Object under all Illumination Conditions

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Problem

What is the set of Images of an Object under all Illumination Conditions?

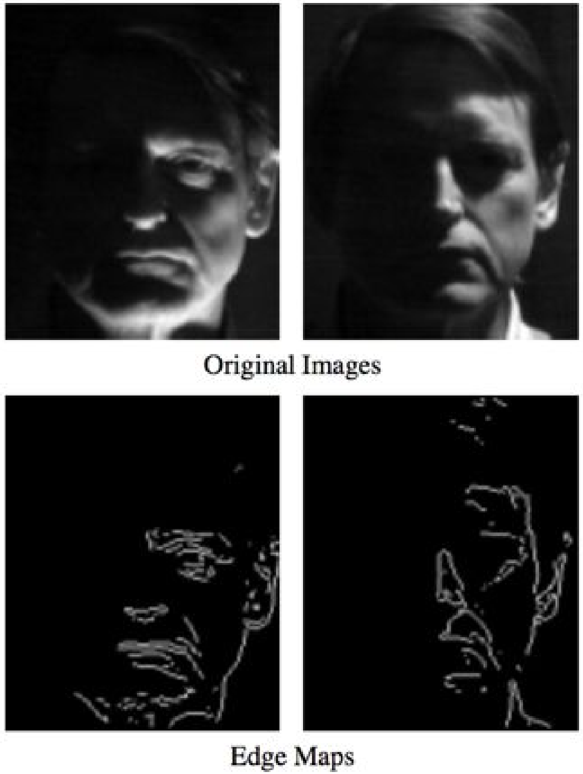
Variability arises from

- Change in lighting conditions
- Change in Viewpoint

How to handle this variability?

Two approaches

- Measure invariant property of the image
- Model the object



what does the paper address?

- what is the set of all images of an object under varying illumination?
- Is this set complex?
- Can a finite number of images characterize this set? If so, how many?

Image as a function

Image = F (Pose, Lighting)

If Lighting = constant, Image-set is 6D

If pose = constant, Image-set is ∞D

Assumptions

1. Object surface has Lambertian reflectance
2. Object surface is convex

Illumination cone

Let,

x - Image of n pixels
B - nx3 matrix with albedo and surface normals
s - the possible illumination

The convex object produces image given by $x = max(Bs, 0)$

For k point light sources $x = \sum_{i=1}^{k} max(Bs_i, 0)$

Illumination Subspace

$$L = \{x \mid x = Bs, \forall\, s \in R^3\}$$

- dimension of L equals the rank of B
- 3D subspace

$$L_0 = L \cap \{x \mid x \in R^n, \text{with all components of x} \geq 0\}\}$$

Positive orthant of L

Lemma 1

The set of images L_0 is a convex cone in R^n

$X \subset R^n$ is convex iff
$x_1, x_2 \in X, \lambda x_1 + (1-\lambda)x_2 \in X$ for every $\lambda \in [0,1]$

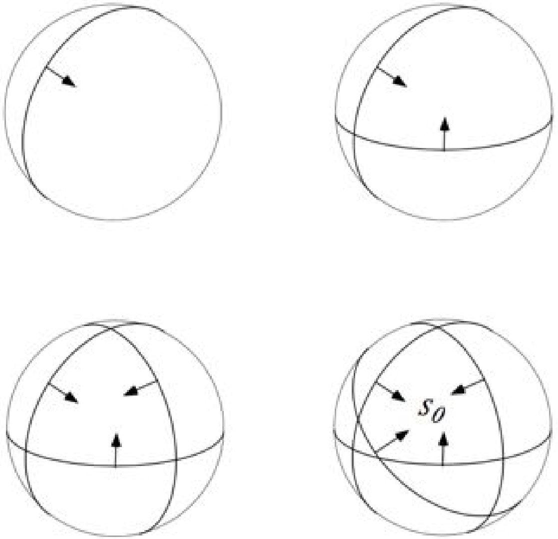
$X \subset R^n$ is a cone iff for any point $x \in X$,
$\alpha x \in X$ for every $\alpha \geq 0$

Illumination Sphere

A sphere representing all possible directions of light source

cell s_i - collection of directions for which same pixels are illuminated

Different cells produce different shadowing configuration



Lemma 2

The set of images $P_i(L_i)$ is a convex cone in R^n

Pi(Li) - projection of point x in Li - such that it ignores negative components

P0(L0) = L0

Theorem 1

“The number of shadowing configurations is at most m(m-1)+ 2, where m <= n is the number of distinct surface normals”

U, set of images by varying direction and strength of single point light source

$$\begin{aligned} \mathcal{U} &= \{\mathbf{x} \mid \mathbf{x} = \max(B\mathbf{s}, \mathbf{0}), \forall \mathbf{s} \in \mathbb{R}^3\} \\ &= \bigcup_{i=0}^{n(n-1)+1} P_i(\mathcal{L}_i). \end{aligned}$$

C, set of images with arbitrary number of light sources

$$\mathcal{C} = \{\mathbf{x} : \mathbf{x} = \sum_{i=1}^{k} \max(B\mathbf{s}_i, \mathbf{0}), \forall \mathbf{s}_i \in \mathbb{R}^3, \forall k \in \mathbb{Z}^+\}$$

Theorem 2

The set of images ***C*** *is a convex cone in* R^n

C is called the Illumination cone

Every object has its own illumination cone

Each point in the cone is an image of the object under a particular lighting configuration

Theorem 3

"The Illumination cone C of a convex Lambertian surface can be determined from as few as three images, each taken under a different, but unknown light source direction"

- Illumination cone C can be determined by Illumination subspace L
- three images from L_0, each taken under different lighting direction, as basis vectors
- Any point can be found as a convex combination of extreme rays

Illumination Subspace Method

- Gather three images without shadowing
- Normalize to unit length
- SVD to estimate best orthogonal basis
- Compute extreme rays defining the cone C

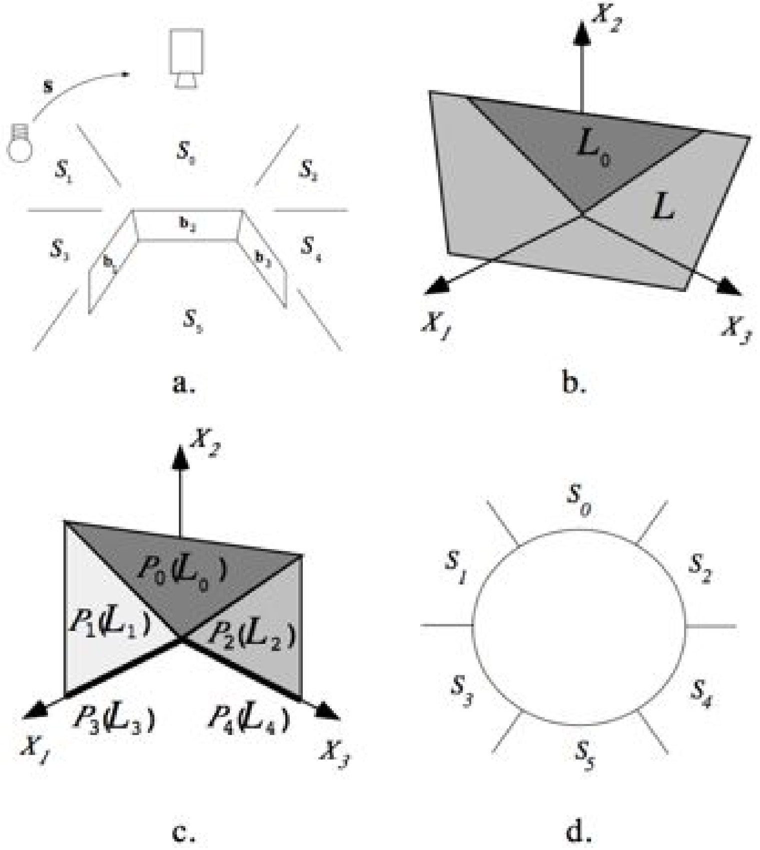
bi and bj be rows of B with (i != j), the extreme rays are,

$$\mathbf{x}_{ij} = \max(B\mathbf{s}_{ij}, \mathbf{0})$$

where

$$\mathbf{s}_{ij} = \mathbf{b}_i \times \mathbf{b}_j.$$

2D example



Theorem 4

The set of n-pixel images of any object, seen under all possible lighting conditions is a convex cone in R^n

- doubtful that Illumination cone can be constructed from three images
- Not because of nonconvexity
  - Nonconvex and Lambertian reflectance can be recovered up to a generalized transformation
- Difficulty is due to that reflectance is unknown
  - could take infinite number of images
- However, Illumination subspace method can be used to approximate

Dimension of the Illumination cone

“The dimension of the Illumination cone C is equal to the number of distinct surface normals”

There exist a cell Sd - which produces a set of images for which all pixels are in shadows

Choose any point $s_b \in S_0$, the point $s_d = -s_b$

connecting half meridian crosses m distinct great circles

since there are only m distinct surface normals

Albedo and Shape

If C_1 *is the illumination cone for an object defined by* $B_1 = R_1 N$ *and* C_2 *is the illumination cone for an object defined by* $B_2 = R_2 N$, *then*

$$C_1 = \{R_1 R_2^{-1} x \;:\; x \in C_2\}$$
$$C_2 = \{R_2 R_1^{-1} x \;:\; x \in C_1\}$$

- the cones of two objects with same geometry but differing albedo differ by a diagonal linear transformation
- True for non convex objects too - since surface normals are same

Shape of the Illumination cone

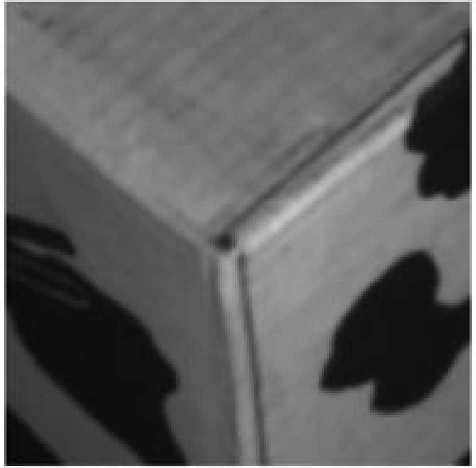
The shape of the cone is “flat”. i.e., most of its volume is concentrated near a low-dimensional subspace.

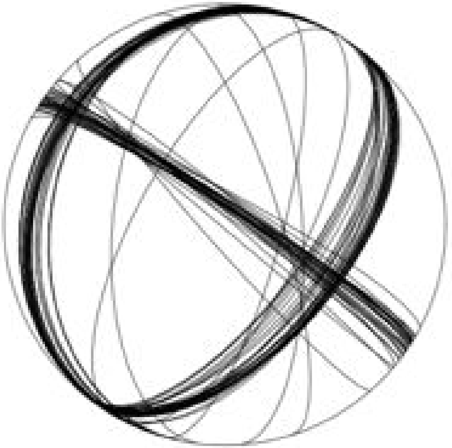
The cone can completely cover positive orthant of R^n

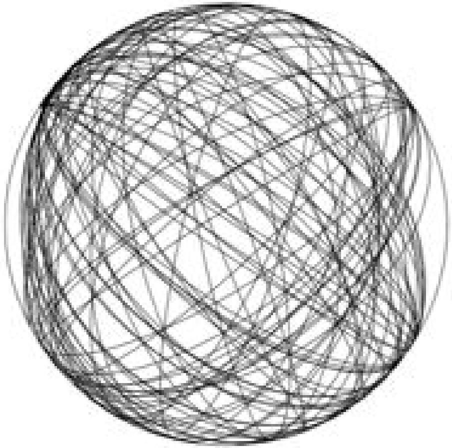
- unlikely

If the cone is small and well separated, then recognition would be possible

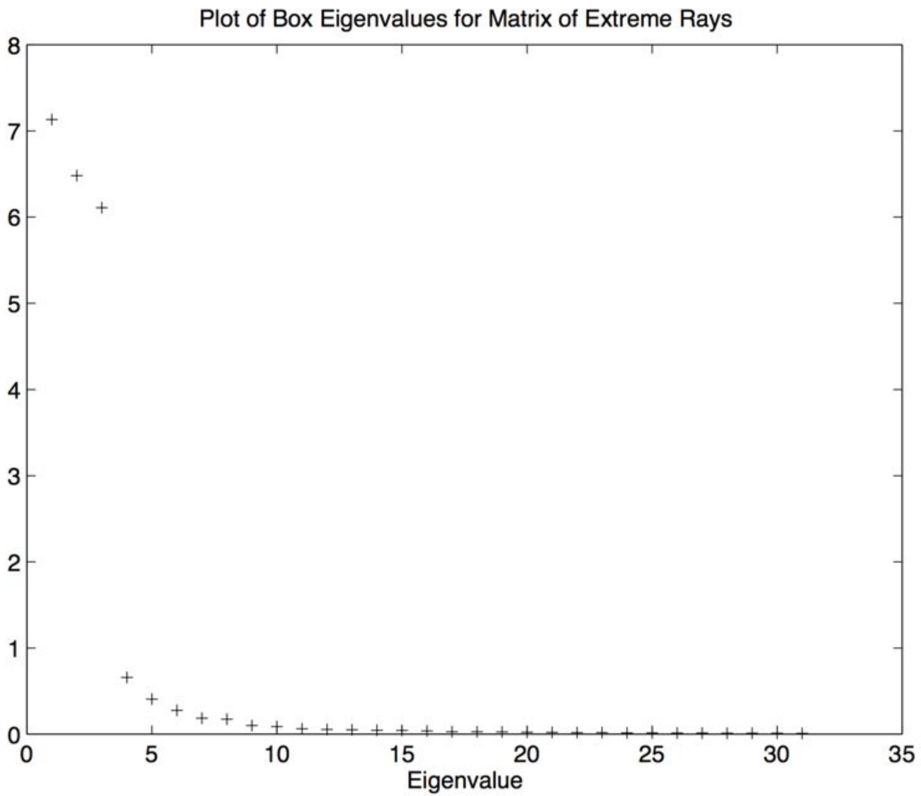
Empirical Investigation

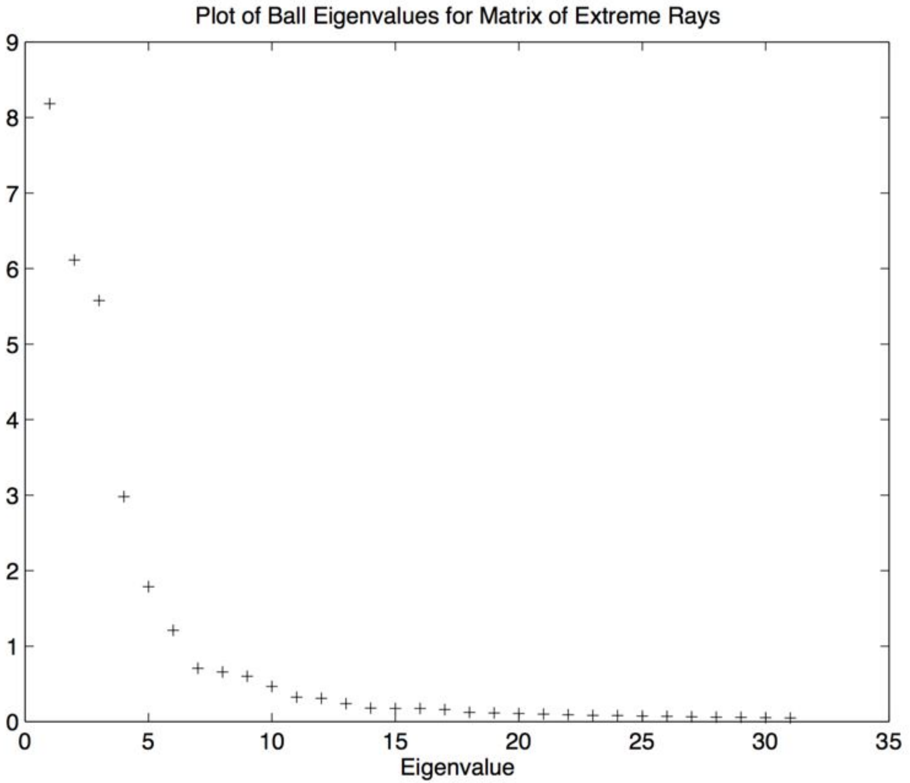






Empirical Investigation





Results

Two different scenes:

1. Human face
2. Desktop still life

Human Face

Original Images

Basis Images

1 Light 2 Lights 3 Lights

Desktop Still Life

Original Images

Basis Images

1 Light 2 Lights 3 Lights

Discussion - Interreflection

$$\mathbf{x}' = (I - RK)^{-1}\mathbf{x}$$

R - albedo diagonal matrix
K - Interreflection kernel

$$B' = (I - RK)^{-1}B$$

- set of shadowing configurations and illumination sphere is generated from B, not B’

Discussion - Effects of Change in Pose

If an object undergoes a rotation or translation, how does the illumination cone deform?

Is there a simple transformation?

Alternatively, is it practical to simply sample the pose space constructing an illumination cone for each pose?

Discussion - Object Recognition

Measuring distance to the illumination cone, rather than distance to the illumination subspace

- non-negative least-squares optimization problem.

Fisherface method for recognizing faces projection directions were chosen to maximize separability of the object classes. A similar approach can be used here.

Score - 1.5

- (Just because the experimental results are not exhaustive)
- It generalized using experimental results rather than theory
- Cited 586 times
- basis for many object recognition work with illumination constraints