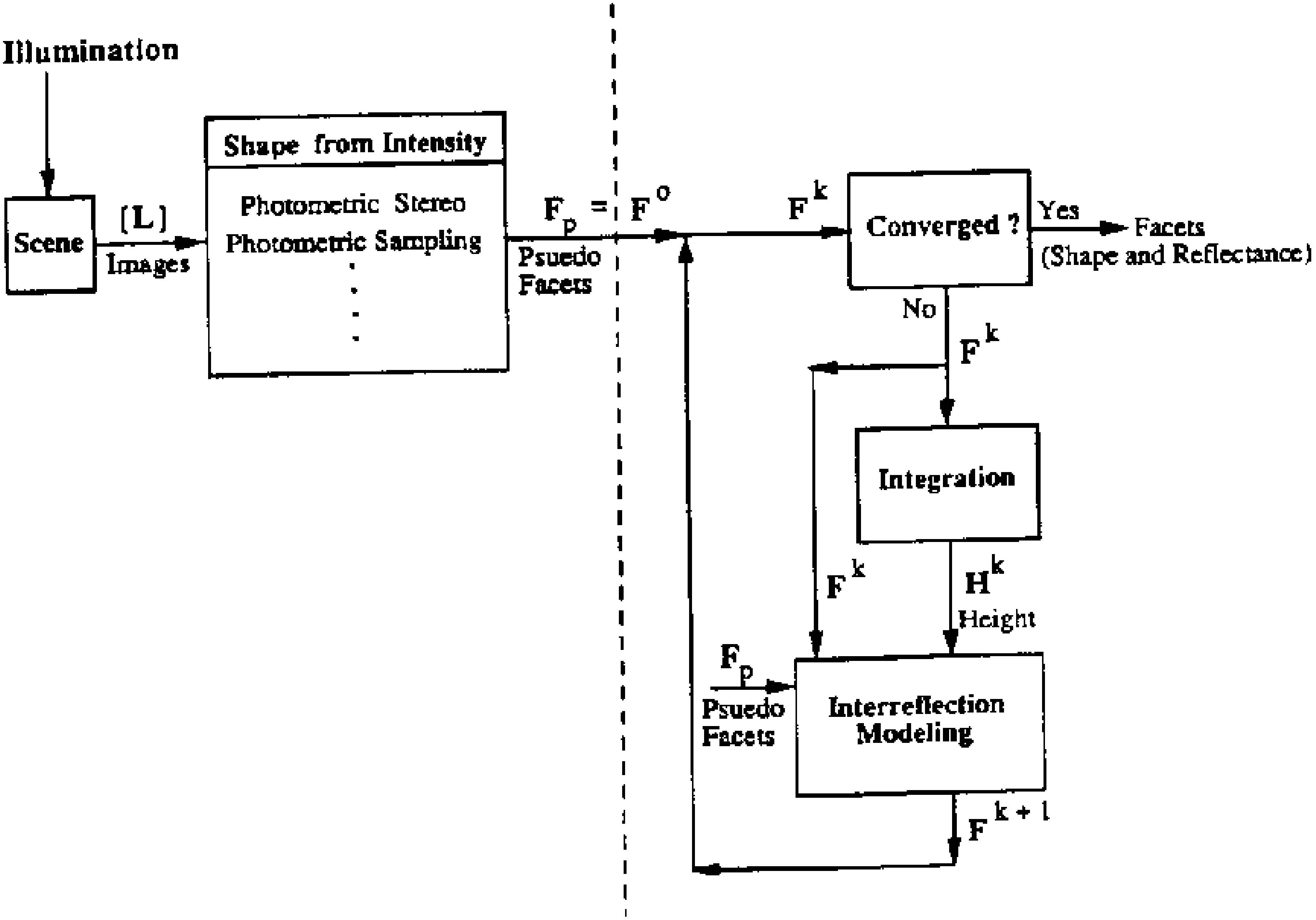
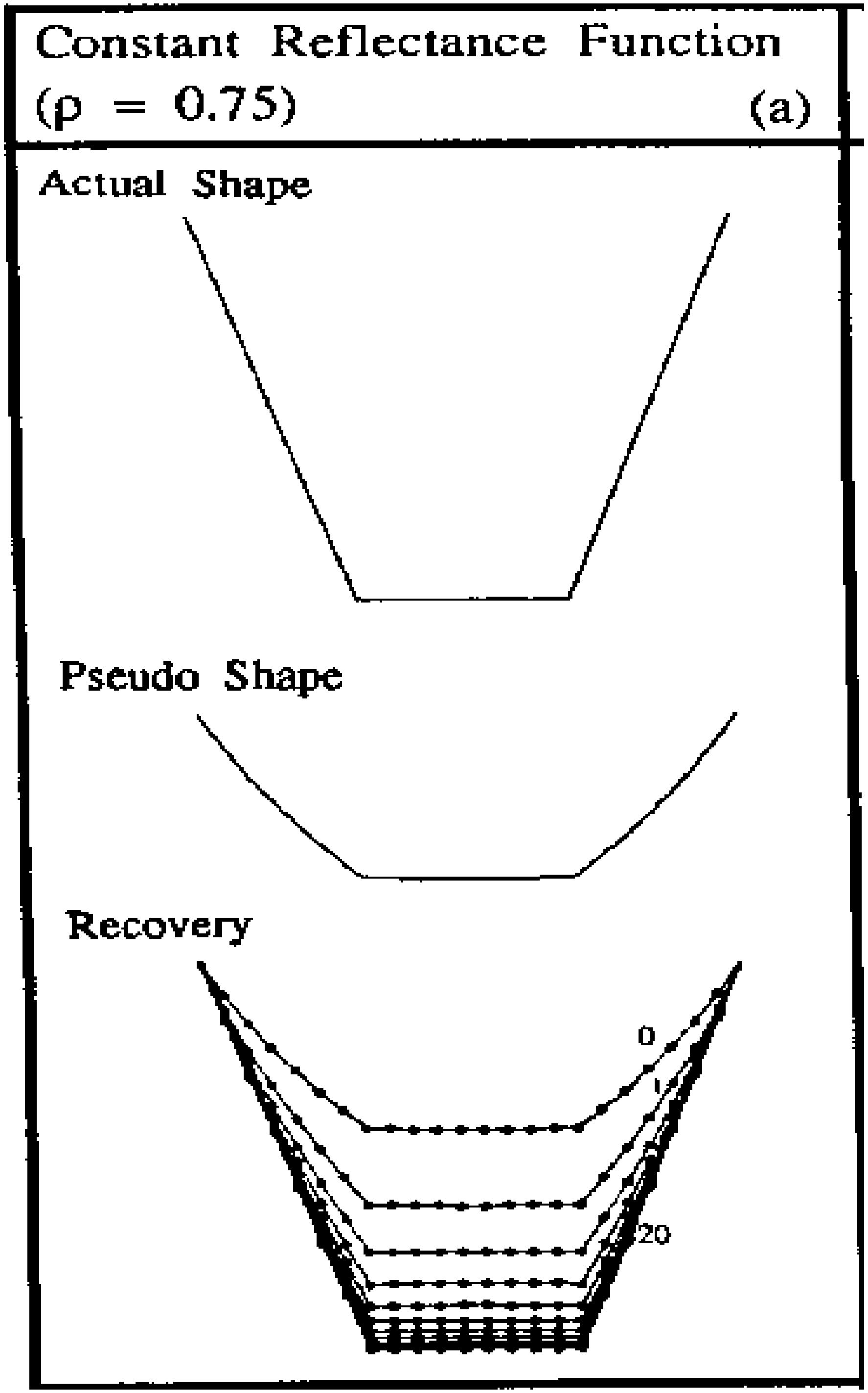
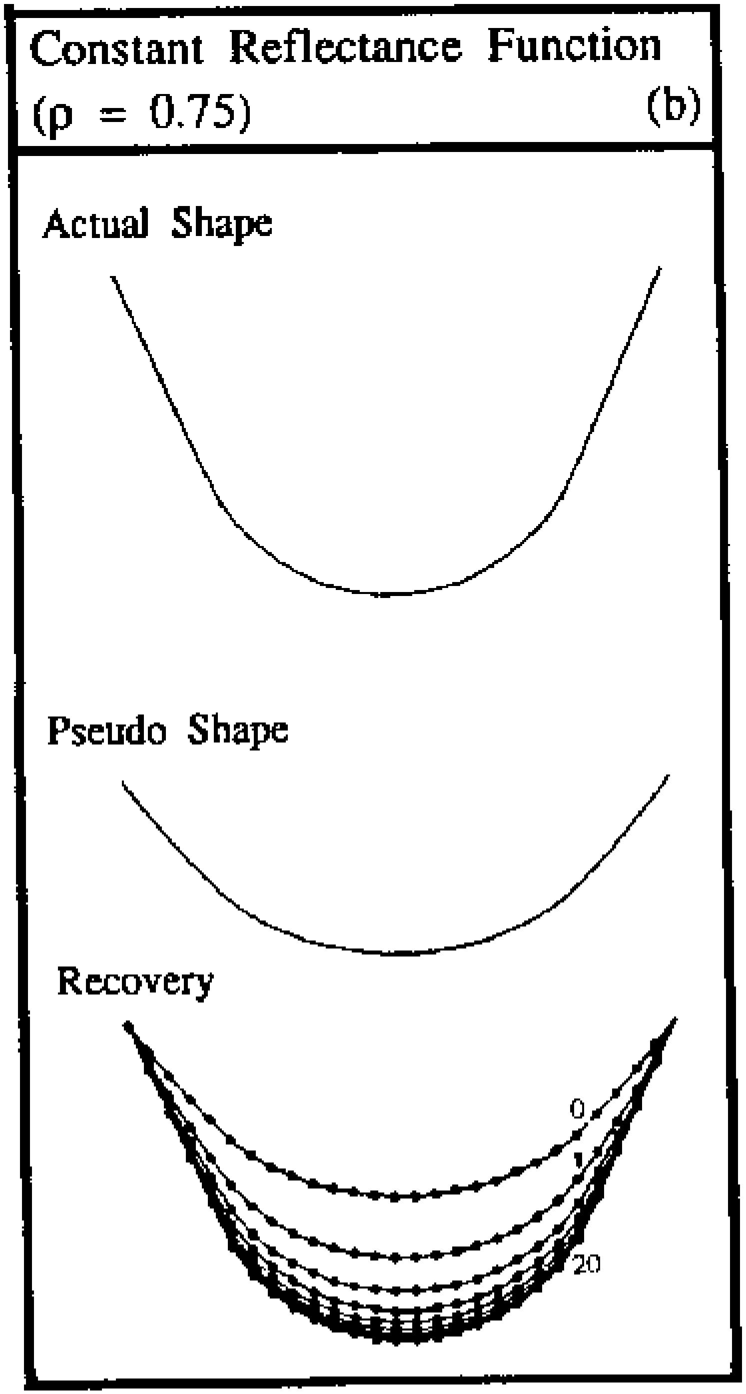


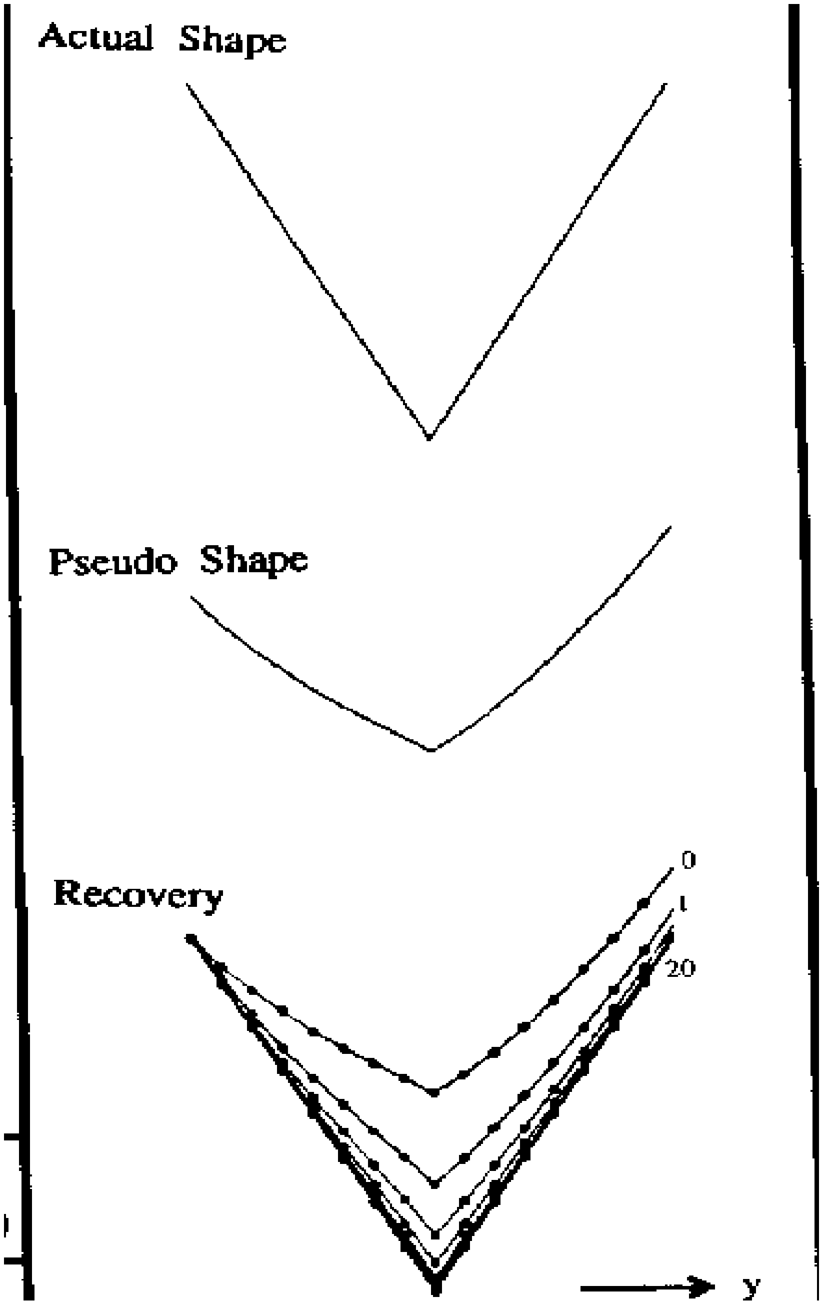
Figure 5: The shape and reflectance recovery algorithm.

Important Assumptions and Observations

- Any shape-from-intensity method can be used.
- Assumes shape is continuous (for integrability).
- All facets contributing to interreflections must be visible to sensor.
- Facets are infinitesimal lambertian patches.
- Complexity: $O(Mn^2)$ (M Iterations, n facets)
- Convergence shown for 2 facets.
- Does not always converge to the right facet for large tilt angles (> 70).


Constant Reflectance Function
(ρ = 0.75)
(a)
Actual Shape
Pseudo Shape
Recovery
0
1
20

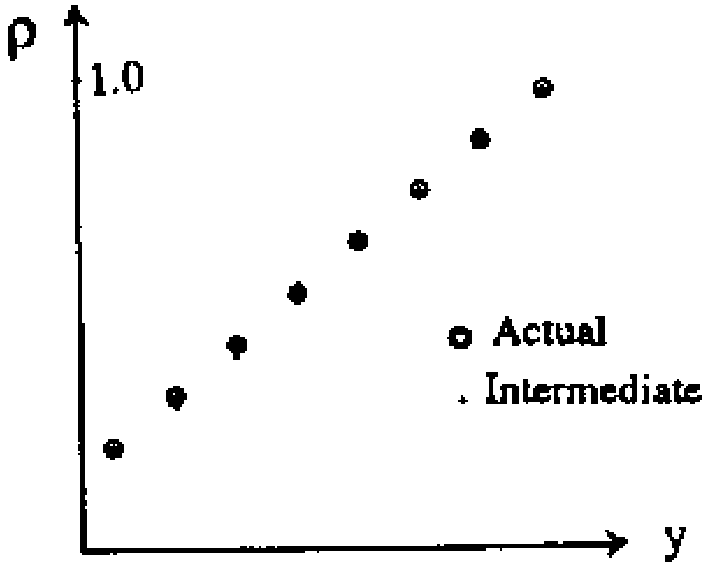

Constant Reflectance Function
(ρ = 0.75)
(b)
Actual Shape
Pseudo Shape
Recovery
0
1
20



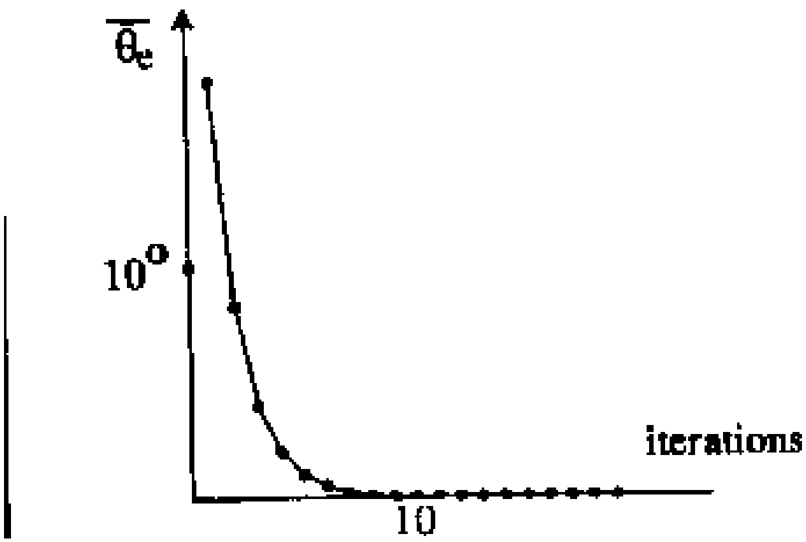
Actual Shape

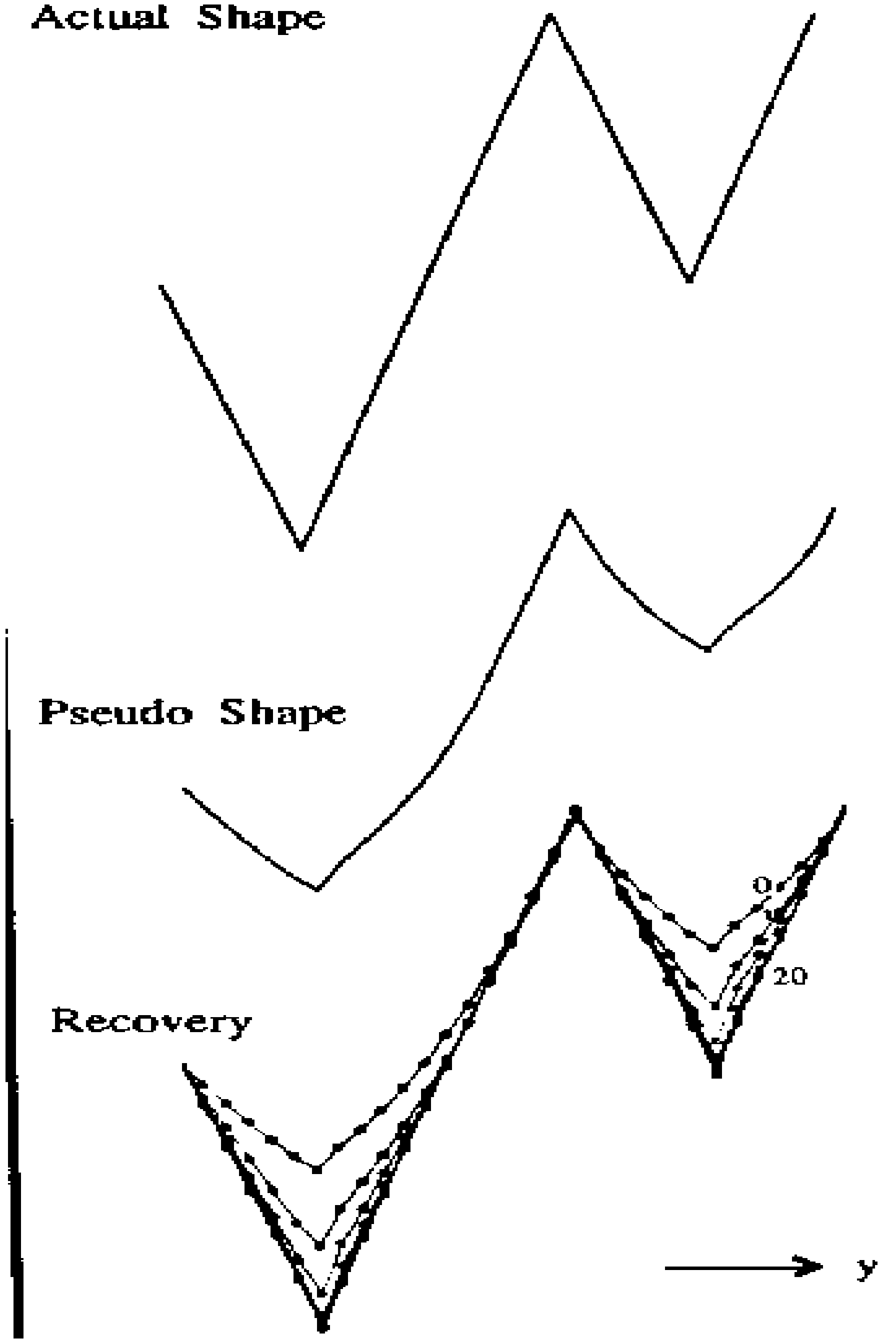
Pseudo Shape

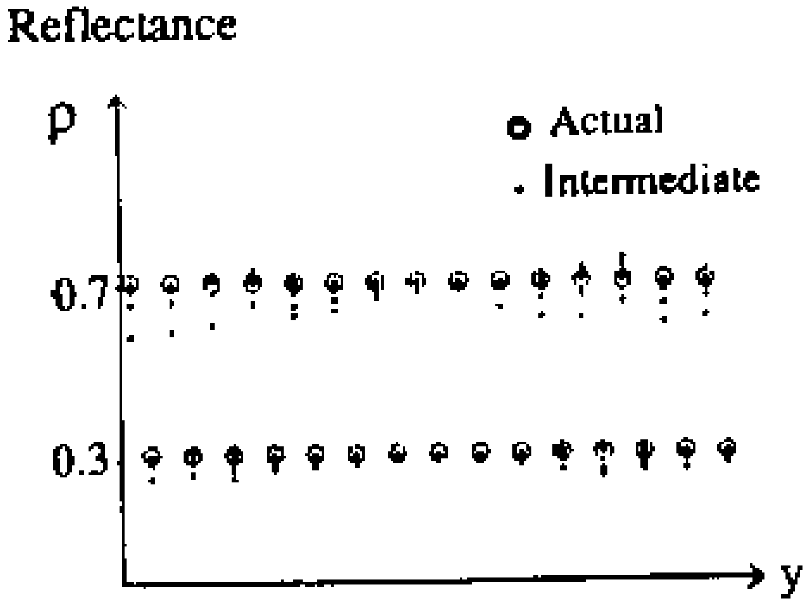
Recovery

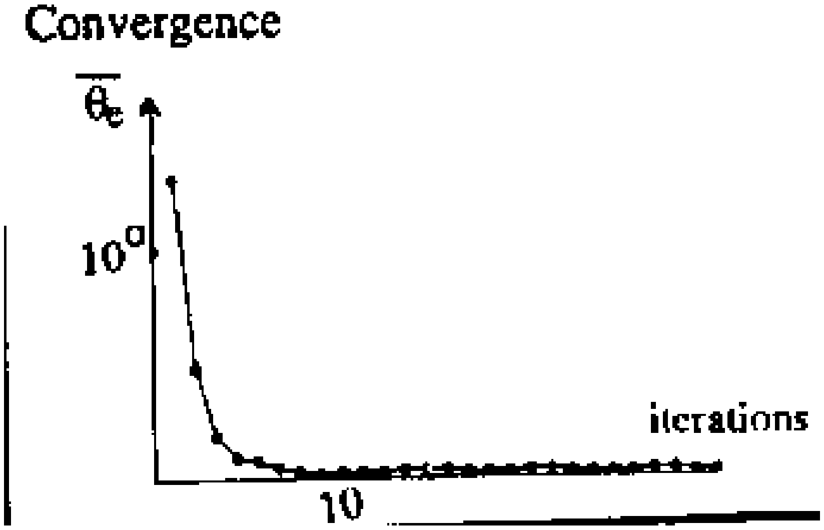


Convergence

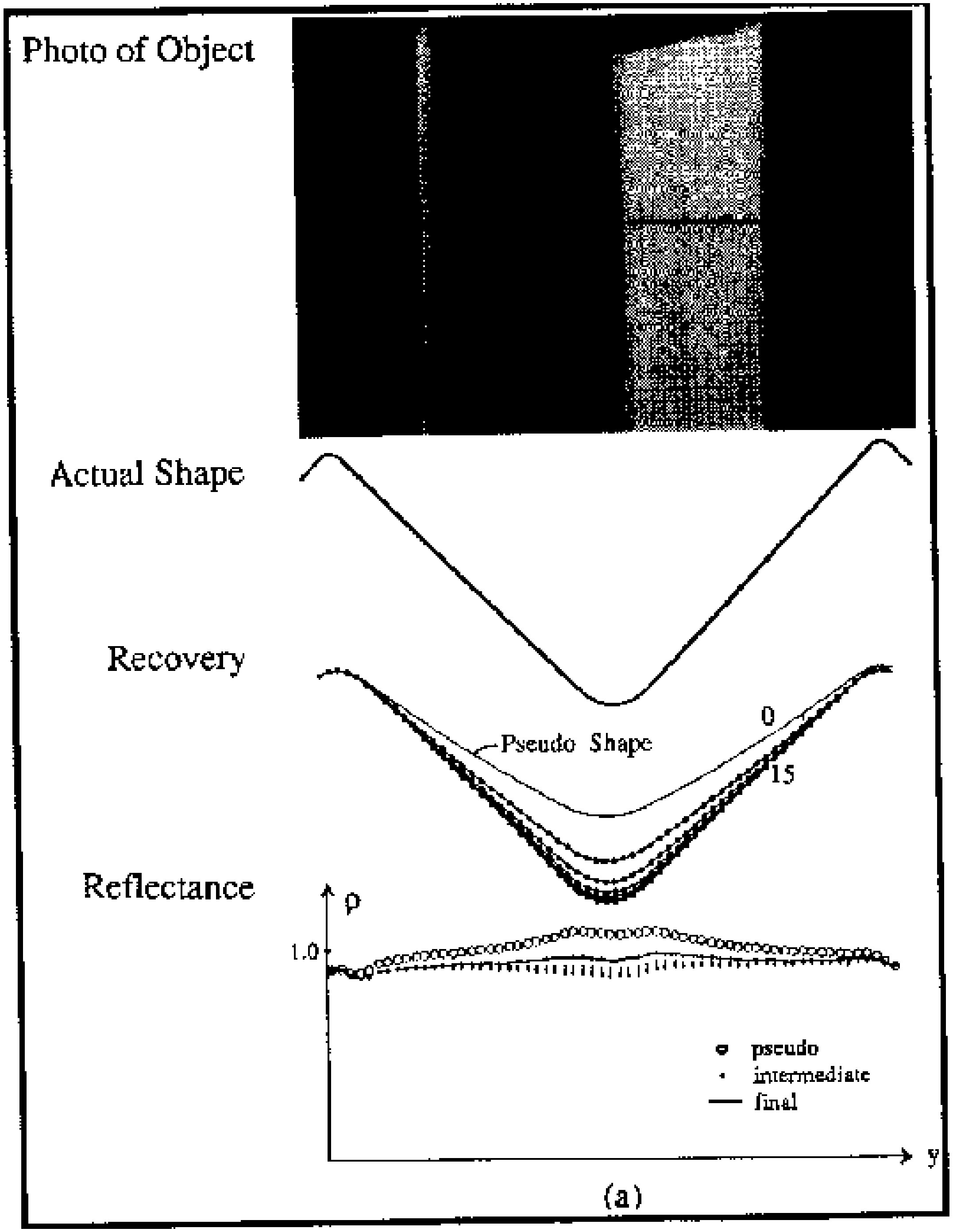



Actual Shape
Pseudo Shape
Recovery
0
20
y


Reflectance
ρ
Actual
Intermediate
0.7
0.3
y

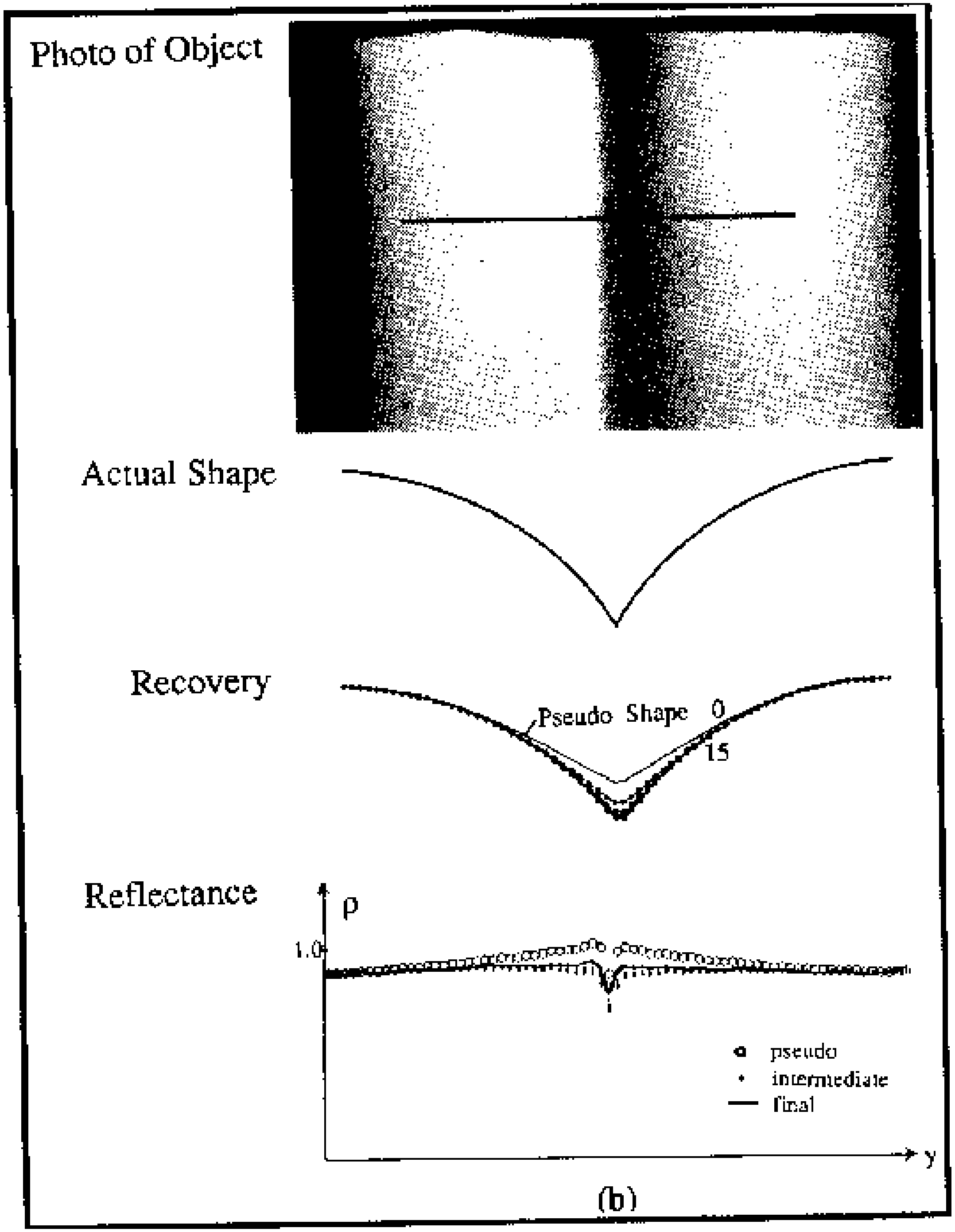

Convergence
$\overline{\theta_e}$
10^0
iterations
10

3D Objects with Translational Symmetry

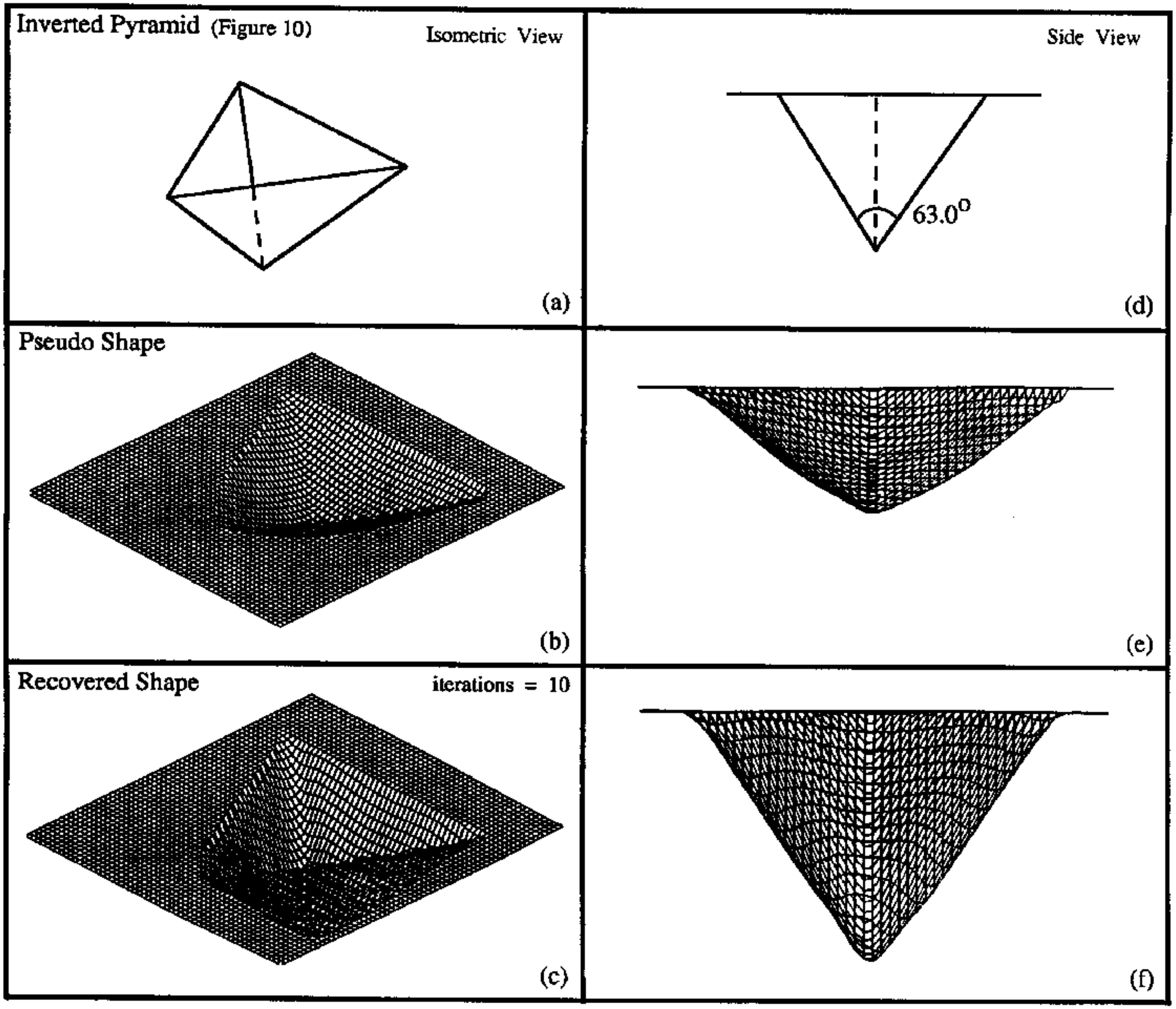


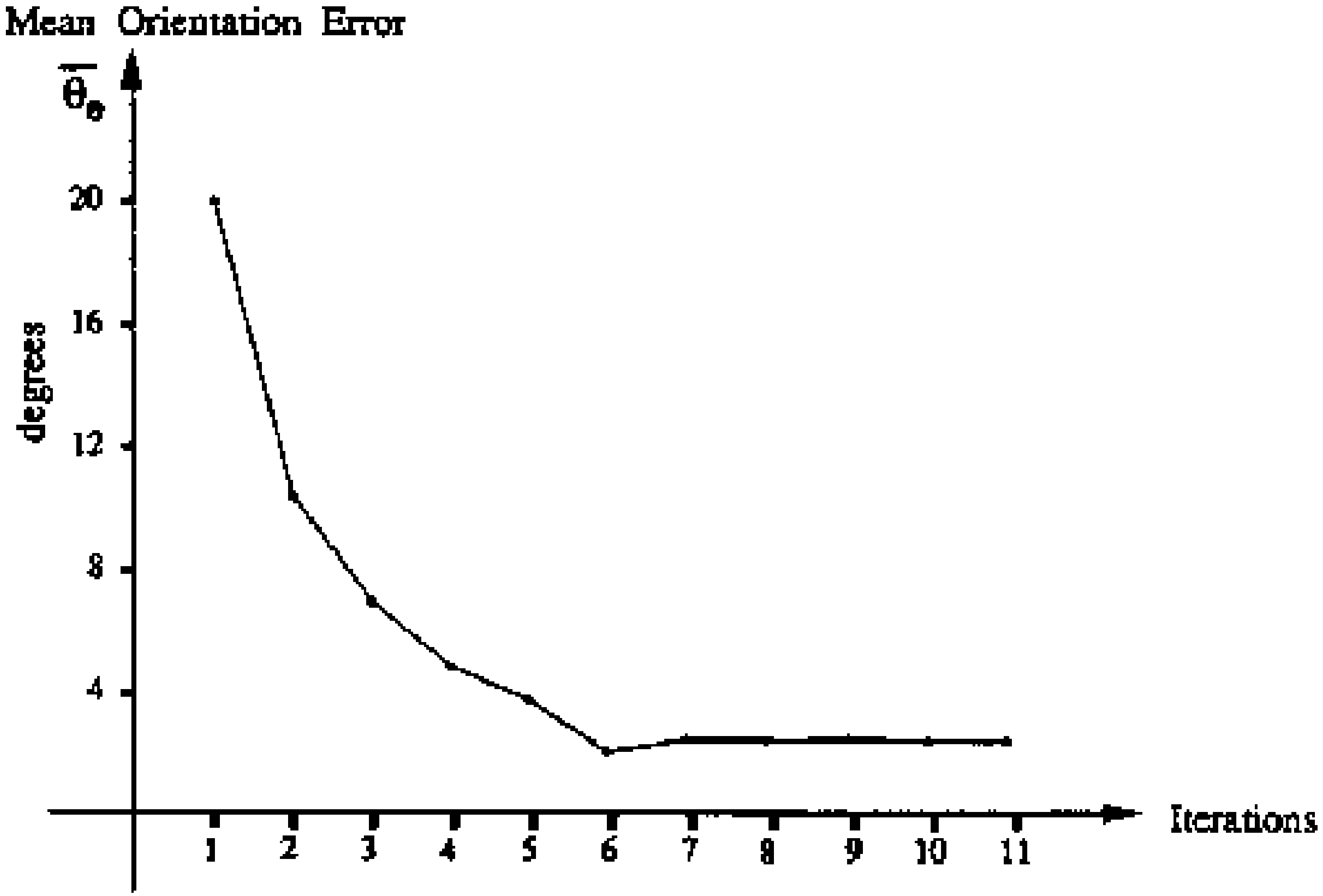
(a)

3D Objects with Translational Symmetry



(b)


Inverted Pyramid (Figure 10)
Isometric View
(a)
Side View
63.0°
(d)
Pseudo Shape
(b)
(e)
Recovered Shape
iterations = 10
(c)
(f)



Mean Orientation Error
$\overline{\theta_e}$
degrees
20
16
12
8
4
1
2
3
4
5
6
7
8
9
10
11
Iterations

Shape and Interreflections: Chicken and Egg

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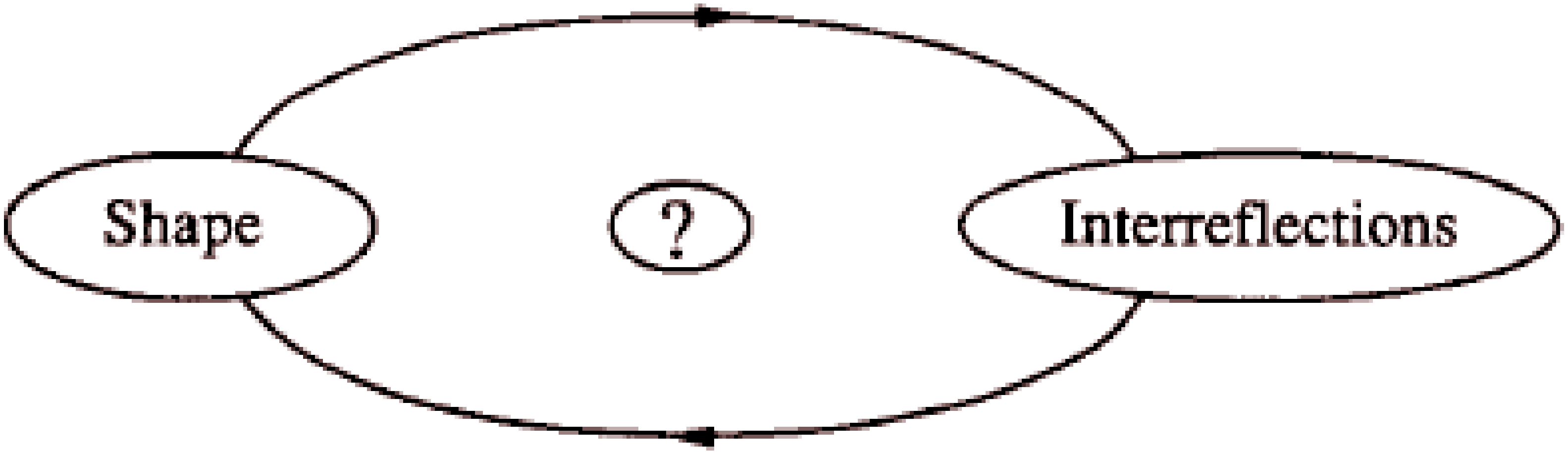
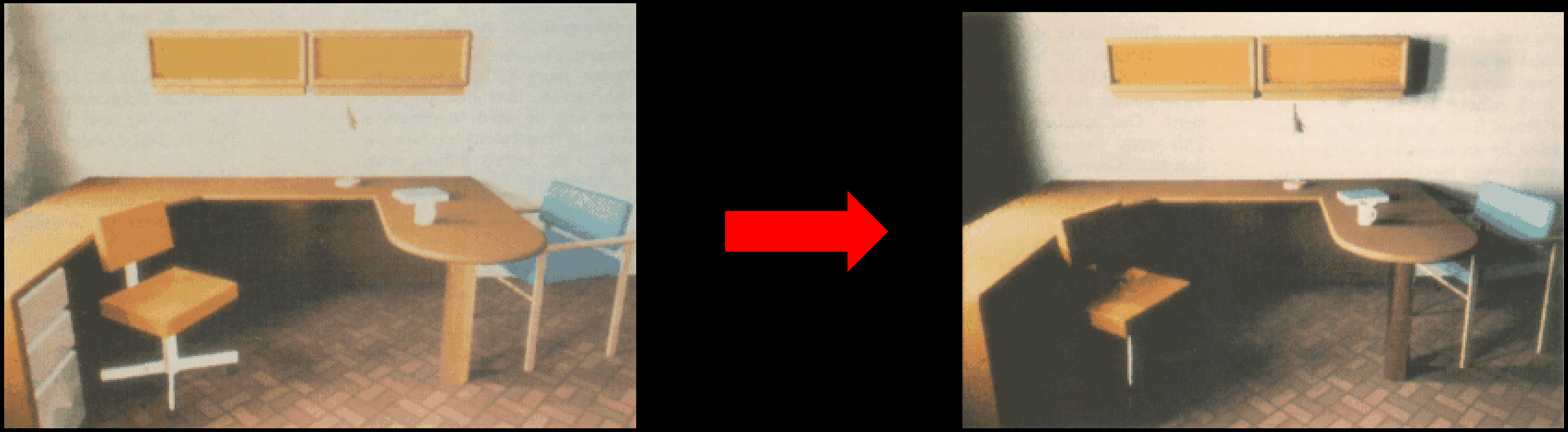
- But, interreflections depend on the shape!!

So, which comes first?

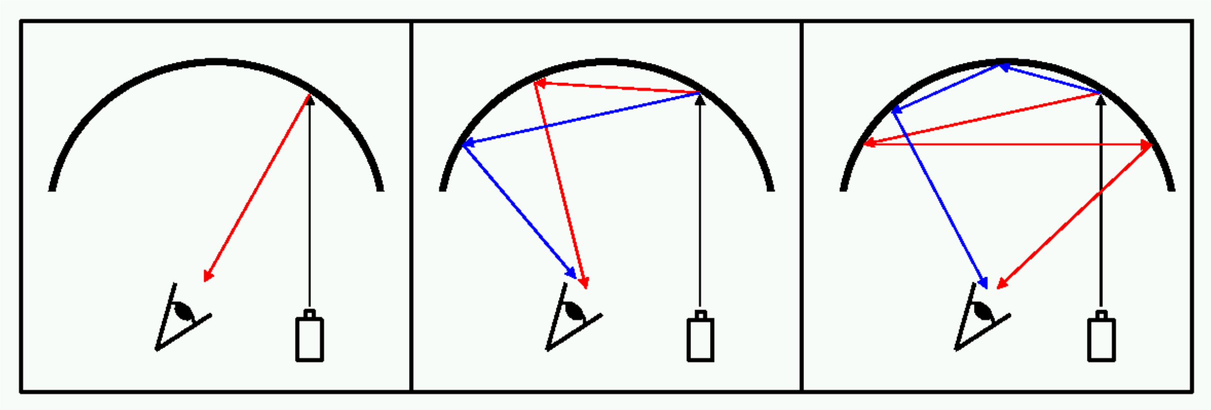


Removing Interreflection



Images by Ward et al., SIGGRAPH 88

Bounce Images



I^1 I^2 I^3

$$I = I^1 + I^2 + I^3 + \ldots$$

Main Results

There exists a matrix C^1 that removes all interreflections in a photograph (or lightfield)

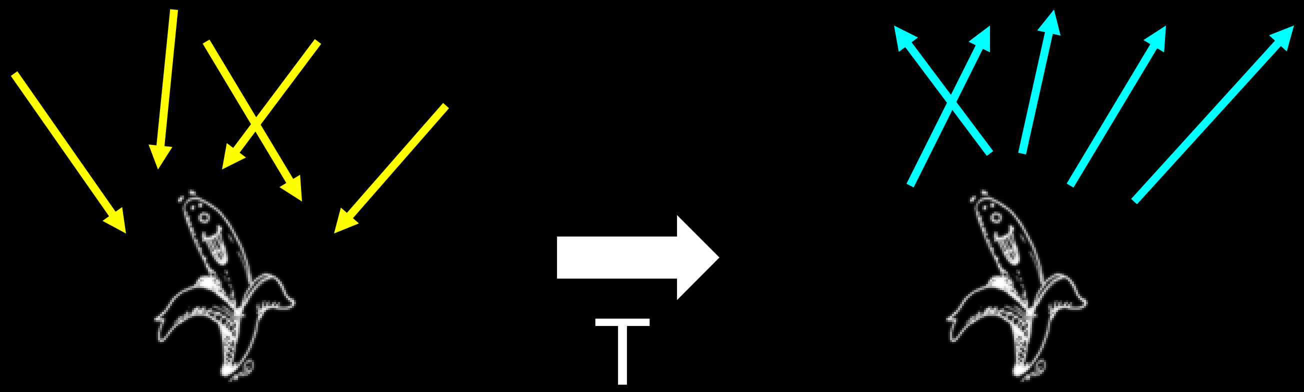
$$I^1 = C^1 I$$

Works for *any* illumination

There is a matrix C^k that retains only the kth bounce

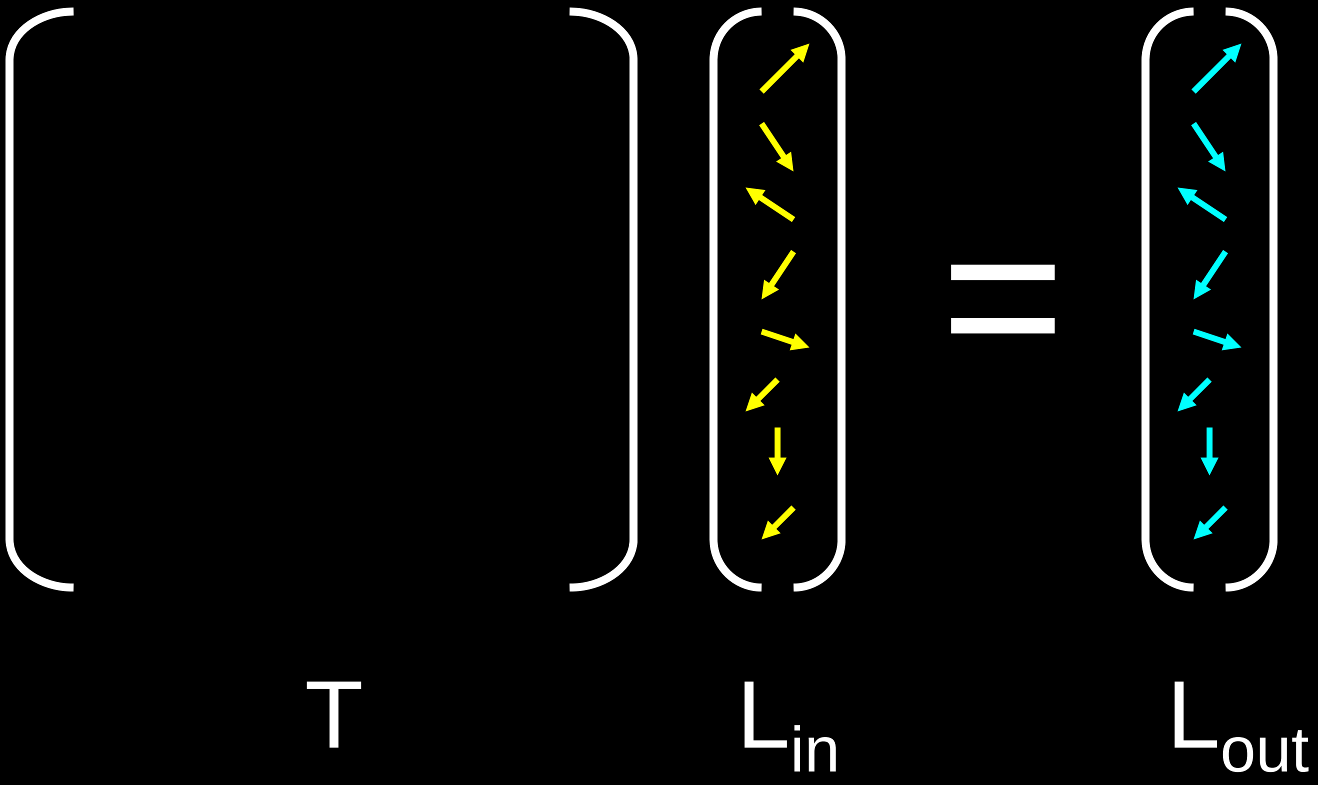
$$I^k = C^k I$$

Light transport



$$\mathbf{L}_{out}[i] = \mathbf{L}^{1}_{out}[i] + \sum_{j} \mathbf{A}[i,j]\mathbf{L}_{out}[j]$$

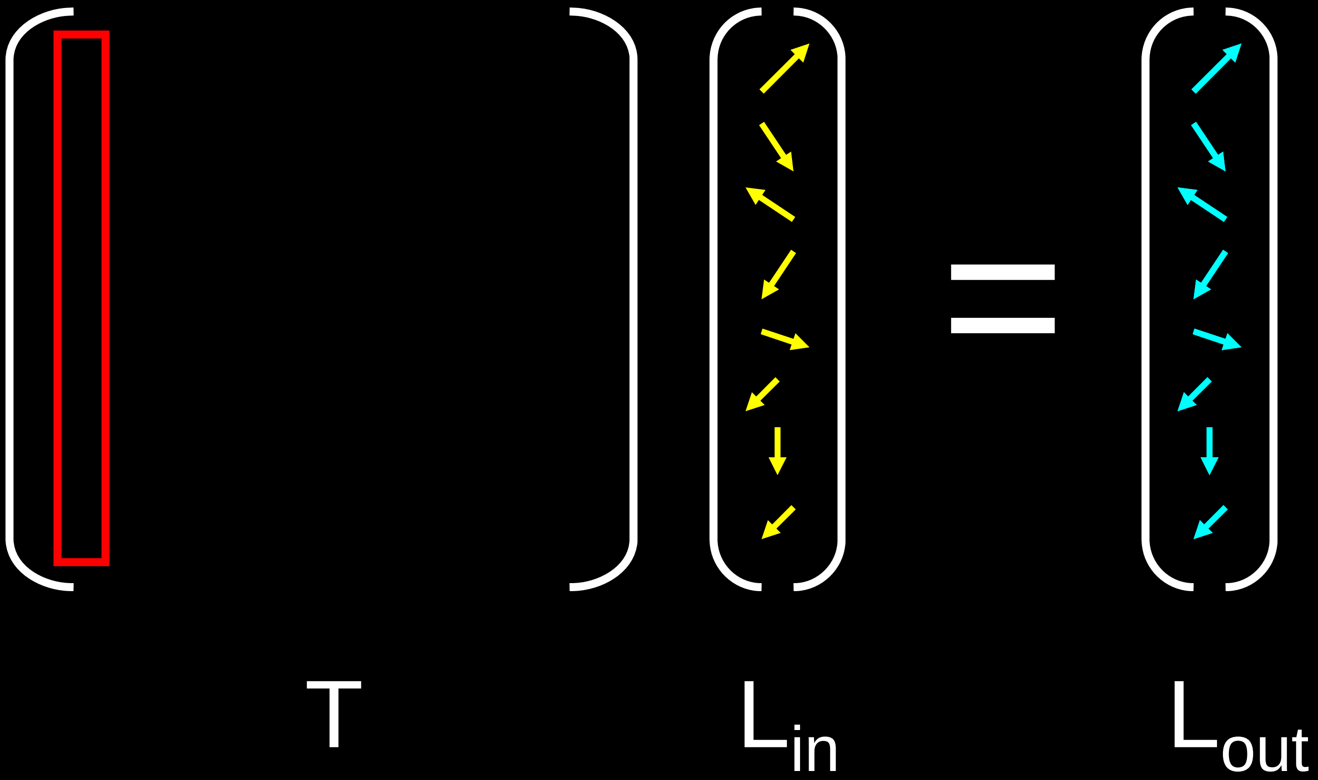
The transport matrix



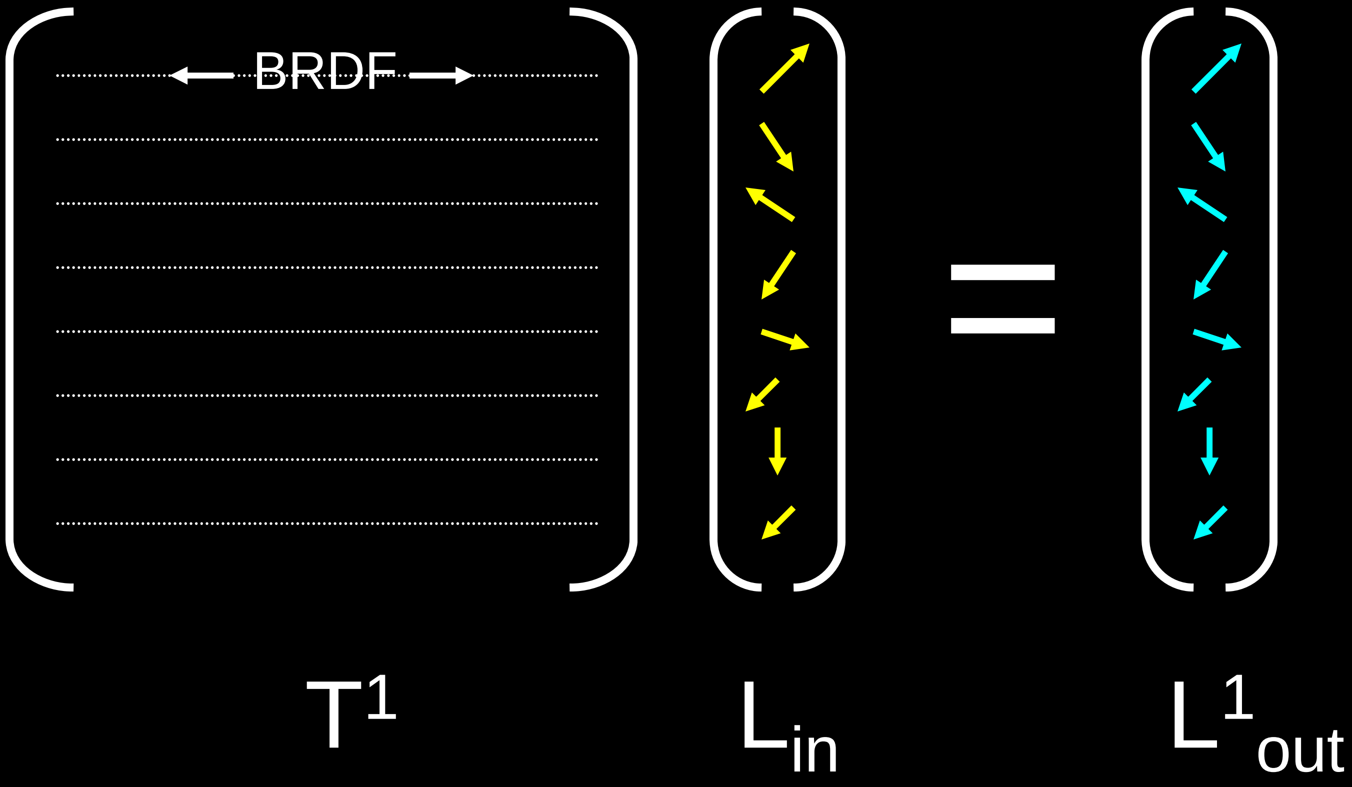
Accounts for

- interreflections, shadows, refraction, subsurface scatter, ...
- [Dorsey 94] [Zongker 99] [Debevec 00] [Peers 03] [Goesele 05] [Sen 05] ...

The transport matrix



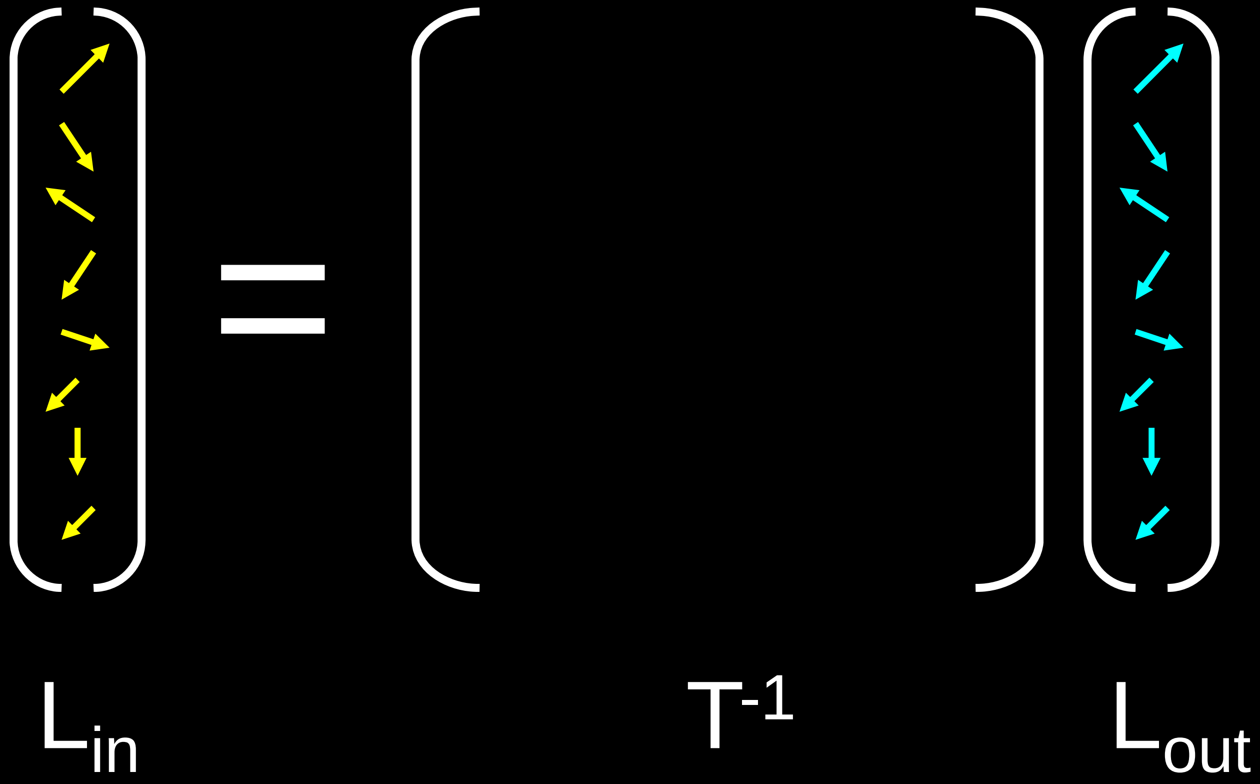
Direct illumination rendering



Single bounce from light to eye

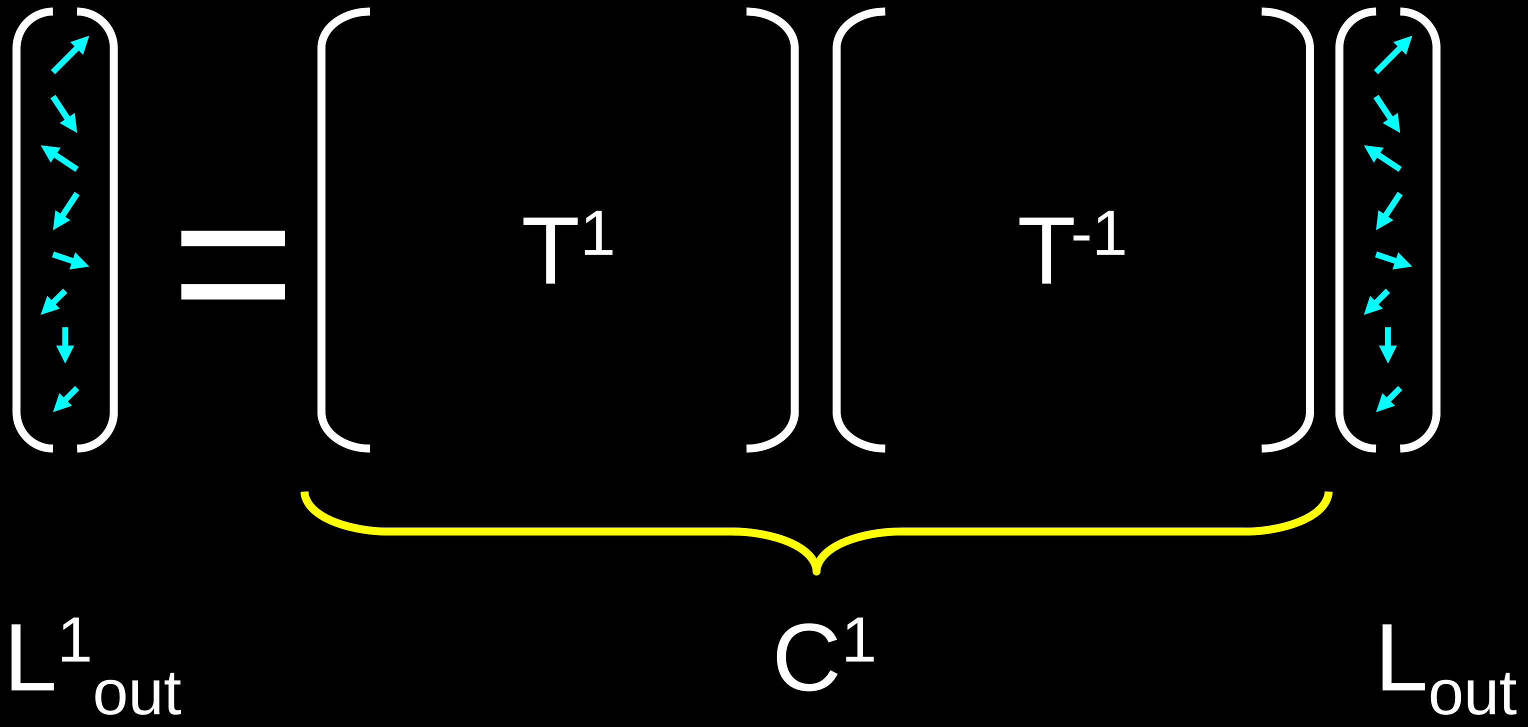
- no interreflections

Inverse rendering



L_{in} T^{-1} L_{out}

Removing Interreflections



$$\underset{L^1_{out}}{\begin{bmatrix} \\ \\ \end{bmatrix}} = \underbrace{\begin{bmatrix} T^1 \end{bmatrix} \begin{bmatrix} T^{-1} \end{bmatrix}}_{C^1} \underset{L_{out}}{\begin{bmatrix} \\ \\ \end{bmatrix}}$$

Second derivation:

- from the rendering equation [Kajiya 86] [Cohen 86]

Cancellation Operators

Recursively define other operators

$(I - C^1)$

- gives interreflected light

$C^2 = C^1 (I - C^1)$

- gives second bounce of light

$C^k = C^1(I - C^1)^{k-1}$

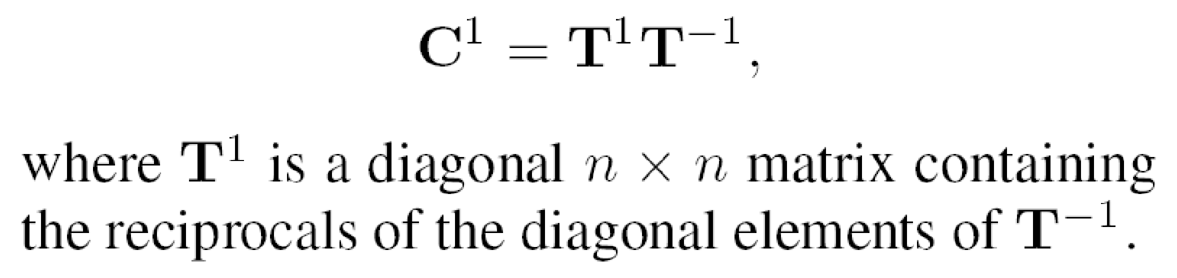
- gives kth bounce of light

Inverse ray tracing!

How to compute C^1

Simplified case

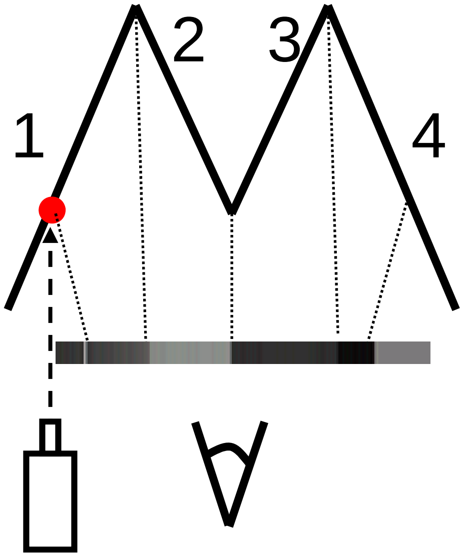
- Lambertian reflectance and fixed viewpoint
- L_{in} and L_{out} are 2D
- Can capture T by scanning a laser

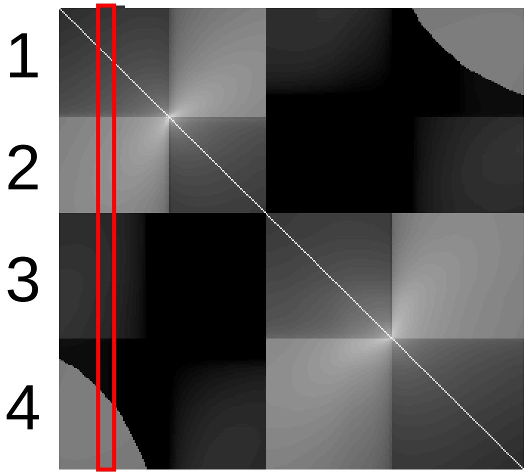


$$\mathbf{C}^1 = \mathbf{T}^1\mathbf{T}^{-1},$$

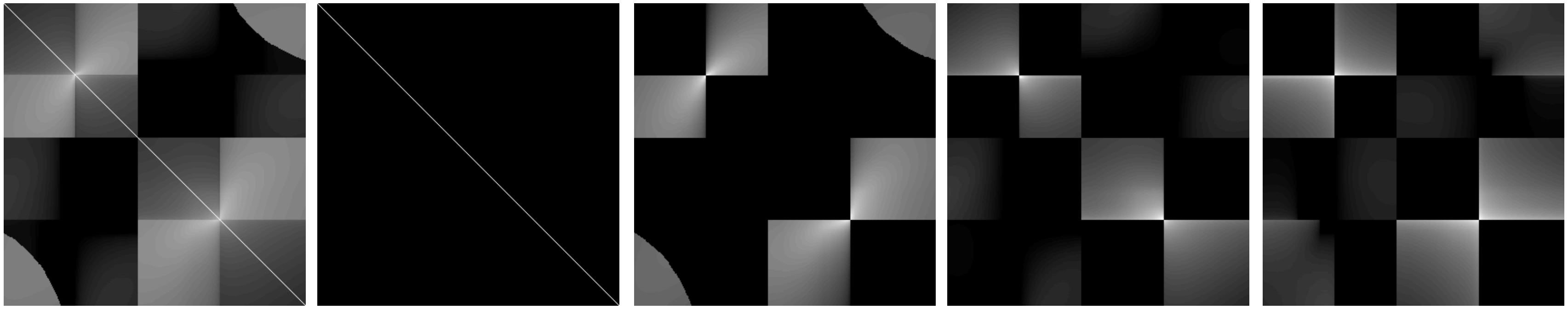
where $\mathbf{T}^1$ is a diagonal $n \times n$ matrix containing the reciprocals of the diagonal elements of $\mathbf{T}^{-1}$.

Synthetic M Scene

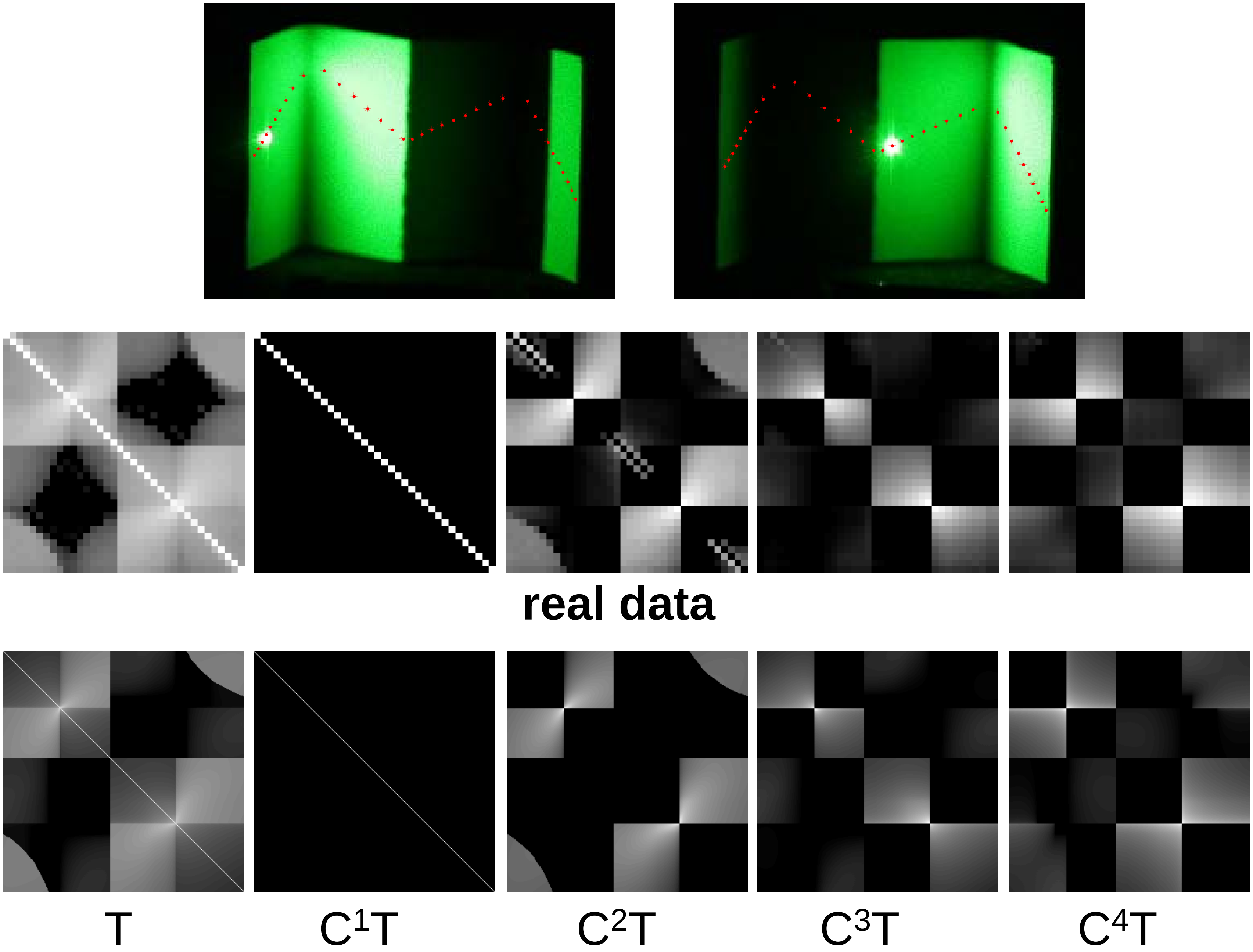


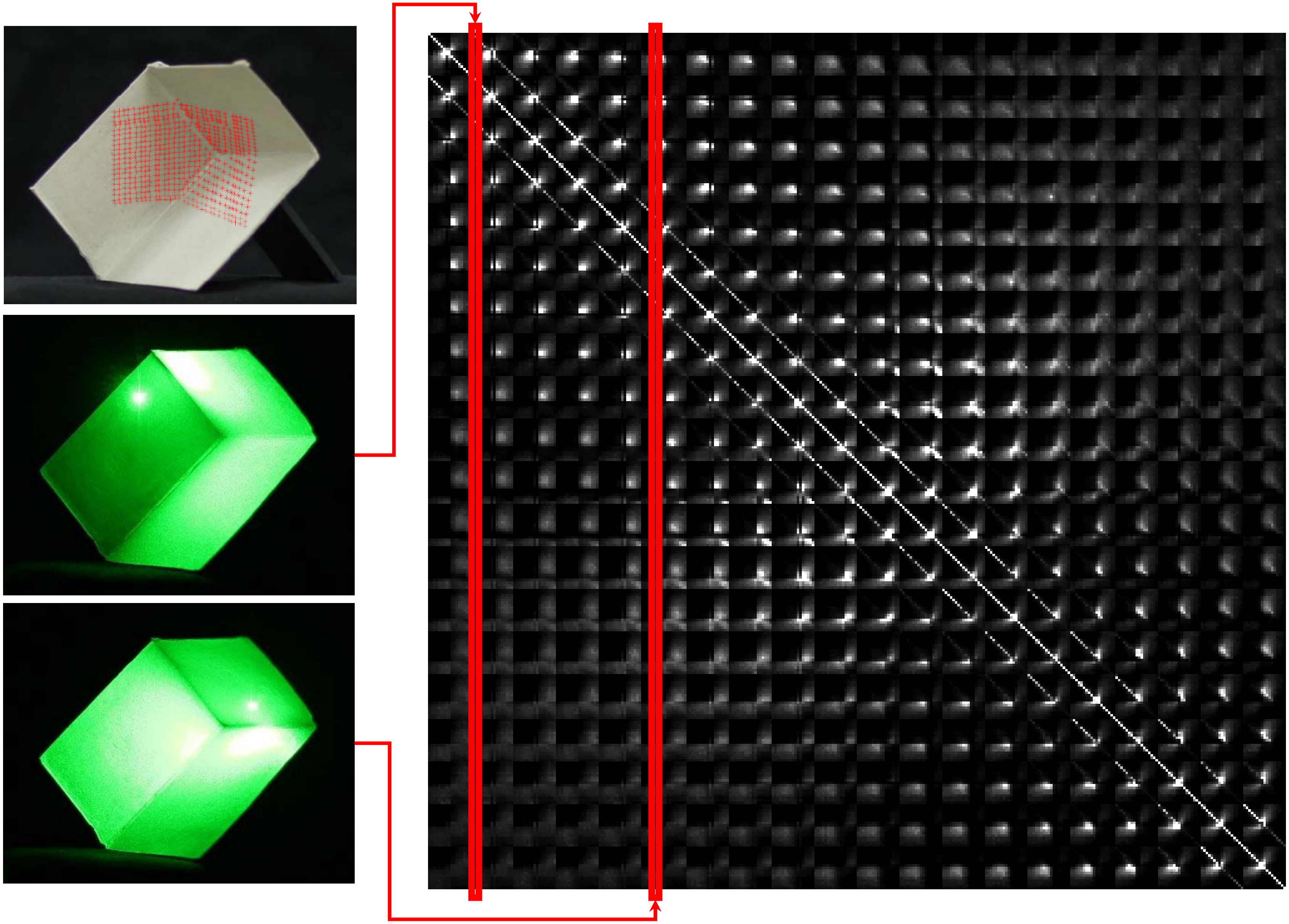


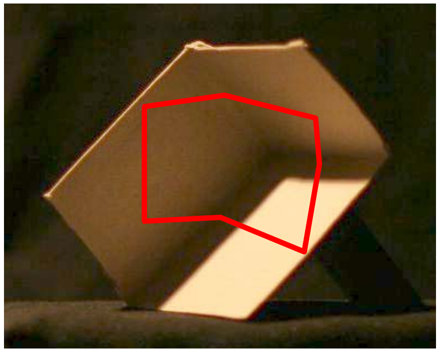
T (log-scaled)

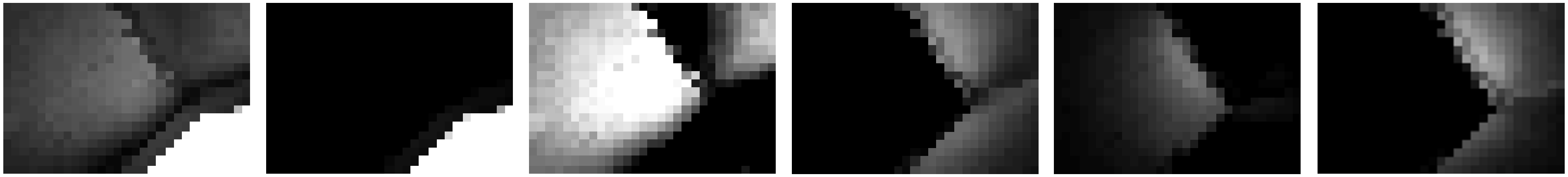


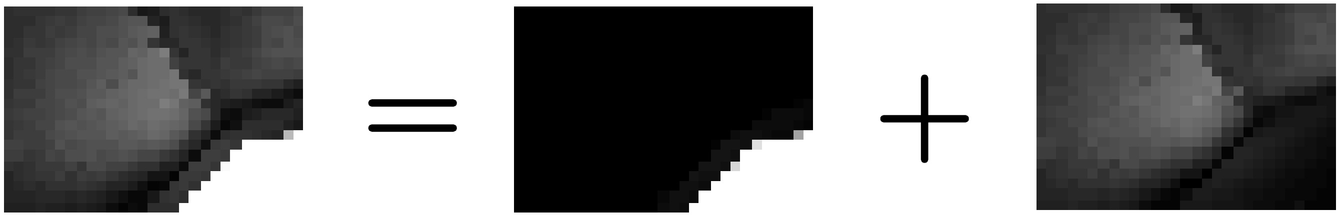
T C^1T C^2T C^3T C^4T

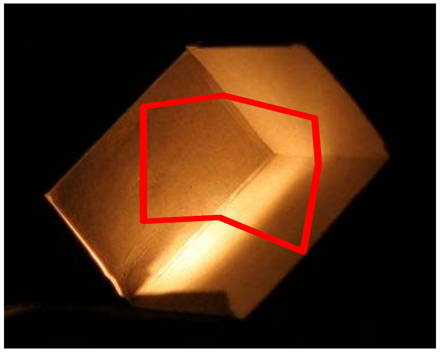

real data
T
C^1T
C^2T
C^3T
C^4T



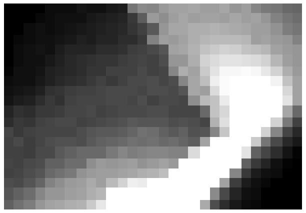



I
I^1
I^2
(x3)
I^3
(x3)
I^4
(x6)
I^5
(x15)

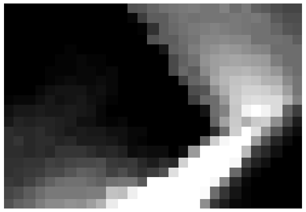

=
+
direct
indirect



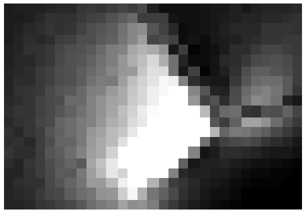
flash light illumination



I

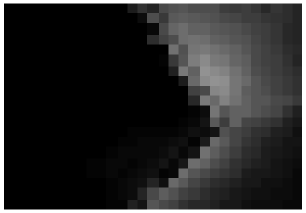


I^1



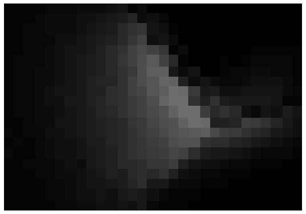
I^2

(x3)



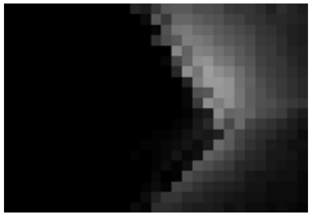
I^3

(x3)



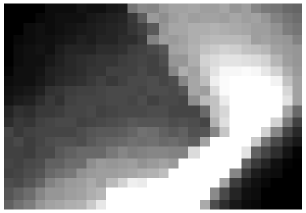
I^4

(x6)

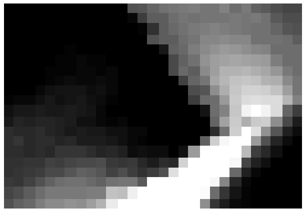


I^5

(x15)

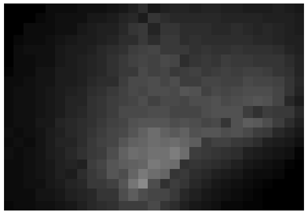


$=$



direct

$+$



indirect

Shape and Interreflections: Chicken and Egg

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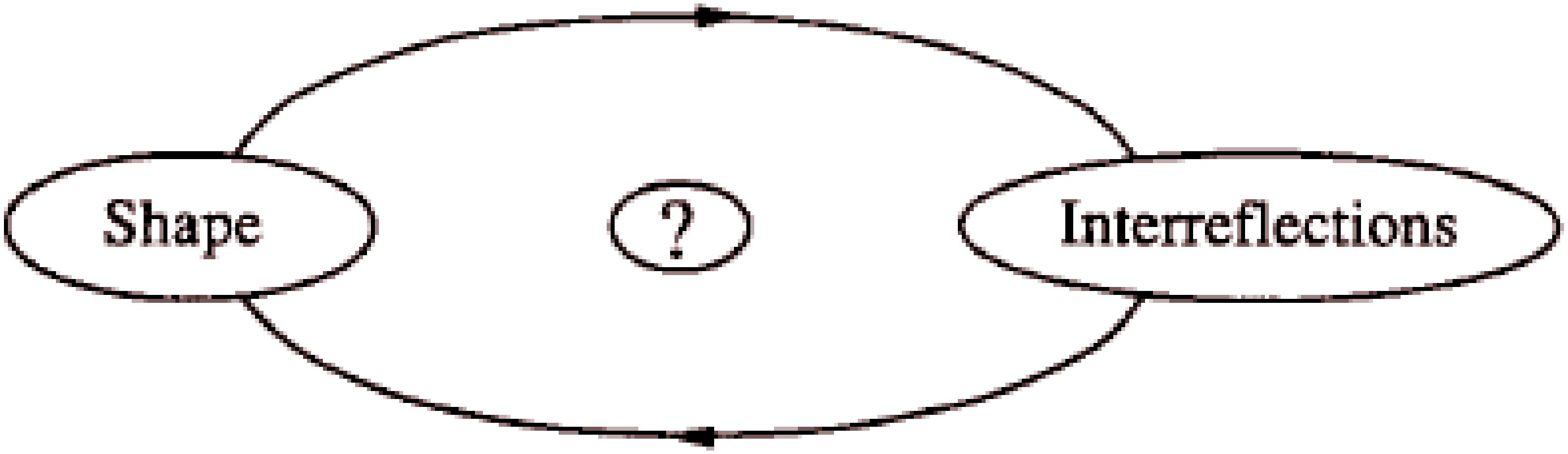
- But, interreflections depend on the shape!!

So, which comes first?

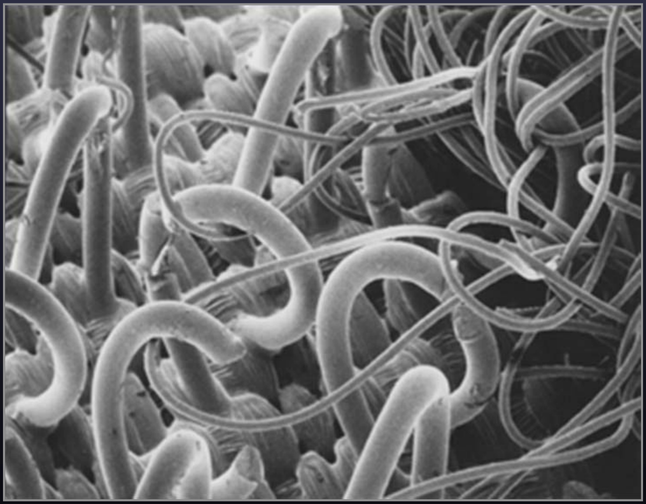


Dual Photography

Pradeep Sen, Billy Chen, Gaurav Garg, Steve Marschner,
Mark Horowitz, Marc Levoy, Hendrik Lensch

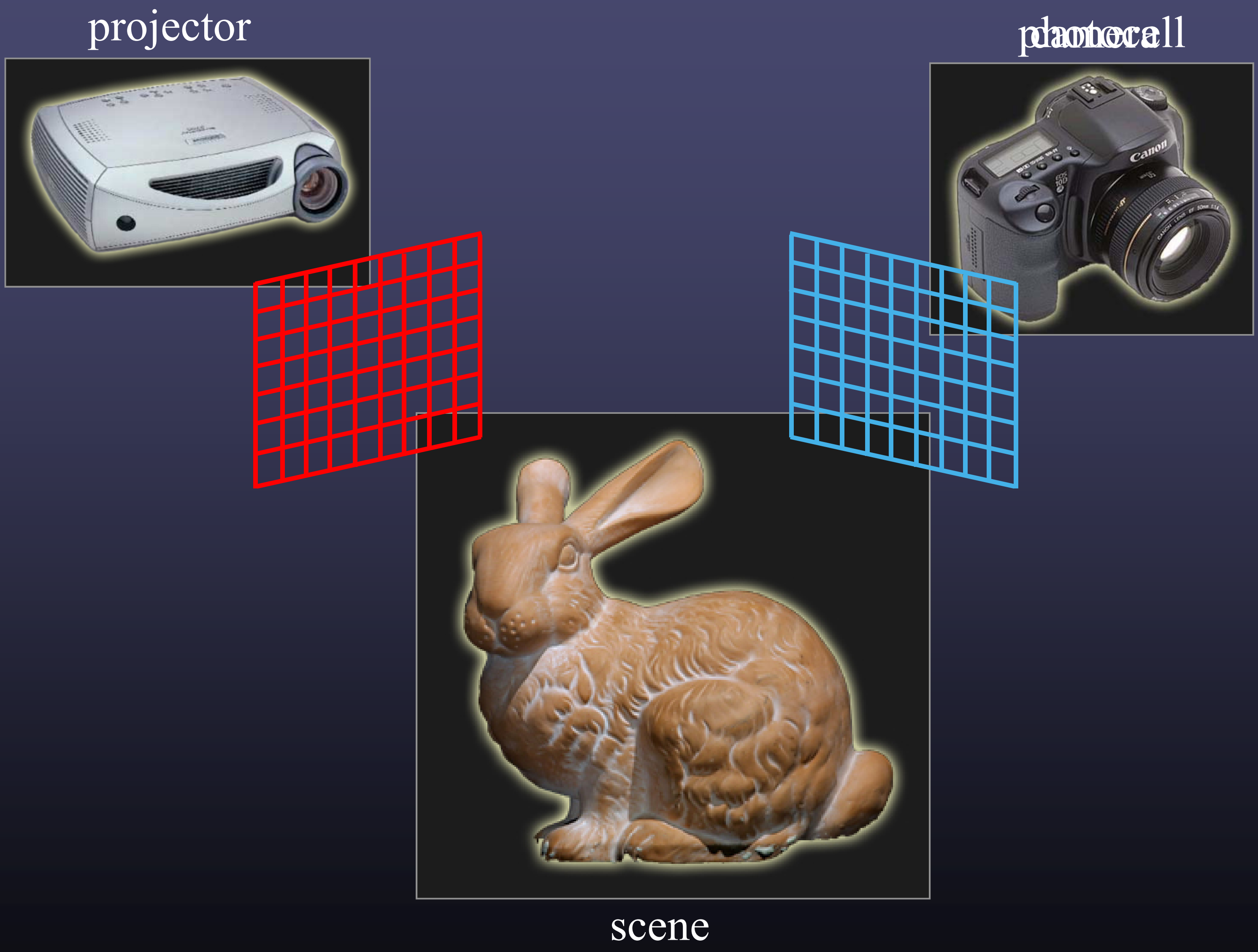
Related imaging methods

- time-of-flight scanner
  - – if they return reflectance as well as range
  - – but their light source and sensor are typically coaxial
- scanning electron microscope

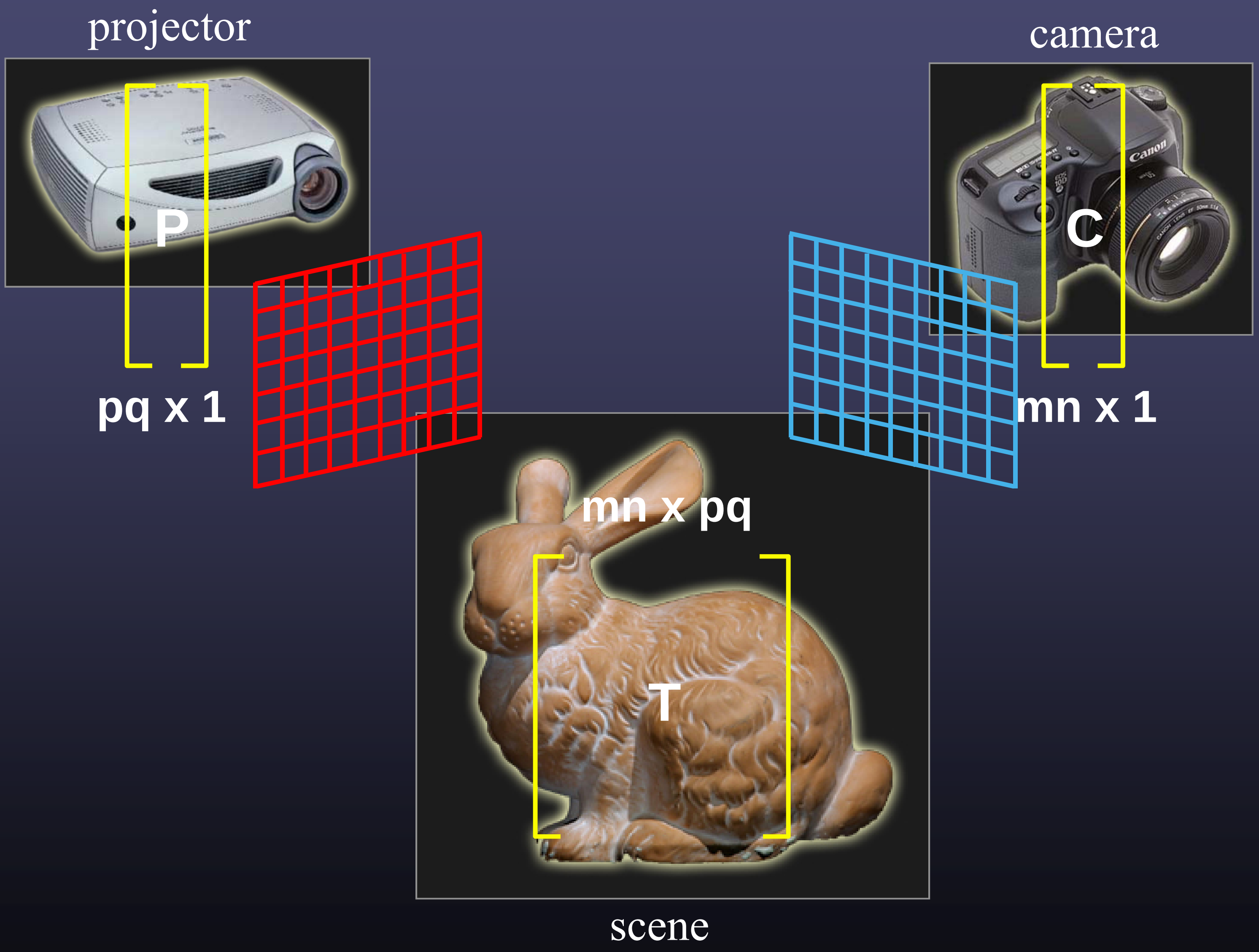


Velcro® at 35x magnification,
Museum of Science, Boston

The 4D transport matrix



The 4D transport matrix



projector

P

pq x 1

camera

C

mn x 1

mn x pq

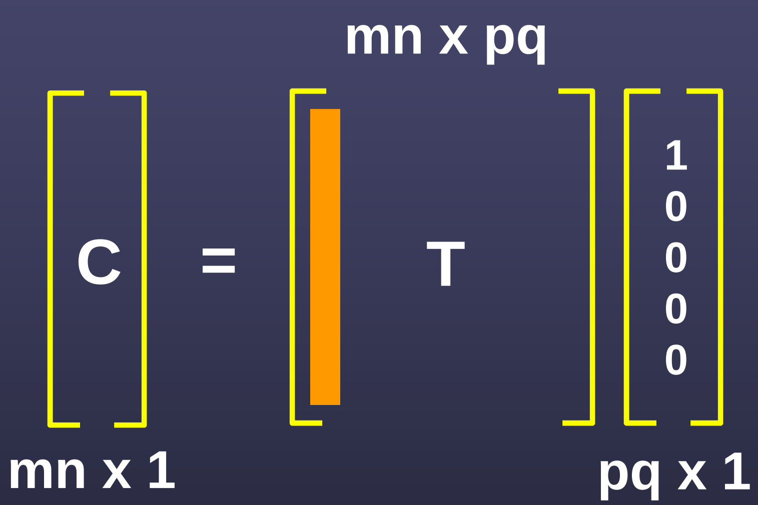
T

scene

The 4D transport matrix

$$\underset{mn\ x\ 1}{\begin{bmatrix} \\ C \\ \\ \end{bmatrix}} = \overset{mn\ x\ pq}{\begin{bmatrix} & & \\ & T & \\ & & \end{bmatrix}} \underset{pq\ x\ 1}{\begin{bmatrix} \\ P \\ \\ \end{bmatrix}}$$

The 4D transport matrix



mn x pq

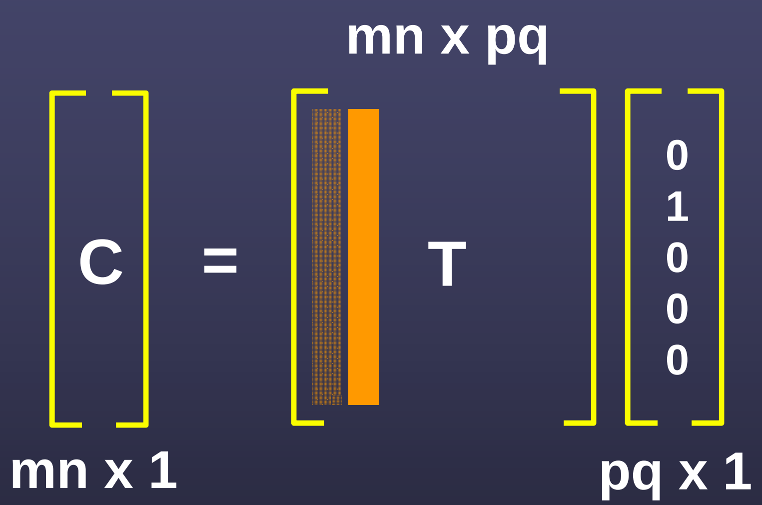
C = T

1
0
0
0
0

mn x 1

pq x 1

The 4D transport matrix



mn x pq

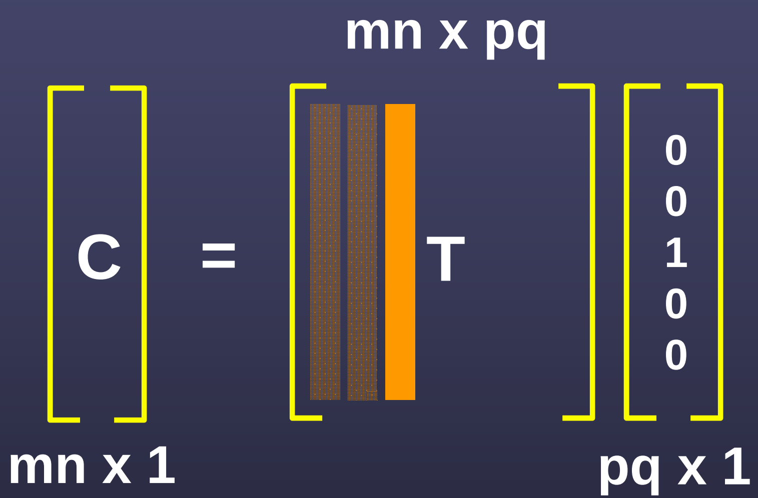
C = T

0
1
0
0
0

mn x 1

pq x 1

The 4D transport matrix



mn x pq

C = T

0
0
1
0
0

mn x 1

pq x 1

The 4D transport matrix

$$\underset{mn \times 1}{\begin{bmatrix} C \end{bmatrix}} = \overset{mn \times pq}{\begin{bmatrix} \quad T \quad \end{bmatrix}} \underset{pq \times 1}{\begin{bmatrix} P \end{bmatrix}}$$

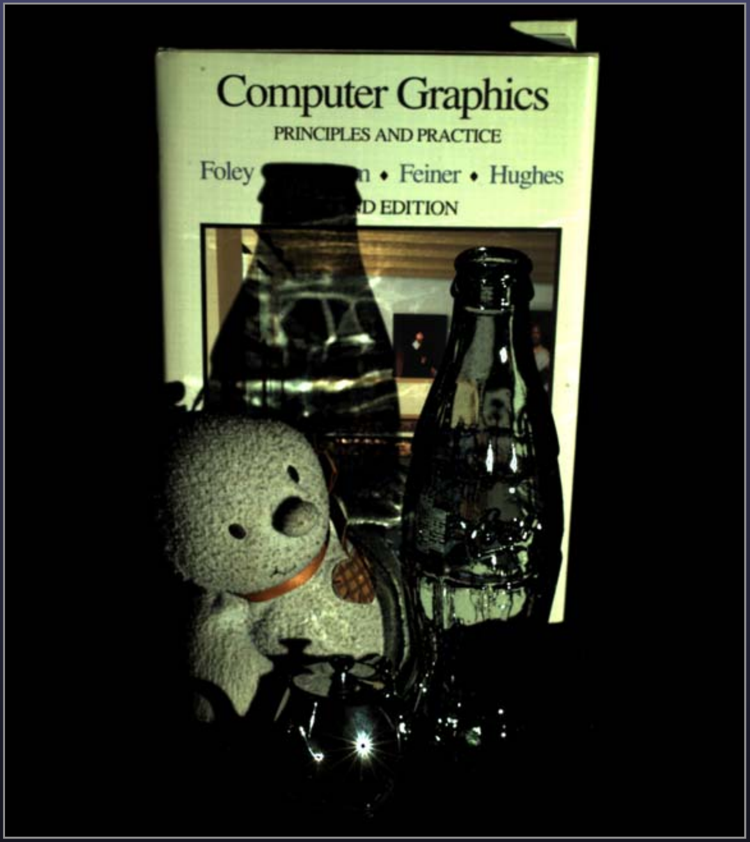
The 4D transport matrix

$$\underset{mn \times 1}{\begin{bmatrix} C \end{bmatrix}} = \overset{mn \times pq}{\begin{bmatrix} \quad T \quad \end{bmatrix}} \underset{pq \times 1}{\begin{bmatrix} P \end{bmatrix}}$$

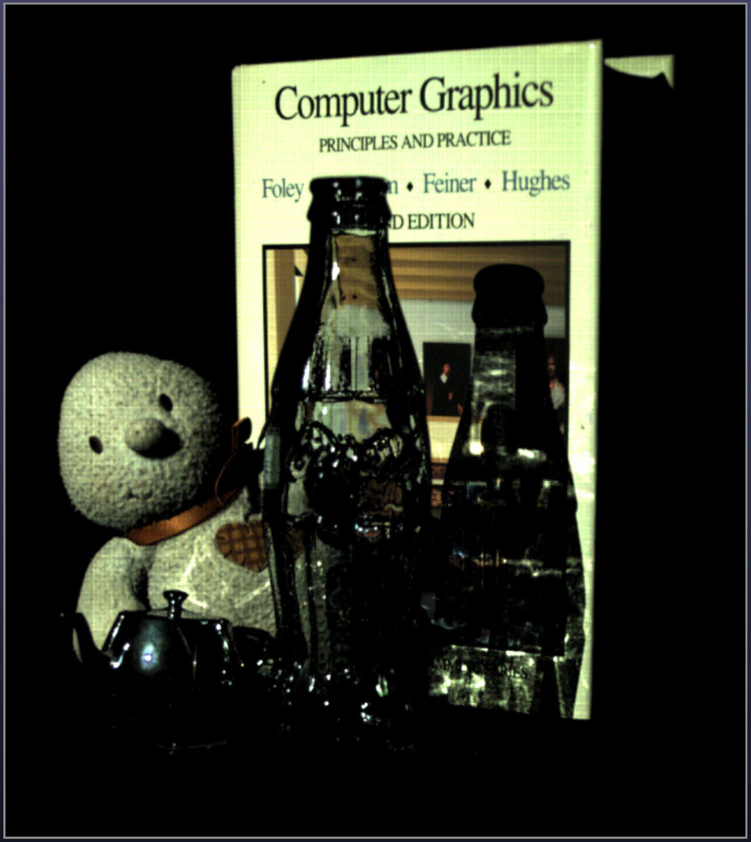
applying Helmholtz reciprocity...

$$\underset{pq \times 1}{\begin{bmatrix} C' \end{bmatrix}} = \overset{pq \times mn}{\begin{bmatrix} \quad T^{T} \quad \end{bmatrix}} \underset{mn \times 1}{\begin{bmatrix} P' \end{bmatrix}}$$

Example



conventional photograph
with light coming from right



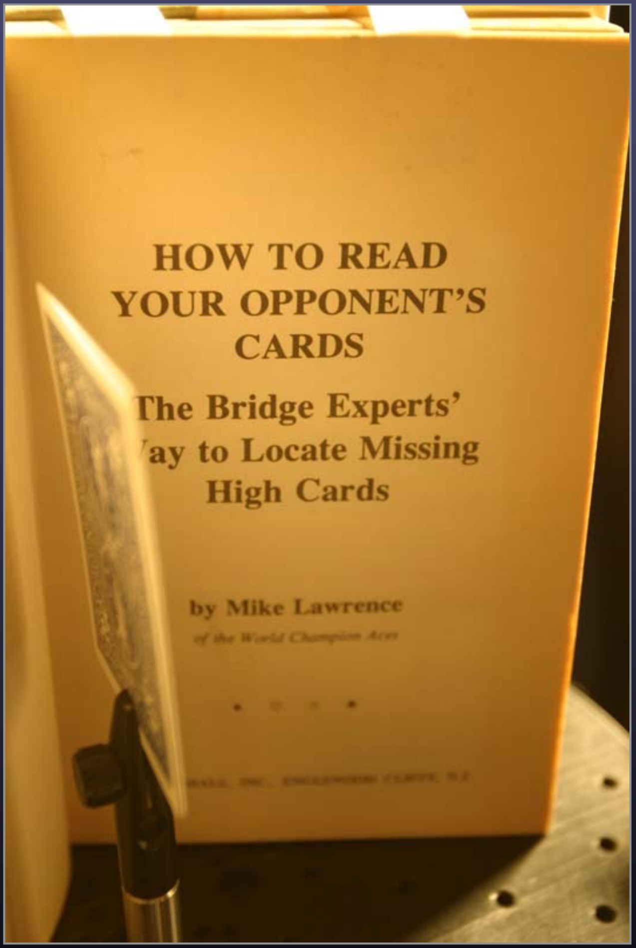
dual photograph
as seen from projector’s position

Properties of the transport matrix

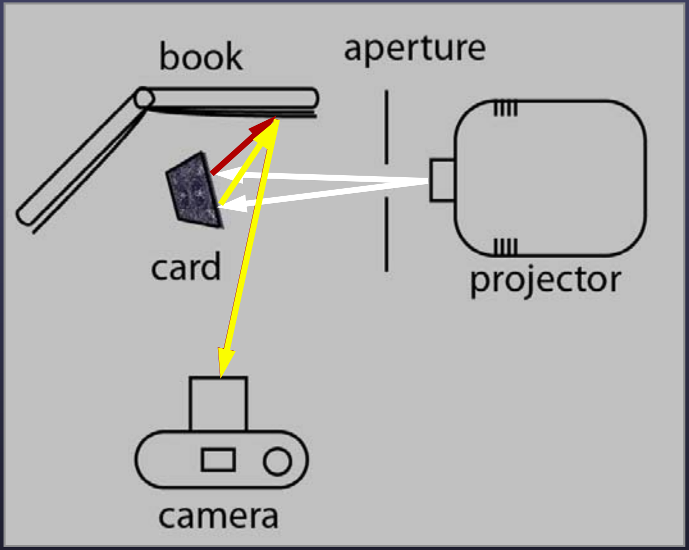
- little interreflection → sparse matrix
- many interreflections → dense matrix
- convex object → diagonal matrix
- concave object → full matrix

Can we create a dual photograph entirely from diffuse reflections?

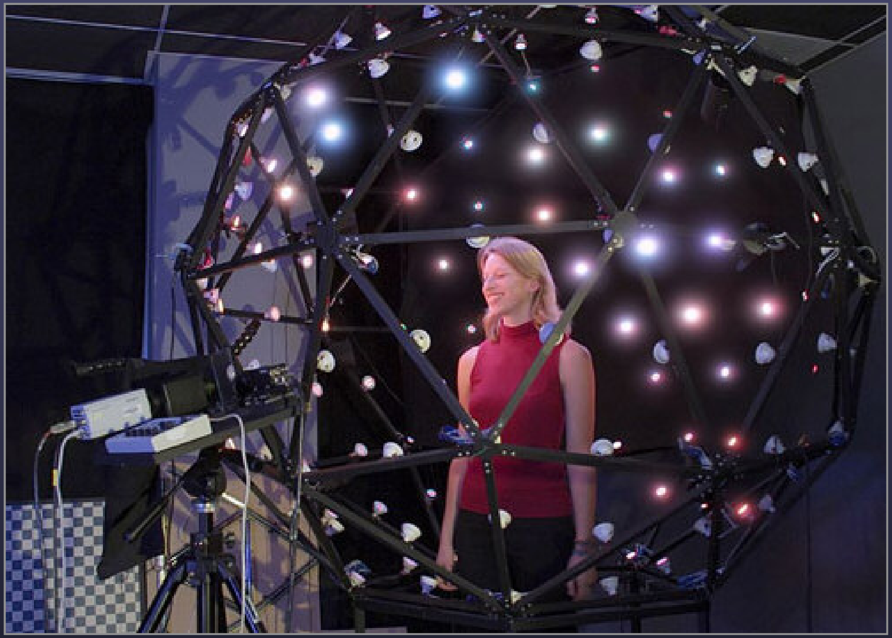
Dual photography from diffuse reflections



the camera's view



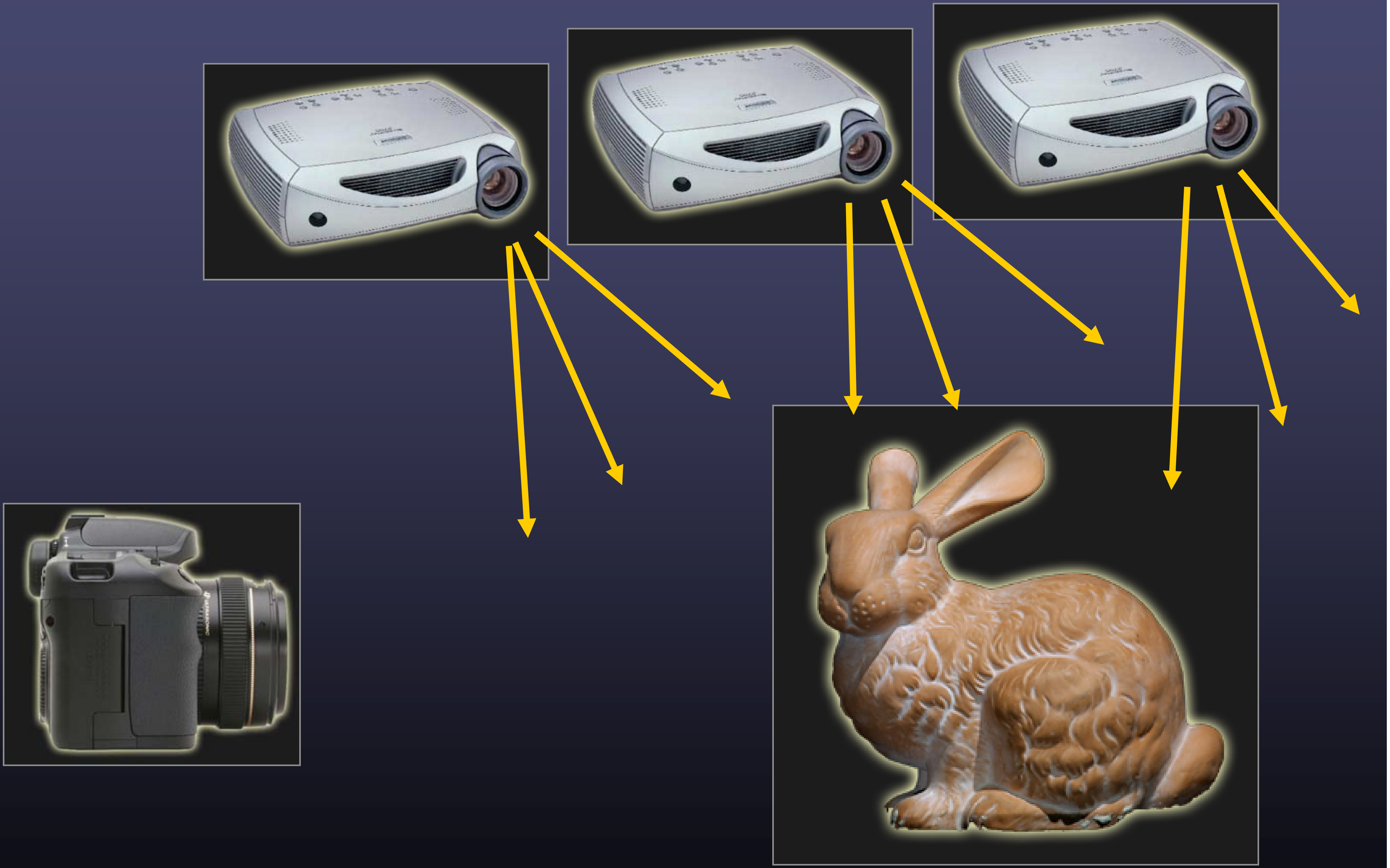
The relighting problem



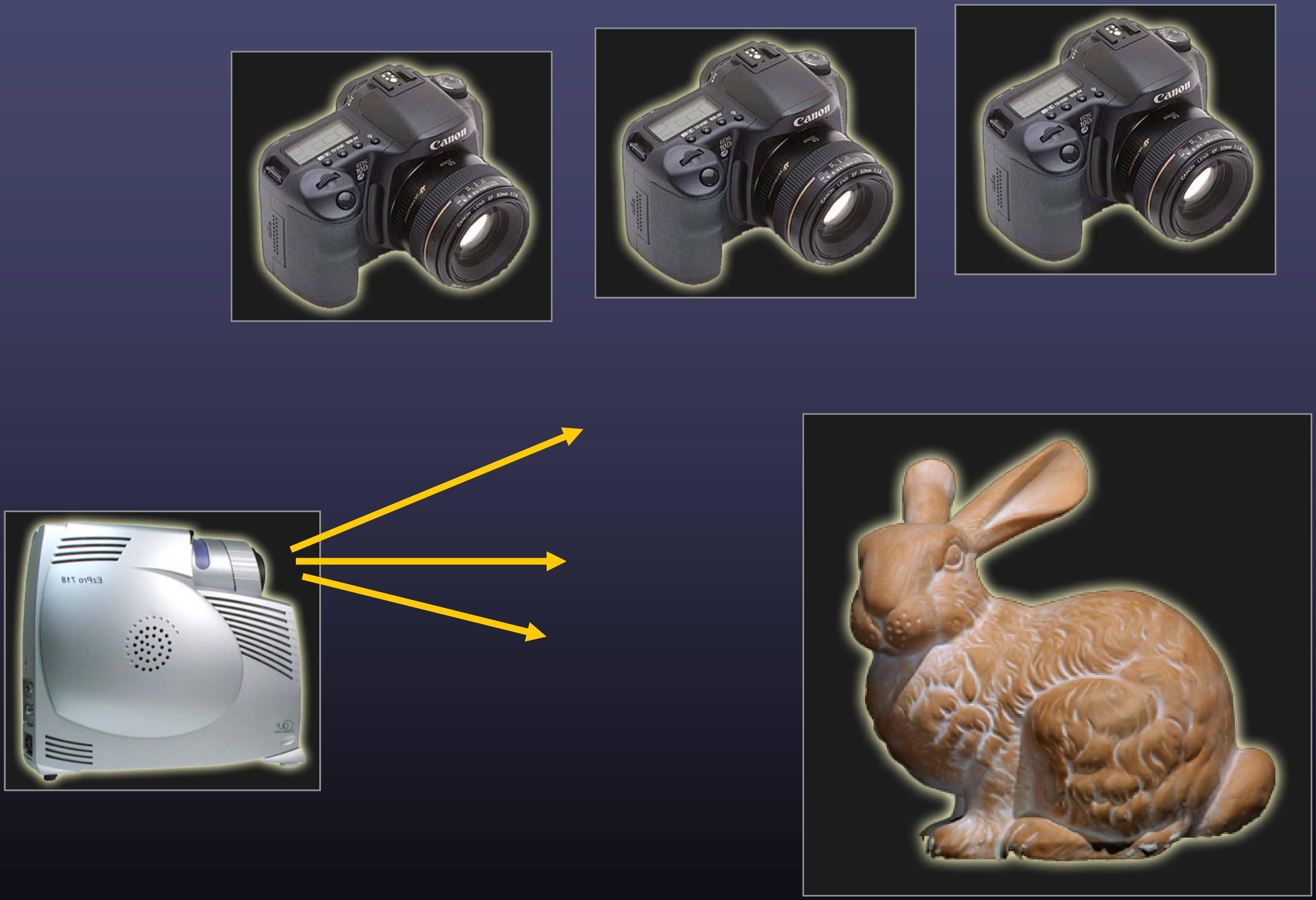
Paul Debevec's
Light Stage 3

- subject captured under multiple lights
- one light at a time, so subject must hold still
- point lights are used, so can't relight with cast shadows

The 6D transport matrix



The 6D transport matrix



The advantage of dual photography

- capture of a scene as illuminated by different lights cannot be parallelized

- capture of a scene as viewed by different cameras can be parallelized

Measuring the 6D transport matrix

projector

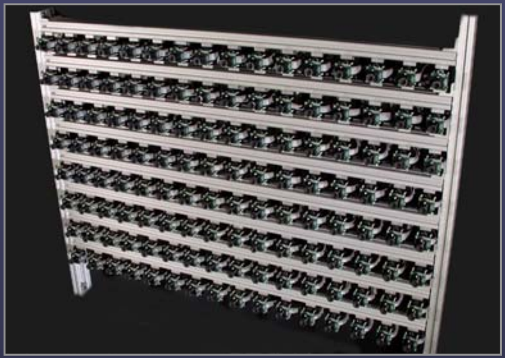
camera array

scene

Relighting with complex illumination

projector

camera array



scene

$$\underbrace{\begin{bmatrix} C' \end{bmatrix}}_{\text{pq x 1}} = \overset{\text{pq x mn x uv}}{\begin{bmatrix} \quad T^T \quad \end{bmatrix}} \underbrace{\begin{bmatrix} P' \end{bmatrix}}_{\text{mn x uv x 1}}$$

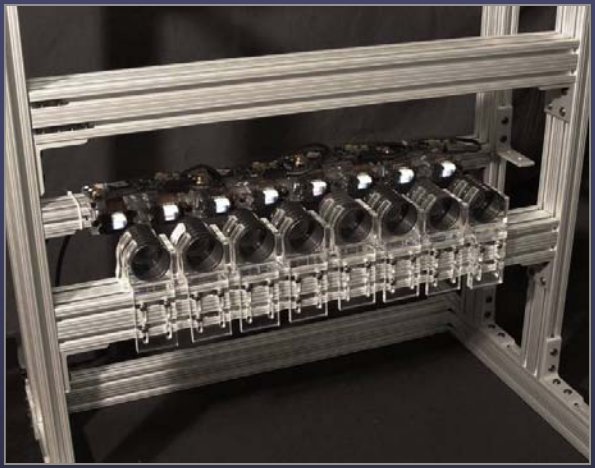
- step 1: measure 6D transport matrix T
- step 2: capture a 4D light field
- step 3: relight scene using captured light field

Running time

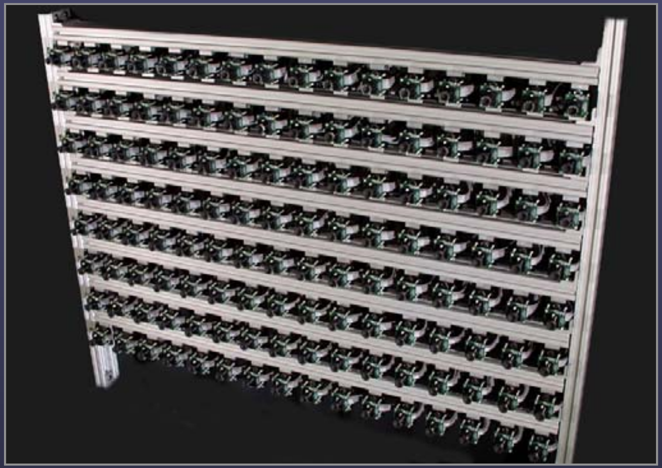
- the different rays within a projector can in fact be parallelized to some extent
- this parallelism can be discovered using a coarse-to-fine adaptive scan
- can measure a 6D transport matrix in 5 minutes

Can we measure an 8D transport matrix?

projector array



camera array



scene