A Dictionary-based Approach for Estimating Shape and Spatially-Varying Reflectance

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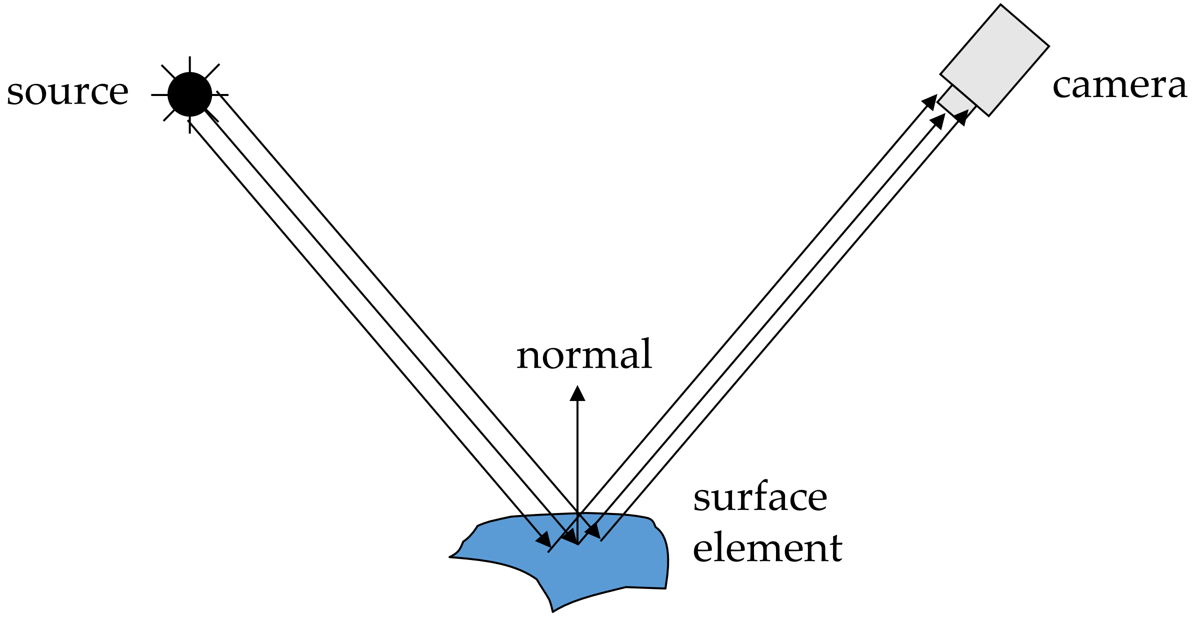
Per-pixel estimation of the shape (*surface normals*) and reflectance (*spatially-varying BRDF*) of an object.

Given images captured with known light sources,

Estimate spatially variant BRDF and surface normals.

Assumptions

- Distant point light source (incidence are parallel), known direction;
- Constant viewing direction $\mathbf{v} \in \mathbb{R}^3$ across scene points
- Convex shape: no shadows, inter-reflection
- Isotropic material



Problem formulation

$$L^{surface}(\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{src}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \sin\theta_i d\theta_i d\phi_i$$

- $\mathbf{p} = (x, y)$, surface normal $\mathbf{n_p}$, BRDF $\rho_\mathbf{p}$, lighting $\mathbf{l}_i$, Intensity value $I_\mathbf{p}^i$

$$I_\mathbf{p}^i = (\mathbf{s}_{\{\mathbf{l}_i, \mathbf{v}; \mathbf{n_p}\}}^\top \rho_\mathbf{p}) \cdot \max\{0, \mathbf{n_p}^\top \mathbf{l}_i\}, \qquad (1)$$

s : sampling function
dot product ~ integration

shading

$\rho_\mathbf{p} \rightarrow$ a point in a $T = 90 \times 90 \times 180$ dimensional space

Problem formulation

$$I_{\mathbf{p}}^{i} = (\mathbf{s}_{\{\mathbf{l}_i,\mathbf{v};\mathbf{n}_{\mathbf{p}}\}}^{\top}\rho_{\mathbf{p}}) \cdot \max\{0, \mathbf{n}_{\mathbf{p}}^{\top}\mathbf{l}_i\}, \qquad (1)$$

- Multiple images $\{I^1, \ldots, I^Q\}$:

  varying lighting direction $\mathbf{l}_i \in \mathbb{R}^3$ in the i-th image I^i

  same viewing direction $\mathbf{v}$

$$\begin{aligned} \mathbf{I}_{\mathbf{p}} &= \begin{pmatrix} I_{\mathbf{p}}^{1} \\ \vdots \\ I_{\mathbf{p}}^{Q} \end{pmatrix} = \begin{bmatrix} \max\{0, \mathbf{n}_{\mathbf{p}}^{\top}\mathbf{l}_1\} \cdot \mathbf{s}_{\{\mathbf{l}_1,\mathbf{v};\mathbf{n}_{\mathbf{p}}\}}^{\top} \\ \vdots \\ \max\{0, \mathbf{n}_{\mathbf{p}}^{\top}\mathbf{l}_Q\} \cdot \mathbf{s}_{\{\mathbf{l}_Q,\mathbf{v};\mathbf{n}_{\mathbf{p}}\}}^{\top} \end{bmatrix} \rho_{\mathbf{p}} \\ &= A(\boxed{\mathbf{n}_{\mathbf{p}}})\boxed{\rho_{\mathbf{p}}} \qquad (2) \end{aligned}$$

Intractable without additional assumptions!

Model for BRDF

- Key assumption

$$\rho_{\mathbf{p}} = D\mathbf{c}_{\mathbf{p}}, \quad \mathbf{c}_{\mathbf{p}} \geq 0,$$

$$D = [\rho^1, \rho^2, \cdots, \rho^M] \qquad \mathbf{c}_{\mathbf{p}} \in \mathbb{R}^M$$

Constrain the BRDF to M-dimension, $M \ll T = 90\text{x}90\text{x}180$

Solution outline

$$\begin{aligned}\mathbf{I}_\mathbf{p} &= \begin{pmatrix} I^1_\mathbf{p} \\ \vdots \\ I^Q_\mathbf{p} \end{pmatrix} = \begin{bmatrix} \max\{0, \mathbf{n}_\mathbf{p}^\top \mathbf{l}_1\} \cdot \mathbf{s}^\top_{\{\mathbf{l}_1, \mathbf{v}; \mathbf{n}_\mathbf{p}\}} \\ \vdots \\ \max\{0, \mathbf{n}_\mathbf{p}^\top \mathbf{l}_Q\} \cdot \mathbf{s}^\top_{\{\mathbf{l}_Q, \mathbf{v}; \mathbf{n}_\mathbf{p}\}} \end{bmatrix} \rho_\mathbf{p} \\ &= A(\mathbf{n}_\mathbf{p})\rho_\mathbf{p} \end{aligned} \tag{2}$$

$$\rho_\mathbf{p} = D\mathbf{c}_\mathbf{p}, \quad \mathbf{c}_\mathbf{p} \geq 0,$$

$$\{\widehat{\mathbf{n}}_\mathbf{p}, \widehat{\mathbf{c}}_\mathbf{p}\} = \arg\min_{\mathbf{n},\mathbf{c}} \|\mathbf{I}_\mathbf{p} - A(\mathbf{n})D\mathbf{c}\|_2^2 + \lambda\|\mathbf{c}\|_1 \qquad \text{s.t} \quad \mathbf{c} \geq 0, \|\mathbf{n}\|_2 = 1 \tag{3}$$

A non-convex optimization problem

Solution Methodology

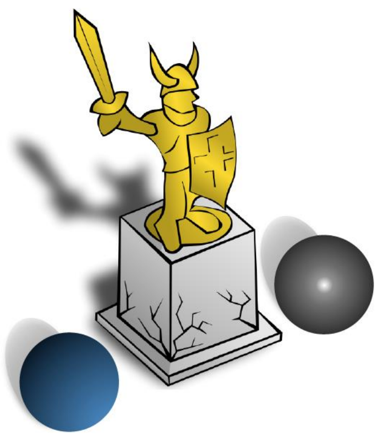
(1) Surface Normal $\mathbf{n_p}$ Estimation

coarse-to-fine search

(2) BRDF estimation

fix $\mathbf{n_p}$, optimize $\mathbf{c_p}$

Surface Normal Estimation



Example-based photometric stereo

- 2 Spheres, one diffuse, one specular: a collection of candidate normals $\mathcal{N}$

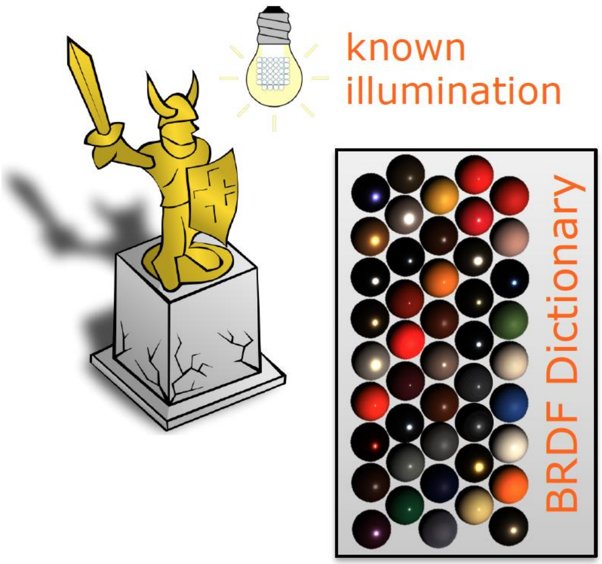
- For $\tilde{\mathbf{n}}$, we have $\mathbf{I}_D(\tilde{\mathbf{n}})$, $\mathbf{I}_s(\tilde{\mathbf{n}})$

$$\mathbf{I}_{\mathbf{p}} = a_1 \mathbf{I}_D(\tilde{\mathbf{n}}) + a_2 \mathbf{I}_s(\tilde{\mathbf{n}})$$

$$\widehat{\mathbf{n}}_{\mathbf{p}} = \arg\min_{\tilde{\mathbf{n}} \in \mathcal{N}} \min_{a_1, a_2 \geq 0} \| \mathbf{I}_{\mathbf{p}} - a_1 \mathbf{I}_D(\tilde{\mathbf{n}}) - a_2 \mathbf{I}_S(\tilde{\mathbf{n}}) \|.$$

A. Hertzmann and S. Seitz. Example-based photometric stereo: Shape reconstruction with general, varying BRDFs. PAMI, 27:1254–1264, 2005.

Virtual example-based Normal Estimation



Proposed technique

$$\mathbf{I_p} = a_1 \mathbf{I}_D(\tilde{\mathbf{n}}) + a_2 \mathbf{I}_s(\tilde{\mathbf{n}})$$

- Render virtual spheres
- lighting $\{\mathbf{l}^1, \ldots, \mathbf{l}^Q\}$, $D = [\rho^1, \ldots, \rho^M]$;

Two spheres -> Q x *M* spheres

$$B(\tilde{\mathbf{n}}) = [b_{ij}(\tilde{\mathbf{n}})] \in \mathbb{R}^{Q \times M}$$

$$b_{ij}(\tilde{\mathbf{n}}) = \max\{0, \tilde{\mathbf{n}}^\top \mathbf{l}_i\} \cdot \mathbf{s}^\top_{\{\mathbf{l}_i, \mathbf{v}; \tilde{\mathbf{n}}\}} \rho^j,$$

Coarse-to-fine search for *n*

$$\{\widehat{\mathbf{n}}_{\mathbf{p}}, \widehat{\mathbf{c}}_{\mathbf{p}}\} = \arg\min_{\mathbf{n},\mathbf{c}} \|\mathbf{I}_{\mathbf{p}} - A(\mathbf{n})D\mathbf{c}\|_2^2 + \lambda\|\mathbf{c}\|_1 \quad \text{s.t} \quad \mathbf{c} \geq 0, \|\mathbf{n}\|_2 = 1 \tag{3}$$

Coarse-to-fine search for *n*

$$B(\tilde{\mathbf{n}}) = A(\tilde{\mathbf{n}})D$$

$$\widehat{\mathbf{n}}_{\mathbf{p}} = \arg\min_{\tilde{\mathbf{n}} \in \mathcal{N}} \min_{\mathbf{c} \geq 0} \|\mathbf{I}_{\mathbf{p}} - B(\tilde{\mathbf{n}})\mathbf{c}\|. \tag{4}$$

$$E(\tilde{\mathbf{n}}) = \min_{\mathbf{c} \geq 0} \|\mathbf{I}_{\mathbf{p}} - B(\tilde{\mathbf{n}})\mathbf{c}\|$$

BRDF Estimation for *c*

$$\{\widehat{\mathbf{n}}_{\mathbf{p}}, \widehat{\mathbf{c}}_{\mathbf{p}}\} = \arg\min_{\mathbf{n},\mathbf{c}} \|\mathbf{I}_{\mathbf{p}} - A(\mathbf{n})D\mathbf{c}\|_2^2 + \lambda\|\mathbf{c}\|_1 \quad \text{s.t} \quad \mathbf{c} \geq 0, \|\mathbf{n}\|_2 = 1 \qquad (3)$$

$$\widehat{\mathbf{n}}_{\mathbf{p}} = \arg\min_{\widetilde{\mathbf{n}}\in\mathcal{N}} \min_{\mathbf{c}\geq 0} \|\mathbf{I}_{\mathbf{p}} - B(\widetilde{\mathbf{n}})\mathbf{c}\|.$$

BRDF Estimation for *c*

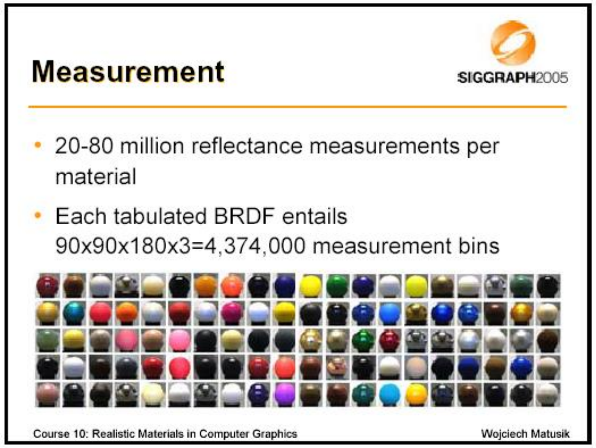
$$\widehat{\mathbf{c}}_{\mathbf{p}} = \arg\min_{\mathbf{c}\geq 0} \|\mathbf{I}_{\mathbf{p}} - B(\widehat{\mathbf{n}}_{\mathbf{p}})\mathbf{c}\|_2^2 + \lambda\|\mathbf{c}\|_1. \qquad (5)$$

A convex optimization problem

$$\widehat{\rho}_{\mathbf{p}} = D\widehat{\mathbf{c}}_{\mathbf{p}}$$

Results

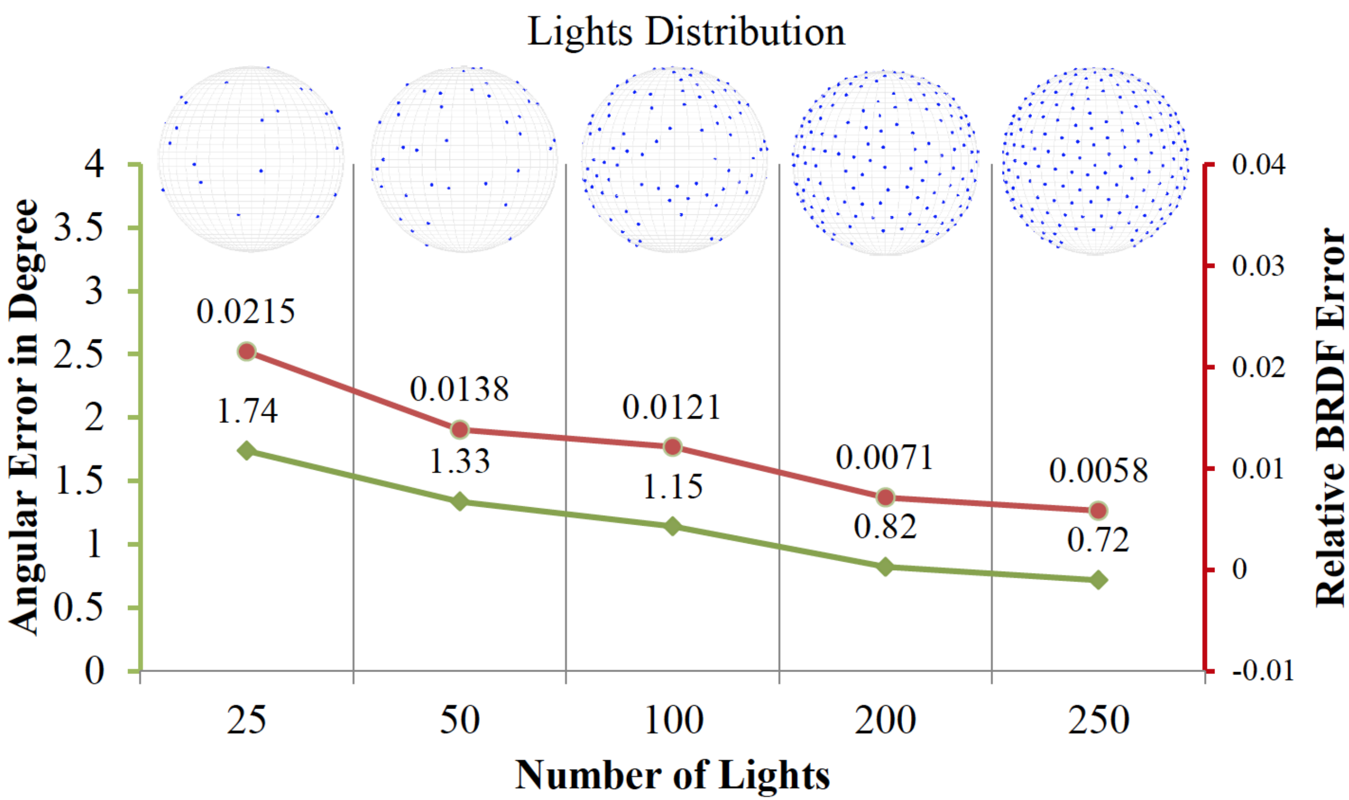
1. Synthetic experiments



- MERL Database [1]
  - leave-one-out scheme: 100 materials in total
- Lighting directions [2] : USC light stage data

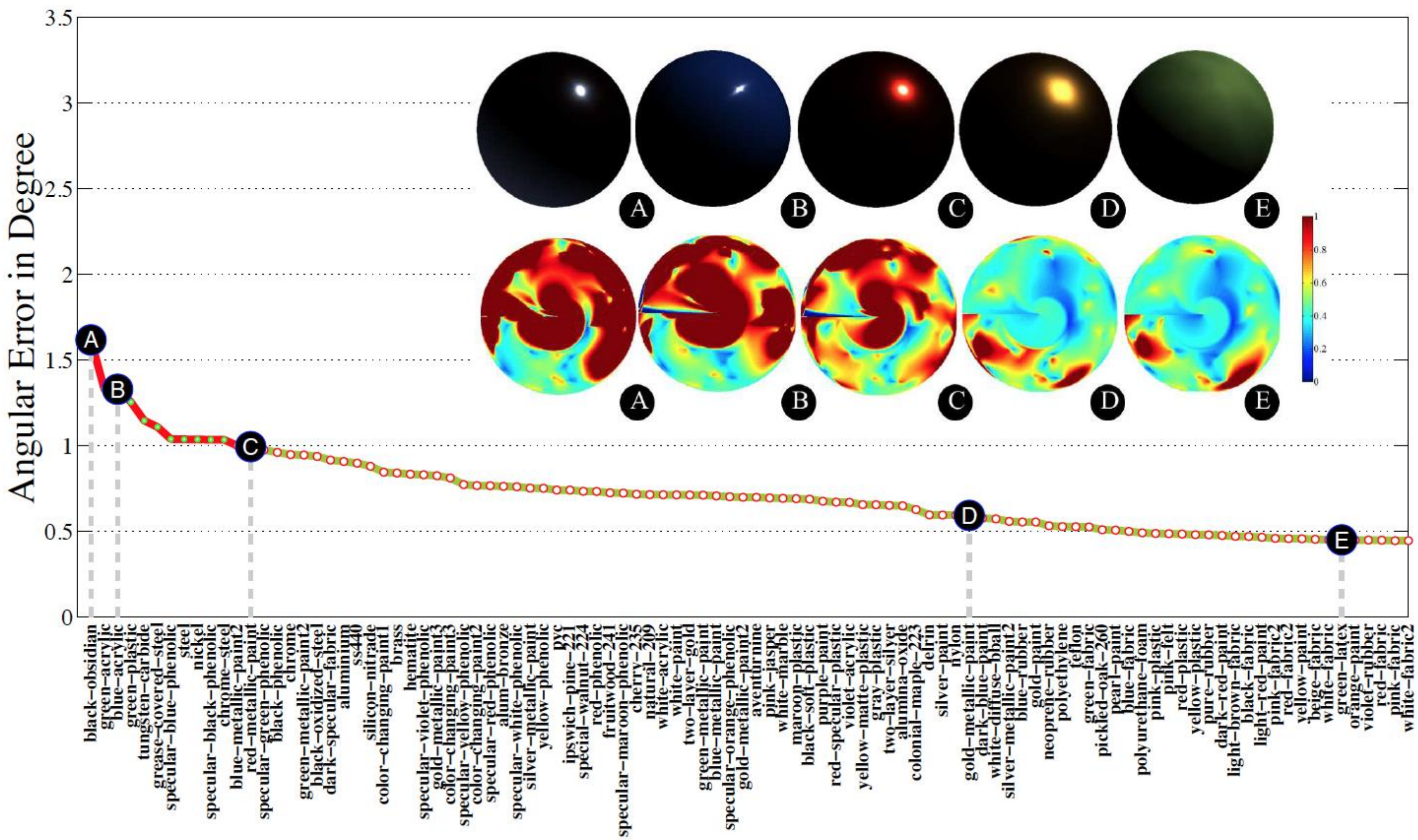
[1] M. . Wojciech, P. Hanspeter, B. Matt, and M. Leonard. A data-driven reflectance model. TOG, 22:759–769, 2003.
[2] P. Einarsson, C. Chabert, A. Jones, W. Ma, B. Lamond, T. Hawkins, M. Bolas, S. Sylwan, and P. Debevec. Relighting human locomotion with flowed reflectance fields. In Rendering techniques, 2006.

Varying number of images



- Surface normal 20,000 random orientations
- 100 materials

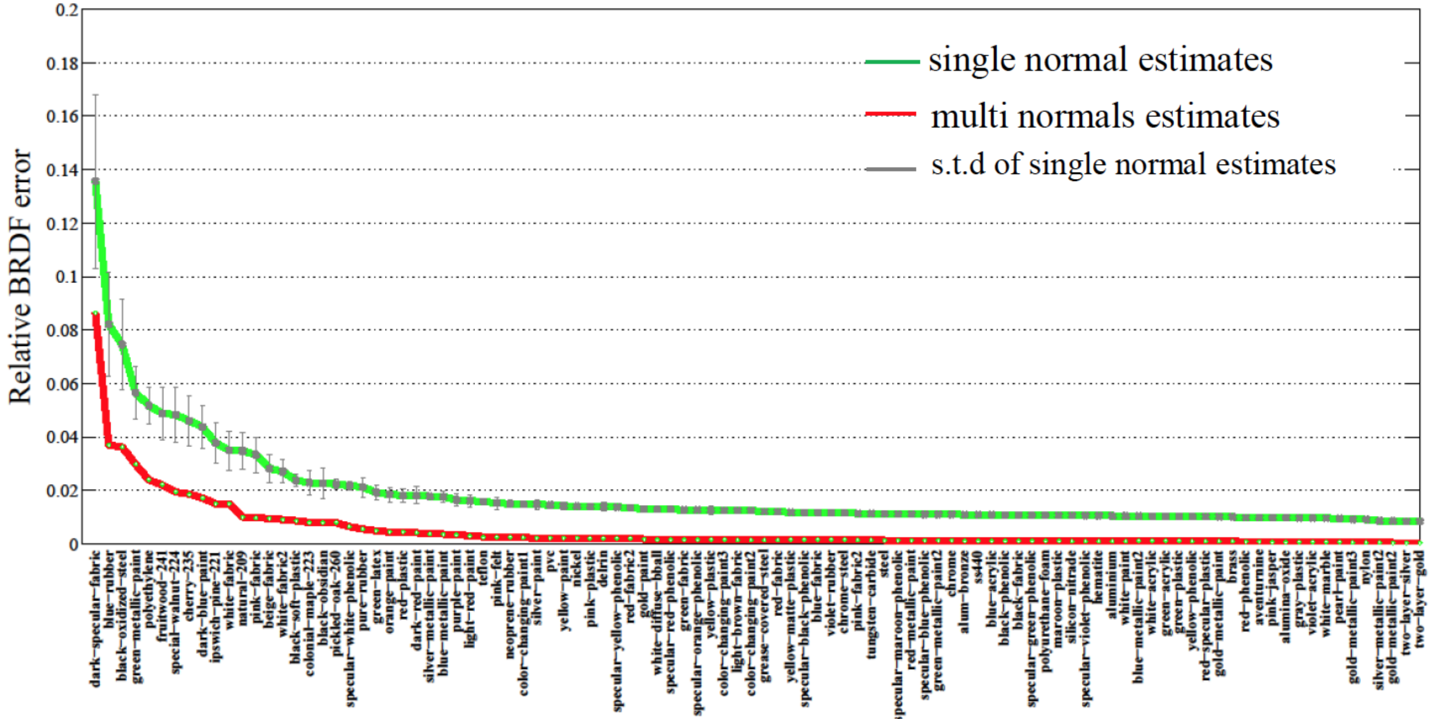
Surface normal estimation for varying BRDF



- Surface normal 50,000 random orientations
- 100 materials
- # of input images = 253

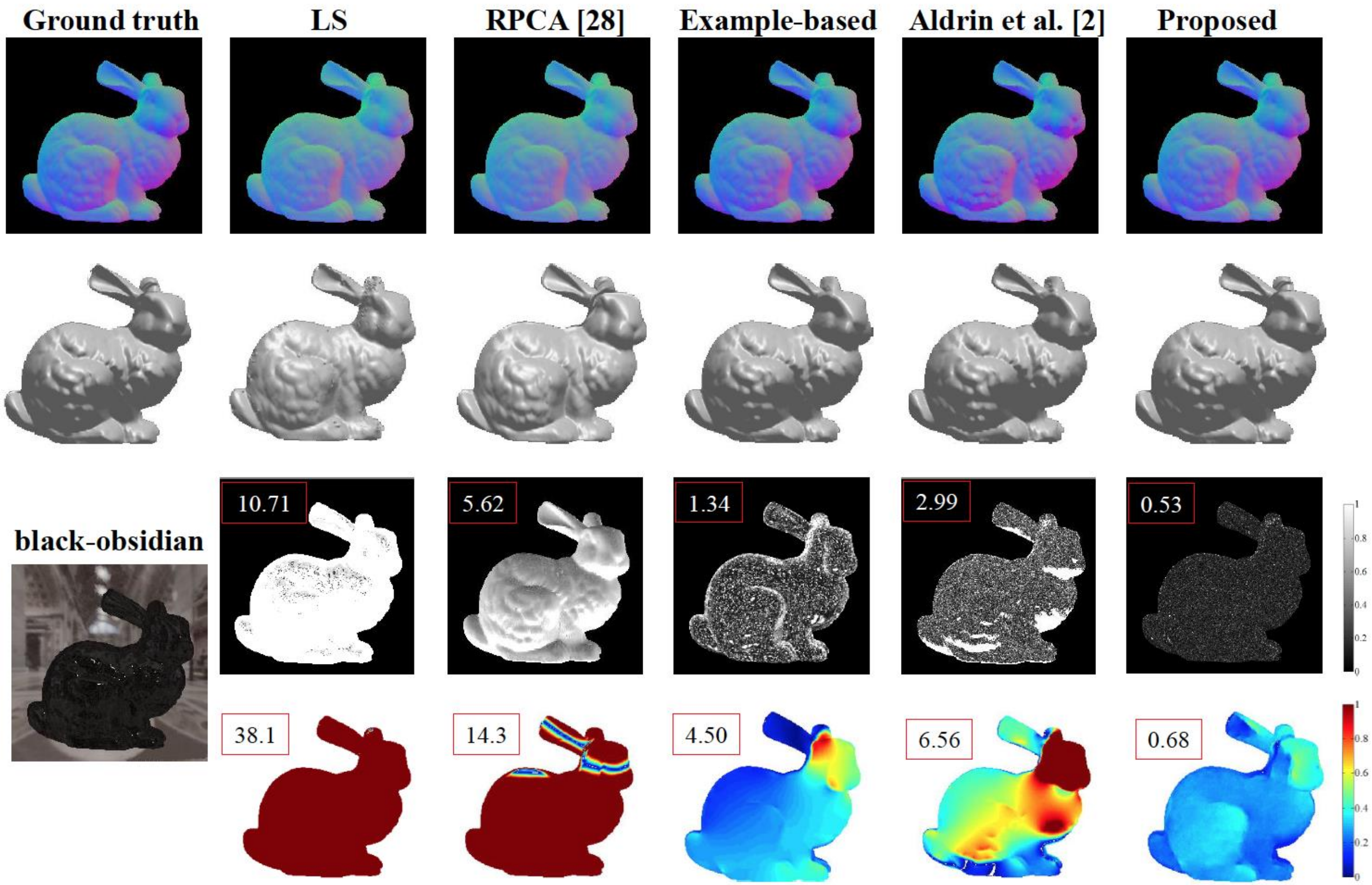
BRDF estimations

BRDF Error
$$\sqrt{\sum_i w_i((\widehat{\rho}(i) - \rho(i)) \cdot \max(0, \cos(\theta_i)))^2 / \sum_i w_i}, \quad (6)$$



- Surface normal 100 random orientations
- 100 materials
- # of input images = 253

Comparison with other methods



Results

2. Real data

- USC Light Stage dataset [1]
- Gourd Dataset from UCSD[2]

[1] P. Einarsson, C. Chabert, A. Jones, W. Ma, B. Lamond, T. Hawkins, M. Bolas, S. Sylwan, and P. Debevec. Relighting human locomotion with flowed reflectance fields. In Rendering techniques, 2006.
[2] N. Alldrin, T. Zickler, and D. Kriegman. Photometric stereo with non-parametric and spatially-varying reflectance. In CVPR, 2008.

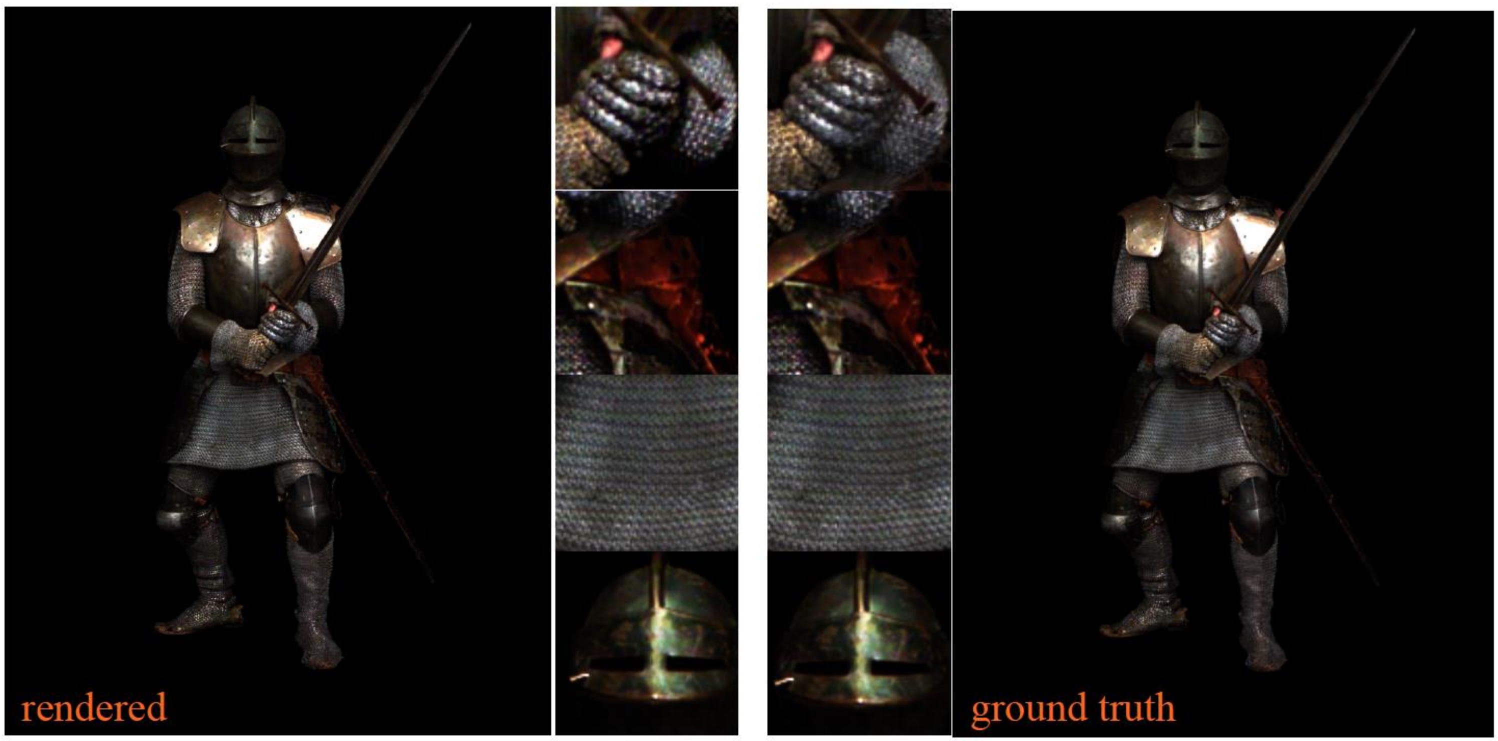
Normal Map   Recovered surface

Relighted   Ground truth   Rendering in natural lighting

- 100 input images

Robustness of per-pixel estimation

Difficulties: various materials, inter-reflections, shadows



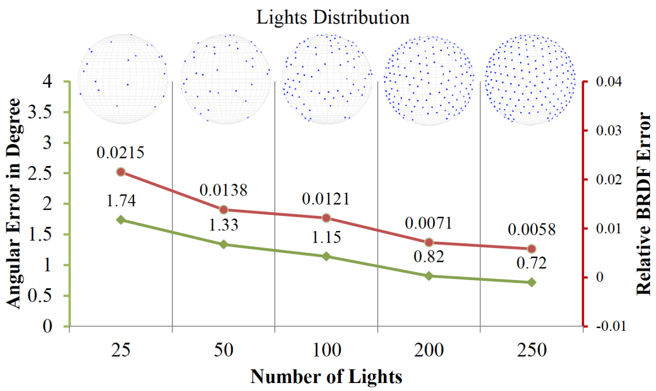
- 250 input images

Pros and Cons

- Pros
  - Use of dictionary simplify the inverse problem
  - Per-pixel normal and BRDF estimation technique, may further speed up
  - Handle arbitrary complex spatial variations in the BRDF
- Cons
  - Time-consuming
  - Noisy estimation: also because of the per-pixel estimation
- Limitations
  - The dictionary needs to be sufficient
  - Require light calibration

Possible future works

- Per-pixel based –> Segments based
  - A more clever way to compromise between smoothness and BRDF estimations
- Experiment: Less input images required; different λ for sparsity



- Maybe estimate the surface normal and BRDF iteratively to get more precise results?

Q & A