Lecture 12
Kinematic models of contact

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Mechanics of Manipulation

Today's outline

Kinematic models of contact

Salisbury

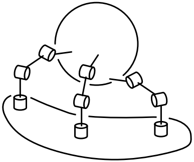
Taxonomy of contacts

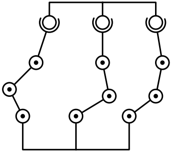
Mobility and connectivity of grasp

In-hand manipulation; subsequent work

Kinematic models of contact

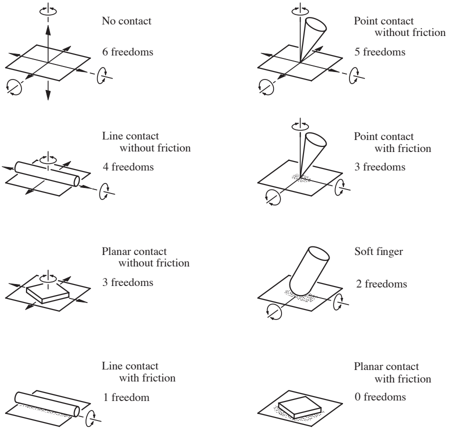
- A grasp is like a kinematic mechanism.
- Assume fingers do not lift or slip.
- Model each contact as a spherical joint.
- Apply Grübler's formula!





Taxonomy of contact types

- In previous slide, contact was modeled as spherical joint. Are there other possibilities?
- Salisbury's PhD thesis, 1982, included a taxonomy.
- Terminology was widely adopted.



Review of mobility and connectivity

- Next several slides are repeated from Lecture 4.

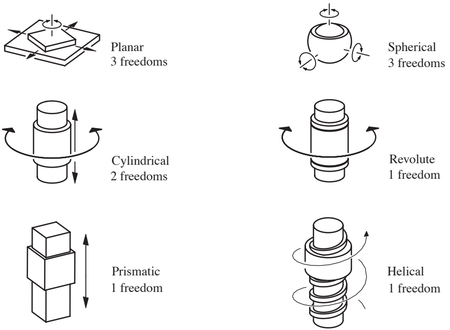
Review: Constraint and kinematic mechanisms

Link: a rigid body;

Joint: imposes one or more constraints on the relative motion of two links;

Kinematic mechanism: a bunch of links joined by joints;

lower pairs joints involving positive contact area.



Review: Mobility and connectivity

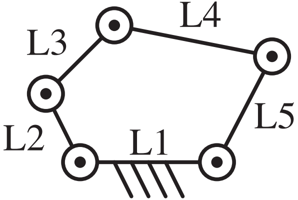
mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

What is the mobility of the five bar linkage at right?

What is the connectivity of

- Link 1 relative to link two?
- Link 3 relative to link 1?
- Link 3 relative to link 4?



Review: Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

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What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

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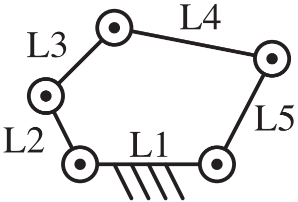

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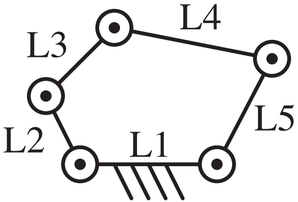

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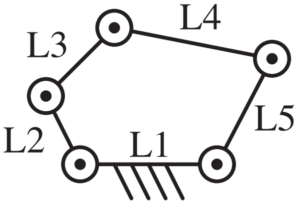

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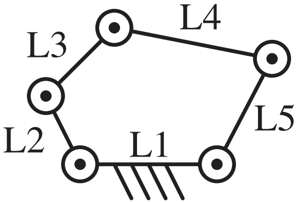


Review: Grübler's formula

Given n links joined by g joints,

with u_i constraints and f_i freedoms at joint i. (Note that $u_i + f_i = 6$.)

Assume one link is fixed and constraints are all independent.

The mobility M is

$$
\begin{aligned}
M &= 6(n-1) - \sum u_i \\
&= 6(n-1) - \sum (6 - f_i) \\
&= 6(n-g-1) + \sum f_i
\end{aligned}
$$

Or, for a planar mechanism:

$$
\begin{aligned}
M &= 3(n-1) - \sum u_i \\
&= 3(n-g-1) + \sum f_i
\end{aligned}
$$

Review: Grübler: special case for loops

The previous formula works (sort of) for all mechanisms.

For loops there is a variant.

One loop: $n = g$, so

$$M = \sum f_i + 6(-1)$$

Two loops: make a second loop by adding k links and $k+1$ joints:

$$M = \sum f_i + 6(-2)$$

Every loop increases excess of joints over links by 1. For l loops:

$$M = \sum f_i - 6l$$

for a spatial linkage, and

$$M = \sum f_i - 3l$$

for a planar linkage.

Review: Common sense

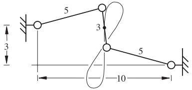
Example: what is the mobility of Watt's linkage?

Planar Grübler's formula:

$$M = 3(n-1) - \sum u_i =$$

$$M = 3(n-g-1) + \sum f_i =$$

$$M = \sum f_i - 3l =$$



Independent constraints is a very strong assumption.

Spatial Grübler's formula:

$$M = 6(n-1) - \sum u_i =$$

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Why?

Review: Common sense

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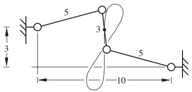

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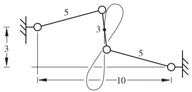

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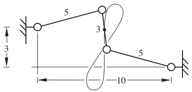

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Spatial Grübler's formula:

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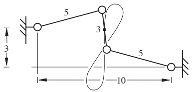
Why?



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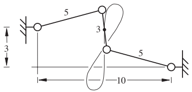

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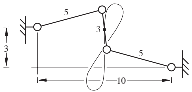

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Applying mobility and connectivity to grasping

Salisbury suggests four measures:

- M Mobility of the entire system with the finger joints free.
- M' Mobility of the entire system, with the finger joints locked.
- C Connectivity of the object relative to a fixed palm, with the finger joints free.
- C' Connectivity of the object relative to a fixed palm, with the finger joints locked.

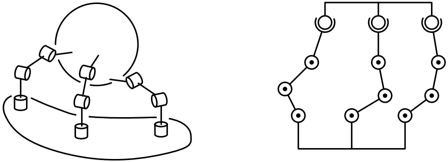
If $C = 6$ then object can make general motions.

If $C' \leq 0$ then hand can immobilize object.

Example: the Salisbury hand

What is C?

What is C'?



- This assumes no finger is in a singular configuration,
- and contacts are not collinear.
- This neglects stability of the grasp. You need statics to even start on grasp stability.
- Salisbury's analysis generalizes nicely: to freely manipulate an object in the hand with point fingers, the hand mechanism needs at least nine DOFs.

Example: the Salisbury hand

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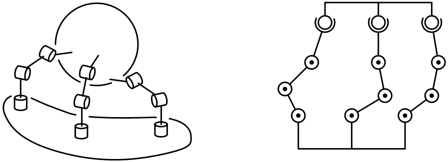

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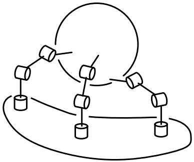

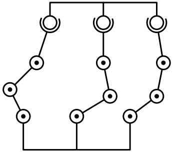

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In-hand manipulation; subsequent work

How to produce in-hand manipulation?

- Salisbury's "dexterous hand" approach: three fingers, each with "point contact with friction". Mason and Salisbury 1985 (!), Salisbury 1982.
- Control, planning, global controllability ... Lots of work, for example see Murray, Li, and Sastry 1994; Marigo and Bicchi 2000.
- Finger gaiting, controlled slip, self throws, ... LOTS of possibilities, a growing research area. One example: "extrinsic dexterity", Chavan-Davle et al, 2014.