Optical Computing for Fast Light Transport Analysis, 2010

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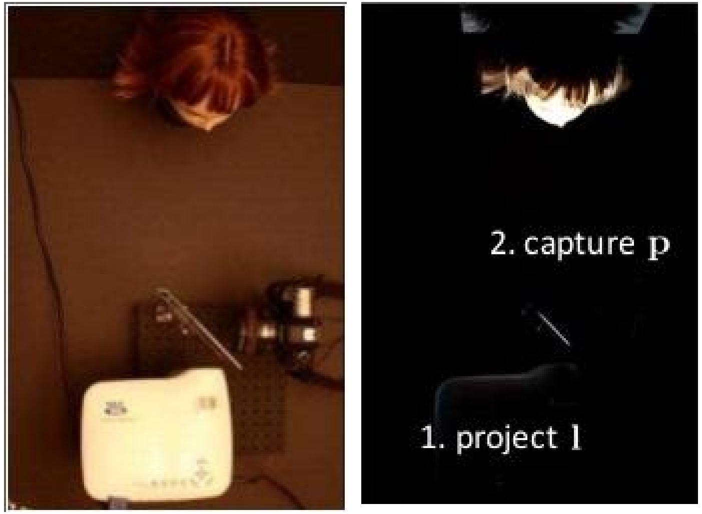
Basic Concept

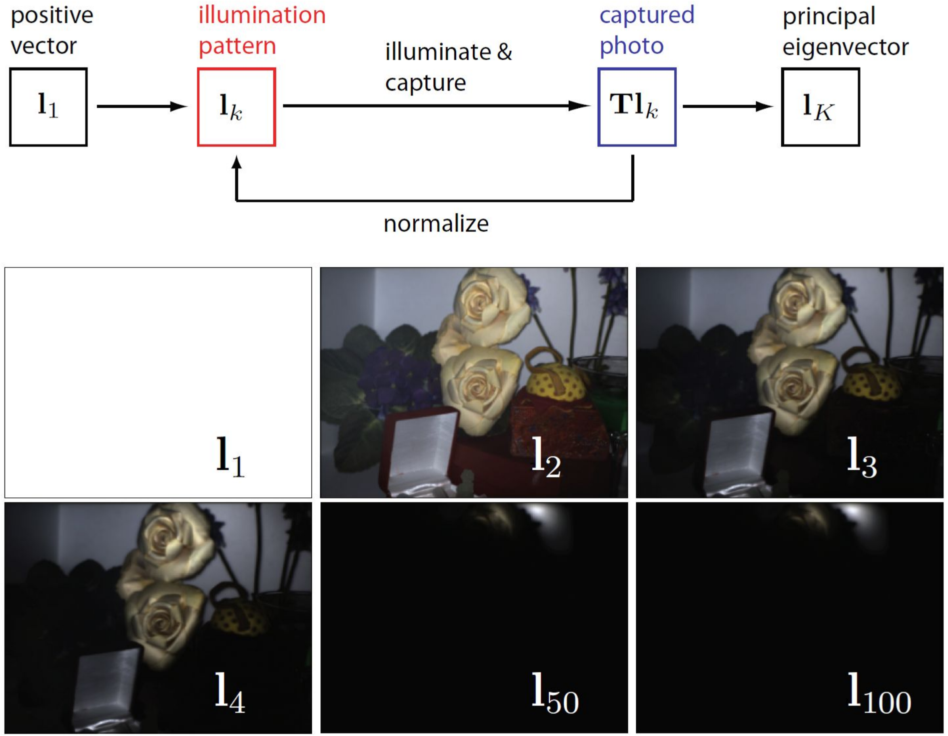
- T-reconstruct pipeline: set up cameras/light sources, come up with sampling scheme, come up with algorithm to reconstruct T

$$\mathbf{p} = \mathbf{T}\,\mathbf{l}$$



- Similar pipeline, estimation of T via "optical computing"
  - Iterative numerical methods for matrix estimation
  - Scene lighting + image capture can replace a matrix-vector in these methods
  - Use this T-estimation as previously seen (scene relighting, light source detection)

Example: Optical Power Iteration



- want to get principal eigenvector of T
- use current photo as basis for illumination of next image capture
- iterate until convergence
- final image is principal eigenvector of T [Trefethen and Bau 1997, non-optical]
- only 1 eigenvector, can be slow

Krylov Subspace Methods

- Power iteration after k steps gives a span of vectors
- This span of vectors is the k-dim Krylov subspace

Algorithm 1 *The power iteration algorithm.*

Numerical Implementation:	**Optical Implementation:**
In: matrix $\mathbf{T}$, iterations K	**In:** iterations K
Out: principal eigenvector of $\mathbf{T}$	**Out:** principal eigenvector of $\mathbf{T}$
$\mathbf{l}_1$ = random vector	$\mathbf{l}_1$ = positive vector
for $k = 1$ to K	**for** $k = 1$ to K
$\boxed{\mathbf{p}_k = \mathbf{T}\mathbf{l}_k}$	illuminate scene with vector $\mathbf{l}_k$ capture photo & store in $\mathbf{p}_k$
$\mathbf{l}_{k+1} = \mathbf{p}_k / \lVert\mathbf{p}_k\rVert_2$	$\mathbf{l}_{k+1} = \mathbf{p}_k / \lVert\mathbf{p}_k\rVert_2$
return $\mathbf{l}_{k+1}$	**return** $\mathbf{l}_{k+1}$

$$\mathbf{l}_1 \quad \mathbf{l}_2 \quad \mathbf{l}_3 \quad \cdots \quad \mathbf{l}_{k+1}$$

$$\Updownarrow \quad \Updownarrow \quad \Updownarrow$$

$$\mathbf{T}\mathbf{l}_1 \quad \mathbf{T}^2\mathbf{l}_1 \quad \cdots \quad \mathbf{T}^k\mathbf{l}_1$$

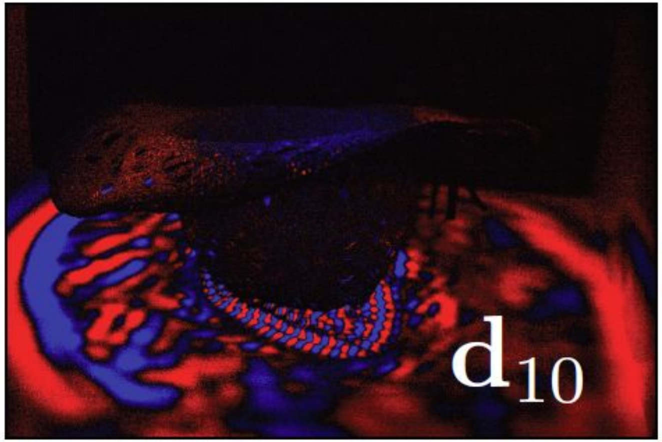
span of vectors given initial illumination and T (optical case)

Krylov Subspace Methods

Optical Matrix-vector products for general vectors

- need to multiply T with negative vectors, can't have negative light
- create a general vector as difference between positive, negative
- positive/negative light can be assigned to 2 different colors, for example

$$\mathbf{l} = \mathbf{l}^p - \mathbf{l}^n$$
$$\mathbf{T}\,\mathbf{l} = (\mathbf{T}\,\mathbf{l}^p) - (\mathbf{T}\,\mathbf{l}^n)\ .$$

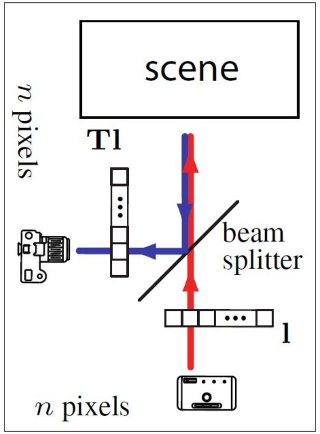


Krylov Subspace Methods

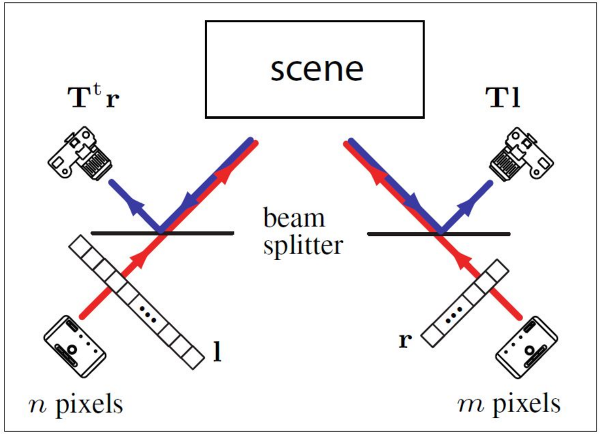
Symmetric vs. non-symmetric transport matrices

- Convergence of some methods dependent on T-symmetry
- 2 ways to enforce symmetry:

$\mathbf{T}$ symmetric



$\mathbf{T}$ non-symmetric



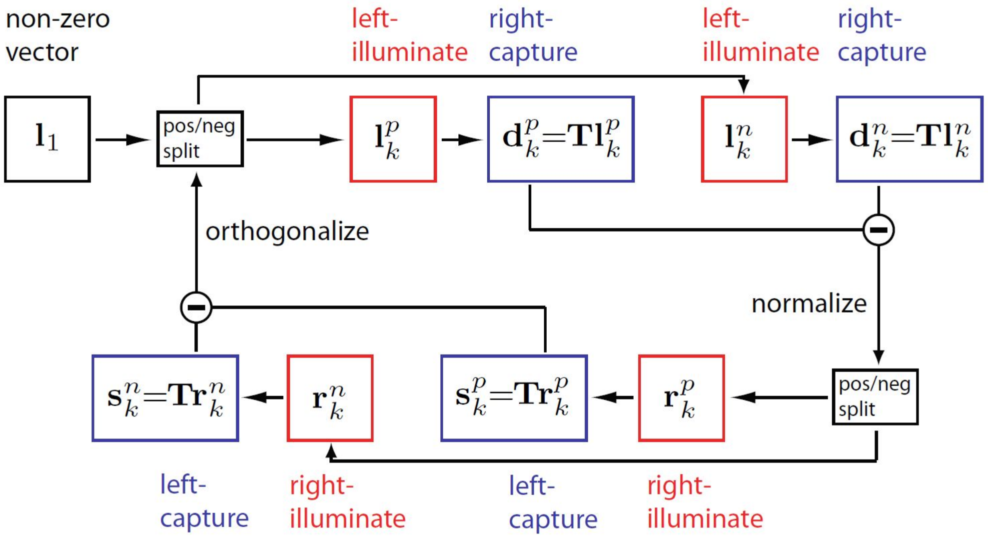
- optically multiply by a different matrix which is symmetric

$$\mathbf{T}^* = \mathbf{T}^t\,\mathbf{T}$$

$$\mathbf{T}^*\,\mathbf{l} = \mathbf{T}^t\,(\mathbf{T}\,\mathbf{l})$$

Transport Acquisition - Optical Arnoldi

Algorithm	Numerical objective	Step 1	Step 4	Step 5
Power iteration (Section 2.1)	estimate principal eigenvector of $\mathbf{T}$	$\mathbf{l}_1$ = positive vector	$\mathbf{l}_{k+1} = \mathbf{p}_k / \|\mathbf{p}_k\|_2$	**return** $\mathbf{l}_{k+1}$
Arnoldi (Section 3)	compute rank-K approximation of $\mathbf{T}$	$\mathbf{l}_1$ = non-zero vector	$\mathbf{l}_{k+1} = \mathrm{ortho}(\mathbf{l}_1, \ldots, \mathbf{l}_k, \mathbf{p}_k)$ $\mathbf{l}_{k+1} = \mathbf{l}_{k+1} / \|\mathbf{l}_{k+1}\|_2$	**return** $[\mathbf{p}_1 \cdots \mathbf{p}_K][\mathbf{l}_1 \cdots \mathbf{l}_K]^{\mathrm{t}}$



- Builds rank k approximation of T

$$T \approx [\mathbf{p}_1 \cdots \mathbf{p}_K][\mathbf{l}_1 \cdots \mathbf{l}_K]^{\mathrm{t}}$$

$$T \approx PL$$

Transport Acquisition - Optical Arnoldi

- Output T is dense, low-rank (good for natural lighting scenes with minimal mirror reflection and sharp shadows)
- reverse of previous methods: eigenvectors of T used to acquire it
- relighting: $$\mathbf{p} = [\mathbf{d}_1 \cdots \mathbf{d}_K][\mathbf{l}_1 \cdots \mathbf{l}_K]^t \, \mathbf{l}$$

Inverse Transport - Optical GMRES

$$\mathbf{l} = \arg\min_{\mathbf{x}} \left\| [\mathbf{p}_1 \cdots \mathbf{p}_K][\mathbf{l}_1 \cdots \mathbf{l}_K]^t \mathbf{x} - \mathbf{p} \right\|_2$$

Least squares problem

- Difference with Arnoldi's: Initial illumination vector is a photo p

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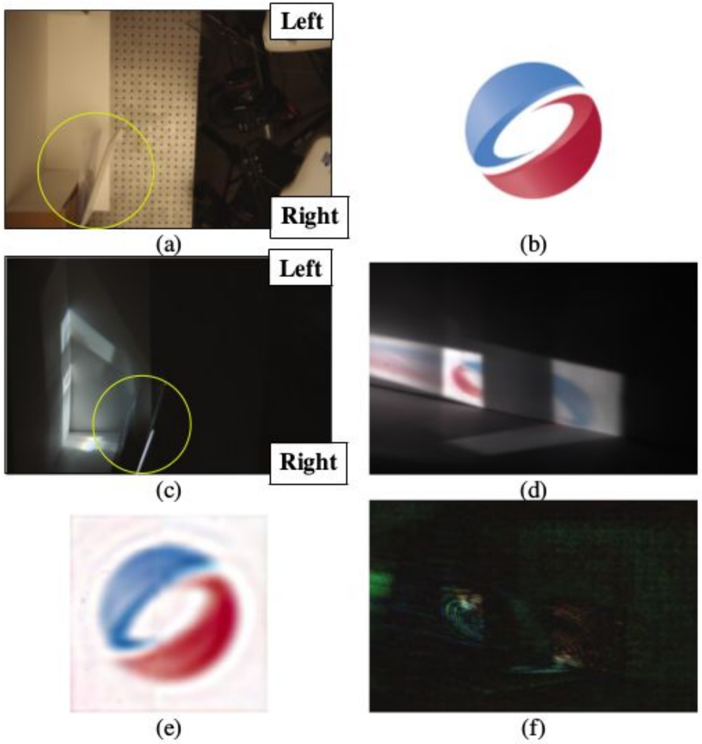
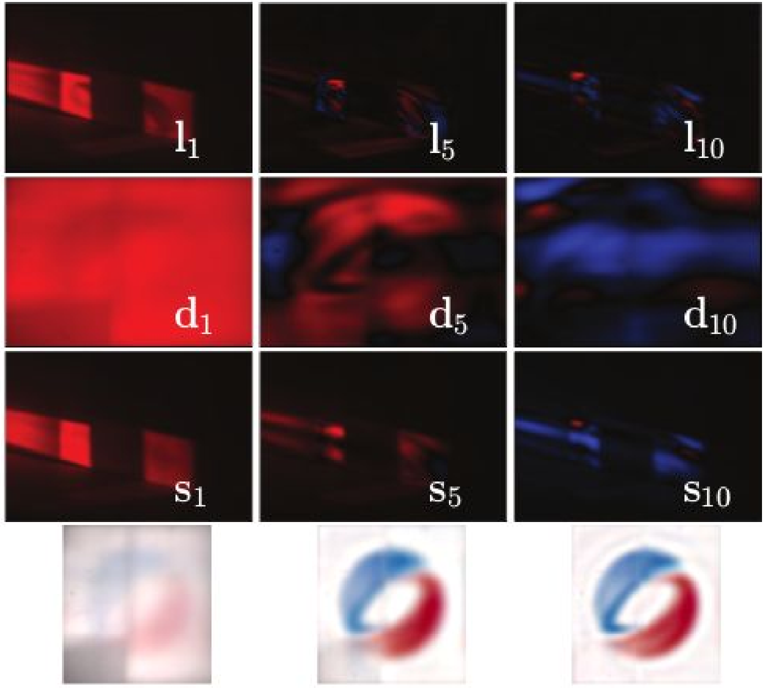
Arnoldi (Section 3)	compute rank-K approximation of $\mathbf{T}$	$\mathbf{l}_1$ = non-zero vector	$\mathbf{l}_{k+1} = \text{ortho}(\mathbf{l}_1, \ldots, \mathbf{l}_k, \mathbf{p}_k)$ $\mathbf{l}_{k+1} = \mathbf{l}_{k+1} / \|\mathbf{l}_{k+1}\|_2$	**return** $[\mathbf{p}_1 \cdots \mathbf{p}_K][\mathbf{l}_1 \cdots \mathbf{l}_K]^t$
Generalized minimal residual (Section 4)	find vector $\mathbf{l}$ such that $\mathbf{p} = \mathbf{T}\,\mathbf{l}$	$\mathbf{l}_1$ = target photo $\mathbf{p}$	$\mathbf{l}_{k+1} = \text{ortho}(\mathbf{l}_1, \ldots, \mathbf{l}_k, \mathbf{p}_k)$ $\mathbf{l}_{k+1} = \mathbf{l}_{k+1} / \|\mathbf{l}_{k+1}\|_2$	**return** $[\mathbf{l}_1 \cdots \mathbf{l}_K][\mathbf{p}_1 \cdots \mathbf{p}_K]^+ \mathbf{p}$

Inverse Transport - Optical GMRES

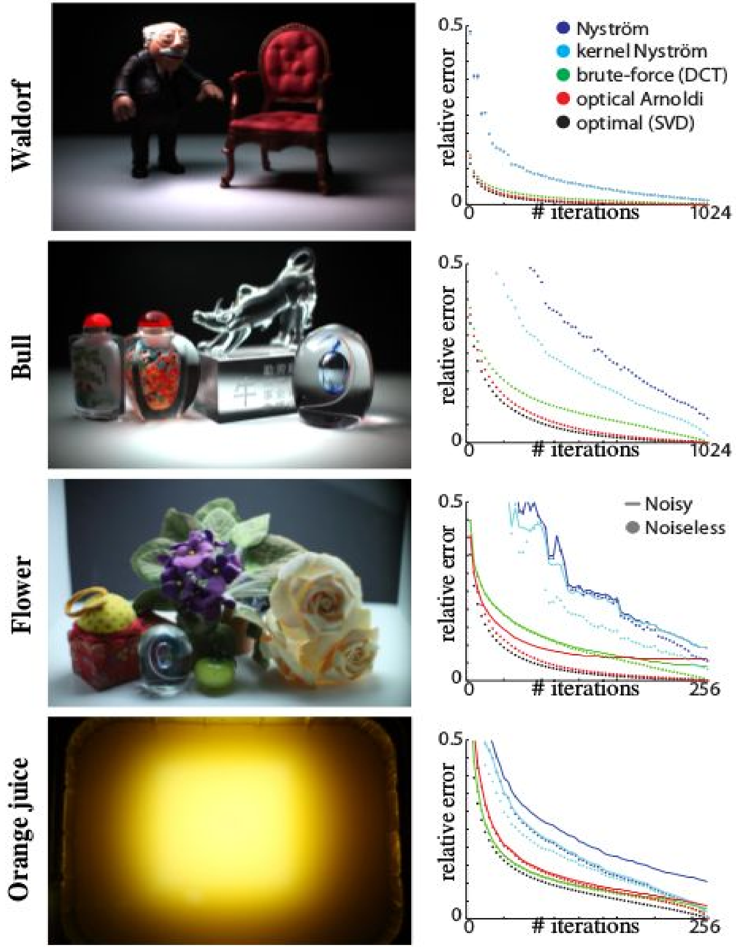




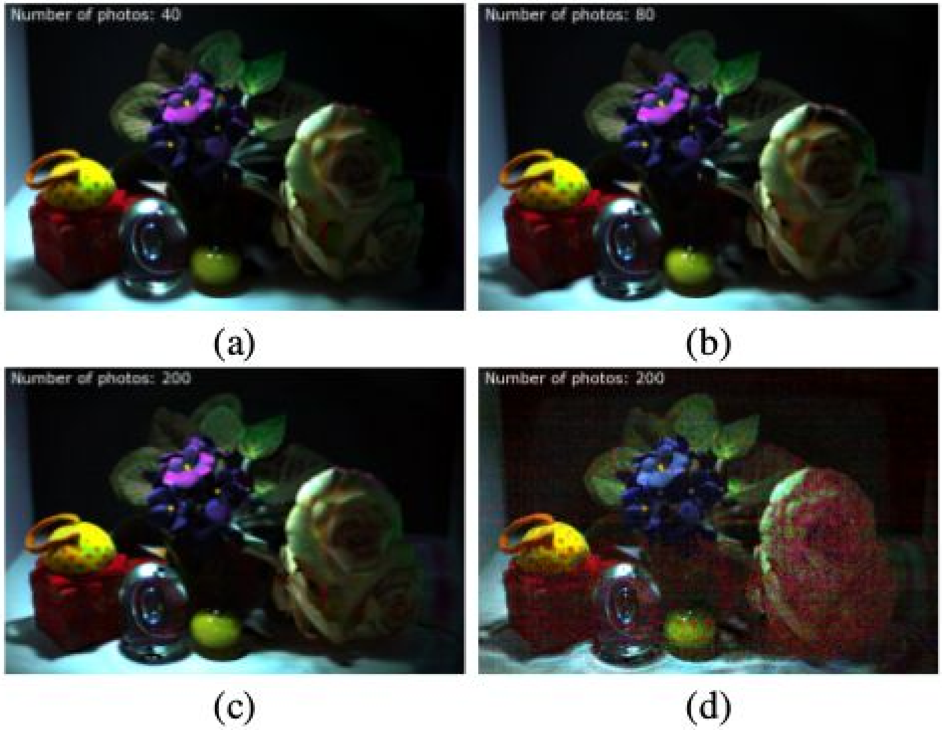
Result-Acquiring Transport Matrices with Optical Arnoldi

$$\epsilon_k = \frac{\|\mathbf{T}_k - \hat{\mathbf{T}}\|_F}{\|\hat{\mathbf{T}}\|_F}$$

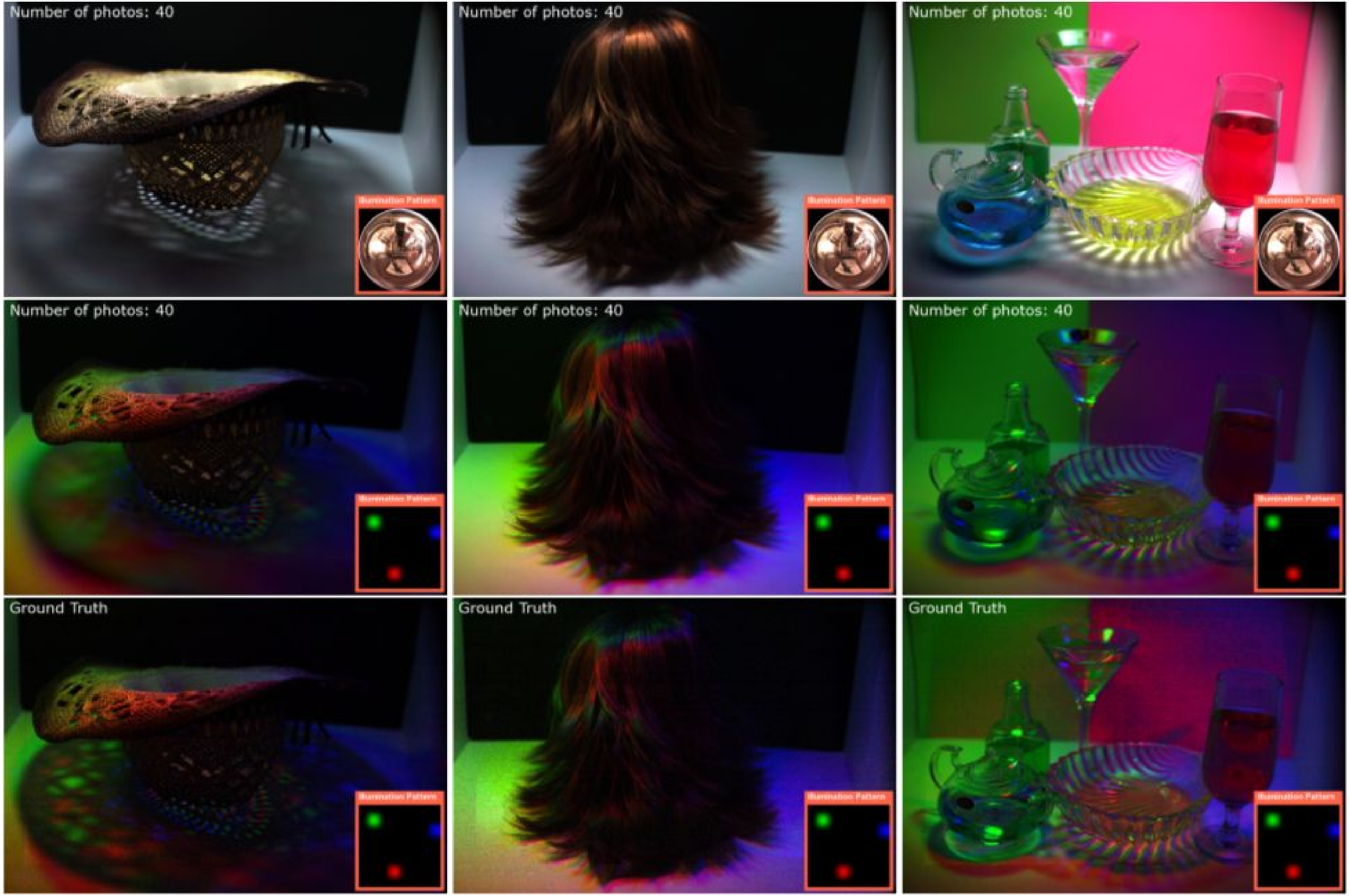
- T: ground truth matrix



Results-Acquiring Transport Matrices with Optical Arnold



(a) 10 iterations for Arnoldi, 200 photos
(b) 20 iterations for Arnoldi, 200 photos
(c) 50 iterations for Arnoldi, 200 photos
(d) Nystrom



rank 10

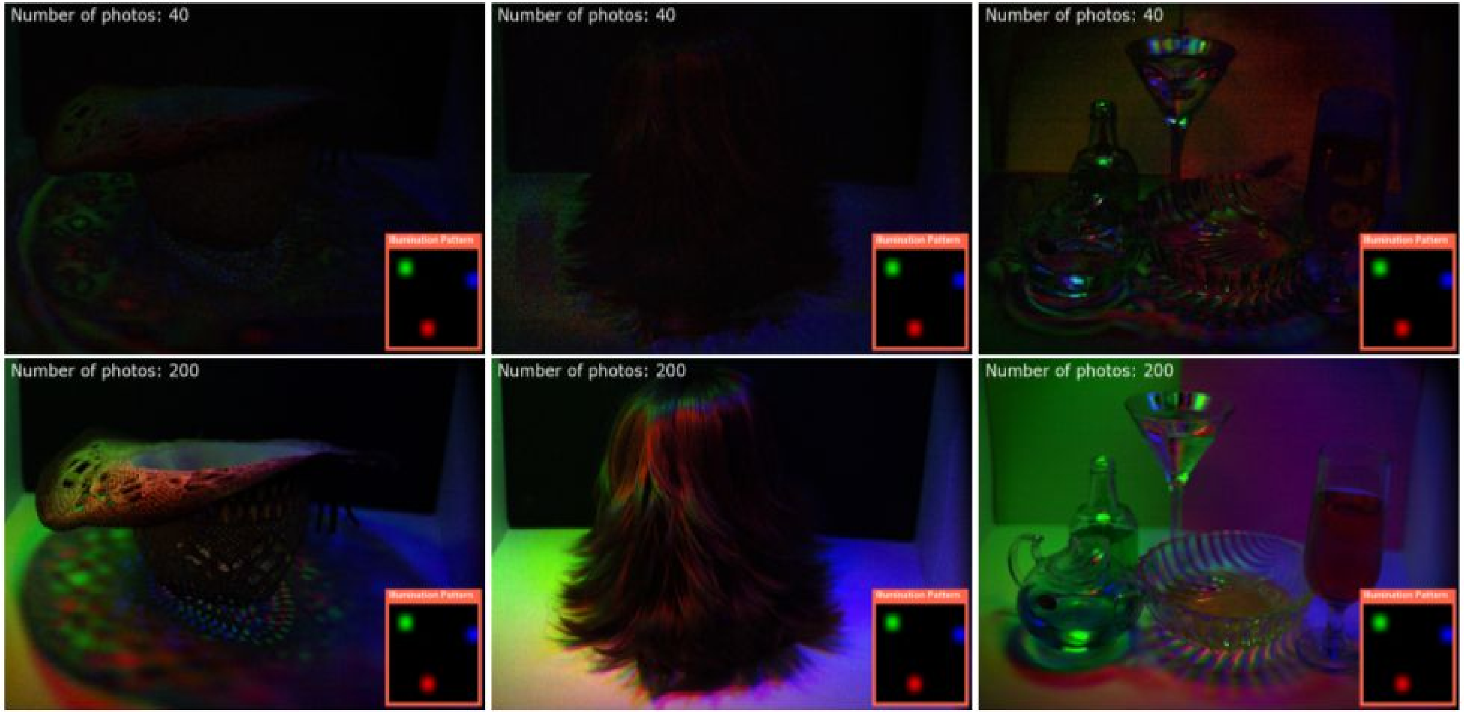
rank 10

spatially -

localized

Actual

photo



Difference between actual photo and relit image

rank 50

Conclusions

- Built optical methods from existing numerical methods
- Have theoretical bounds on convergence rate
- Methods applied easily to large matrices (i.e. T)

Limitations:

- Optical Arnoldi performs poorly for high-rank matrices
- Optical GMRES inverts only 1 image (at a time)

Pros and Cons

Grade: 1.1

Pros:

- simple, intuitive method requires relatively few photos
- is fast for relighting/inverse transport

Cons:

- leave most technical details in previous works

Questions?