Interreflections and Radiosity :
The Forward Problem

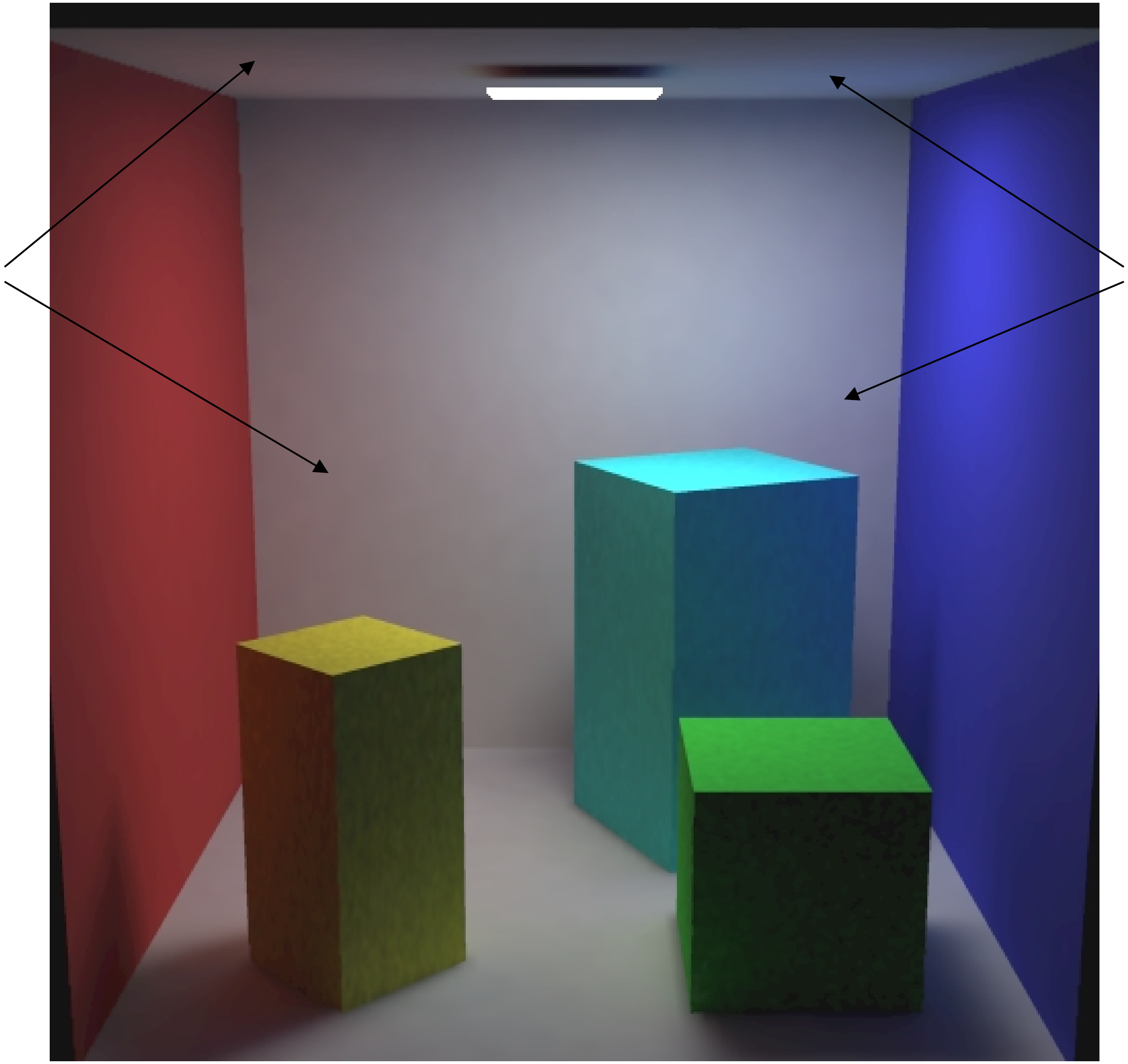
Lecture #11

Thanks to Kavita Bala, Pat Hanrahan, Doug James, Ledah Casburn

Mies Courtyard House with Curved Elements

Modeling: Stephen Duck; Rendering: Henrik Wann Jensen

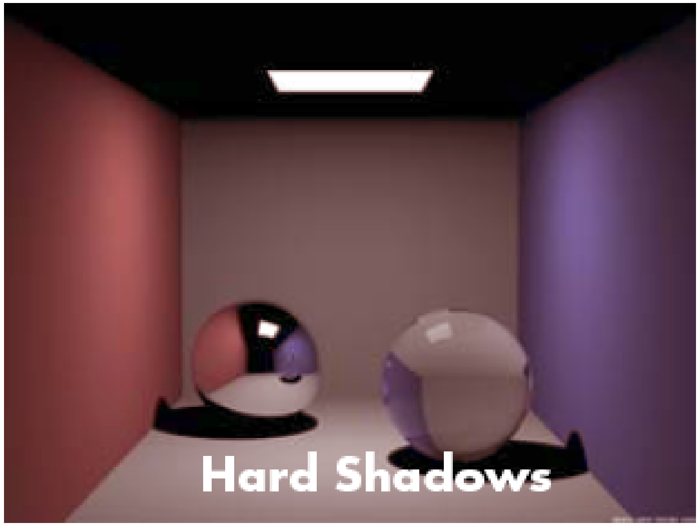
Cornell Box

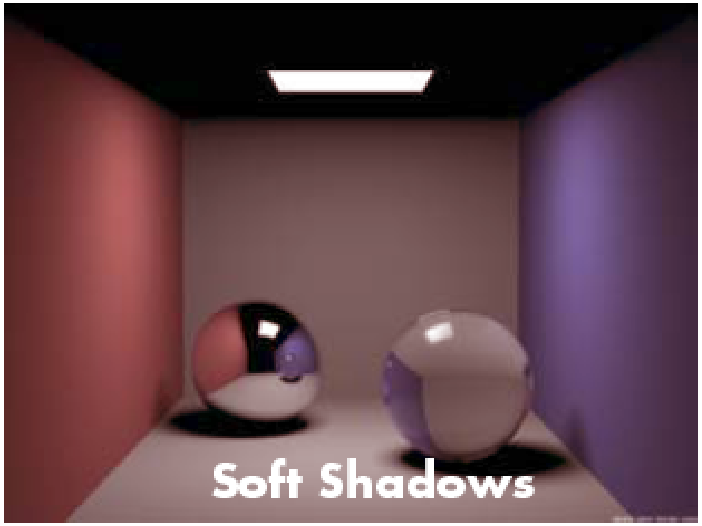


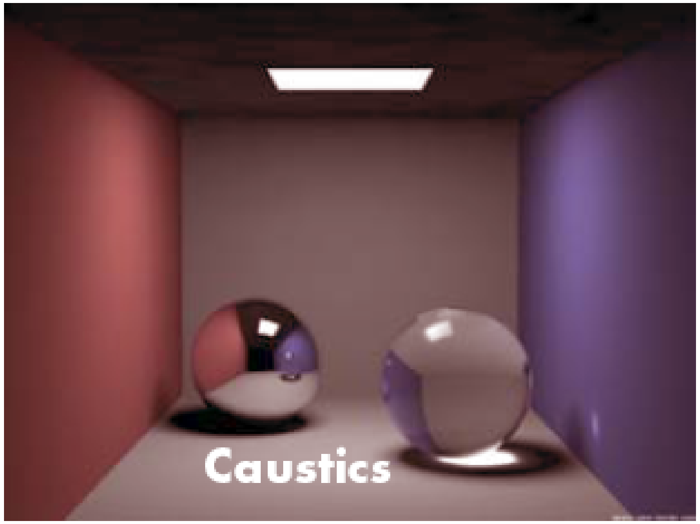
red hue

blue hue

Lighting Effects



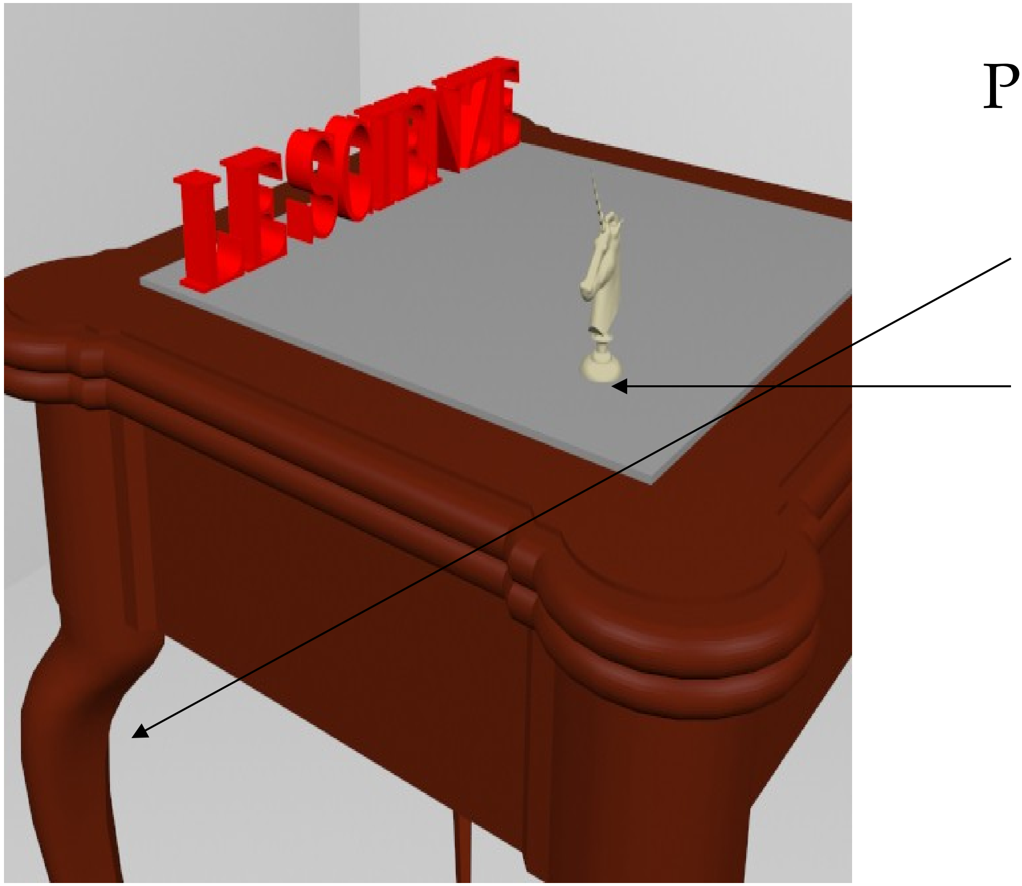




The ambient lighting in the upper-right image is approximated by a constant value. This is typical of most scanline algorithms. The middle and lower-left images were rendered with a ray tracing global illumination algorithm.

The middle image was rendered with no ambient light calculations. The lower-left image was rendered with several levels of diffuse re-reflection to give a better approximation of the ambient light in this scene.

Phong Shading



Plastic looking scene

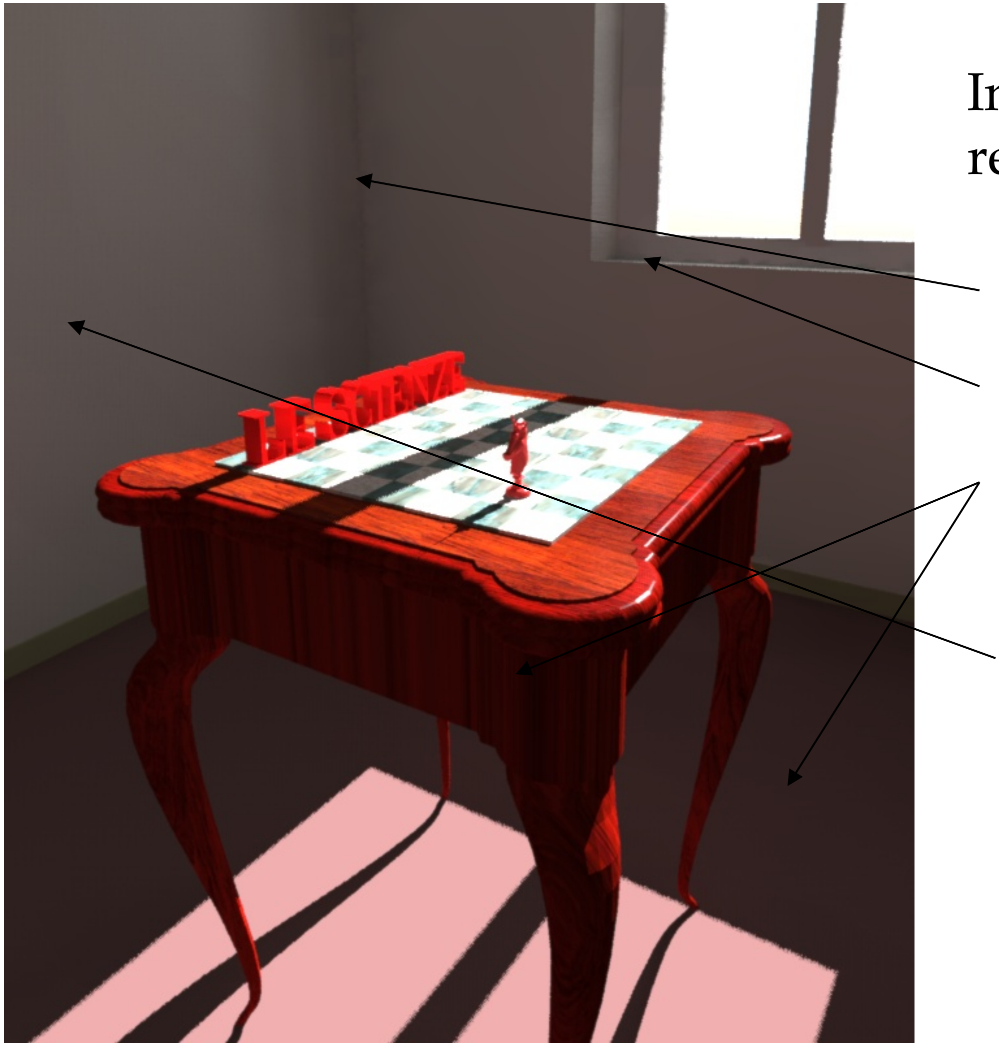
- no object interactions
- no shadows

Ray Tracing

Scene doesn’t look realistic enough.

- where is the corner of room?
- is window flush with wall?
- is the carpet and wood supposed to be this dark?

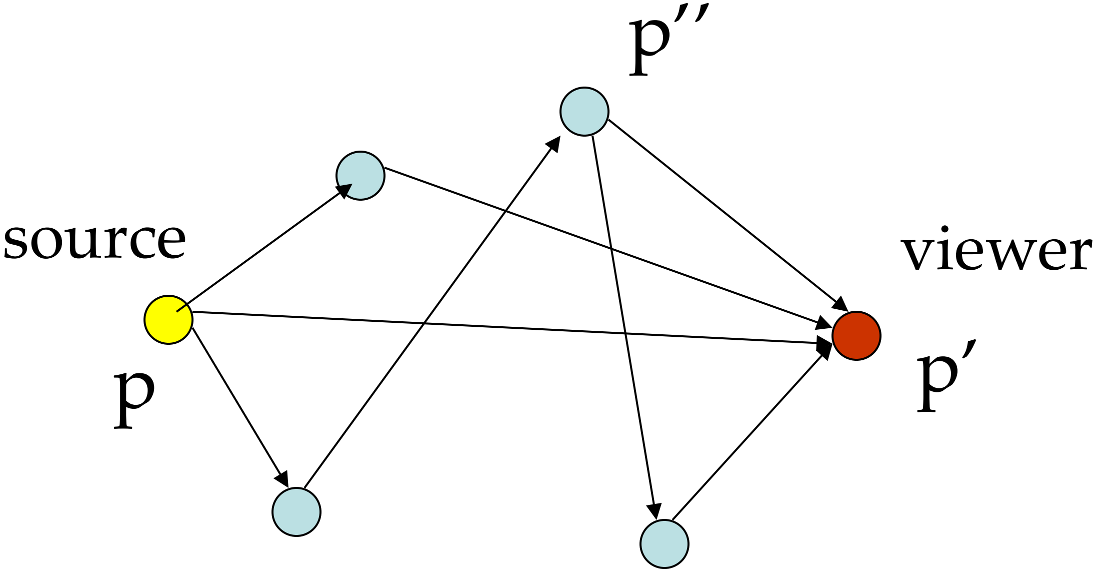
Radiosity – today's topic



Indirect lighting affects realism.

- room has a corner
- window has depth
- carpet and wood on table is lighter
- walls look more pink

The Rendering Equation – Graph Style



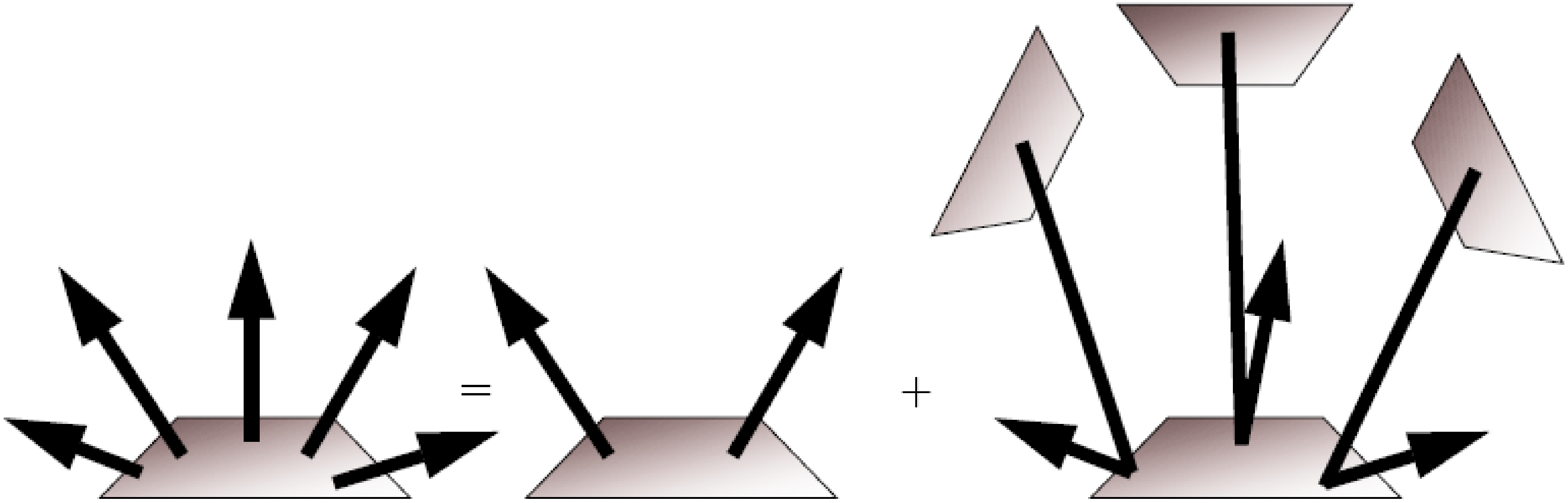
$$i(\mathbf{p},\mathbf{p}') = \nu(\mathbf{p},\mathbf{p}')(\epsilon(\mathbf{p},\mathbf{p}') + \int \rho(\mathbf{p},\mathbf{p}',\mathbf{p}'')i(\mathbf{p}',\mathbf{p}'')d\mathbf{p}'')$$

Visibility
(shadows)

Emission
(light source)

Reflectance from
Surfaces

Conservation of Energy



Emitted power = self-emitted power + received & reflected power

Diffuse Interreflections - Radiosity

- Consider lambertian surfaces and sources.
- Radiance independent of viewing direction.
- Consider total power leaving per unit area of a surface.
- Can simulate soft shadows and color bleeding from diffuse surfaces.
- Used abundantly in heat transfer literature

Irradiance, Radiosity

- Irradiance E is the power **received** per unit surface area
  - Units: W/m^2

- Radiosity
  - Power per unit area **leaving** the surface (like irradiance)

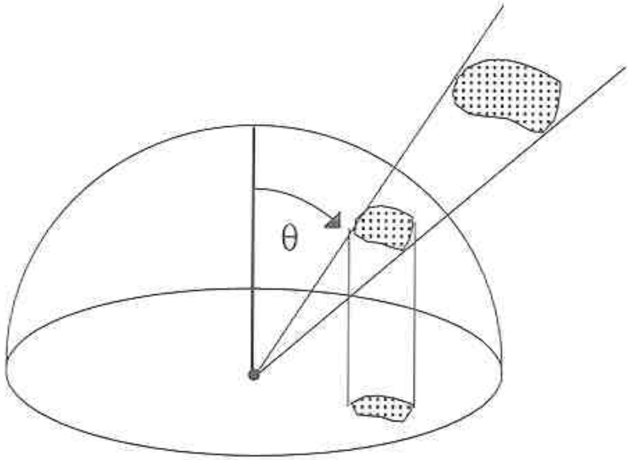
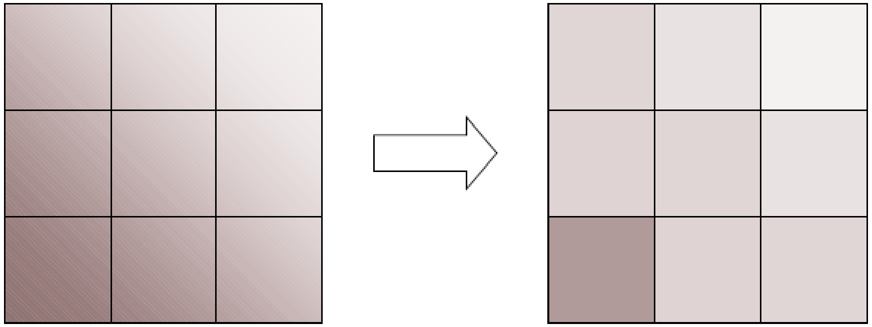


Figure 2.8: *Projection of differential area.*

Planar piecewise constancy assumption

- Subdivide scene into small “uniform” polygons



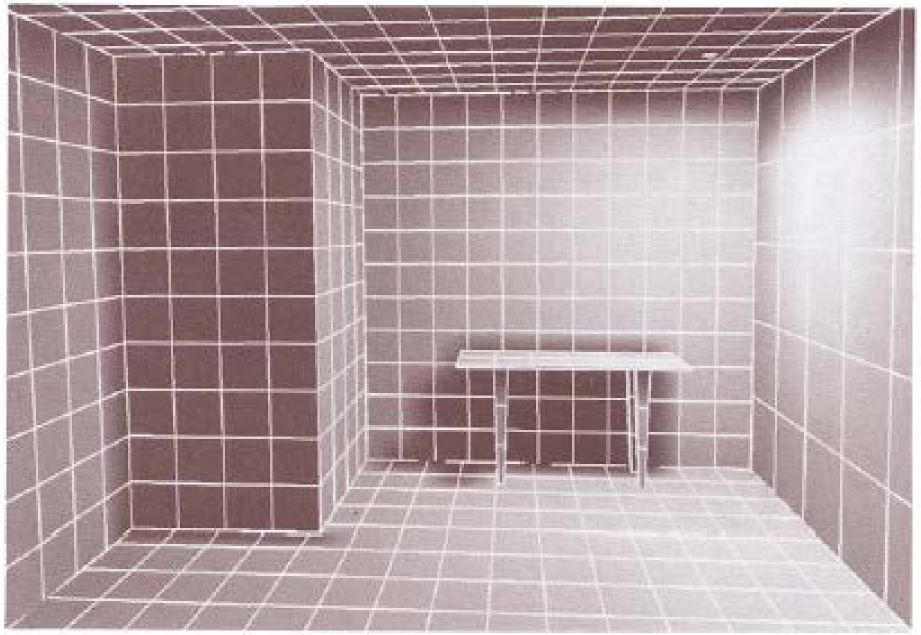


Table in room sequence from Cohen and Wallace

Power Equation

- Power from each polygon:

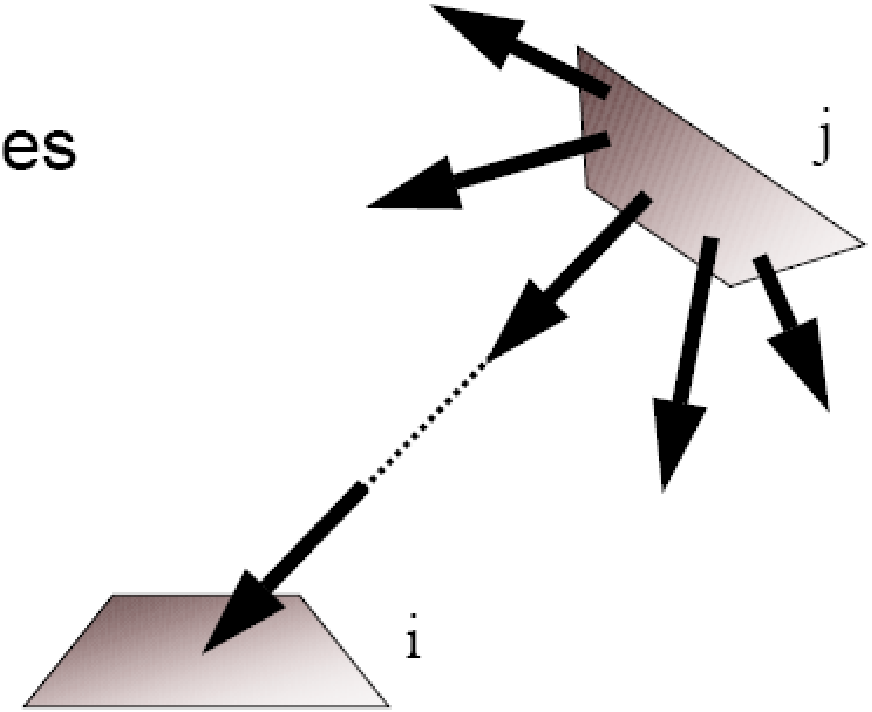
$$\forall i : \Phi_i = \Phi_{ei} + \rho_i \sum_{j=1}^{N} \Phi_j \; F(i \rightarrow j)$$

- Linear System of Equations:

  - Φ_i : power of patch i (unknown)
  - $\Phi_{e,i}$: emission of patch i (known)
  - ρ_l : reflectivity of patch i (known)
  - $F(j \rightarrow i)$: form-factor (coefficients of matrix)

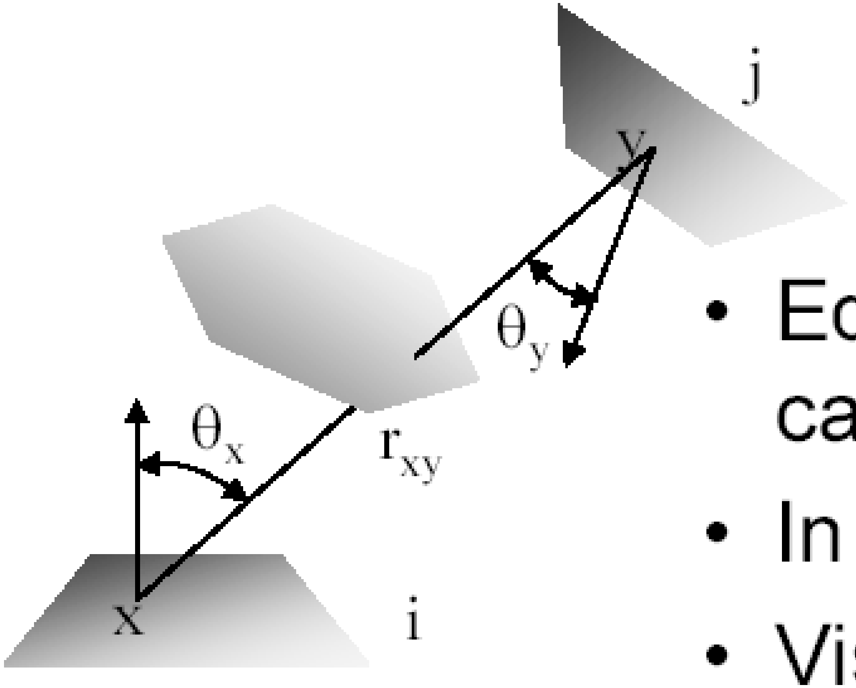
Form Factor

- $F_{j \rightarrow i}$ = the fraction of power emitted by j, which is received by i
- <u>Area</u>
  - – if i is smaller, it receives less power
- <u>Orientation</u>
  - – if i faces j, it receives more power
- <u>Distance</u>
  - – if i is further away, it receives less power



Form Factor

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y)\, dA_y\, dA_x$$



- Equations for special cases (polygons)
- In general hard problem
- Visibility makes it harder

Form Factors Invariant

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x, y)\, dA_y\, dA_x$$

$$F(i \rightarrow j) = \frac{1}{A_i} \int_{A_j} \int_{A_i} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x, y)\, dA_x\, dA_y$$

$$F(i \rightarrow j) A_i = F(j \rightarrow i) A_j$$

Form Factor Computation

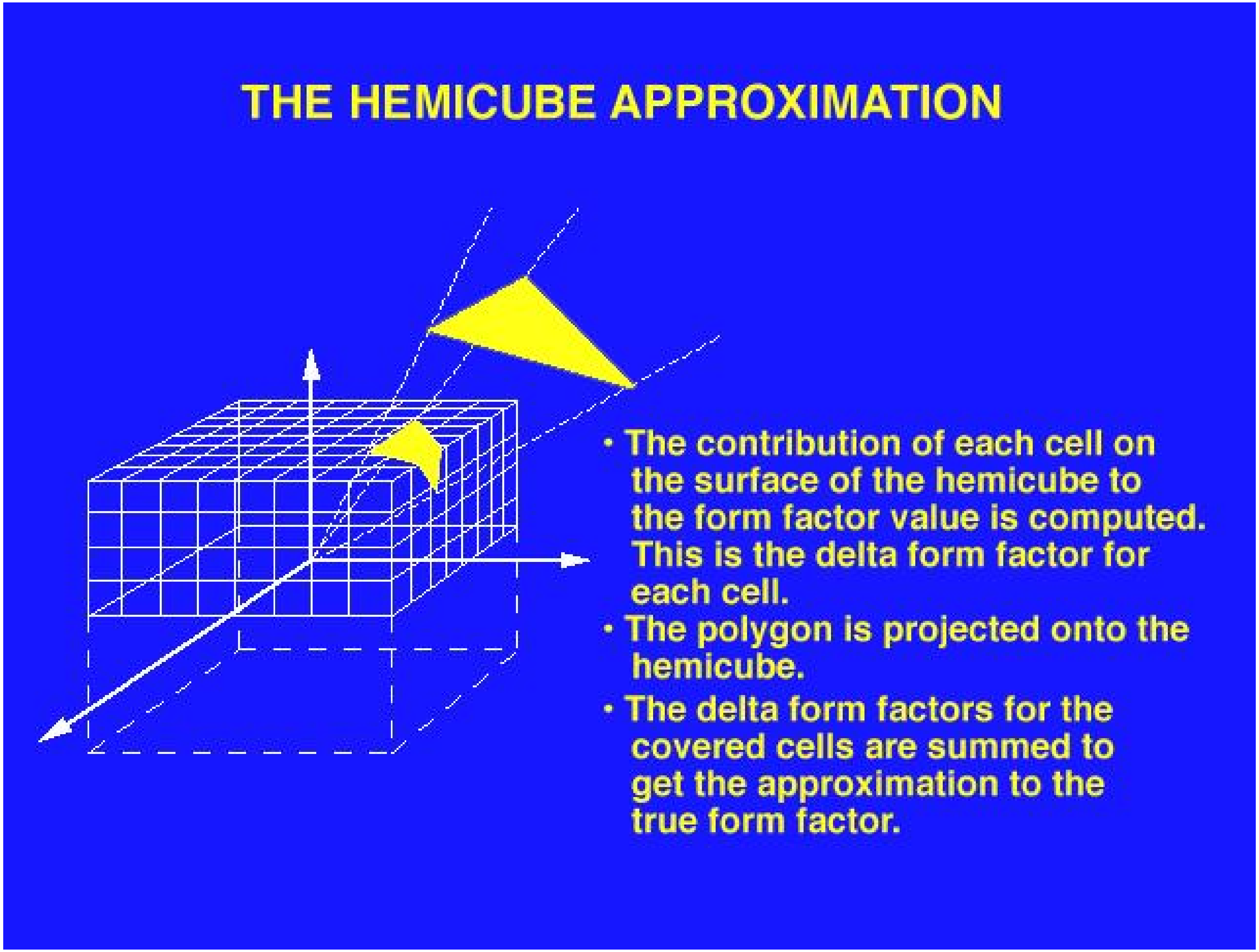
$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x, y)\, dA_y\, dA_y$$

- Schroeder and Hanrahan derived an analytic expression for polygonal surfaces.

- In general, computing double integral is hard.

- Use Monte Carlo Integration.

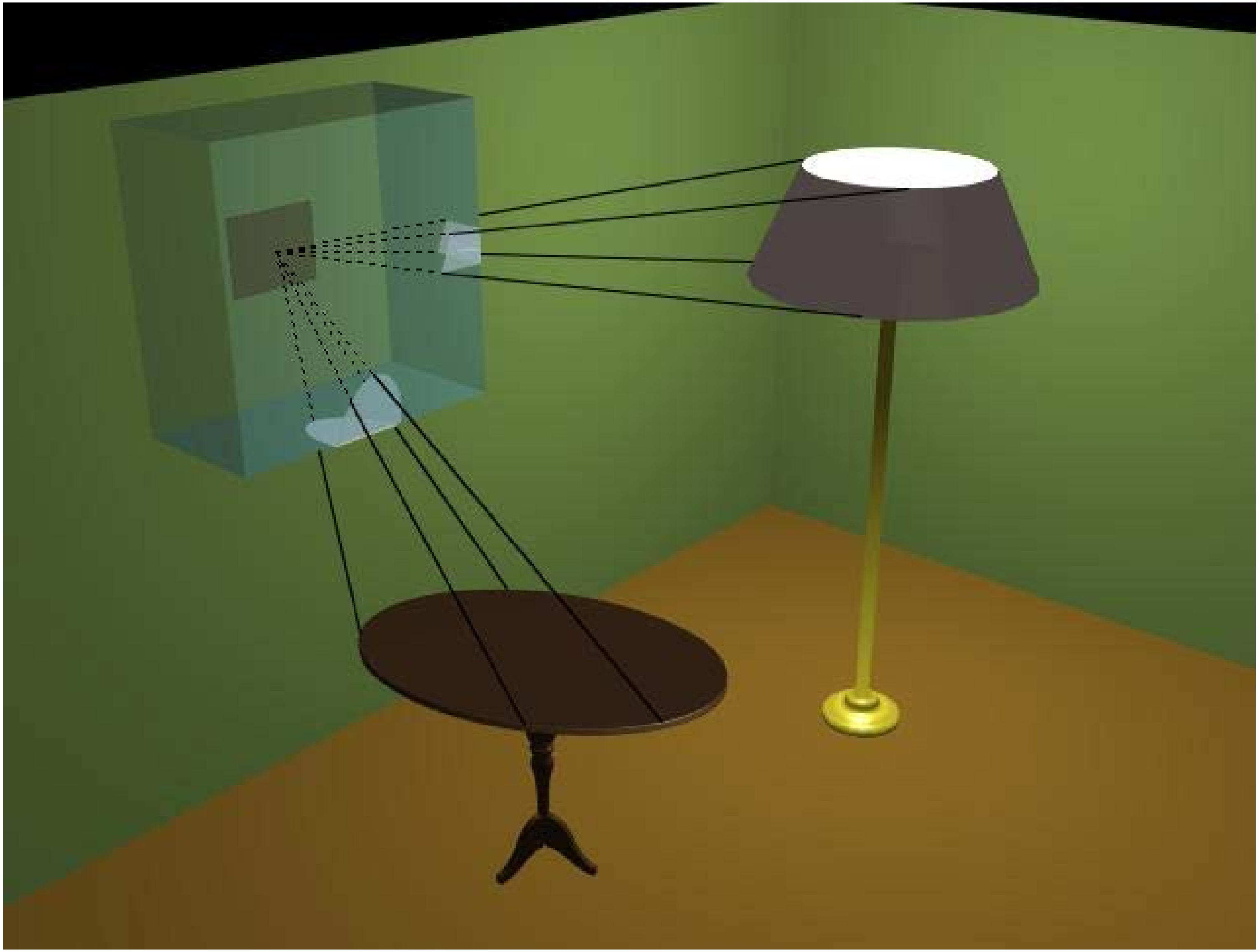
Form Factor Computation



THE HEMICUBE APPROXIMATION

- The contribution of each cell on the surface of the hemicube to the form factor value is computed. This is the delta form factor for each cell.
- The polygon is projected onto the hemicube.
- The delta form factors for the covered cells are summed to get the approximation to the true form factor.

Form Factor Computation



Power →Radiosity

$$\Phi_i = \Phi_{e,i} + \rho_i \sum_{j=1}^{N} \Phi_j F(j \to i)$$

$$\Big\downarrow \text{ Divide by } A_i$$

$$\frac{\Phi_i}{A_i} = \frac{\Phi_{e,i}}{A_i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(j \to i)}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j \dfrac{F(i \to j) A_i}{A_j}}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(i \to j)}{A_j}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \to j)$$

Linear System of Radiosity Equations

$$\forall \text{patches i:} \qquad B_i = B_{ei} + \rho_i \sum_j F_{i \to j} B_j$$

$$\underset{\downarrow \atop \text{Known}}{\begin{bmatrix} 1 - \rho_1 F_{1 \to 1} & -\rho_1 F_{1 \to 2} & \dots & -\rho_1 F_{1 \to n} \\ -\rho_2 F_{2 \to 1} & 1 - \rho_2 F_{2 \to 2} & \dots & -\rho_2 F_{2 \to n} \\ \dots & \dots & \dots & \dots \\ -\rho_n F_{n \to 1} & -\rho_n F_{n \to 2} & \dots & 1 - \rho_n F_{n \to n} \end{bmatrix}} \underset{\downarrow \atop \text{Unknown}}{\begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_n \end{bmatrix}} = \underset{\downarrow \atop \text{Known}}{\begin{bmatrix} B_{e1} \\ B_{e2} \\ \dots \\ B_{en} \end{bmatrix}}$$

- Matrix Inversion to Solve for Radiosities.

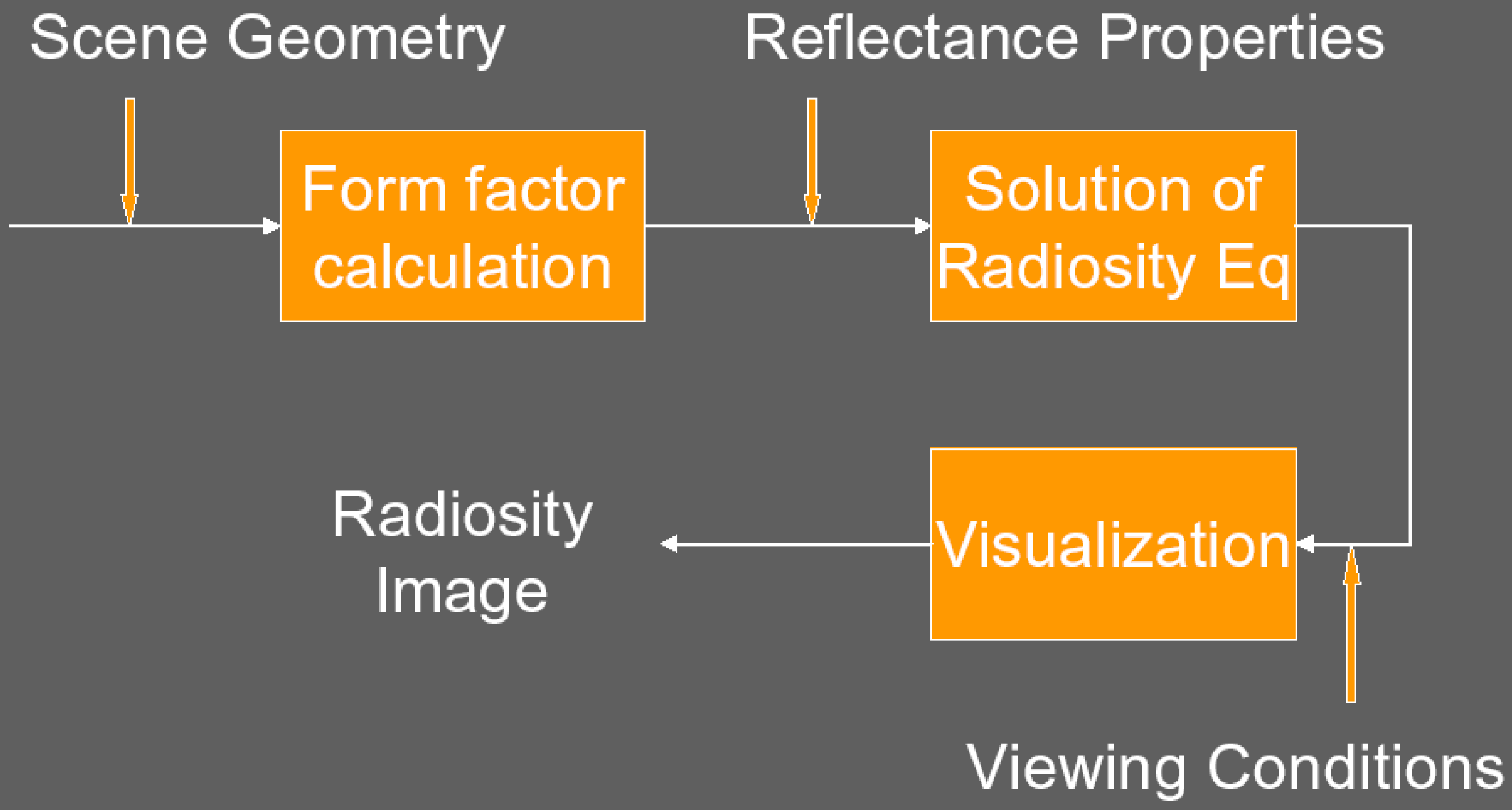
Iterative approaches

- Jacobi iteration
- Start with initial guess for energy distribution (light sources)
- Update radiosity/power of all patches based on the previous guess

$$\underset{\text{new value}}{\underset{\swarrow}{B_i}} = B_{e,i} + \rho_i \sum_{j=1}^{N} \underset{\text{old values}}{\underset{\searrow}{B_j}} F(i \rightarrow j)$$

- Repeat until converged

Radiosity "Pipeline"



Scene Geometry

Reflectance Properties

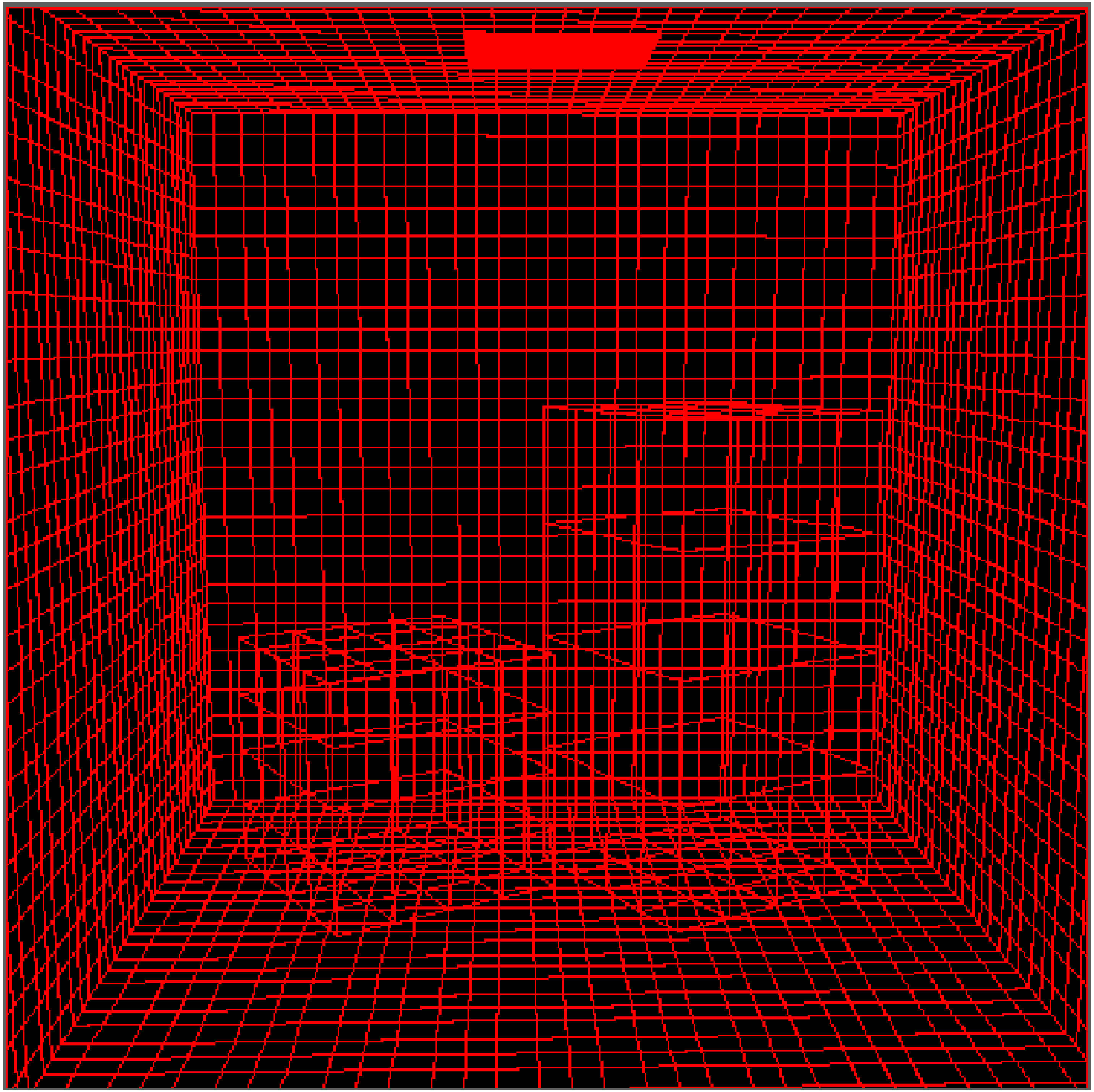
Form factor calculation

Solution of Radiosity Eq

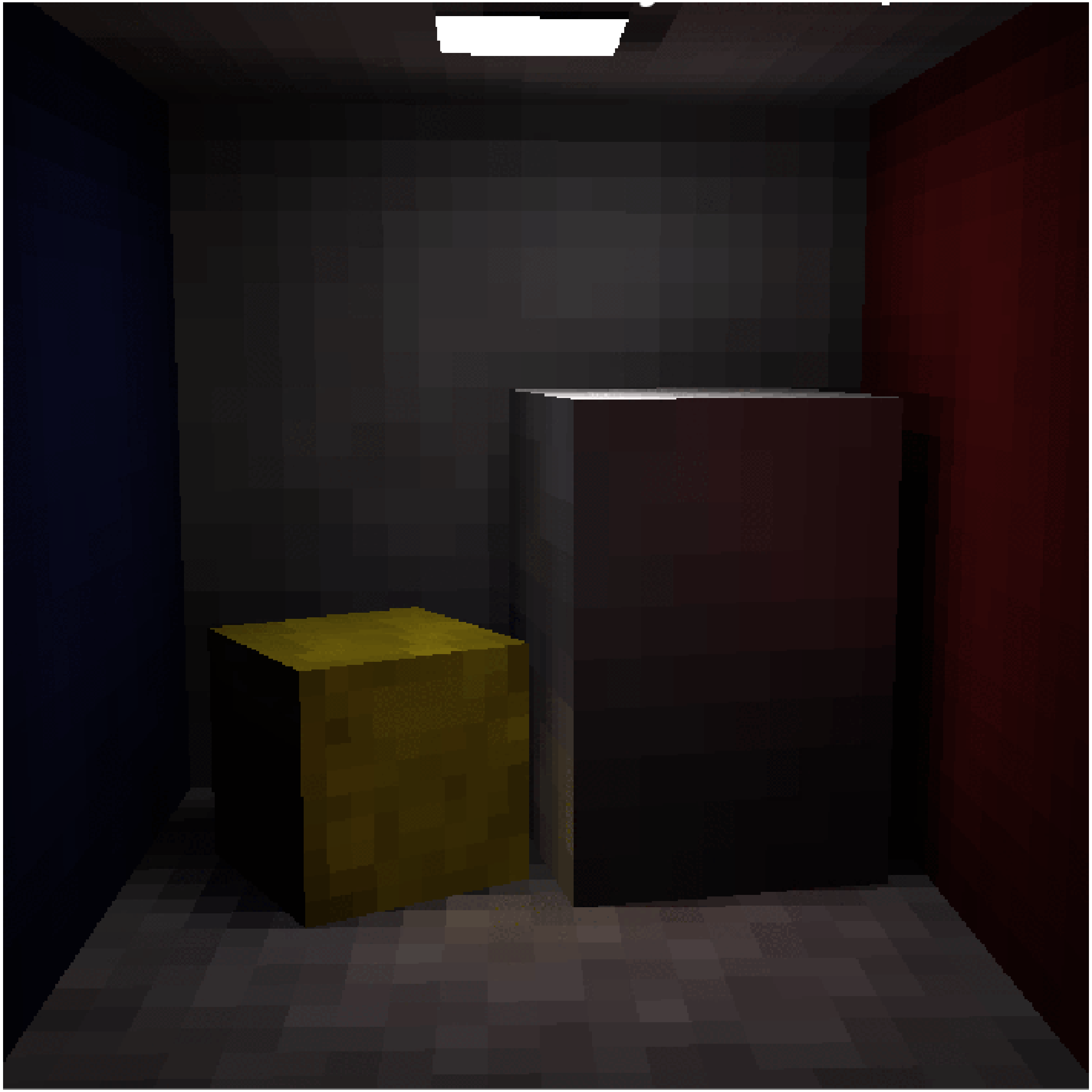
Radiosity Image

Visualization

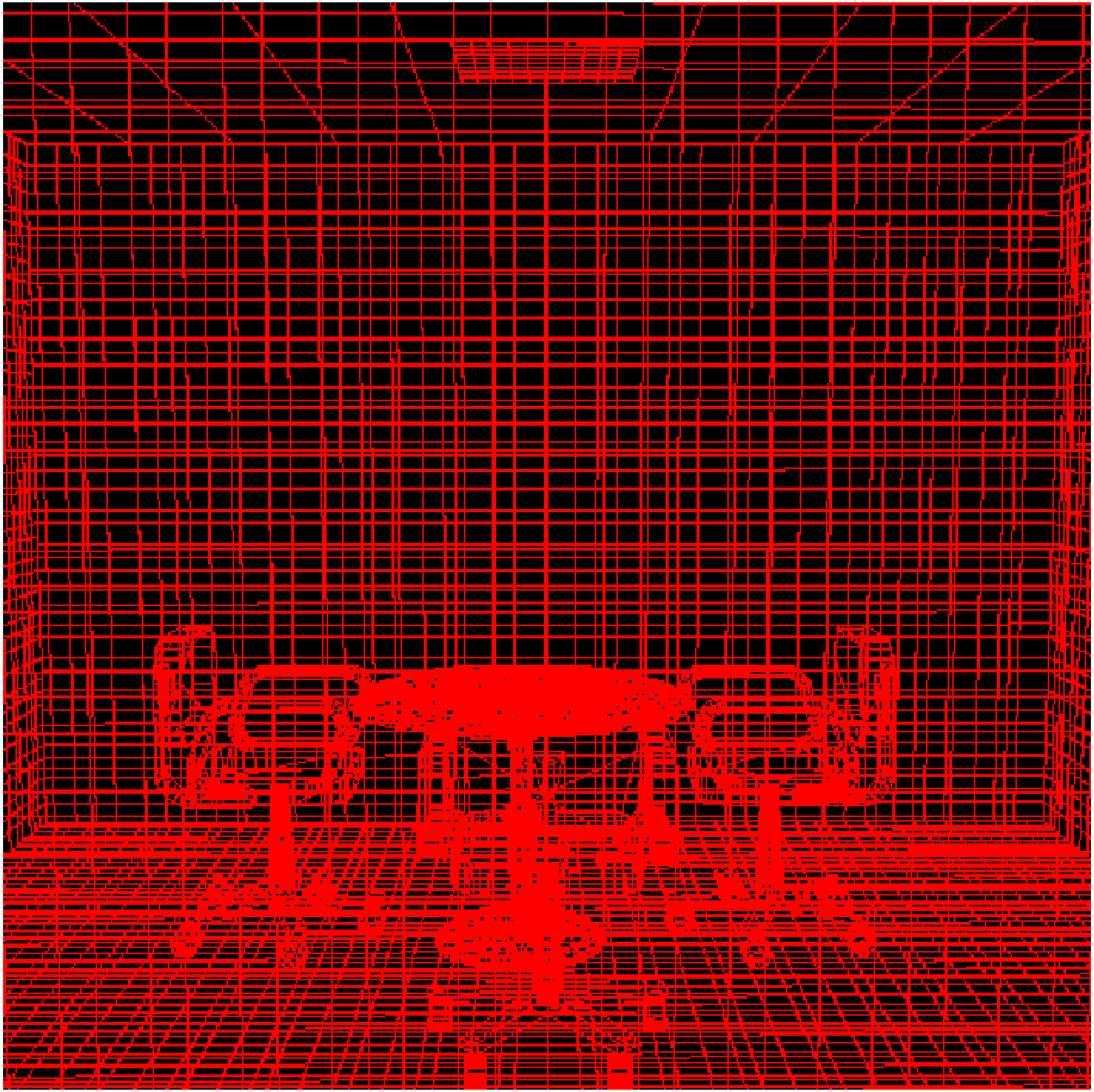
Viewing Conditions



Wireframe



- Classical Approach
- No Interpolation



Wireframe

- Classical Approach
- Low Res

- Classical Approach
- High Res
- More accurate

- Classical Approach
- High Res
- Interpolated

PROGRESSIVE SOLUTION

The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.

Sample Scenes

Sample Scenes

From Cohen, Chen, Wallace and Greenberg 1988

Sample Scenes

Sample Scenes

Sample Scenes

Summary

Classic radiosity = finite element method

Assumptions

- Diffuse reflectance
- Usually polygonal surfaces

Advantages

- Soft shadows and indirect lighting
- View independent solution
- Precompute for a set of light sources
- Useful for walkthroughs

Image vs. Object Space

- Image space: Ray tracing
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)
- Object space: Radiosity
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL

Two Pass Solution

- First Pass: Diffuse Interreflections

  View independent, global diffuse illumination computed with radiosity.

- Second Pass: Specular Interreflections

  View dependent, global specular illumination computed with ray-tracing.

- Combine strengths of radiosity and ray-tracing.

Interreflections :

The Inverse Problem

Lecture #12