19.
Force Dual

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Mechanics of Manipulation
Spring 2010

Today's outline

Review representation of PCCs.

Duality between points and lines.

Working in the oriented plane.

Using force dual technique

Examples.

Review: how did cones arise?

- Why polyhedral convex cones in wrench or twist space?
  - Wrenches applied by frictional or frictionless contacts.
    - Each normal or friction edge is a wrench $\mathbf{w}_i$.
    - Total wrench must be in positive linear span: $\text{pos}(\{\mathbf{w}_i\})$.
    - Edge representation of a cone.
  - Twists consistent with kinematic constraints.
    - Each contact normal $\mathbf{c}_i$ describes a half-space of feasible twists, reciprocal or repelling to the normal.
    - Twist must be in intersection $\cap \text{half}(\mathbf{c}_i)$.
    - Face representation of a cone.

Review: representing cones

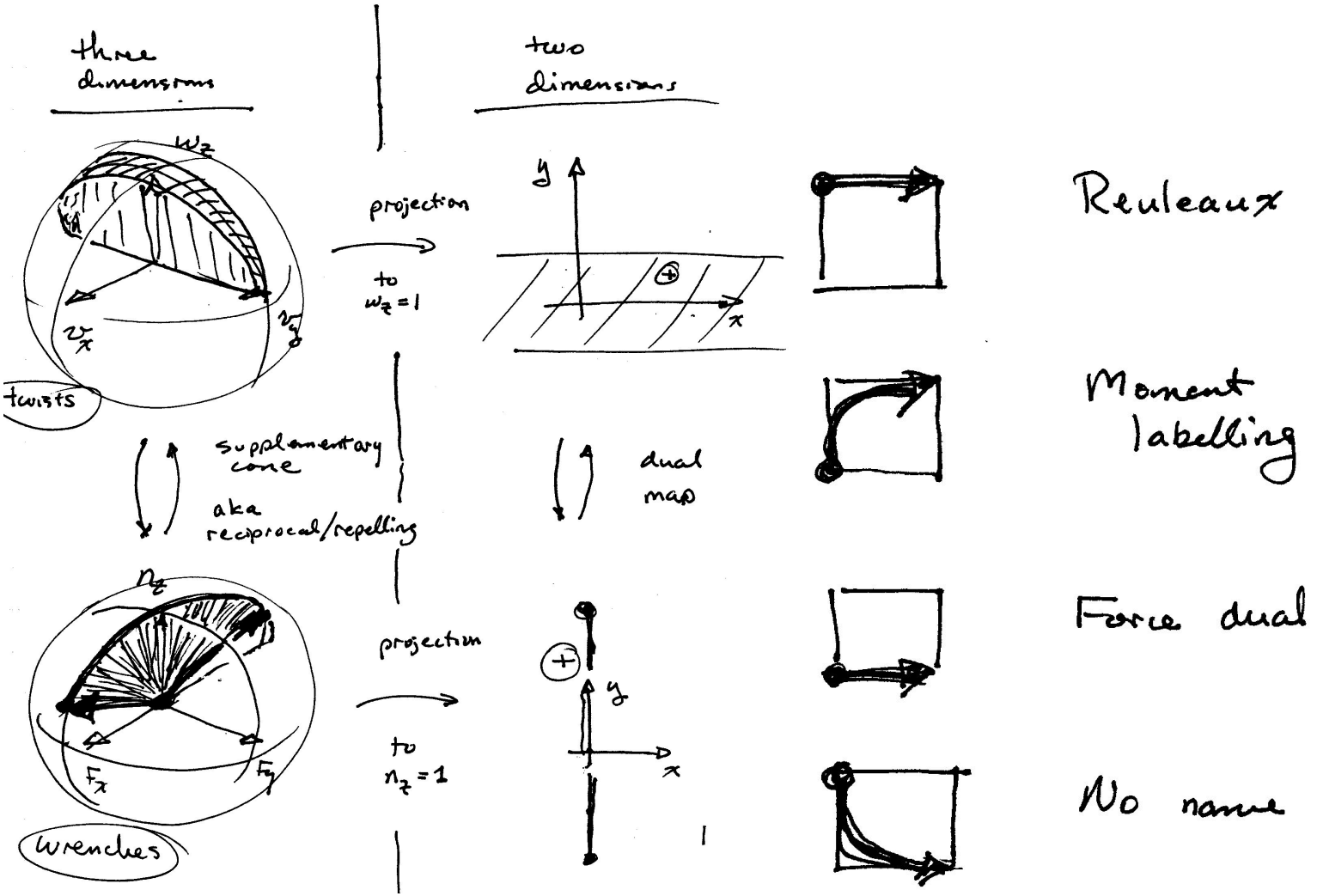
- For 3 space (6D wrench or twist space) represent them by the edges or by the face normals.
- For the plane (3D wrench or twist space) we can use 2D graphical techniques:
  - Reuleaux's method:
    - Label rotation centers, keep consistent labellings.
    - Equivalent to projection of twists to oriented plane.
  - Moment labeling.
    - Label moments, keep consistent labellings.
    - Equivalent to projection of *reciprocal or repelling twists* to the oriented plane.
  - Force dual.
    - Map each line of action (contact normal or edge of friction cone) to *dual point* in oriented plane.
    - Construct convex hull in oriented plane.
    - Equivalent to projection of wrenches to oriented plane.

Do we really need another graphical method

Why another method?

- Moment labelling is sort of strange!
  - Doesn't map an element to an element.
  - Empty set maps to universe, and vice versa.
  - Cannot represent non-convex cones.
  - Cannot represent set of all contact normals of an object.
  - Not easily combined with other techniques.
- Force dual remedies these things

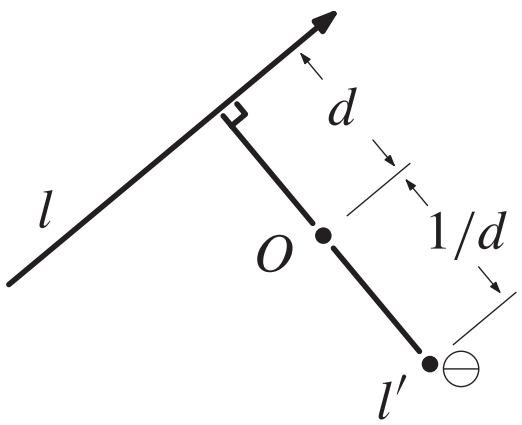
Roadmap to graphical techniques



Duality in the projective plane

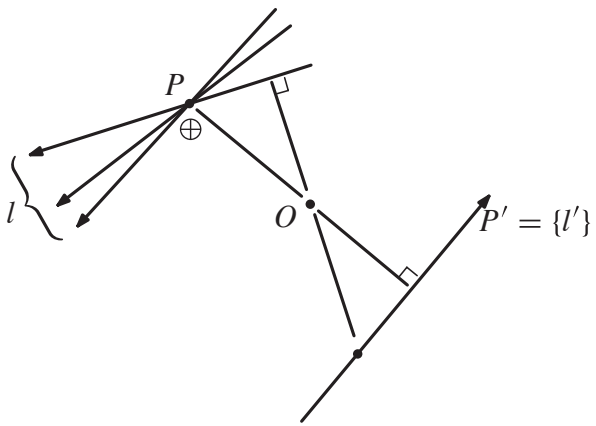
- Recall that for the projective plane there is a duality between *point* and *line*
- We can make that concrete by defining a mapping D.
  - Define $D(l)$ of a line l to be the point p such that $Op \ldots$
  - Define $D(p)$ of a point p to be the locus of $D(l)$ for all l through p.
  - Note $D(p)$ is a line, and $D(D(p))$ is p.
  - Note what happens at infinity.
  - Note it depends totally on choice of scale and origin.
- Check out the movies.

Dual of a directed line



- Given a directed line, and an origin;
- Construct perpendicular through origin;
- Take point on perpendicular, at distance inverse to moment arm;
- Note the sign of the moment.

Dual of a signed point



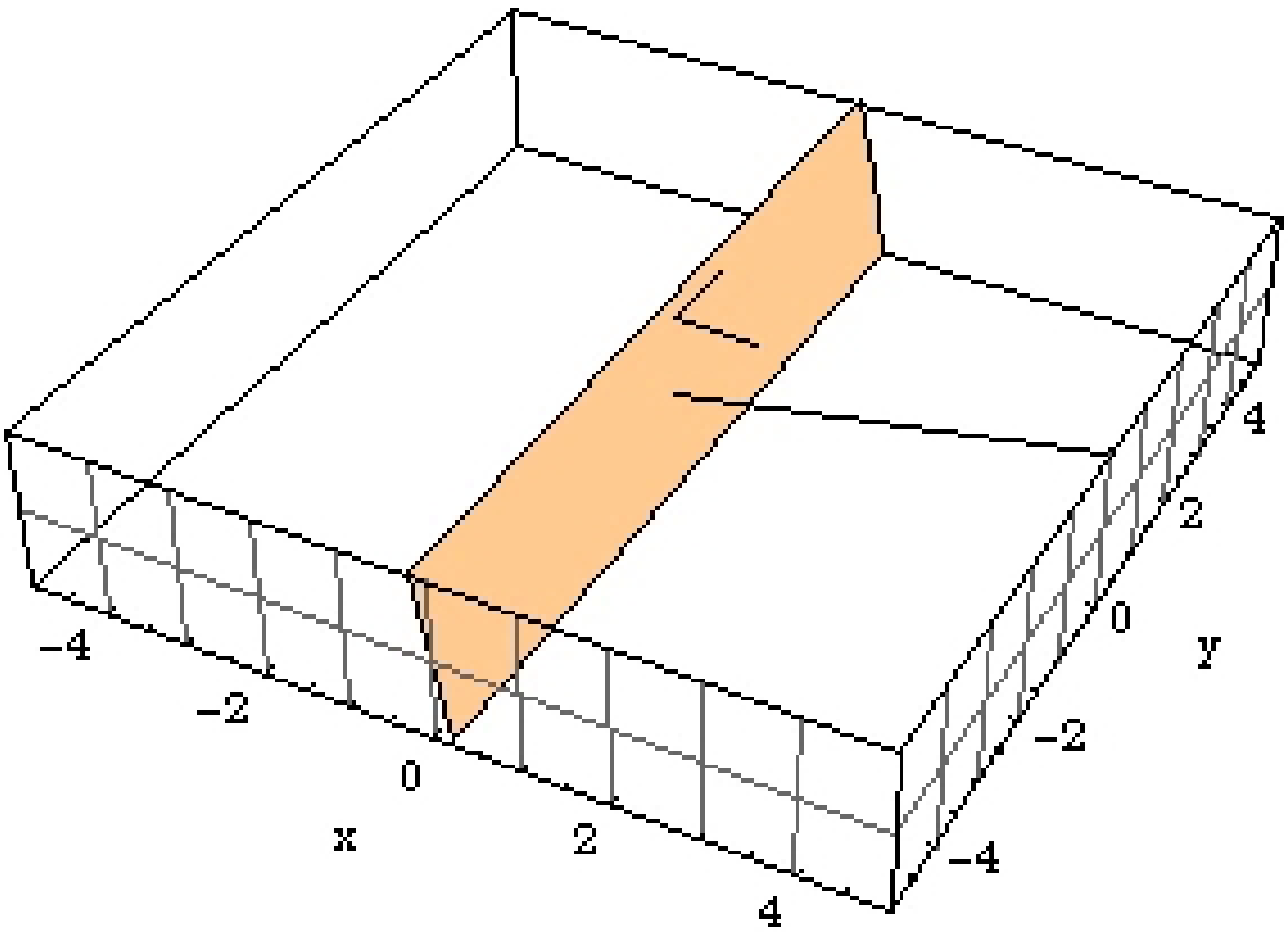
- Given signed point P, let $\{l\}$ be the set of directed lines through P.
- Define P' to be $\{l'\}$, with a direction determined by the sign of P.
- A simple geometric proof shows that:
  - P' is a directed line;
  - $P'' = P$.

  Hence the transformation is *dual*.

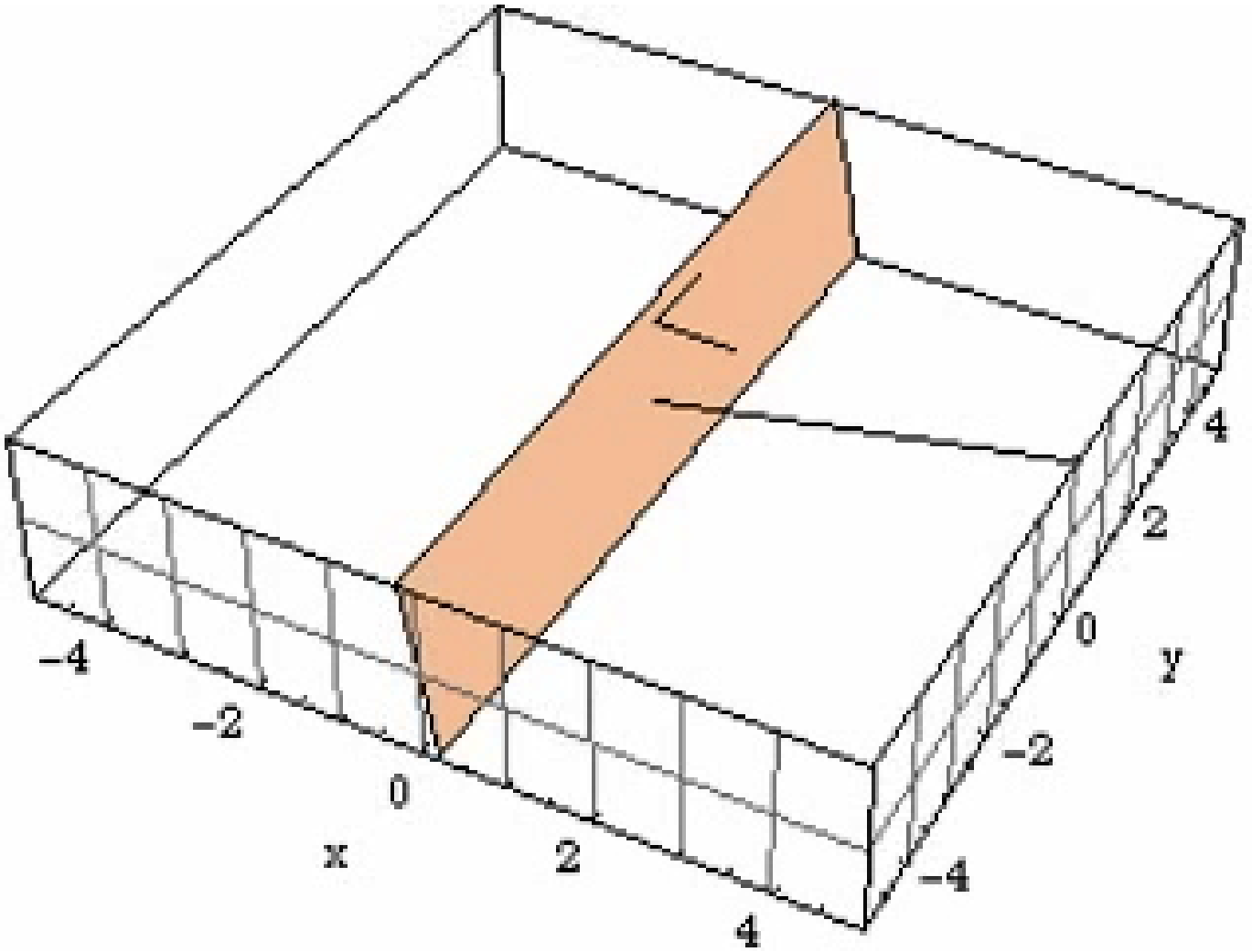
About the dual map in the oriented plane

- This dual map in the projective plane (i.e. without the sign) is widely used.
- The dual map preserves geometric incidence relationships.
  - If points A and B determine a line l, then lines A' and B' intersect at point l'.
- The origin is dual to the line at infinity.

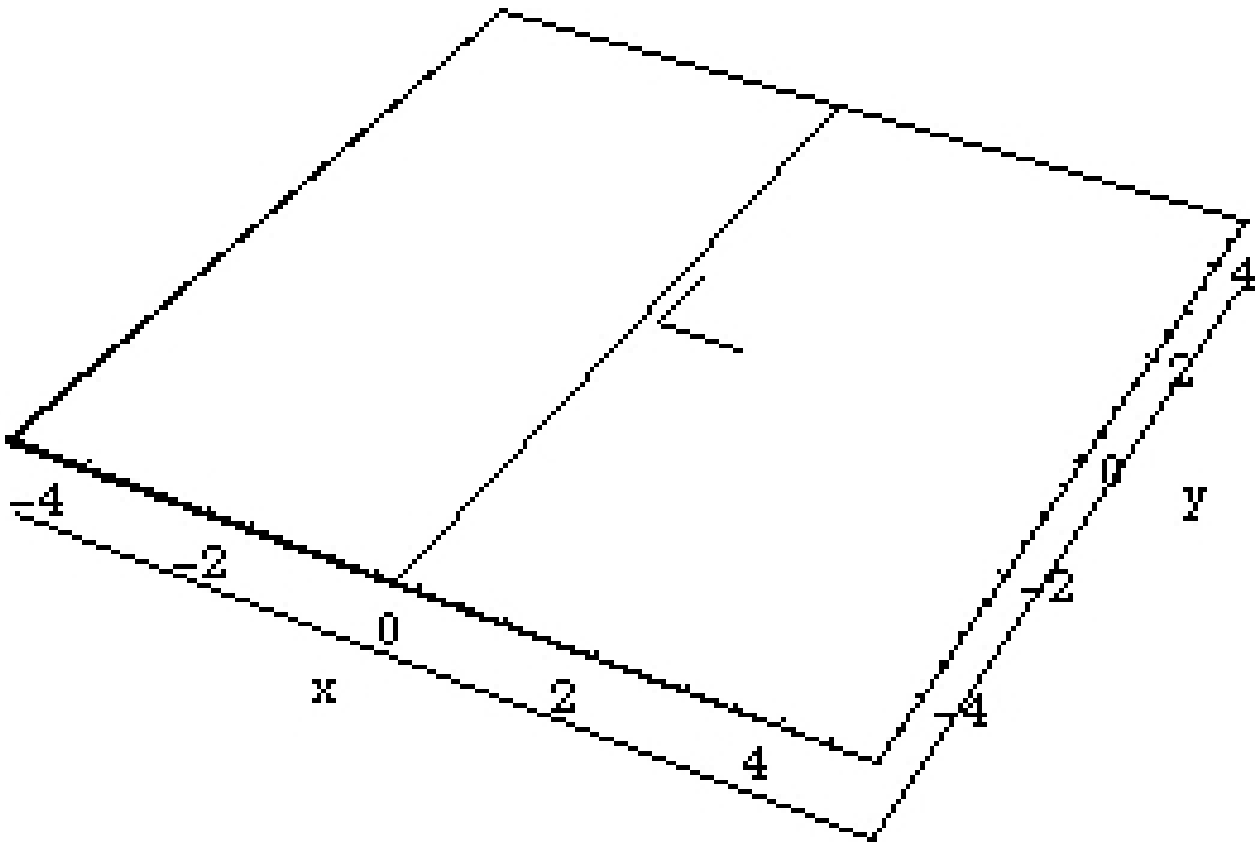
Movie1



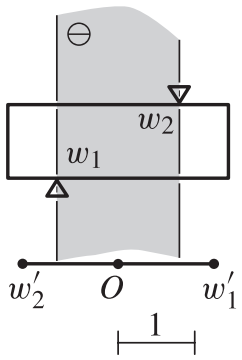
Movie2

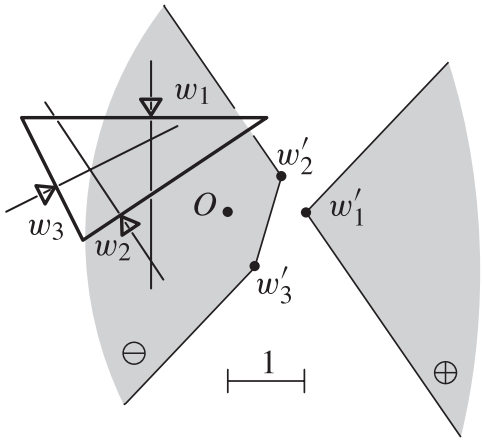


Movie3



Representing wrench cones



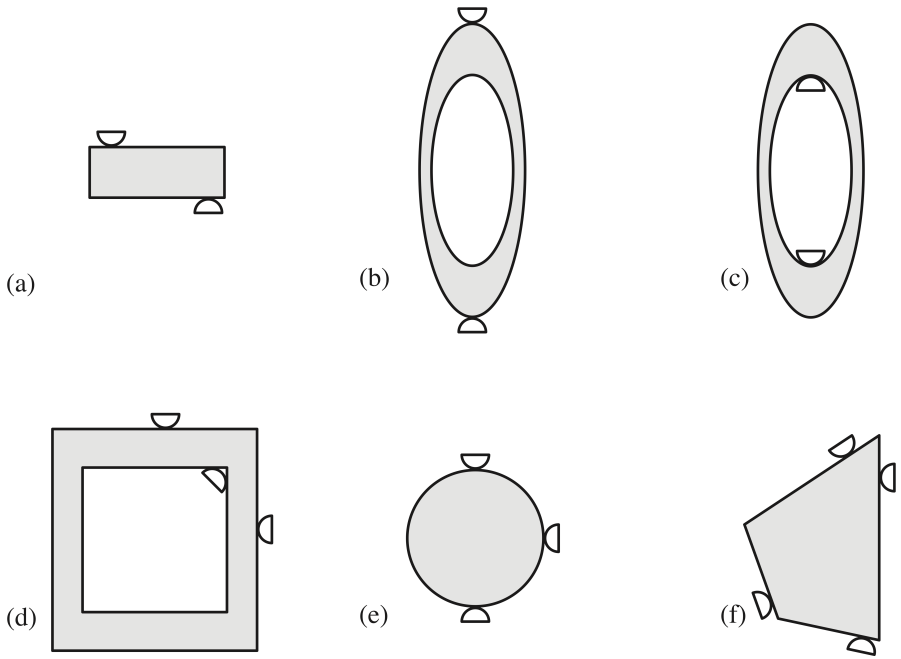


The method:

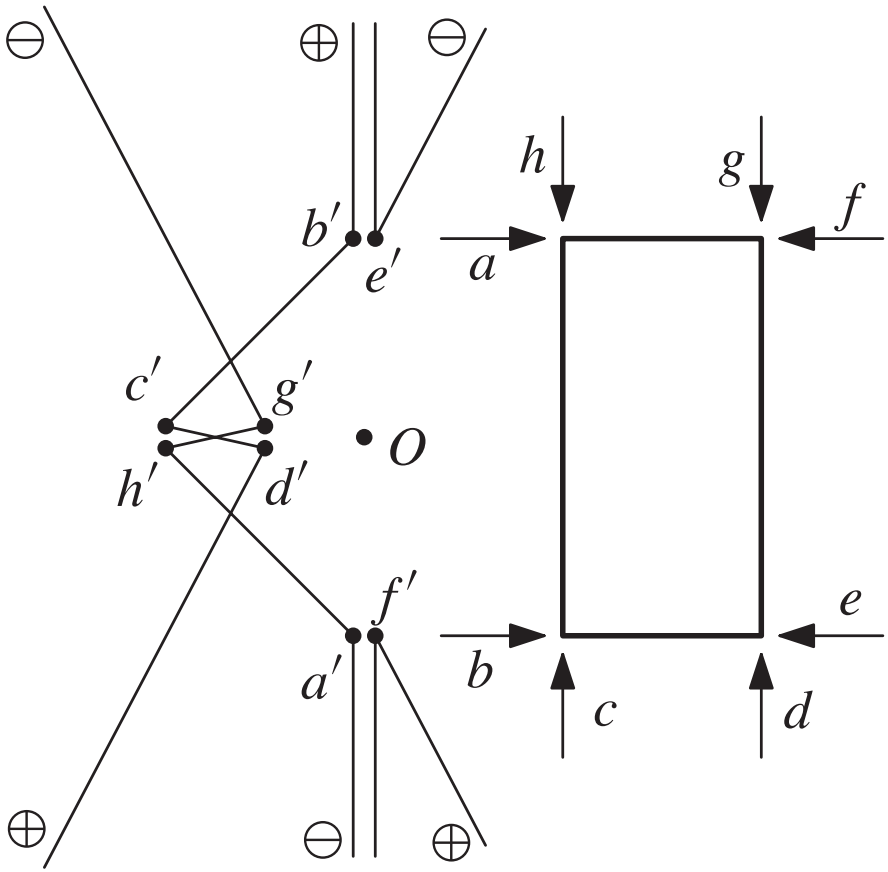
1. Choose origin and unit length.
2. Construct dual of each line of action.
3. Take the convex hull.

Examples

- Single contact.
- Two contacts.
- Friction cone.
- Parallel or anti-parallel contacts.



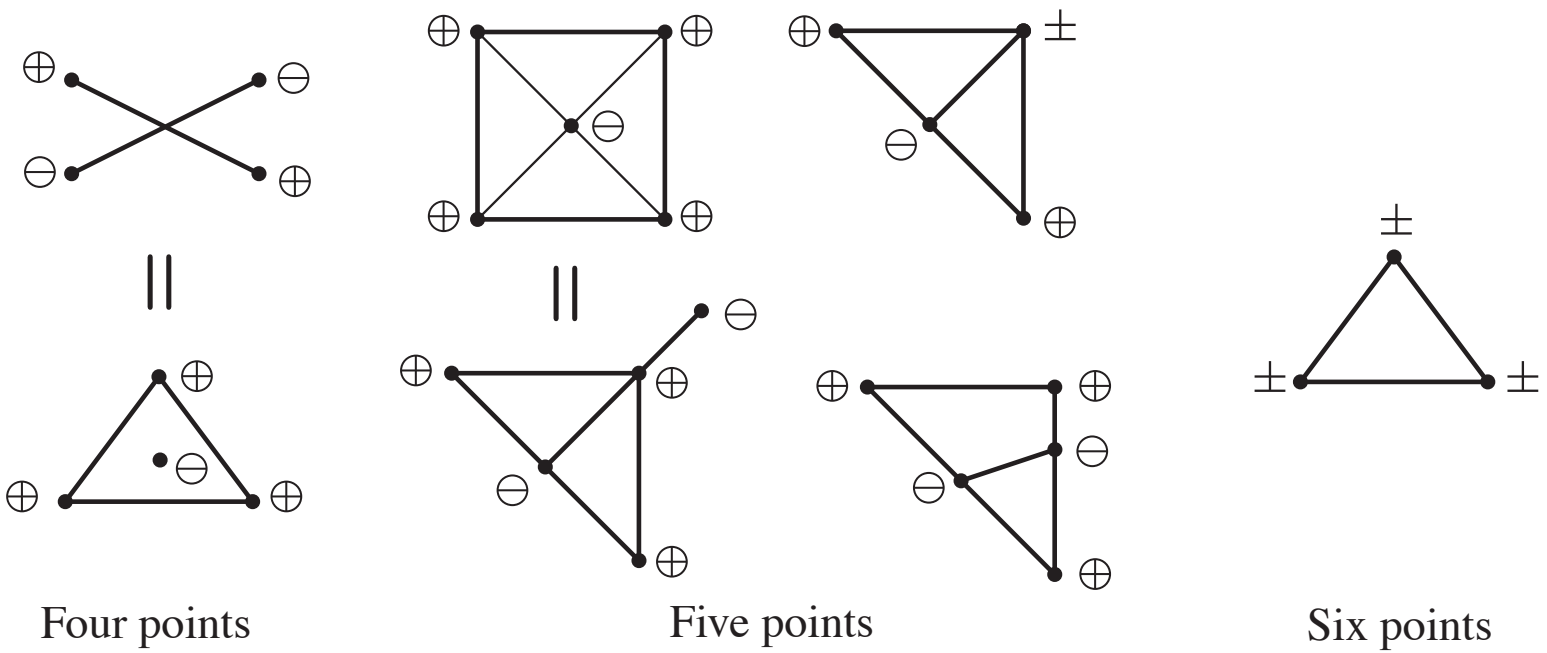
Zigzag locus



- Force dual can represent *non-convex cones*!
- Example: The set of contact normals.
- Also known as the set of frictionless contact forces.
- Force dual is called the *zigzag locus*.

Irreducible arrangements

- Suppose you run the Mishra, Schwartz, and Sharir grasp planning algorithm. It terminates with an irreducible set of contact normals. How many such arrangements are there?



Four points

Five points

Six points