# Refractive and Specular Light-Path Triangulation

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# Objective

How to reconstruct an object that does not has its own appearance

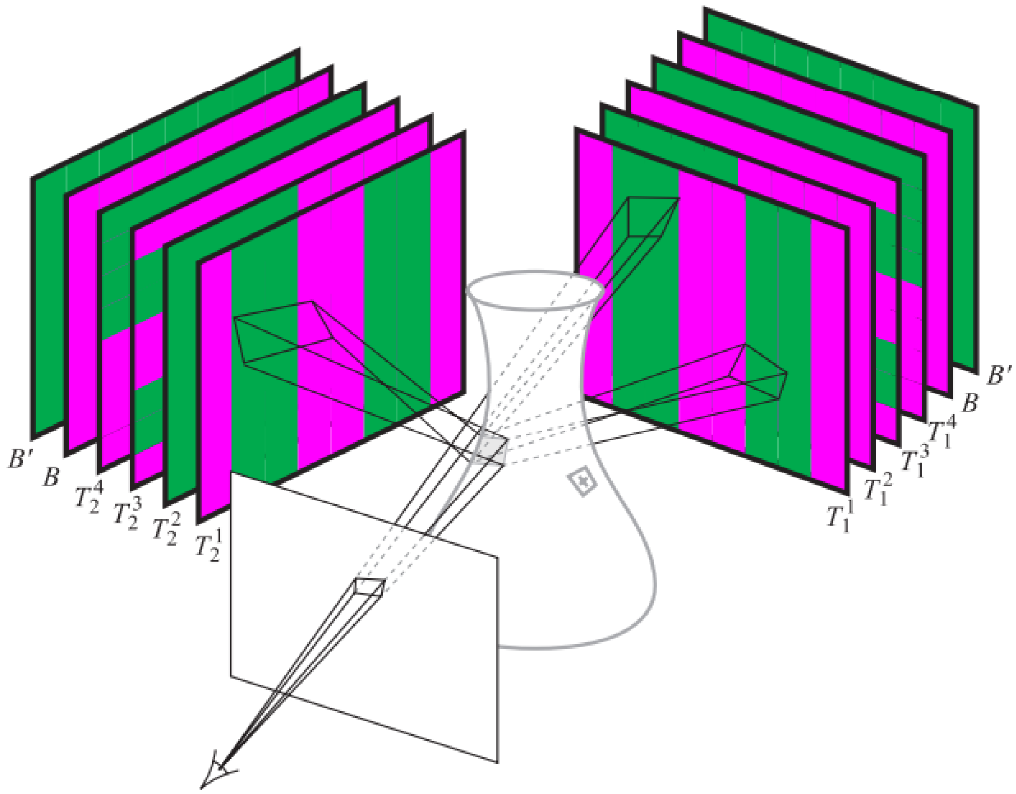
# Failure of stereo matching

# Shape recovery with models

- Limited to very simple objects.

Ben Ezra and Nayar, “What does motion reveal about transparency?”

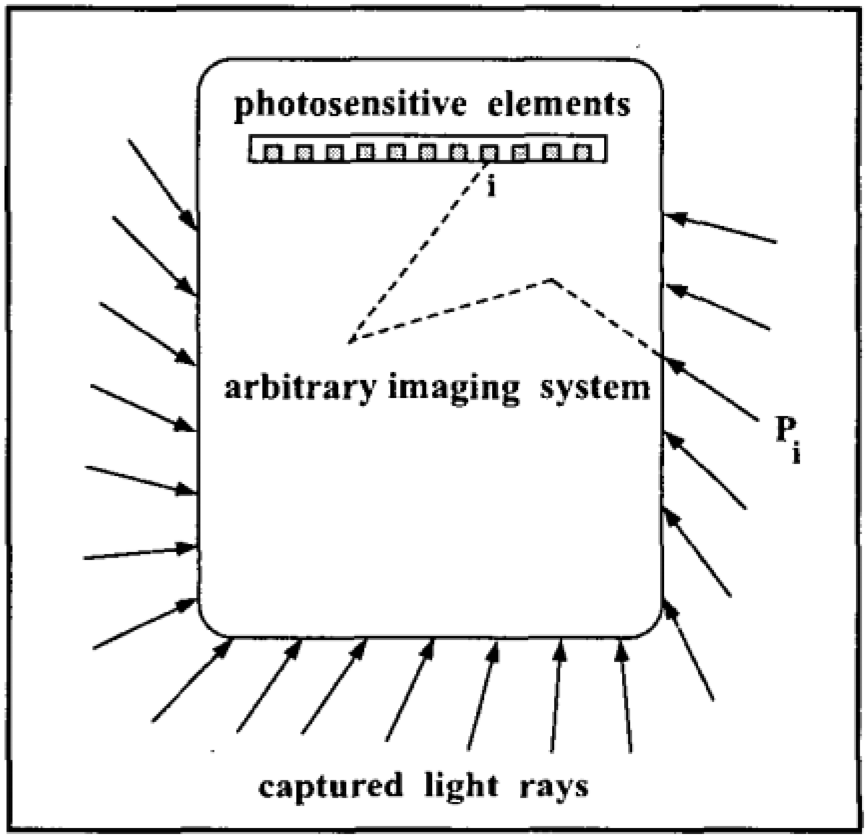
# Environmental matting




**Arrangement of cameras, mirrors and lenses and unknown. Estimate the mapping from environment to image.**

Zonker et al., "Environment matting and compositing."

# Generalized imaging models




**Non-perspective mapping of rays. Assumes correspondence between rays and sensor elements.**

Grossberg and Nayar, “A general imaging model and a method to find its parameters.”

# In short ...

- All previous methods use simple models or structured light.
- Black box assumption of scene settings.
- No single image reconstruction algorithms.

# A Theory of Refractive and Specular 3D Shape by Light-Path Triangulation

- Provides a general framework for analyzing refraction and reflection.
- Characterize the light path.
- Provide reconstruction algorithms for some cases.

# Light-Path Triangulation

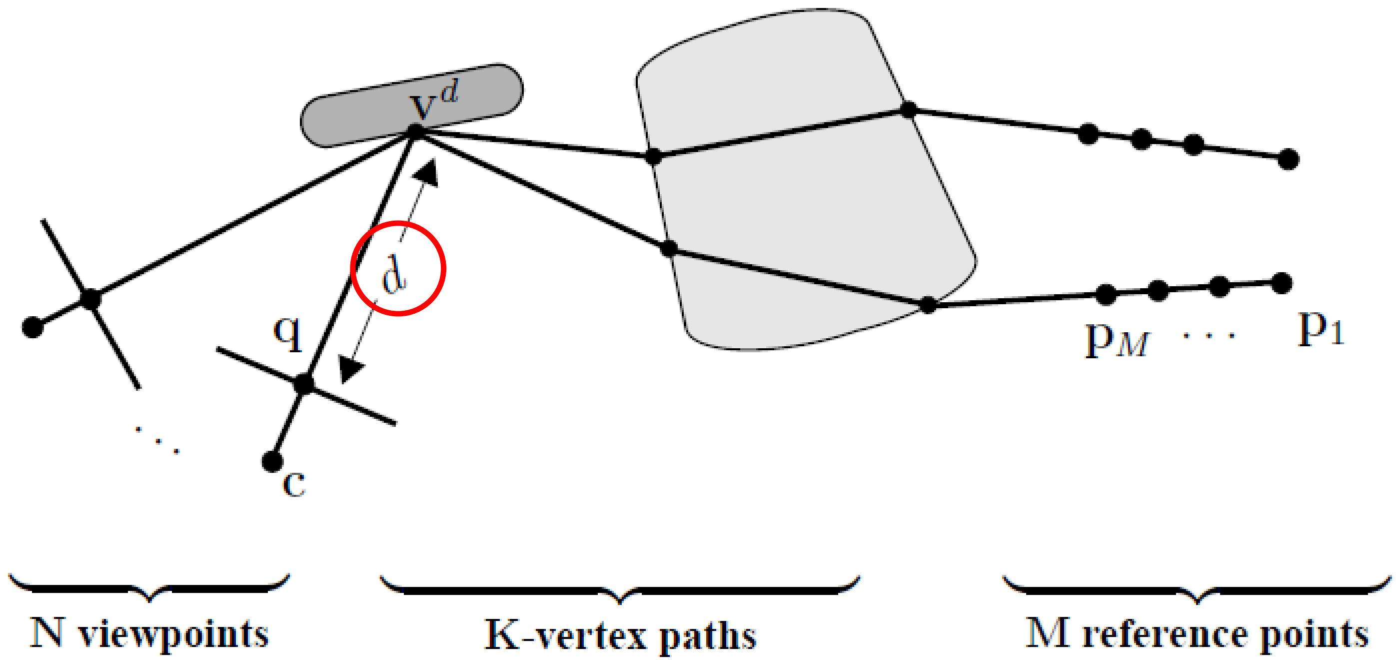
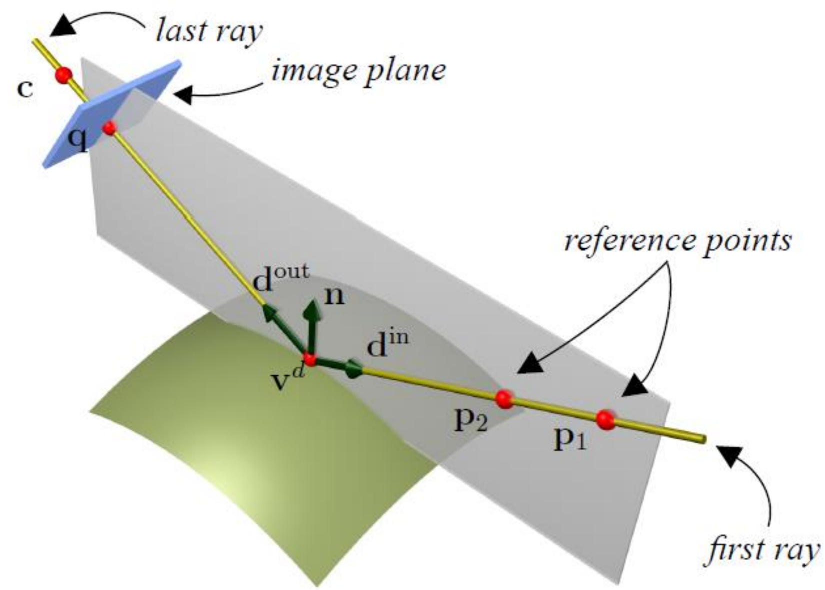



Figure 3: Basic geometry of $\langle N, K, M \rangle$-triangulation.

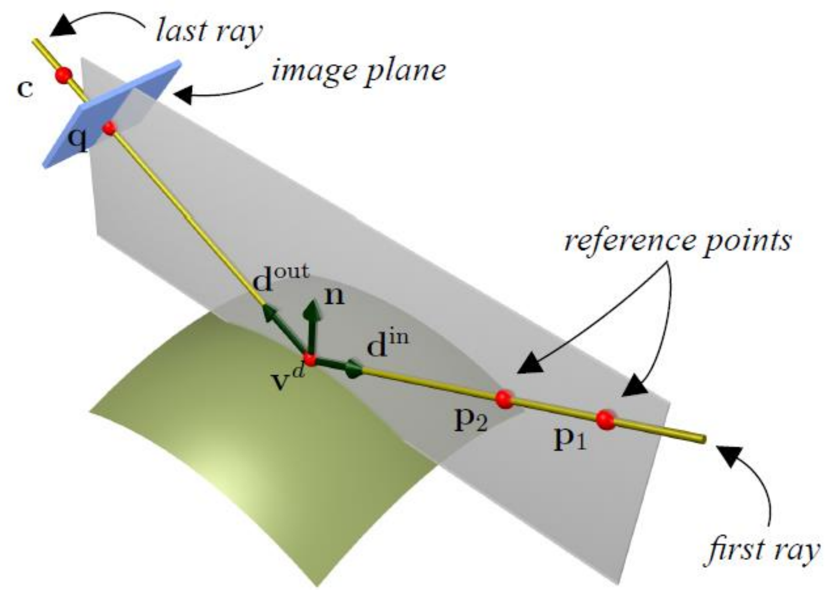
# Three light path properties.

1. $\mathbf{n}, \mathbf{d}^{\mathrm{in}}, \mathbf{d}^{\mathrm{out}}$ are coplanar.



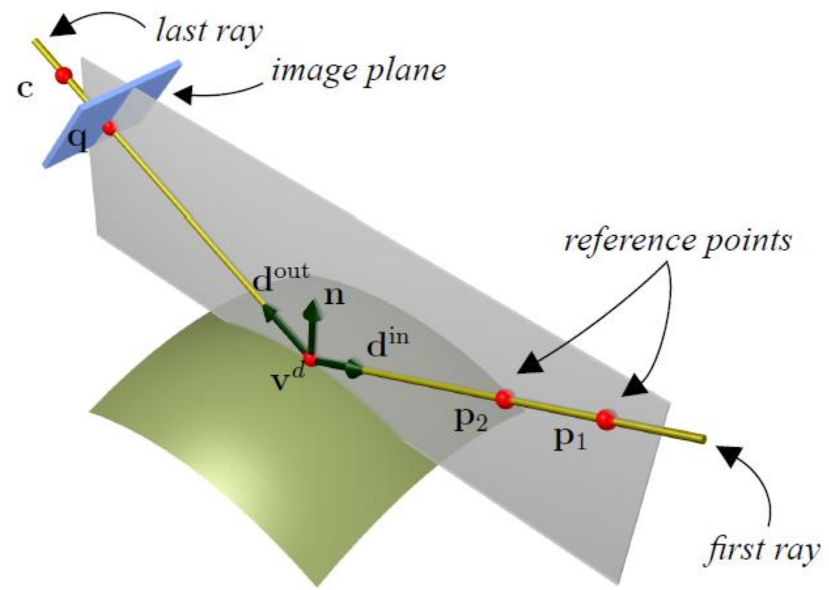
# Three light path properties.

1. $\mathbf{n}, \mathbf{d}^{\mathrm{in}}, \mathbf{d}^{\mathrm{out}}$ are coplanar.
2. With known refractive index and any two of $\mathbf{n}, \mathbf{d}^{\mathrm{in}}, \mathbf{d}^{\mathrm{out}}$, the third can be determined.

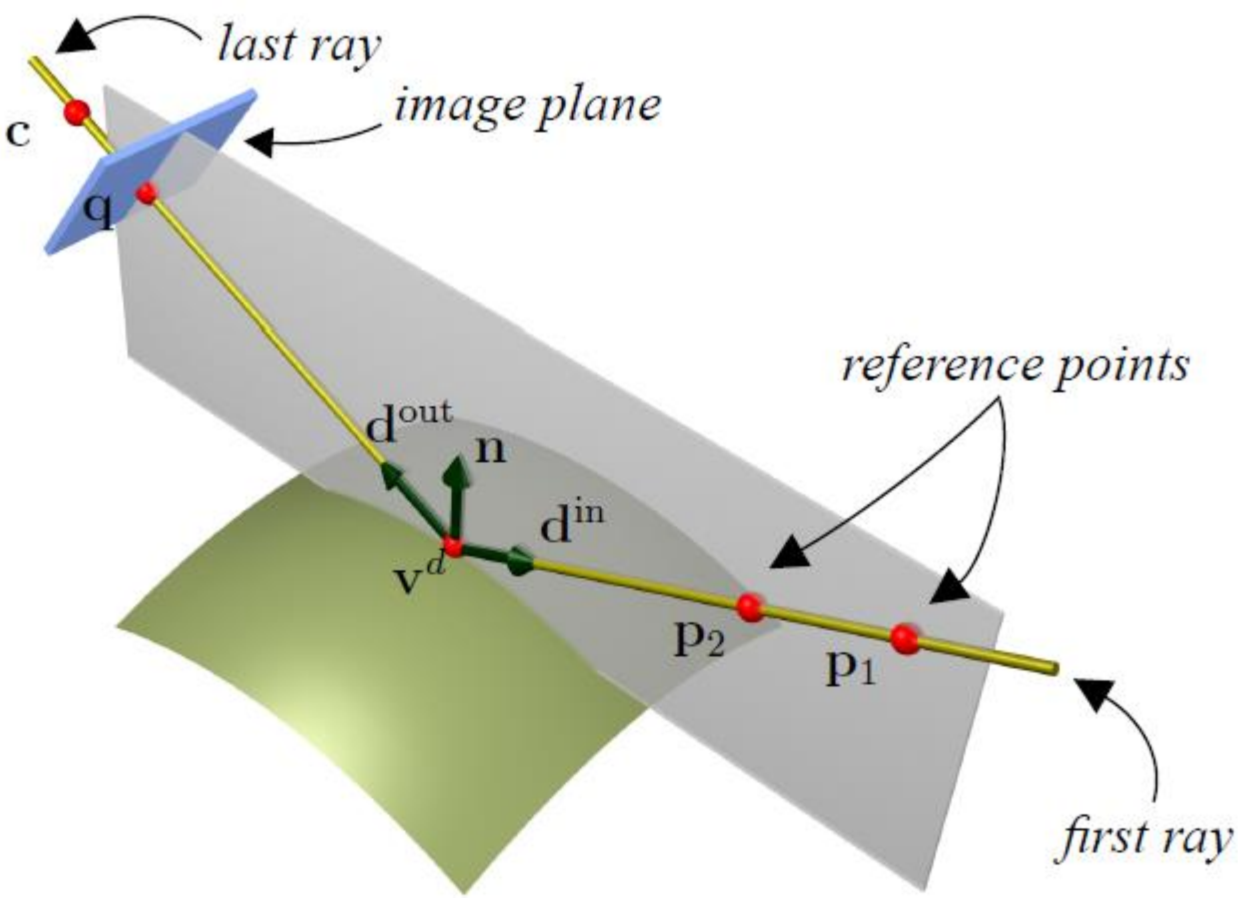


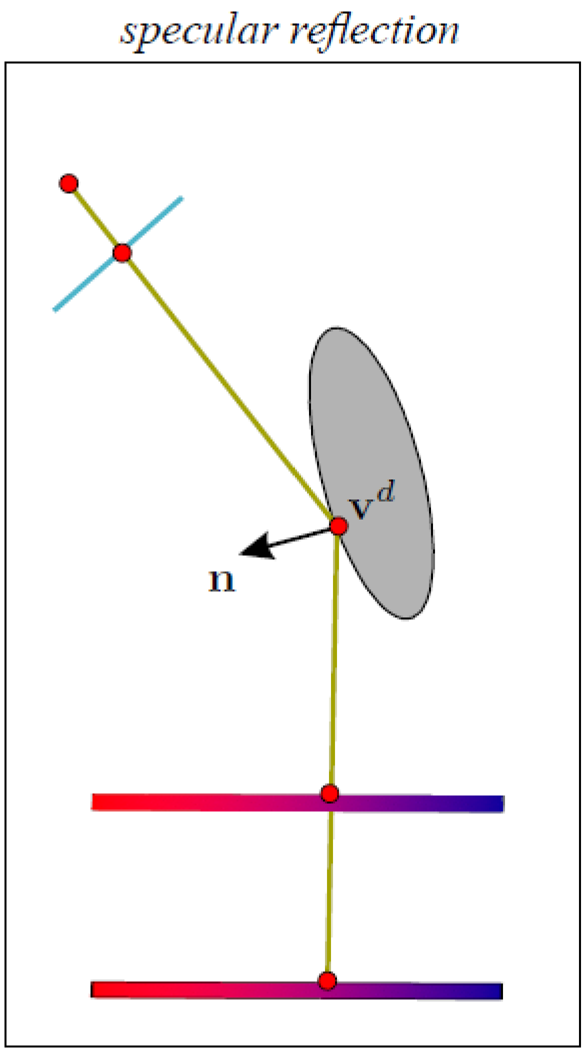
# Three light path properties.

3. Reference point corresponding to same image point will be on the same first ray.



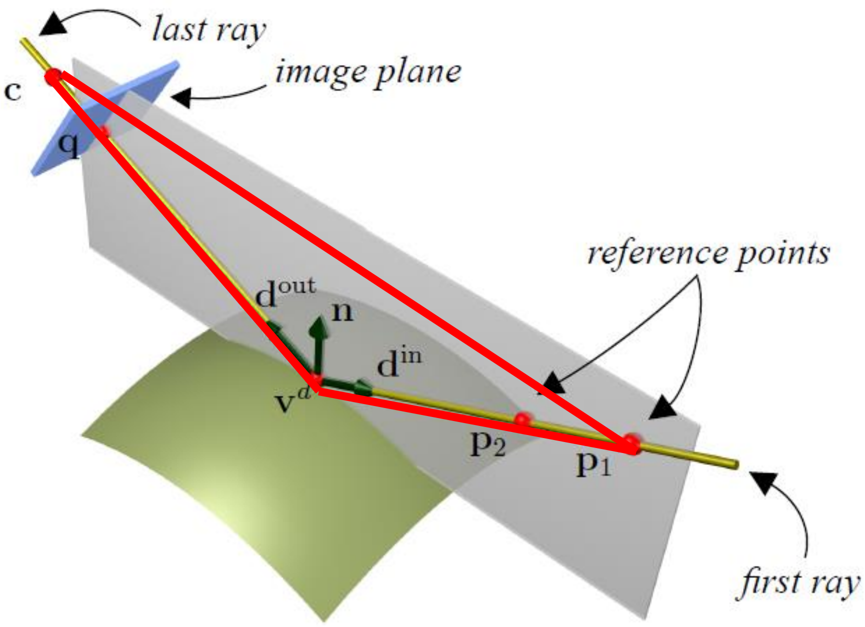
# (1,1,2)-triangulation



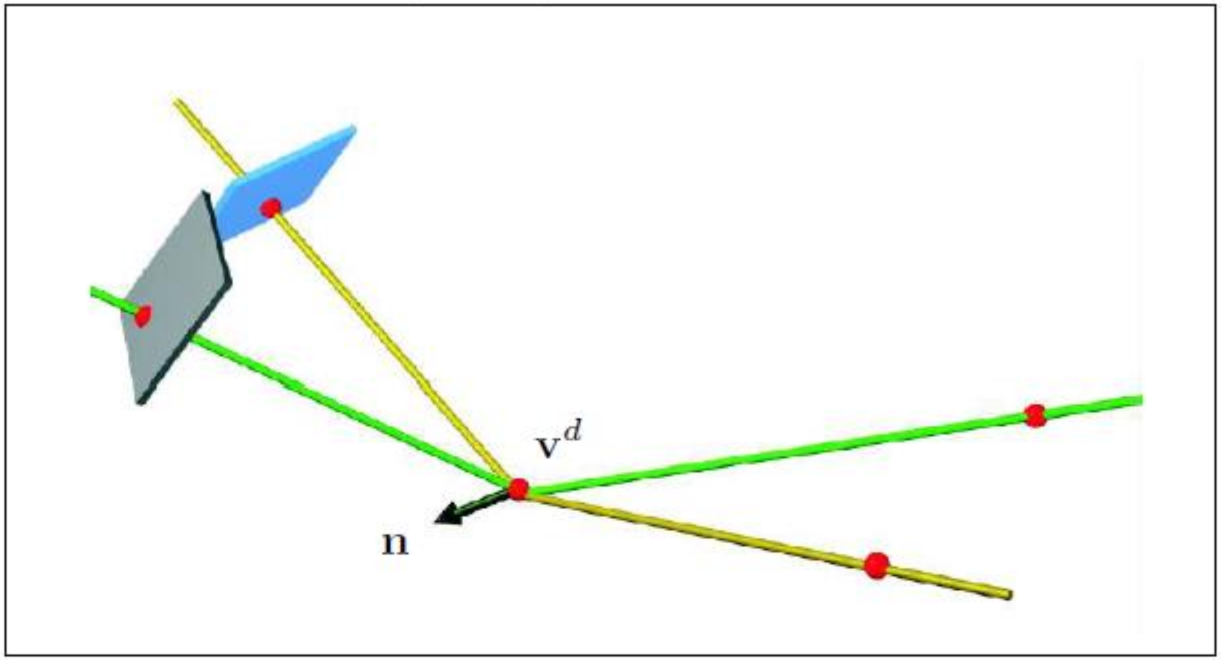
# (1,1,2)-reconstruction

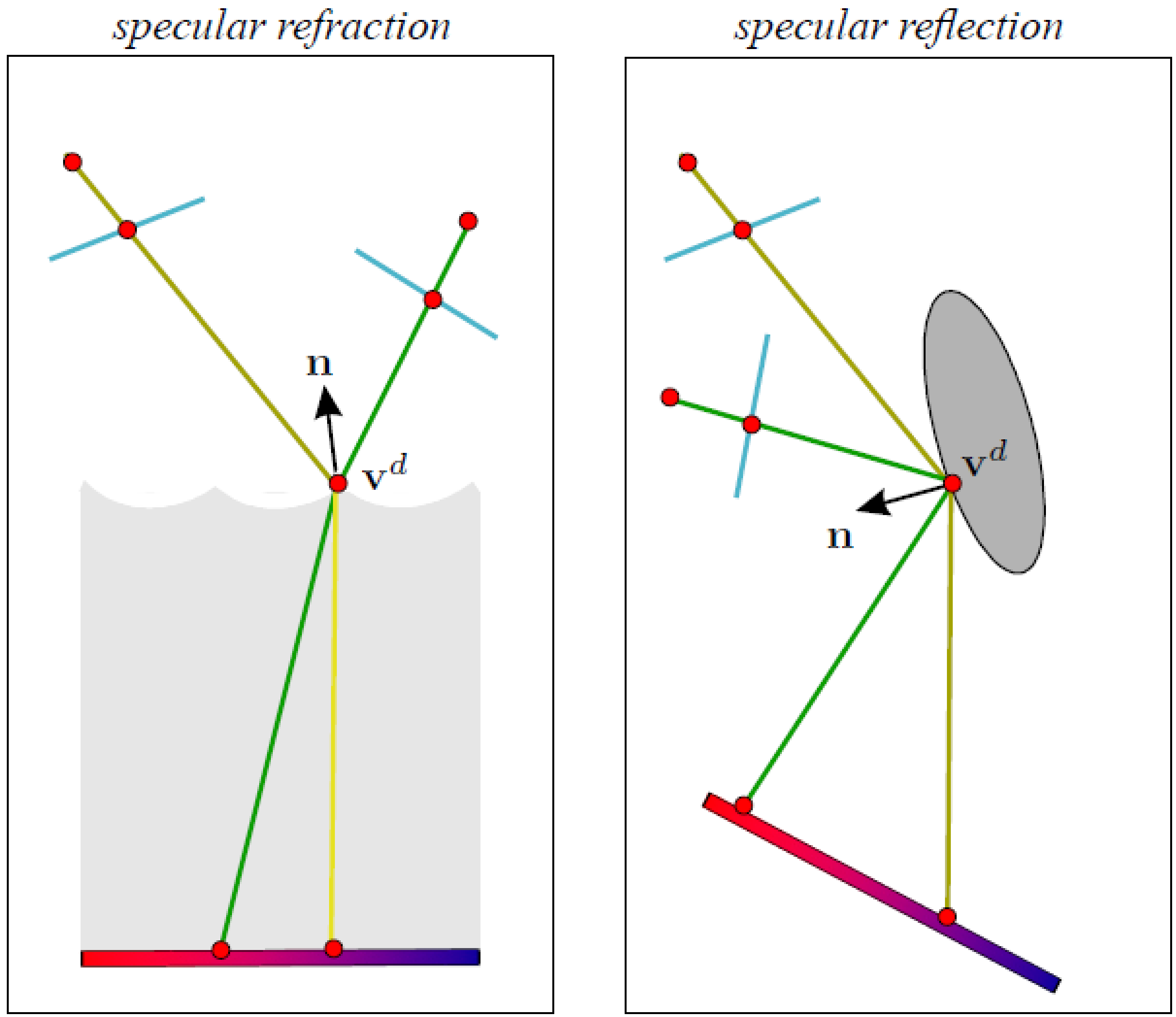
- Law of sine



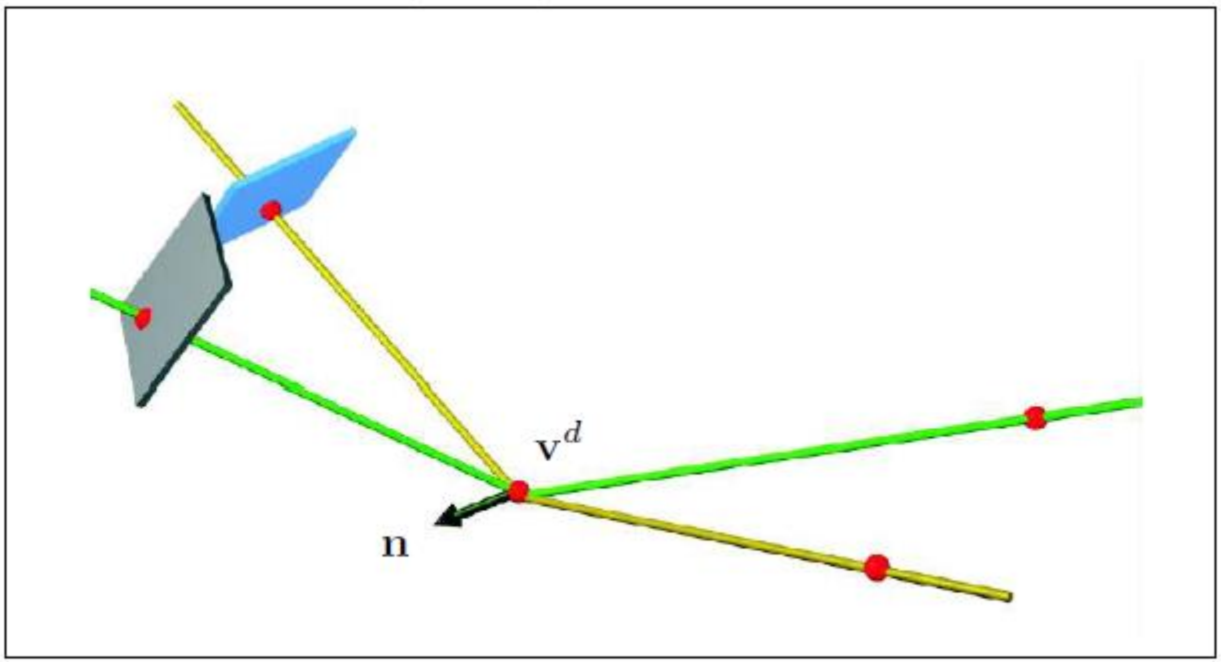

$$d = \frac{\|(\mathbf{p}_1 - \mathbf{c}) \times \mathbf{d}^{\text{in}}\|}{\|\mathbf{d}^{\text{out}} \times \mathbf{d}^{\text{in}}\|}$$

# (2,1,1)-triangulation



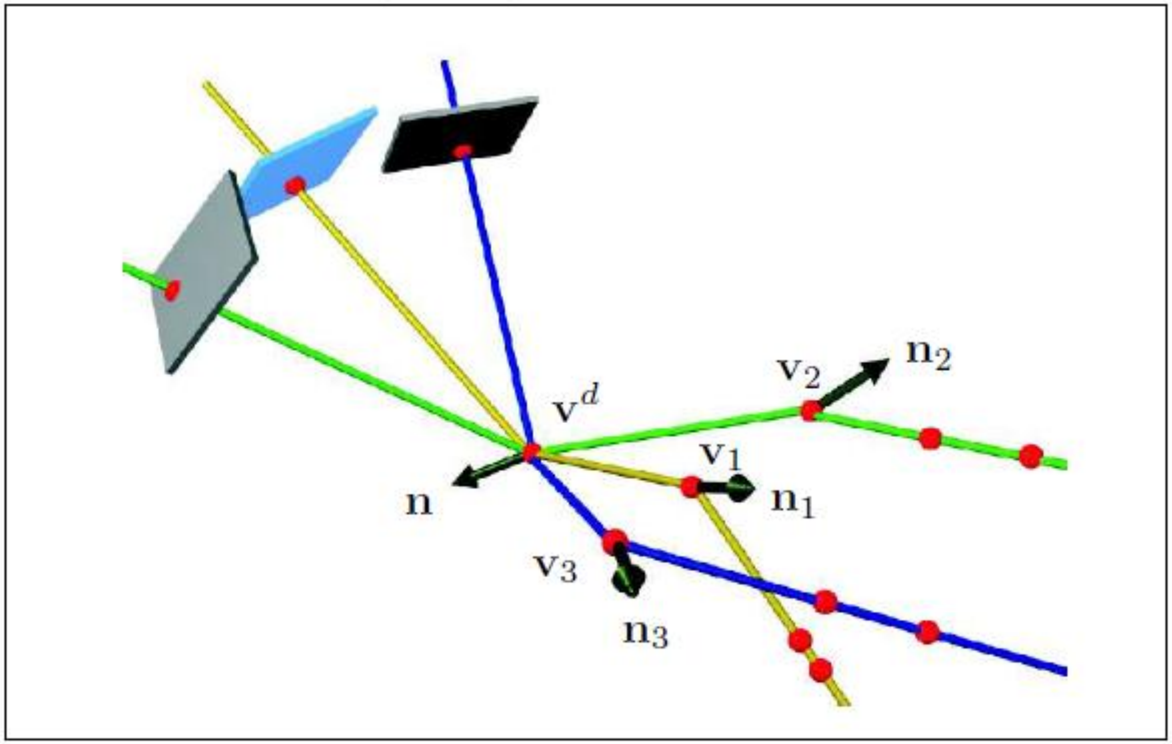

specular refraction
specular reflection
n
$\mathbf{v}^d$
n
$\mathbf{v}^d$

# (2,1,1)-triangulation

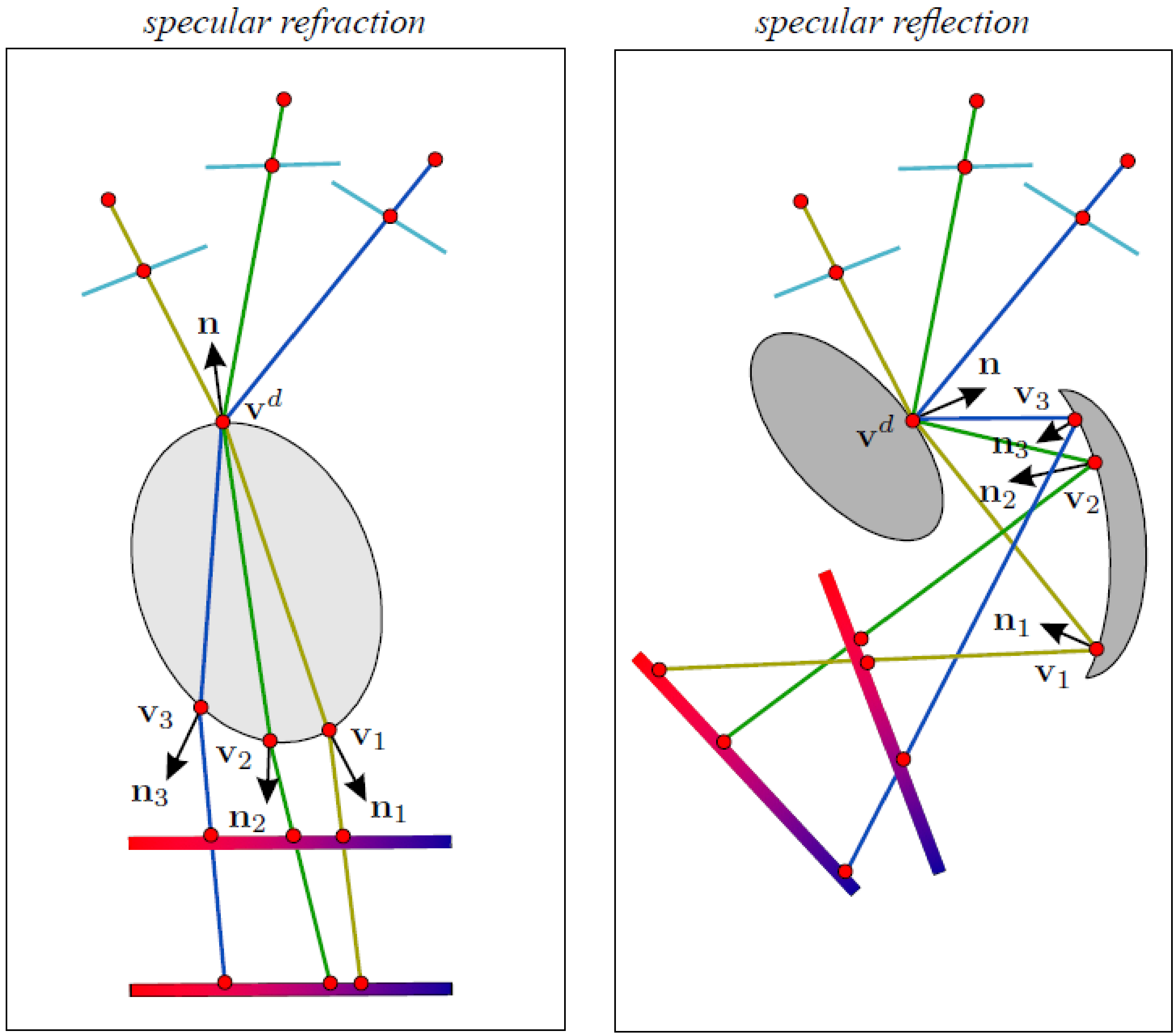



d is used for reconstruction

# (3,2,2)-triangulation

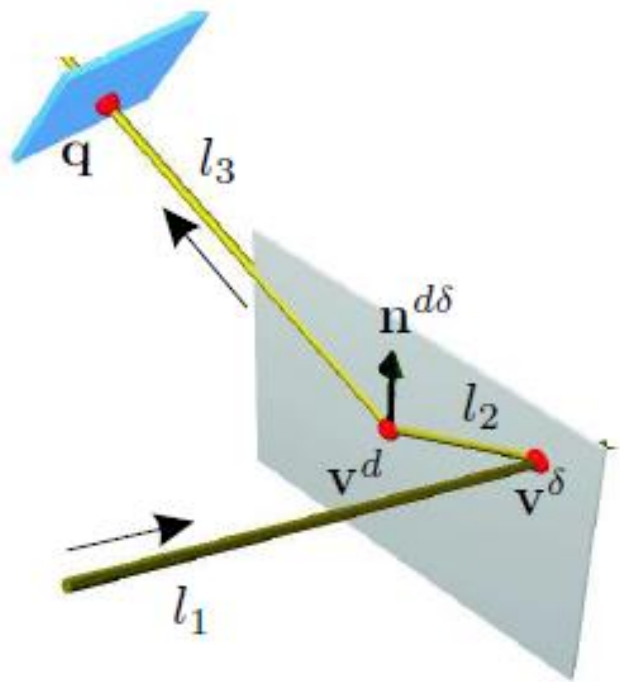


# (3,2,2)-triangulation


specular refraction
specular reflection
n
v^d
v_3
v_2
v_1
n_3
n_2
n_1

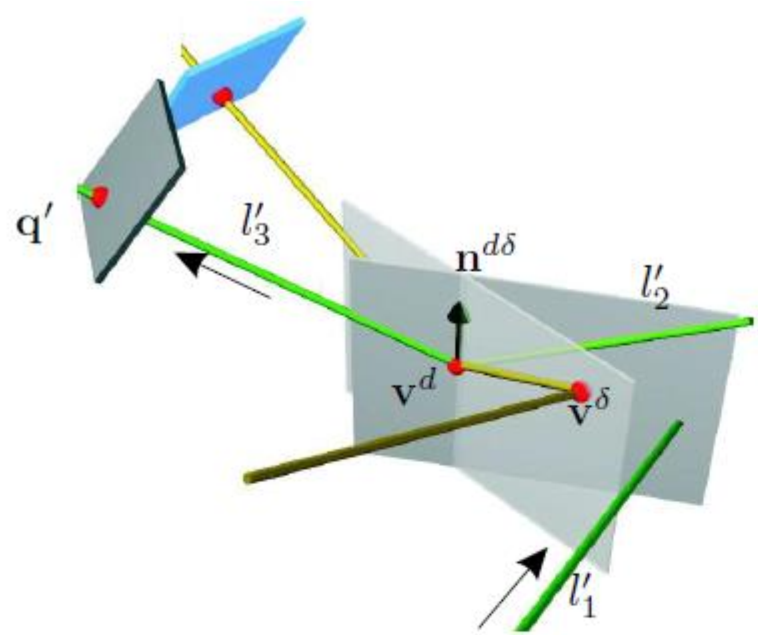
# (3,2,2)-reconstruction

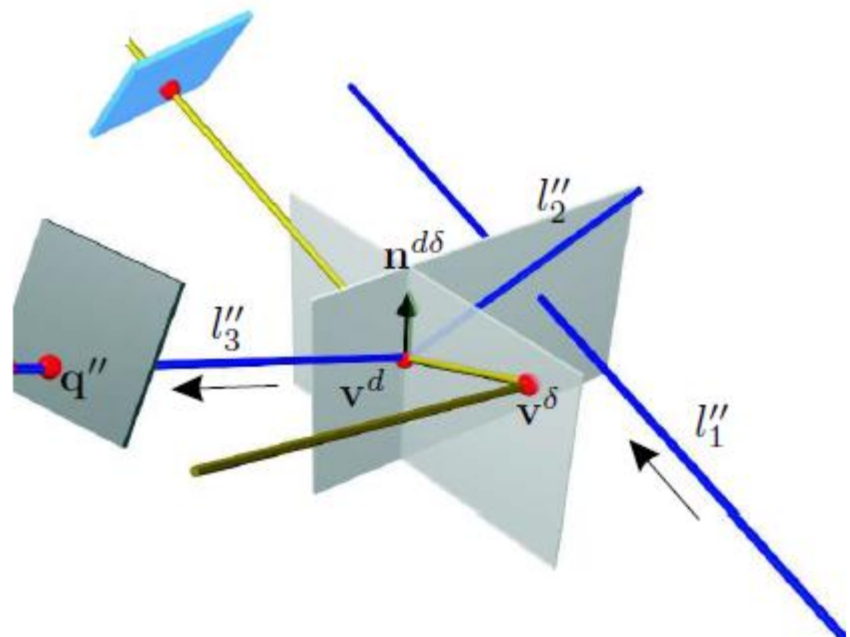
Use d and δ for analyzing the light path, and assume a refractive index




Searching through (d, δ) plane to find discrete set of solution.

# (3,2,2)-reconstruction



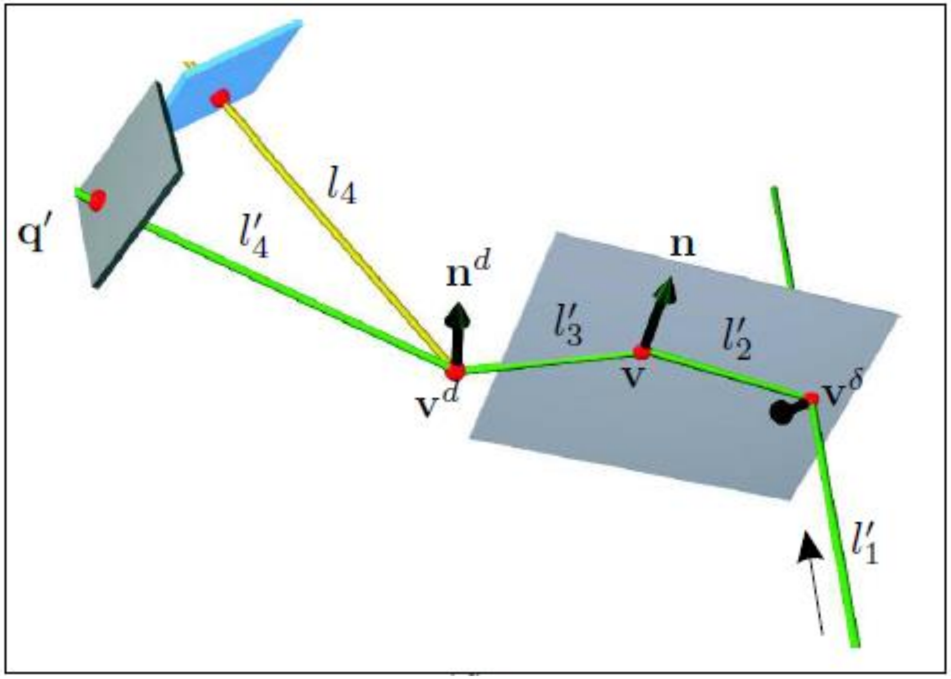



# Ambiguity

- There exist a discrete set of $(d, \delta)$ for each refractive index.

- (3,2,2)-triangulation is tractable only when the refractive index is known

# (N,3,2)-triangulation




Intractable, because of infinite combination of d and δ.

# Tractable cases

**Theorem 1** *The only tractable $\langle N, K, M \rangle$-triangulations are shown in the tables below:*

| **One reference point** $(\mathrm{M} = 1)$ | | | |
|---|---|---|---|
| | $K = 1$ | $K = 2$ | $K \geq 3$ |
| $N = 1$ | | | |
| $N \geq 2$ | ✓ × | | |

| **Two or more reference points** $(\mathrm{M} \geq 2)$ | | | |
|---|---|---|---|
| | $K = 1$ | $K = 2$ | $K \geq 3$ |
| $N = 1$ | ✓ × | | |
| $N = 2$ | ✓ × | | |
| $N = 3$ | ✓ × | ✓ | |
| $N \geq 4$ | ✓ × | ✓ × | |

*where '✓' marks tractable problems where the scene is either known to be a mirror or its refractive index is known; '×' marks tractable problems where the refractive index (or whether it is a mirror) is unknown; and blanks correspond to intractable cases.*

# possible ways to deal with K>=3 case

- If every scene point is intersected by at least 3(K-1) light paths of length <=K, and if the first and the last ray of the path are known, the location of the scene point is constrained to a 0-dimensional solution manifold.

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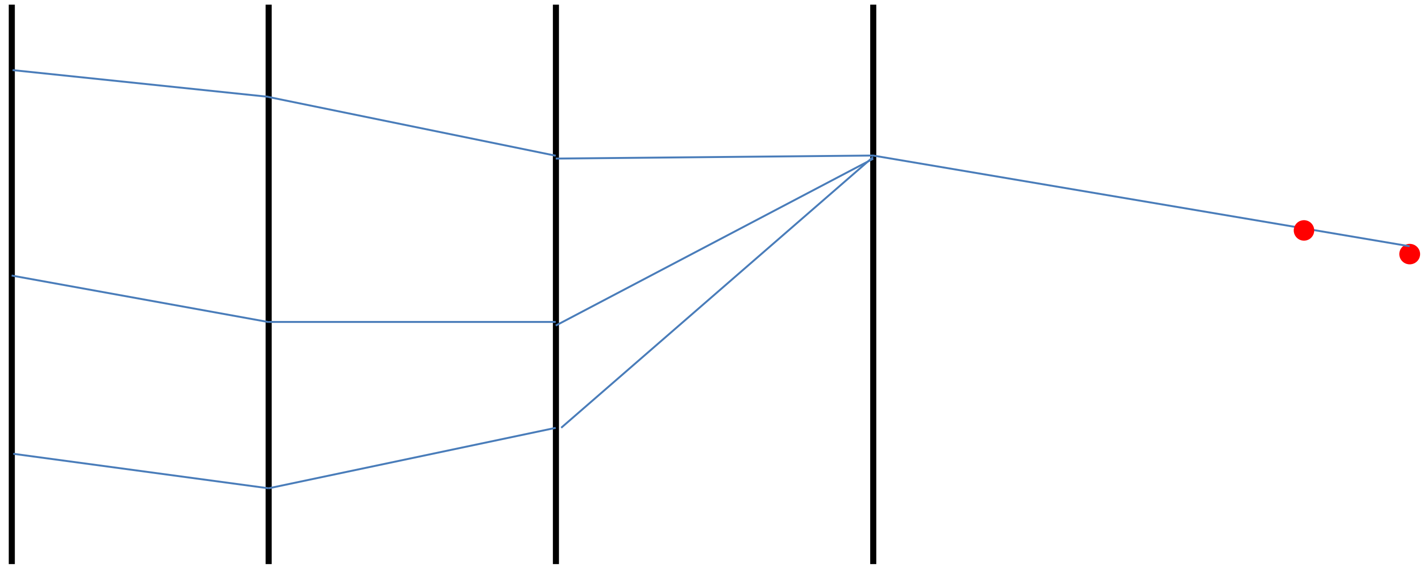
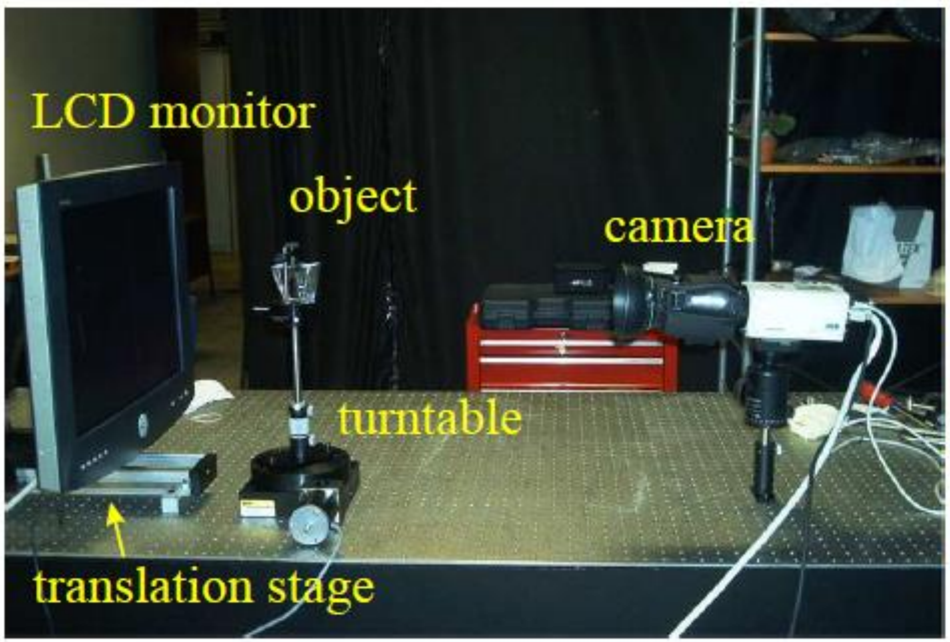
# possible ways to deal with K>=3 case

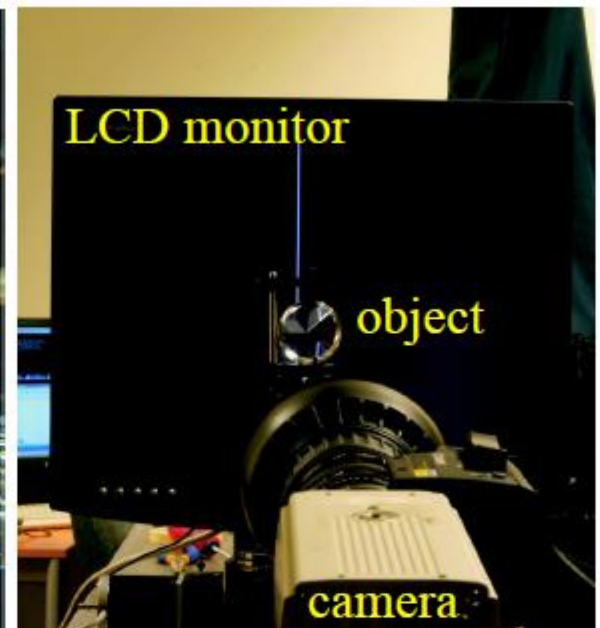
- Intuition    K surface, K-1 connecting rays



# Experiment setup

- Calibrated camera and scene



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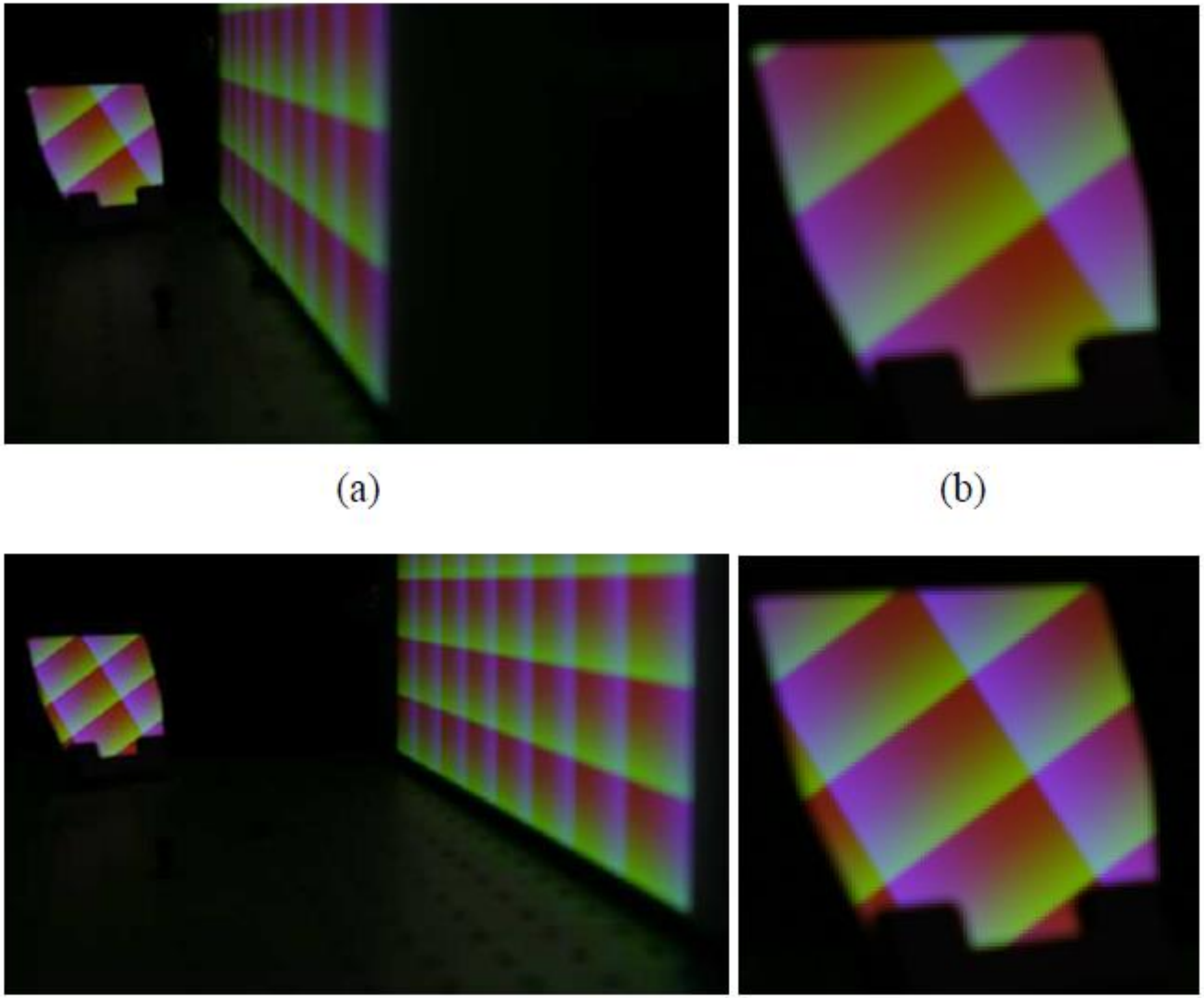
(a) (b) (c) (d)

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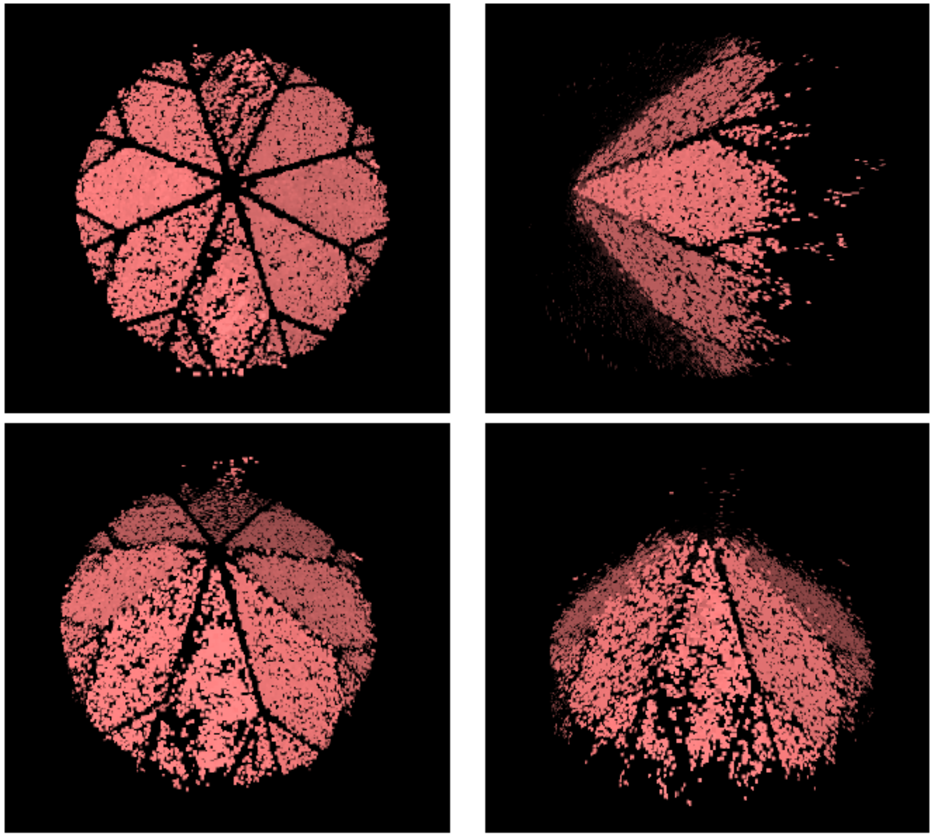
# Results

## Point-wise reconstruction



# score

Score: 1.5

Pros

- A systematic framework to analyze specular and refractive object reconstruction.
- Well written paper.

Cons

- Very cumbersome data collection procedure.
- Only two objects