Humans:
The Final Frontier

source: http://www.gimartex.es/myfiles/Ballet-dancer_01.jpg

Adrien Treuille

Overview

- State of the art.
- Body models.
- Animation
- Questions

source: 3dscience.com

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State of the Art

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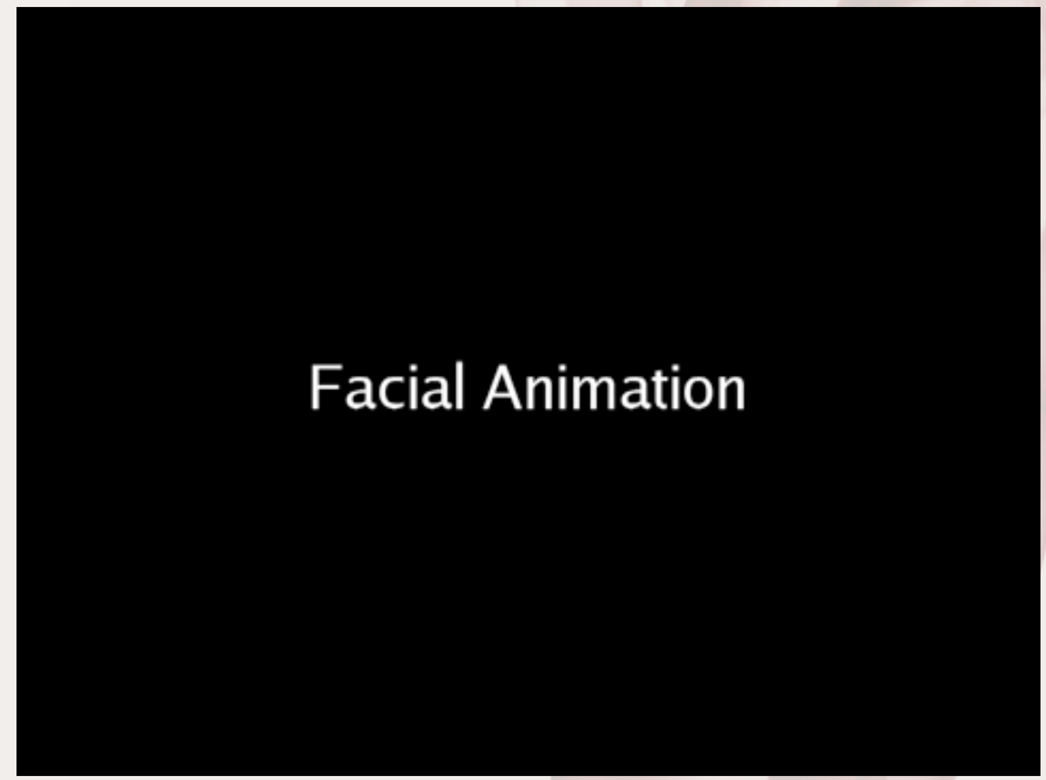

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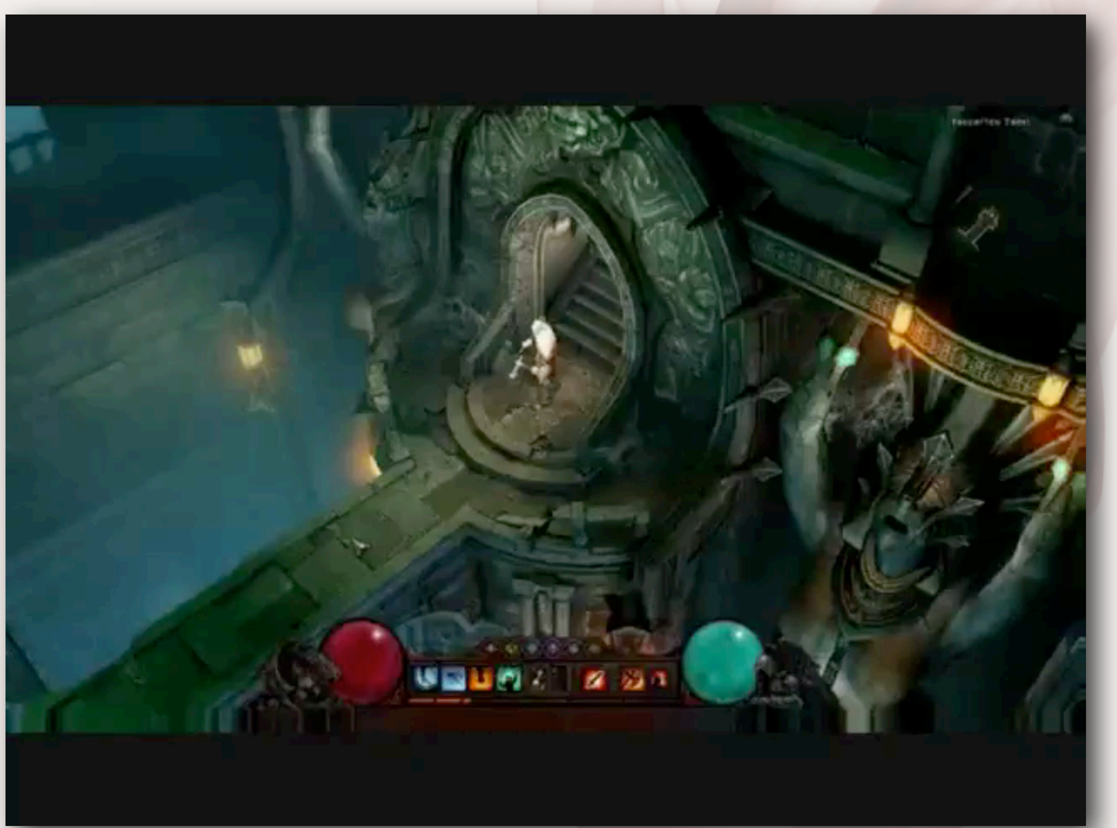

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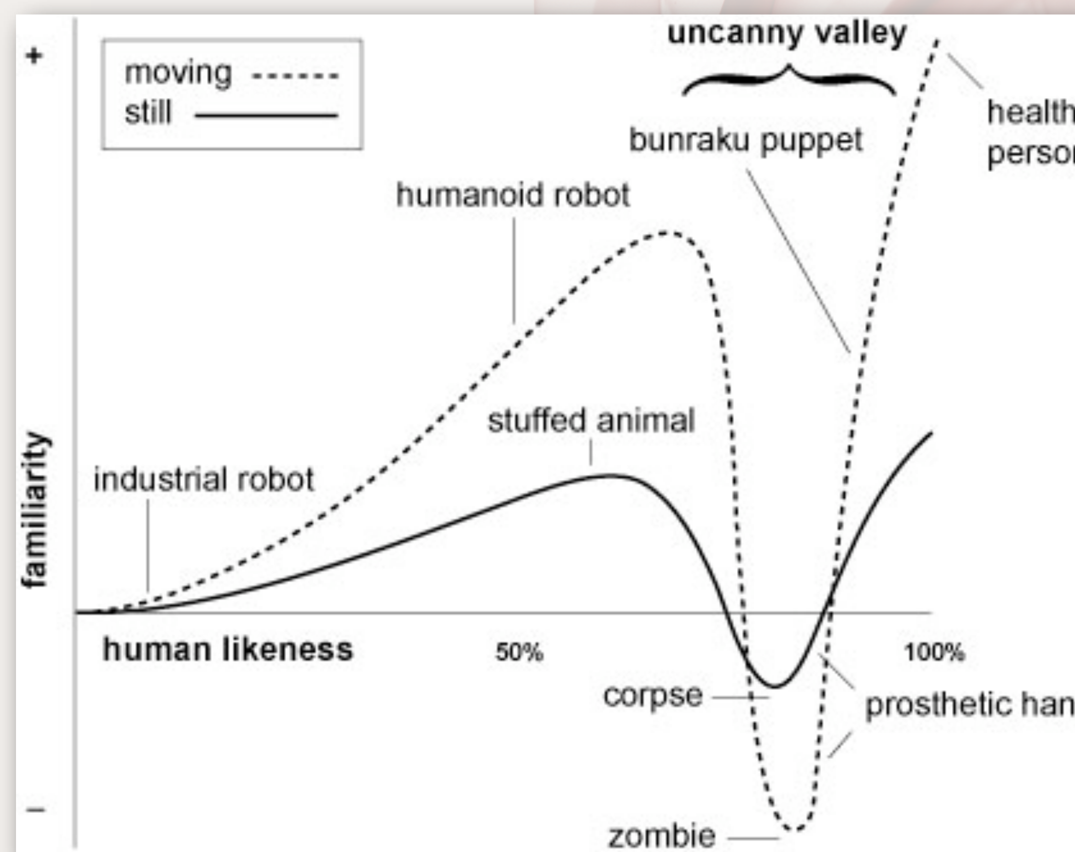

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State of the Art

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    - (As usual, rendering ahead of animation.)
- State of the art for animation production.
- State of the art for games.
- Uncanny Valley.
- Facial Animation.
- Most human animation is *data driven*.

(Like saying that graphics is solved by the camera.)

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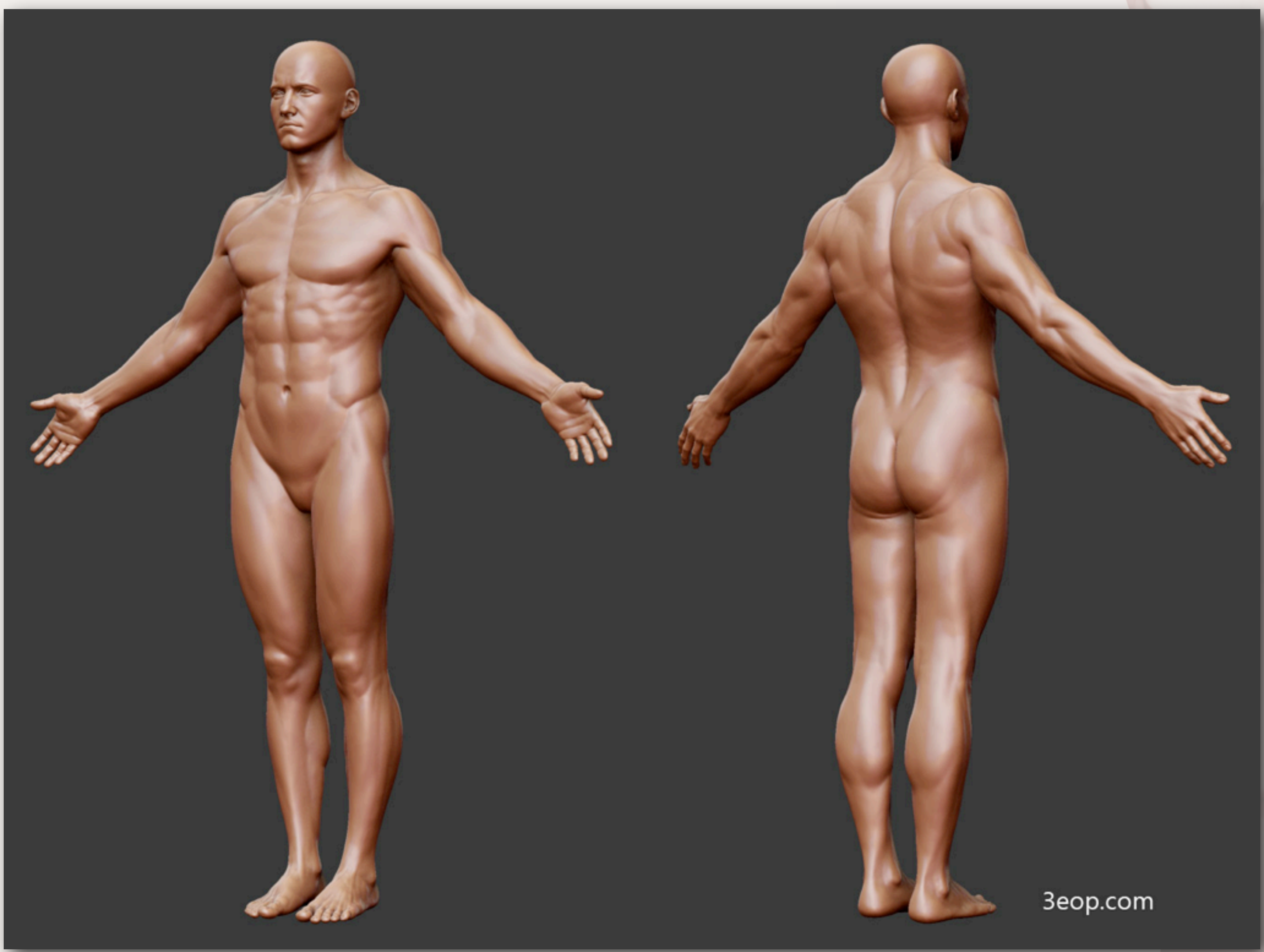
Body Representation

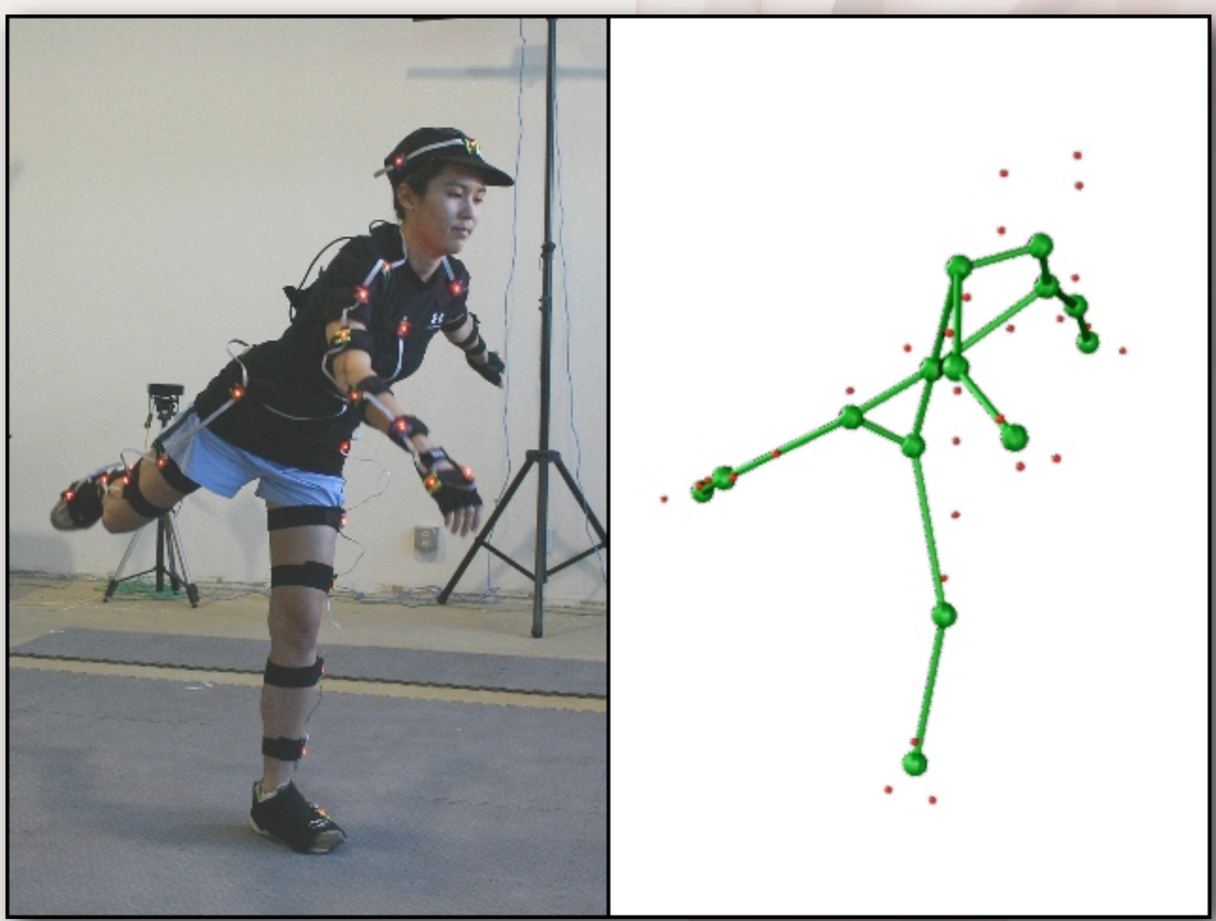


source: http://www.3eop.com/data/3d/images/08_05_26_anatomy_study_male.jpg

How to represent a human body on a computer?

Body Representation

- **Kinematic Skeleton**



source: https://buffy.eecs.berkeley.edu/PHP/resabs/resabs.php?f_year=2005&f_submit=advgrp&f_advid=10917651

Body Representation

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- Anatomical

source: http://physbam.stanford.edu/~fedkiw/

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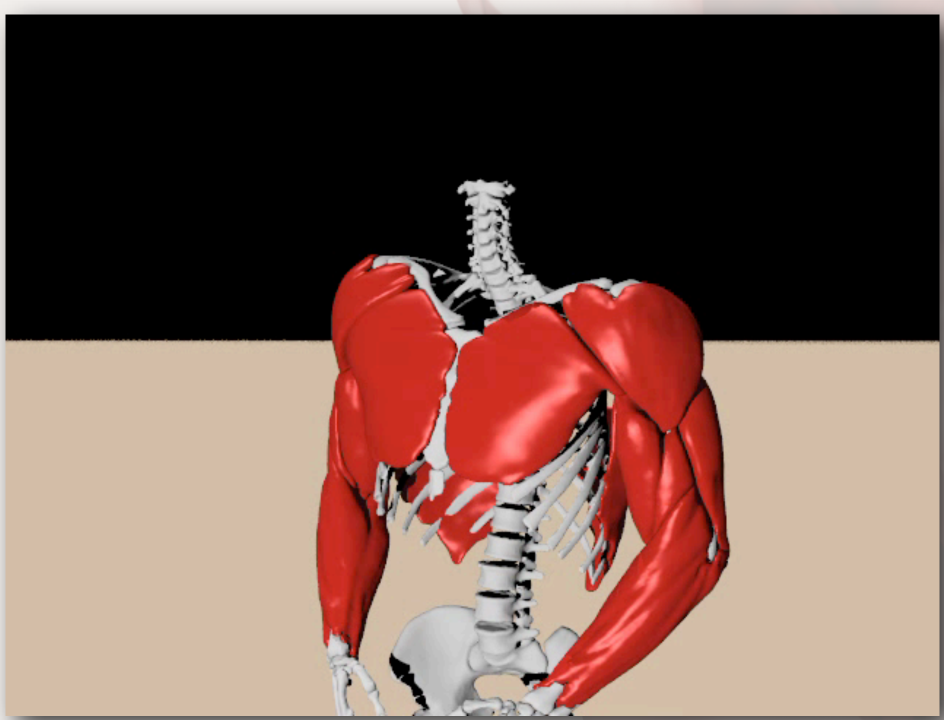

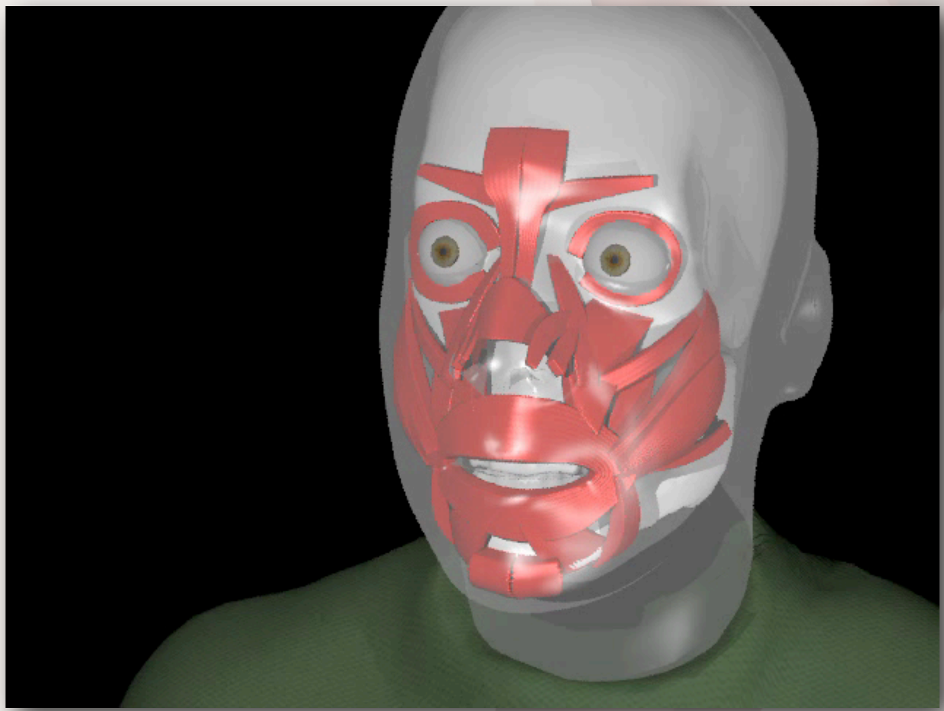

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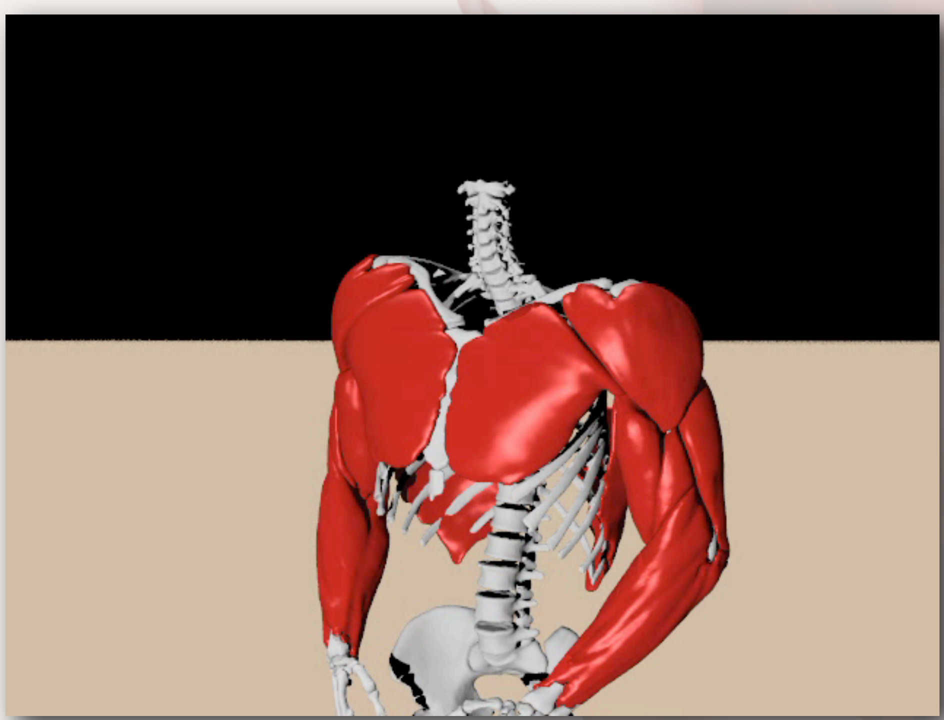

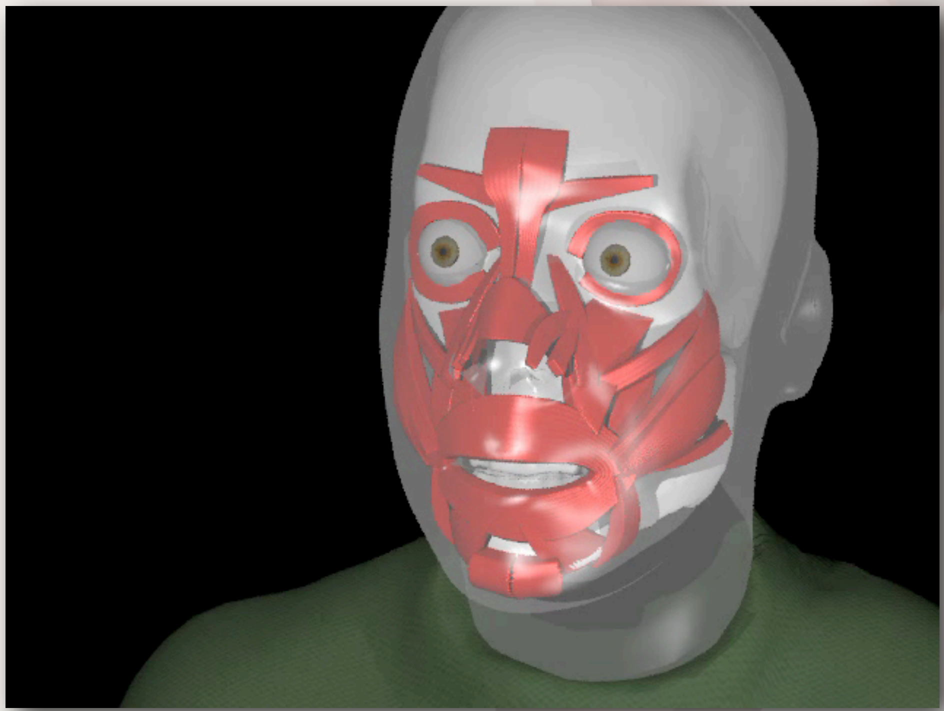

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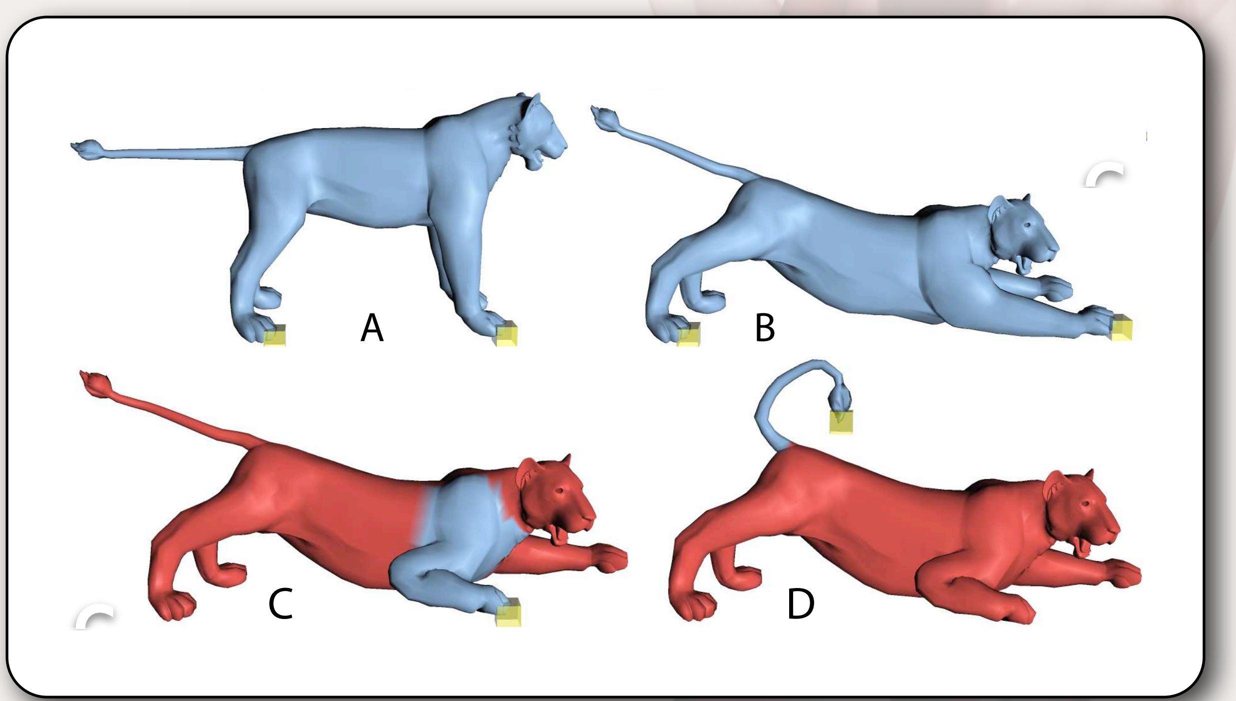
Body Representation

- Kinematic Skeleton
- Anatomical
- Pure Mesh



source: http://people.csail.mit.edu/sumner/research/meshik/

Body Representation

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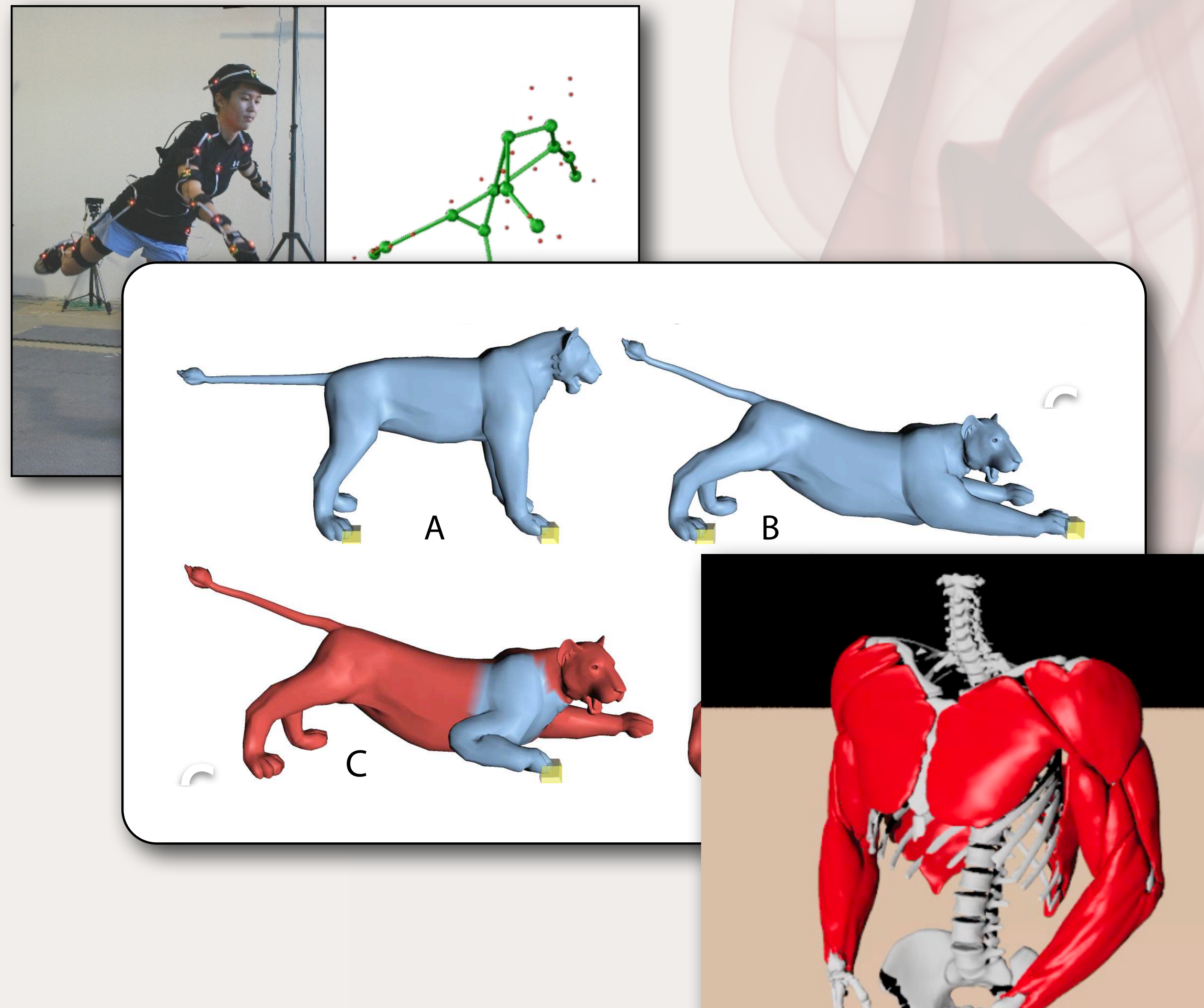
- **What are the advantages and disadvantages?**



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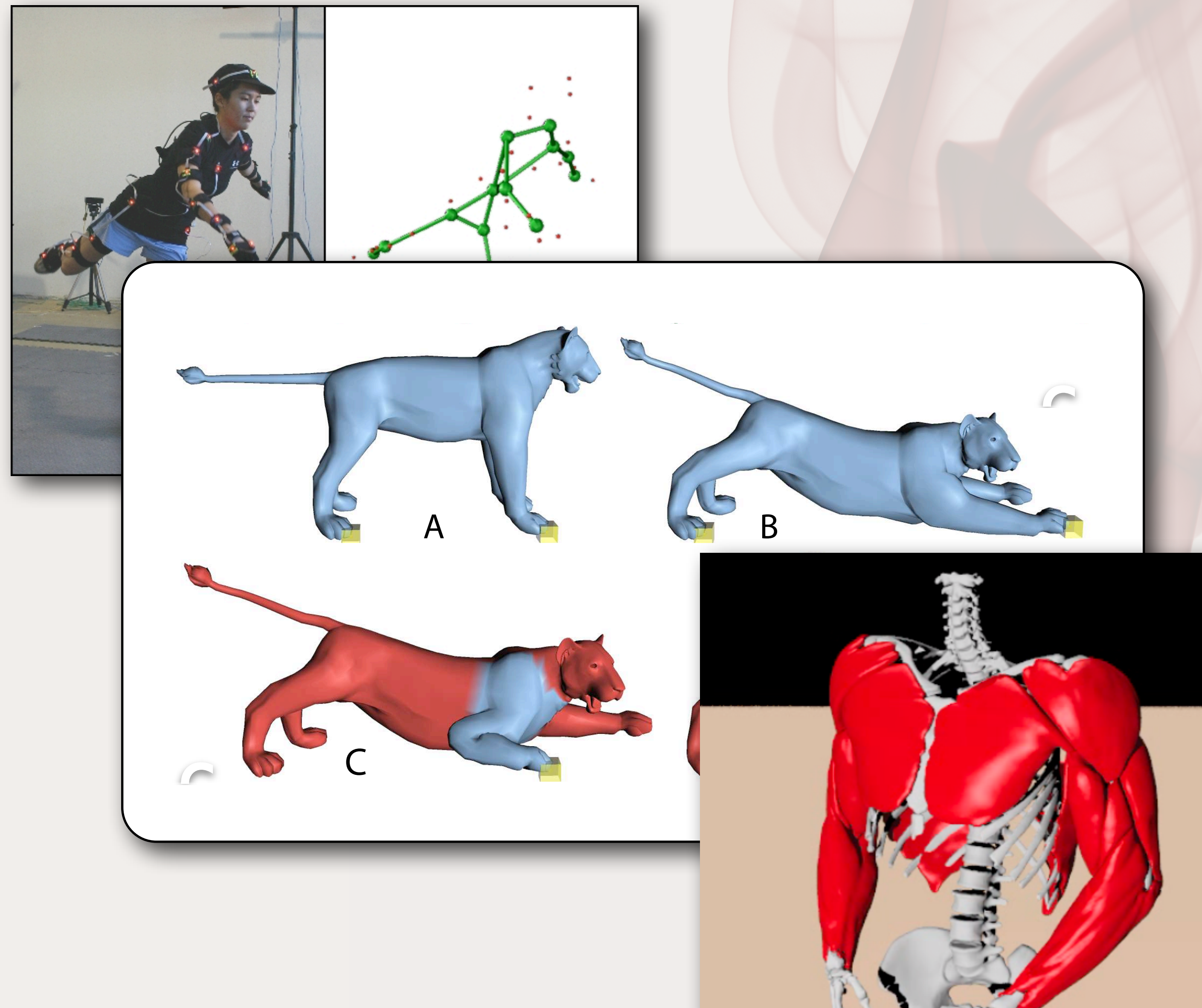
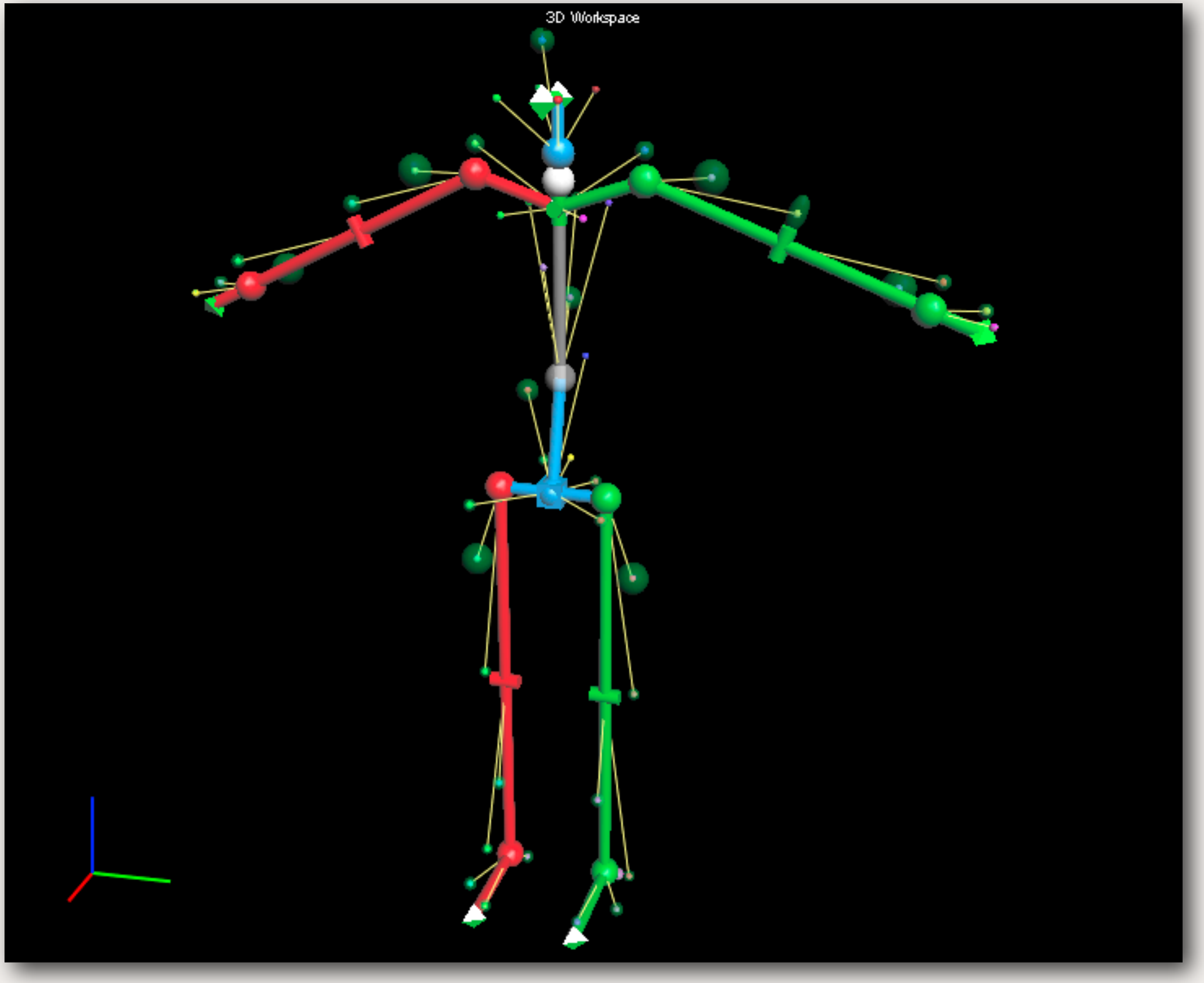


Skeleton Representation



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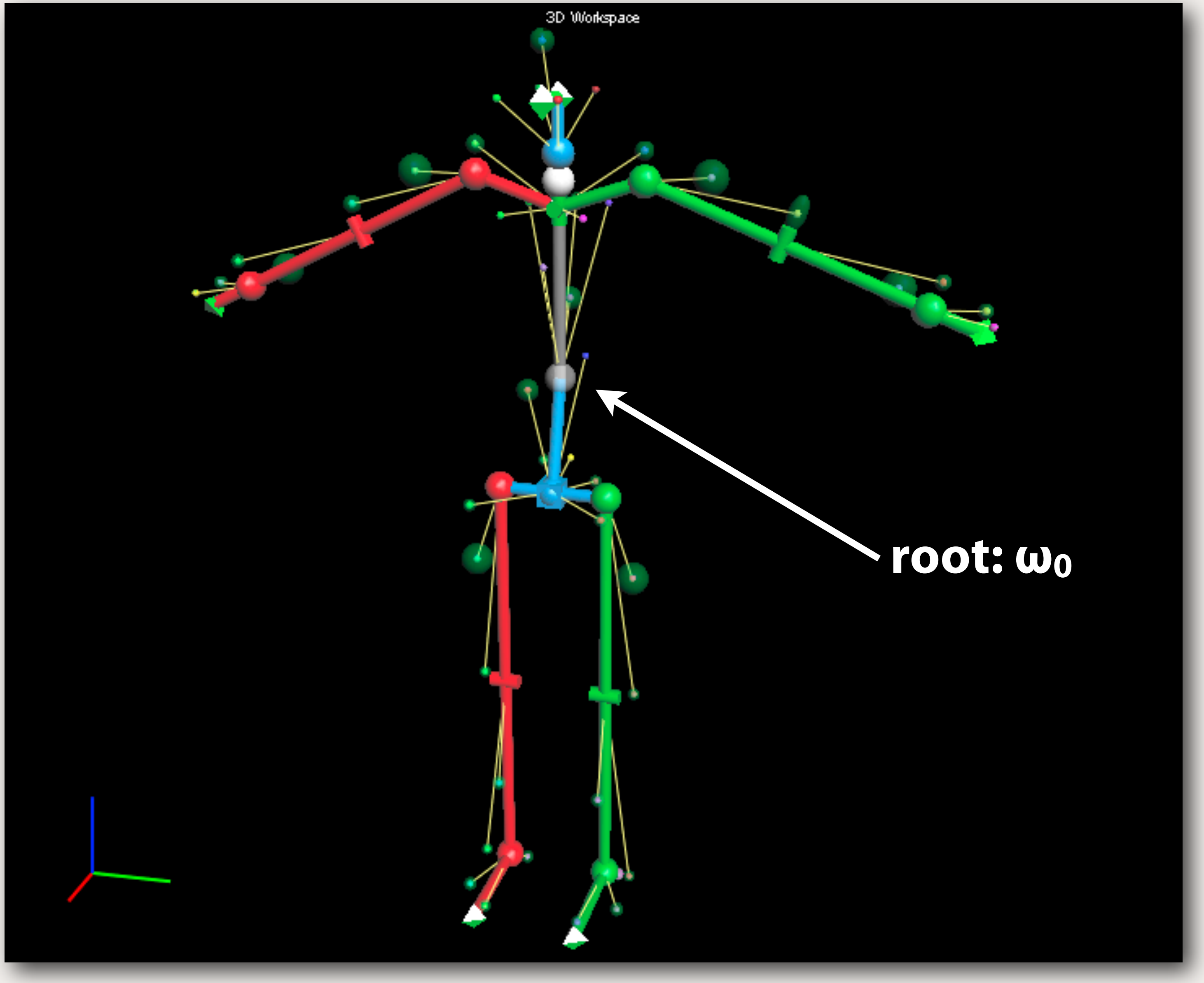
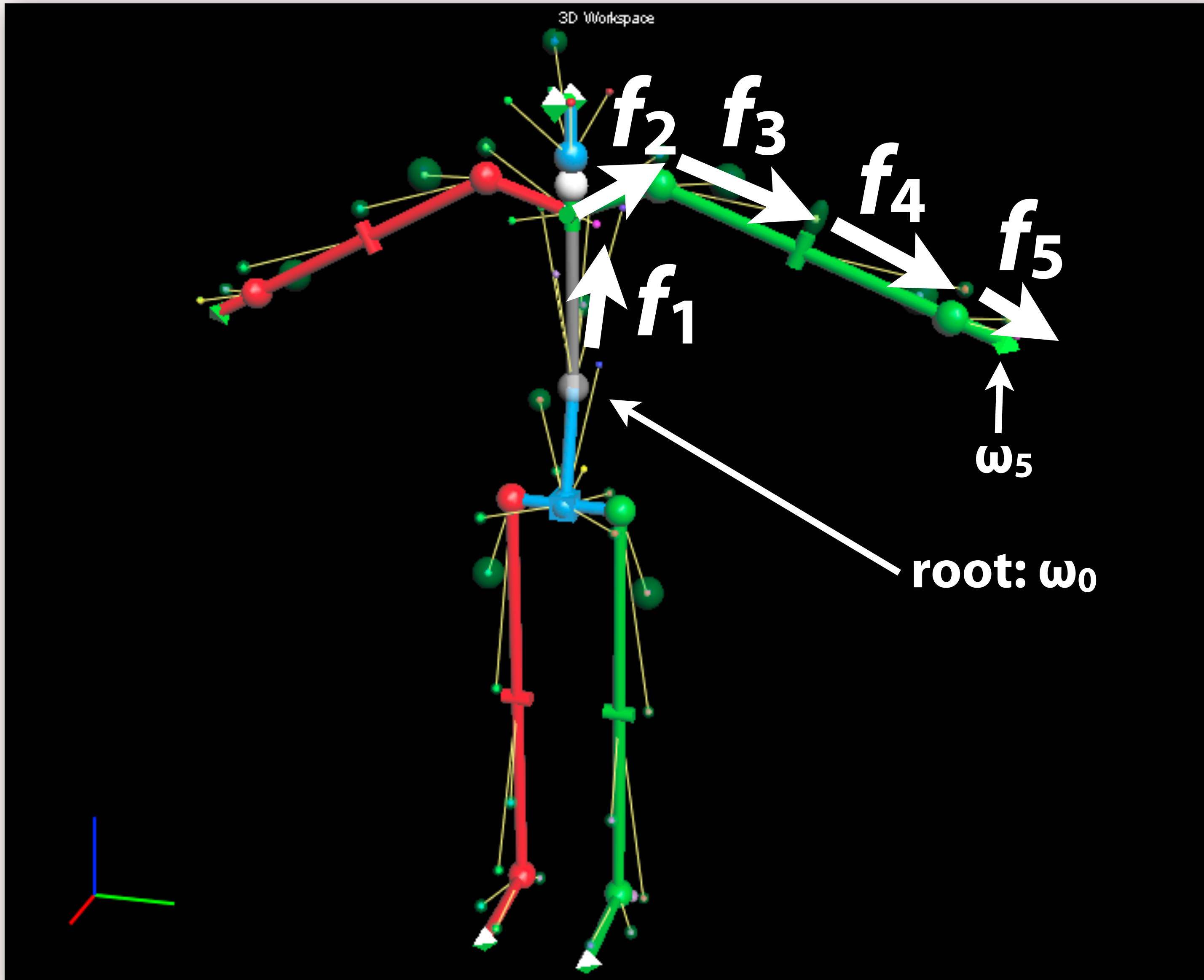


$$\omega_0 = [\mathbf{x}_0, \theta_0] \in \mathbf{R}^6$$

Skeleton Representation



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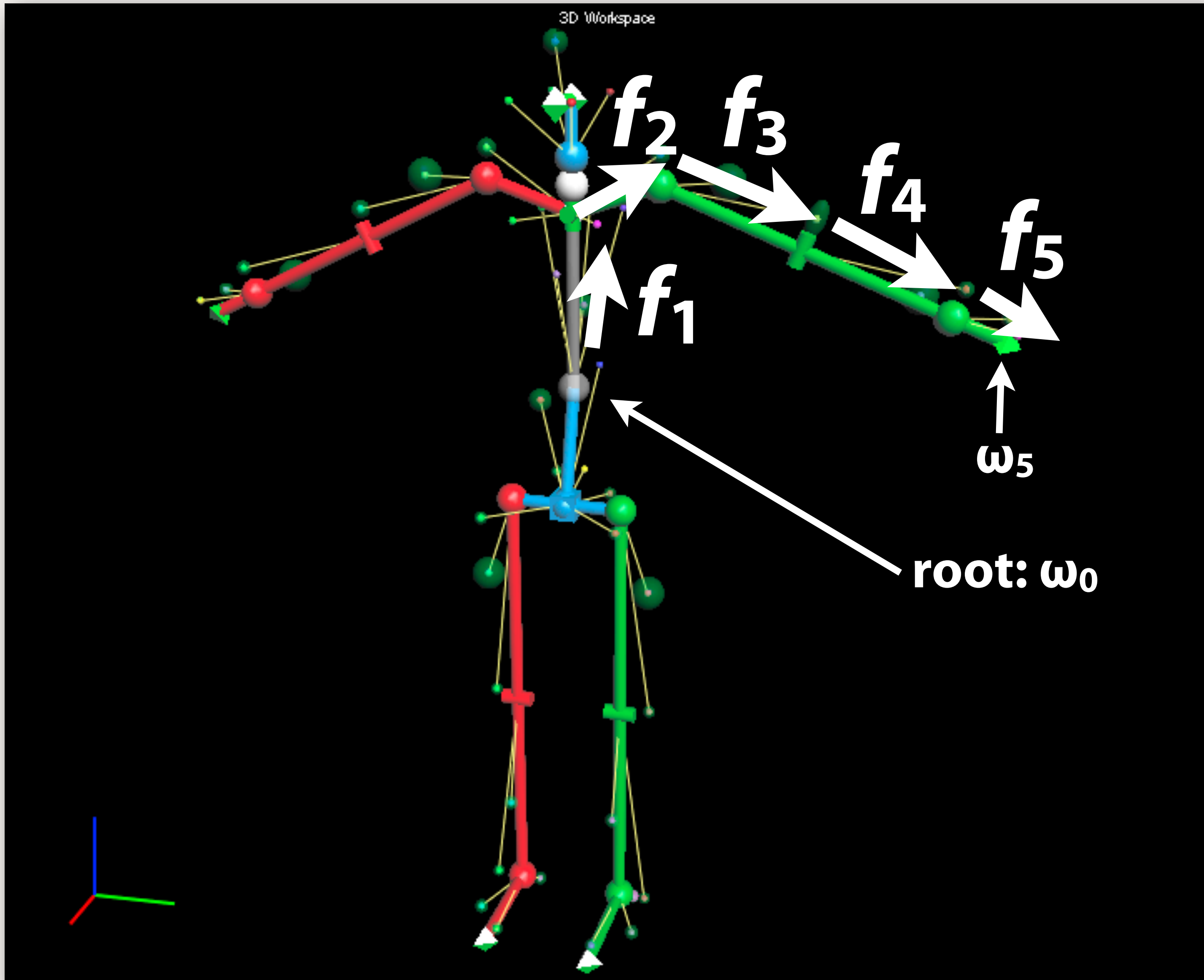

Ω is the vector of *internal* joint angles, i.e. shoulders, hips, etc.

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Motion Capture

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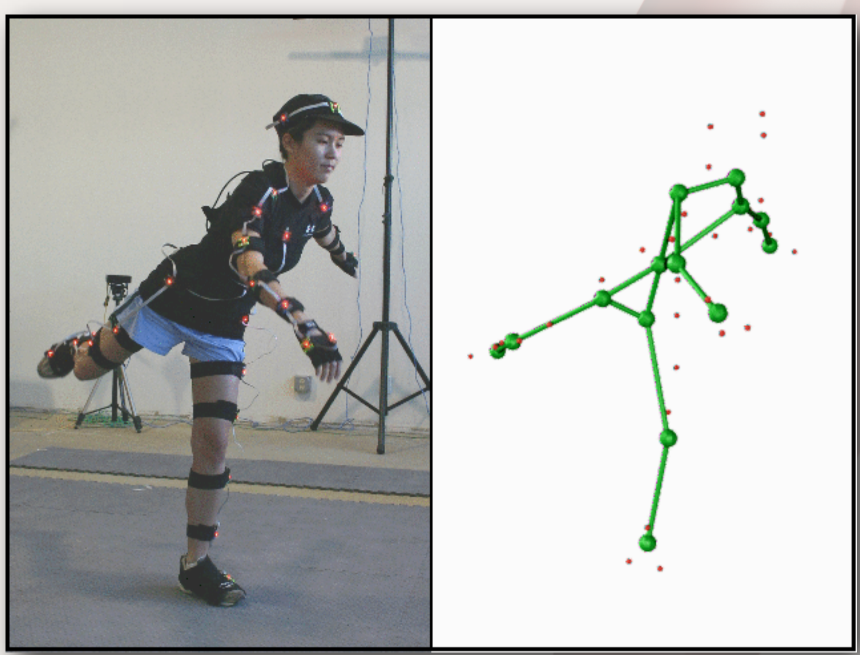
- **Attach markers to a humans body.**



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- **Attach markers to a humans body.**
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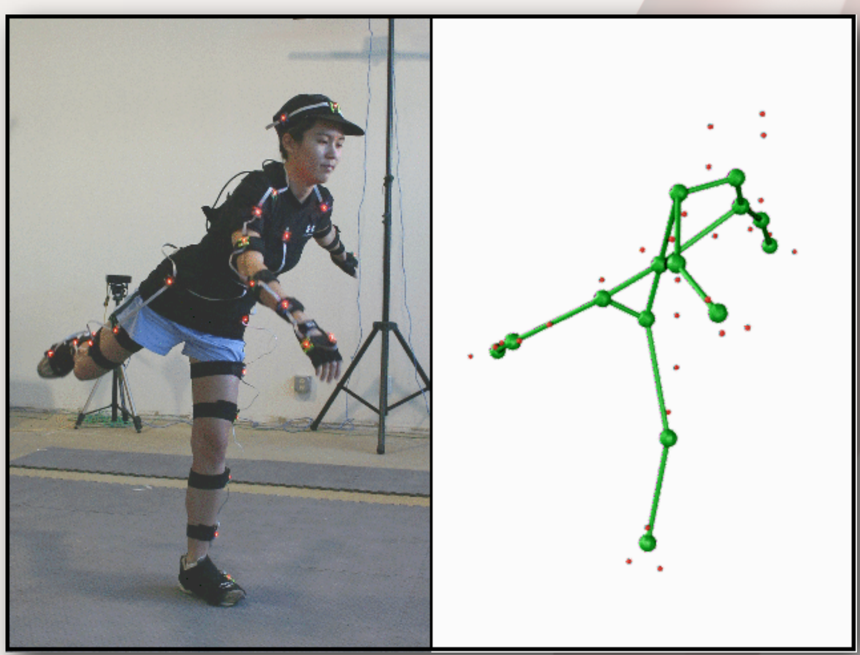

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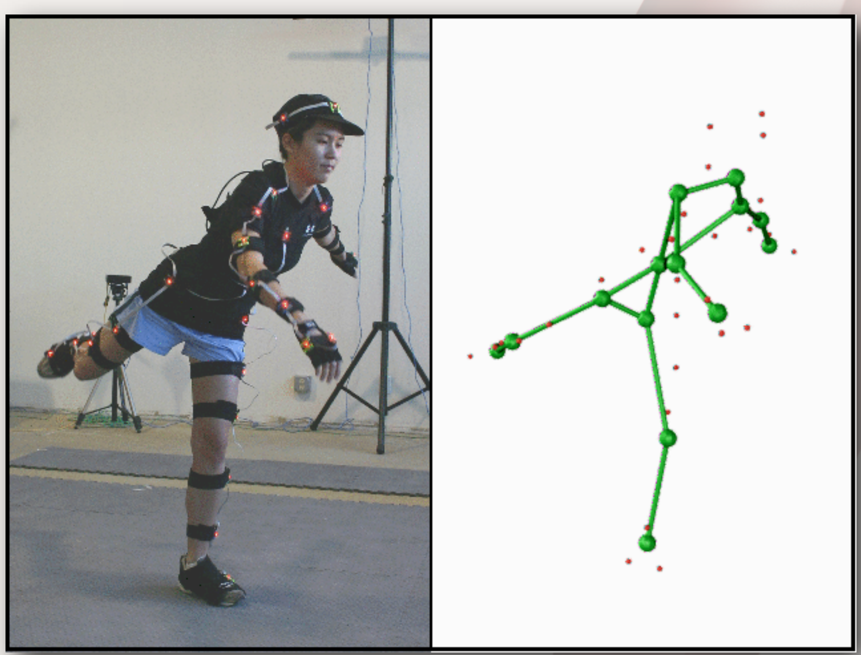
- Cameras capture 2D markers positions.



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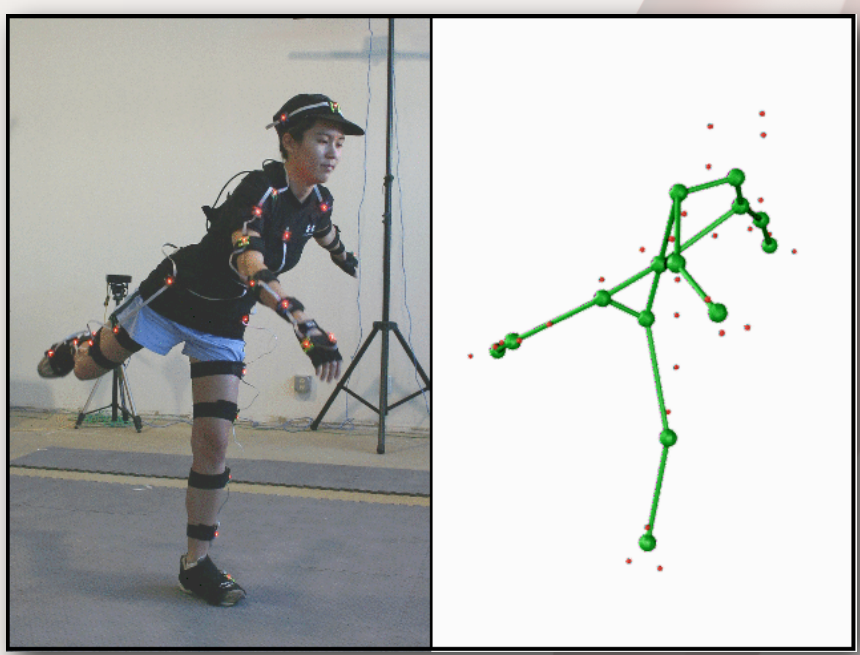

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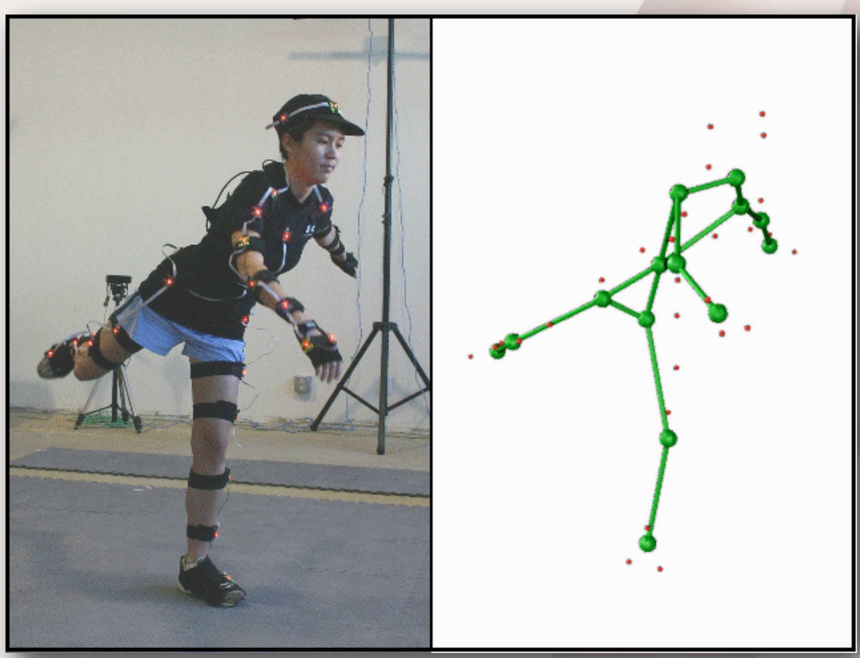

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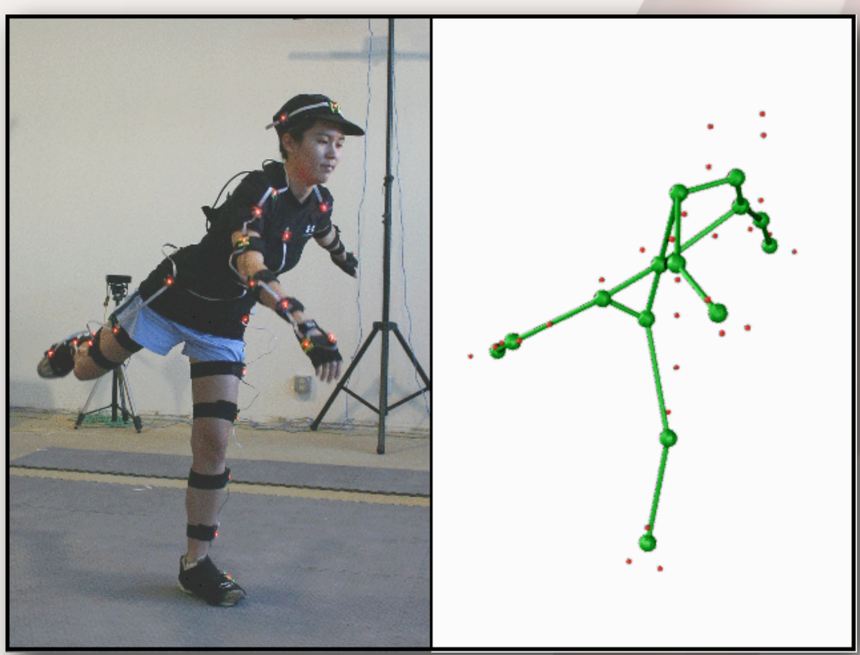
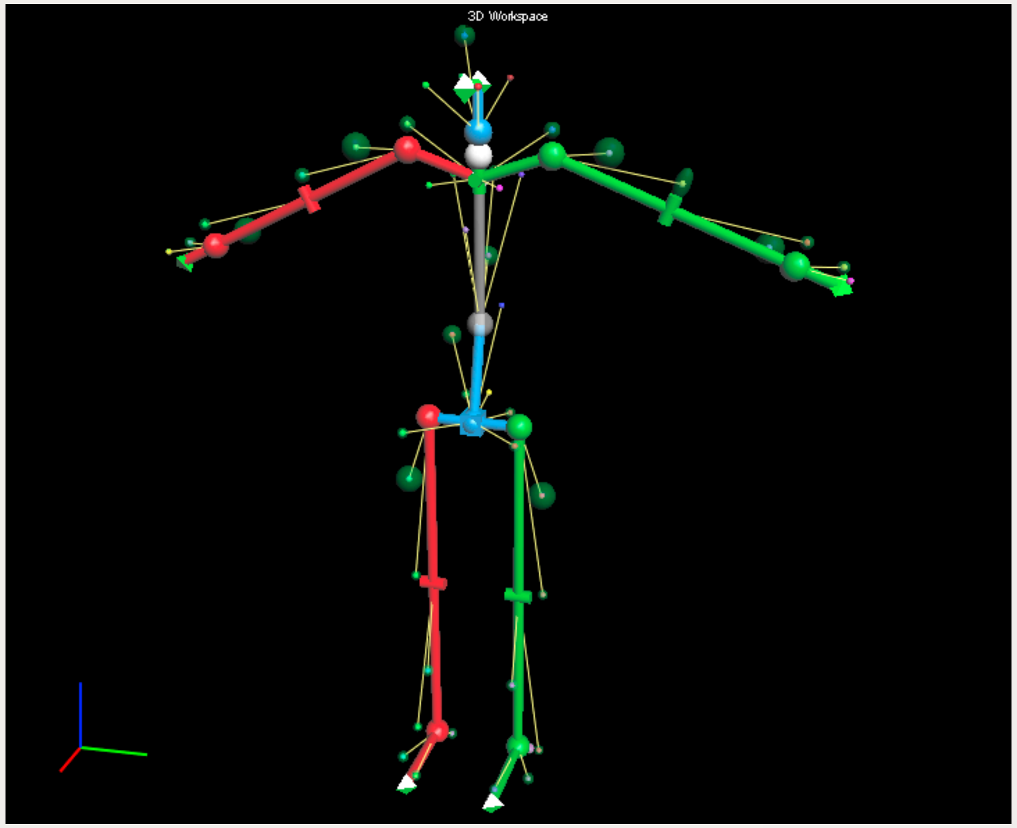


Marker Energy Function



$$\omega_i = f_{i,\Omega}(\omega_{i-1})$$

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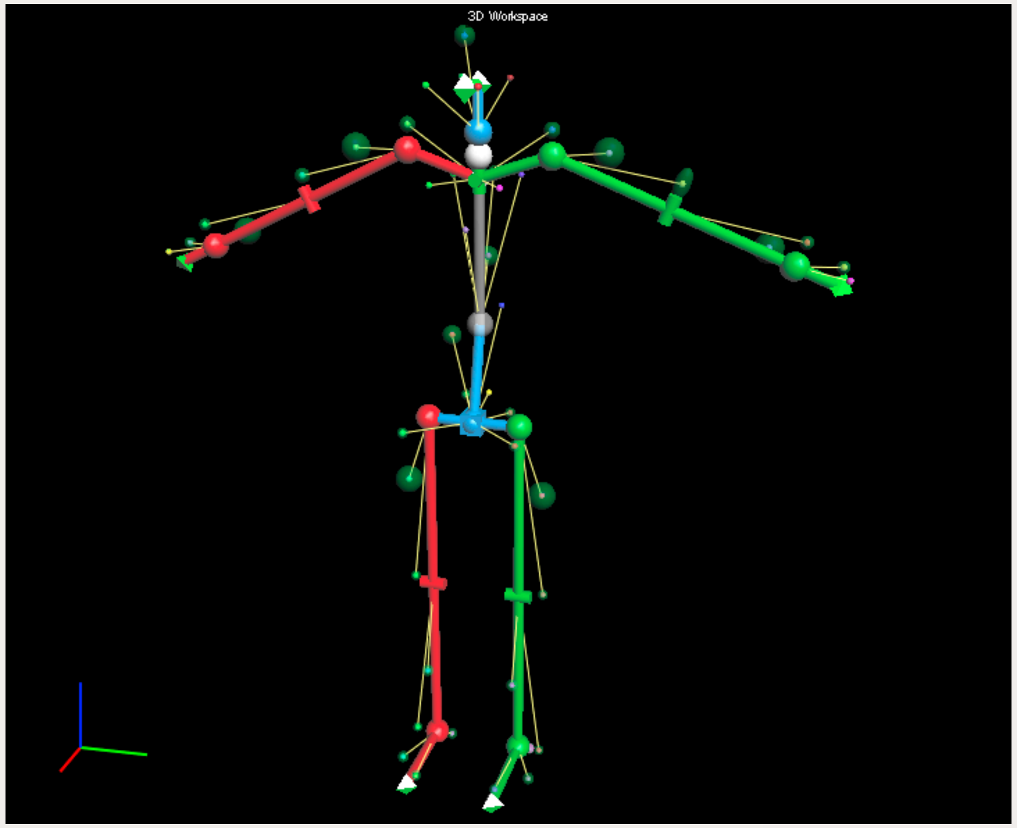

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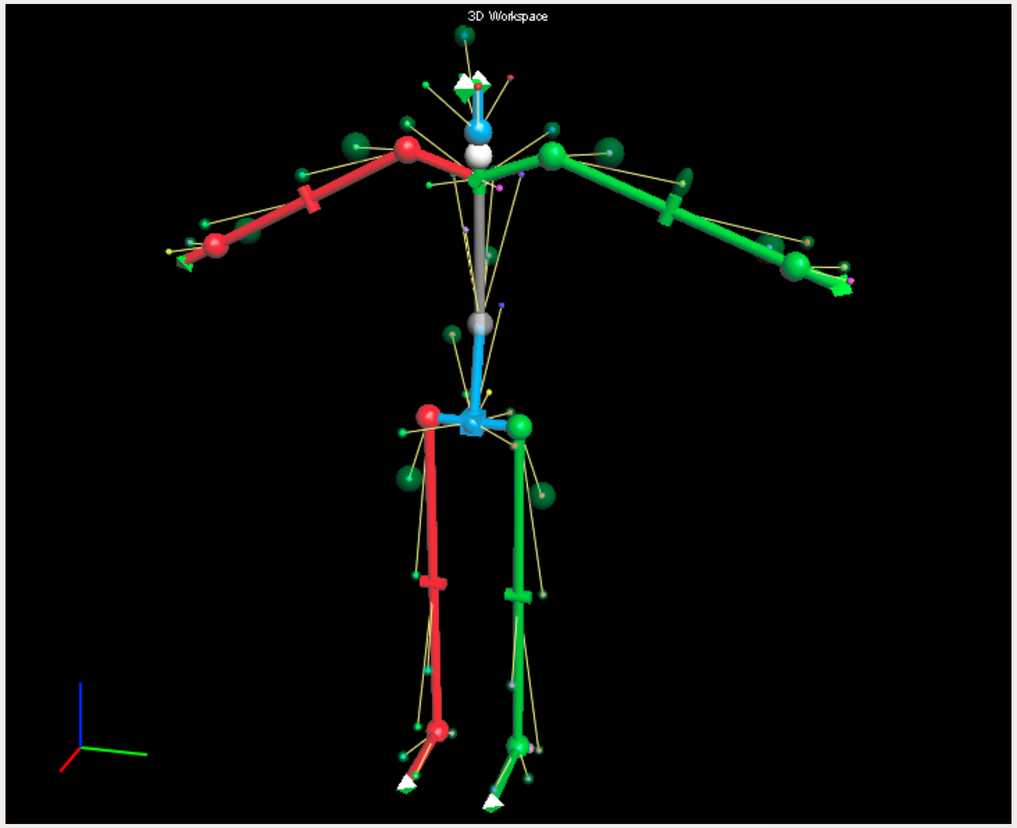

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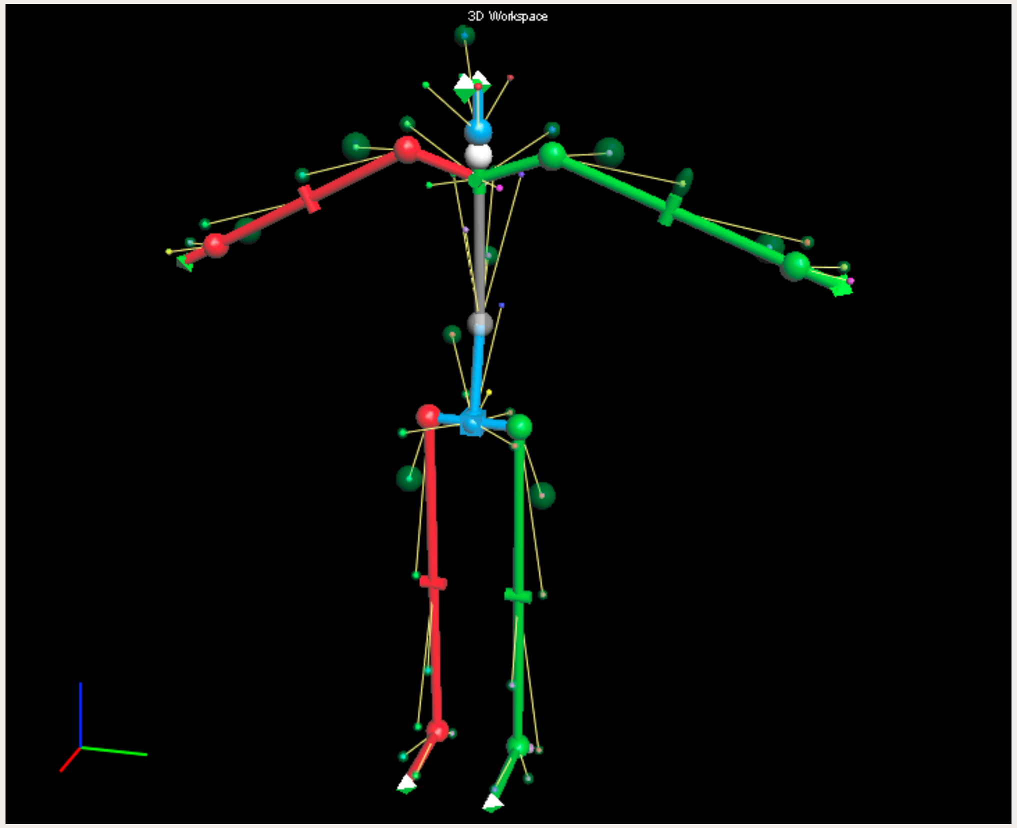

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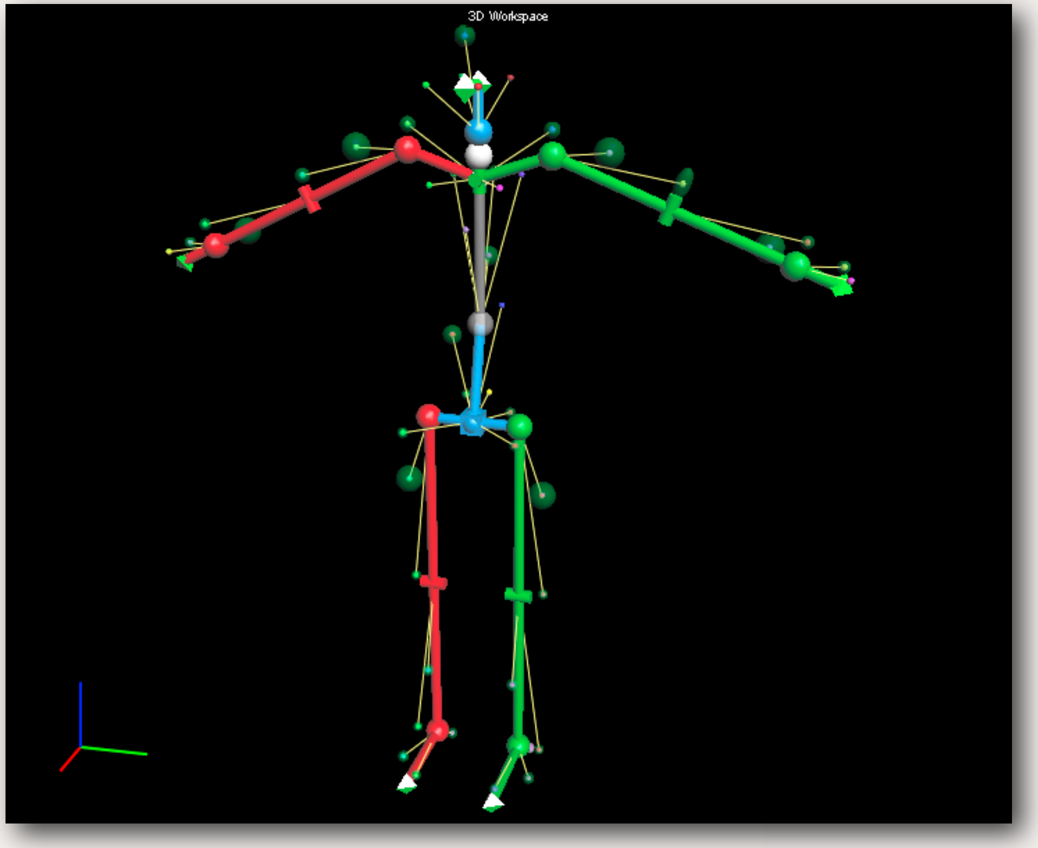
$$\boxed{\frac{dE}{d\Omega}}$$

Derivatives



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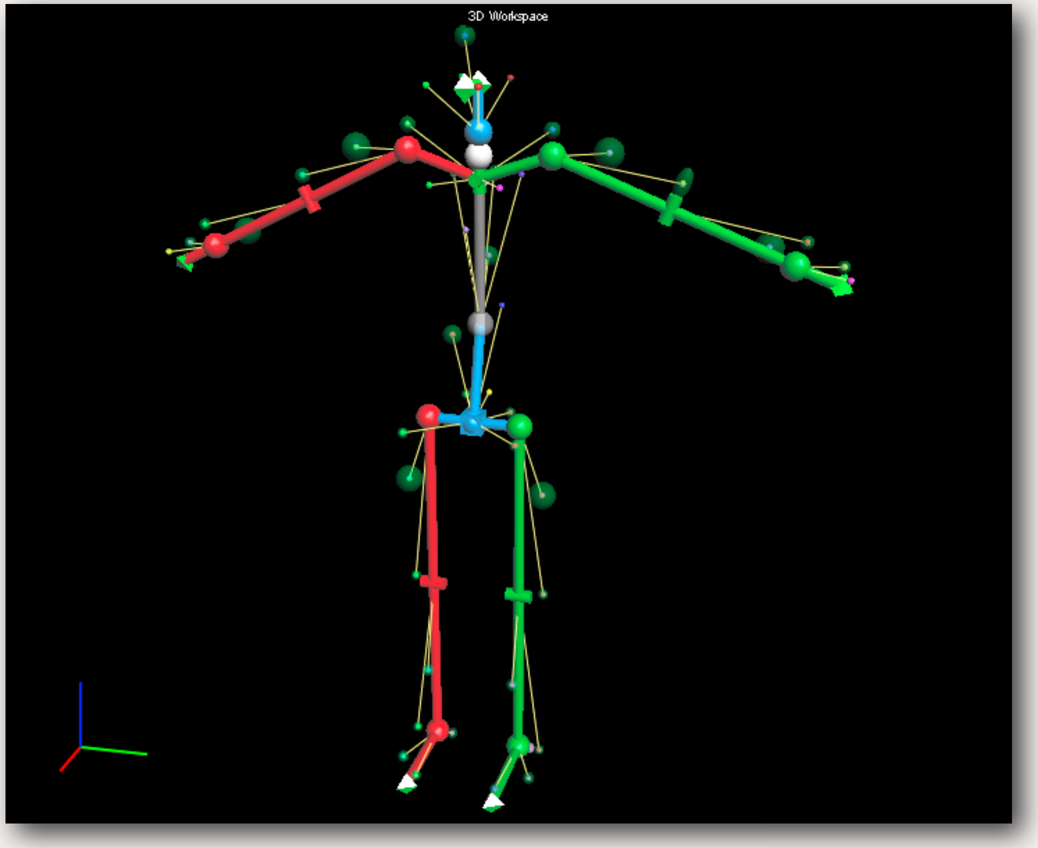

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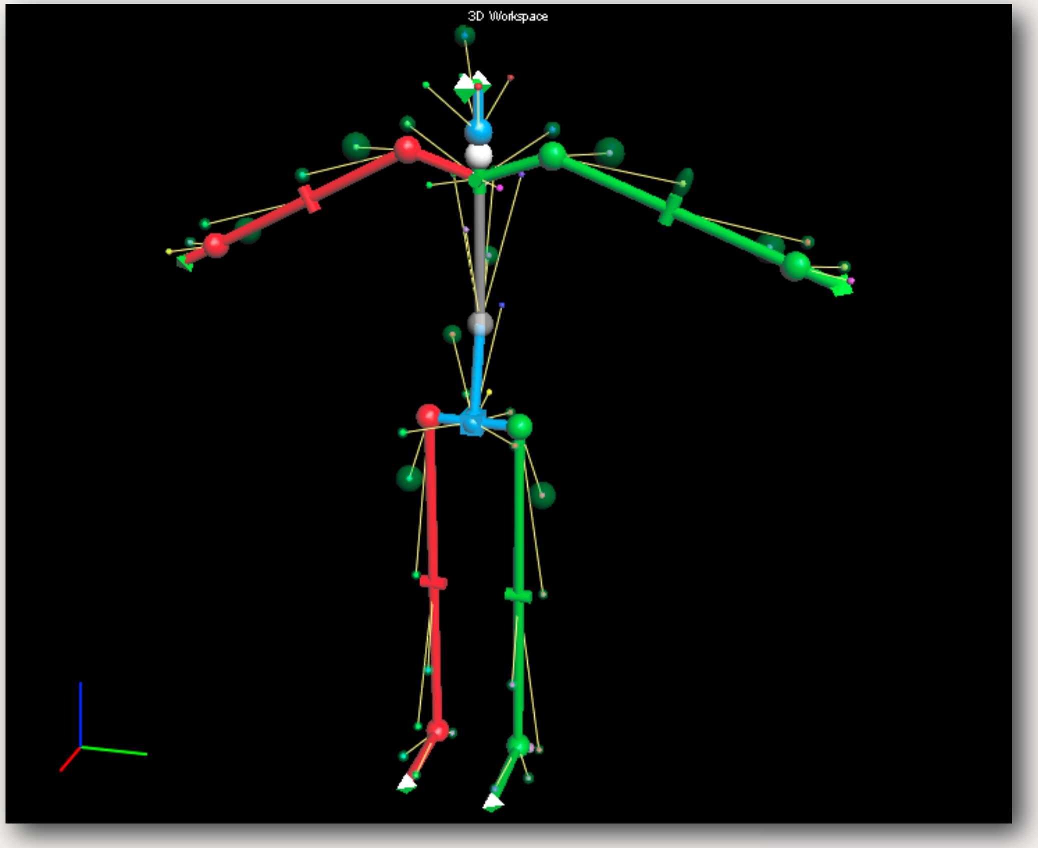

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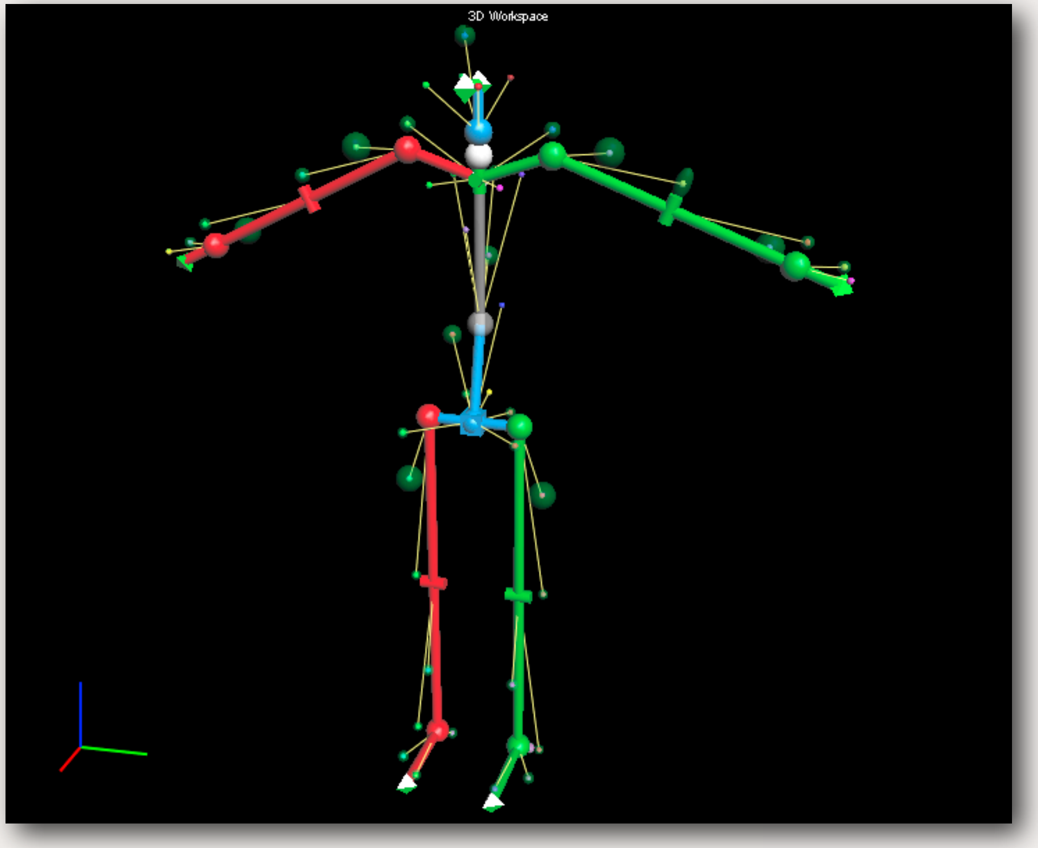

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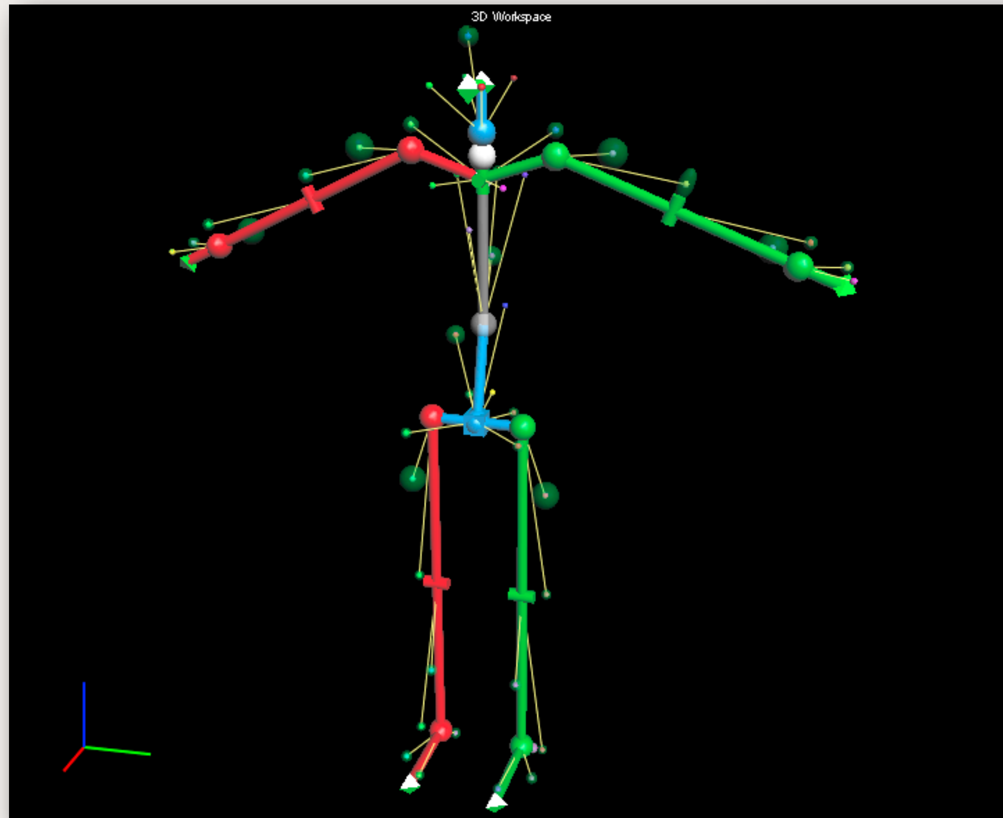

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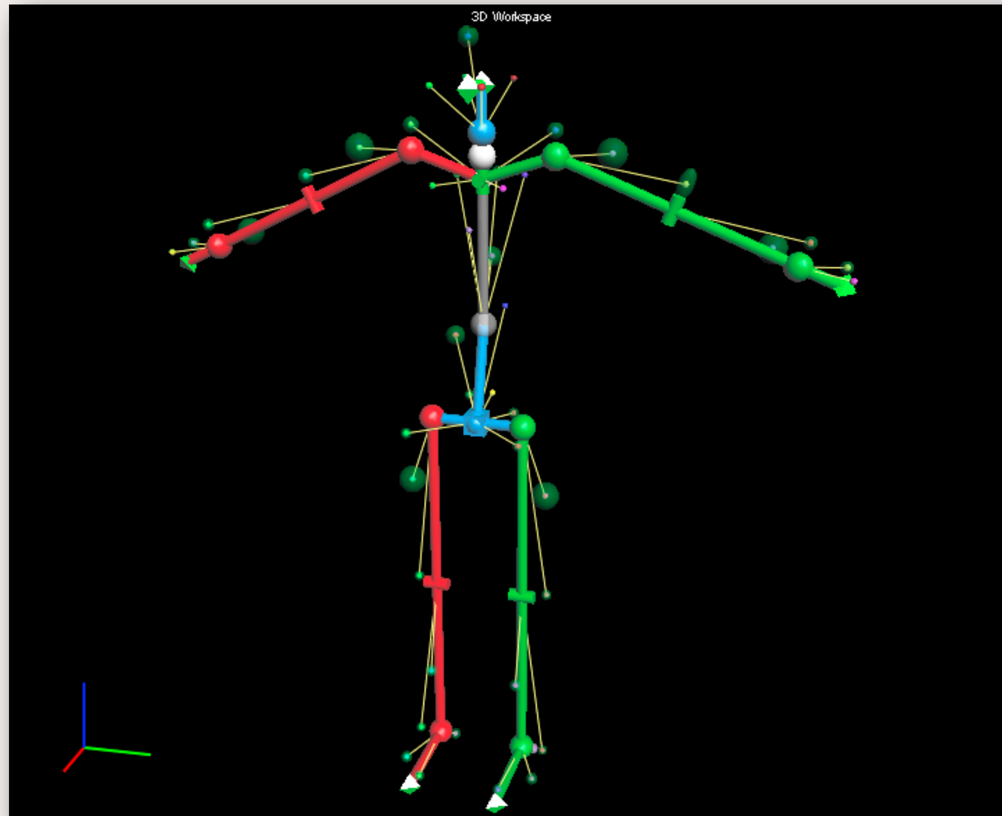

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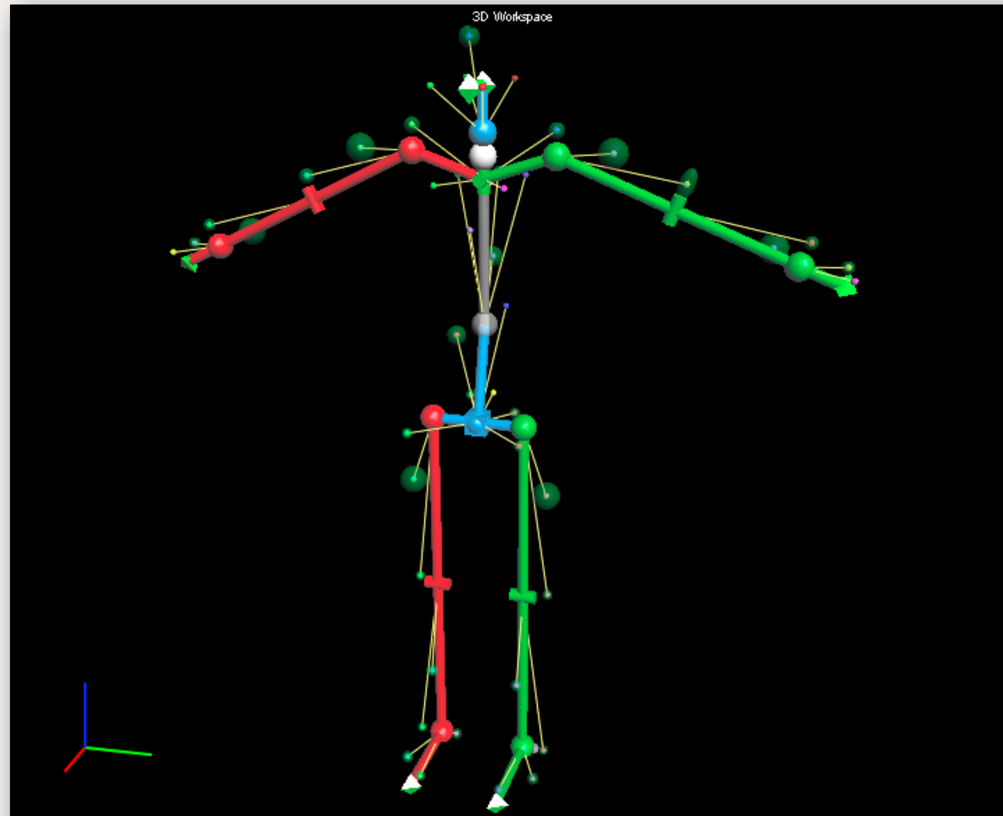

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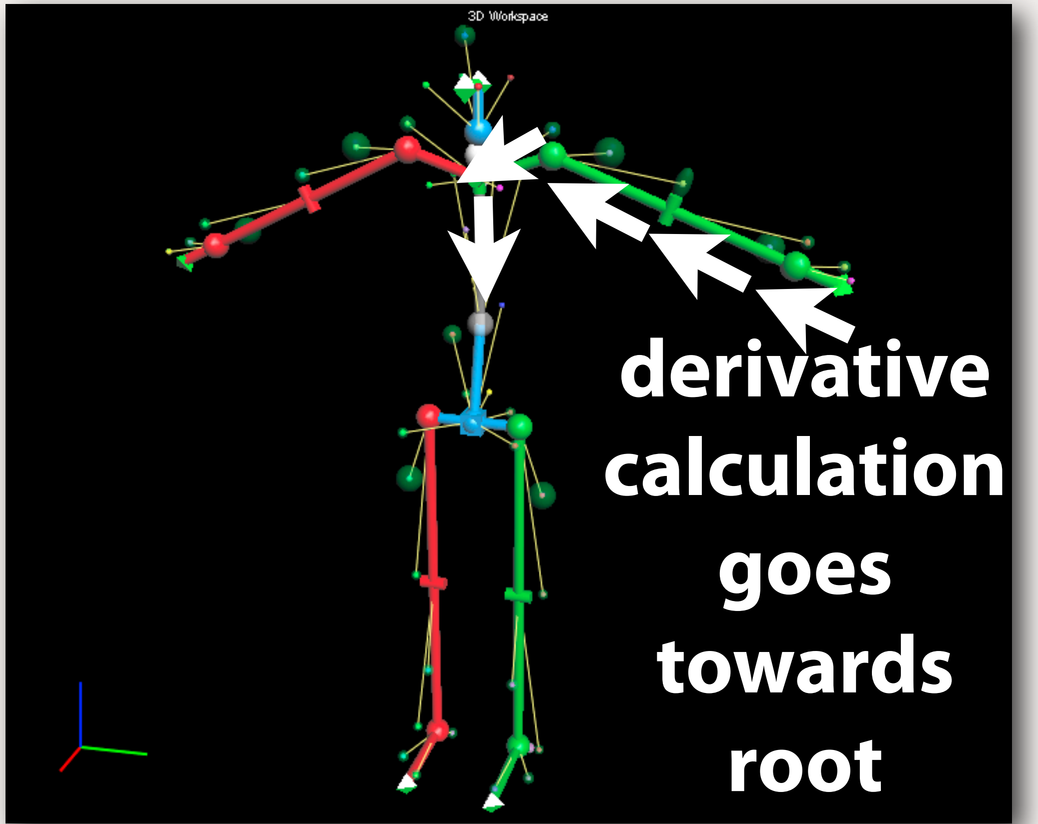
Derivatives



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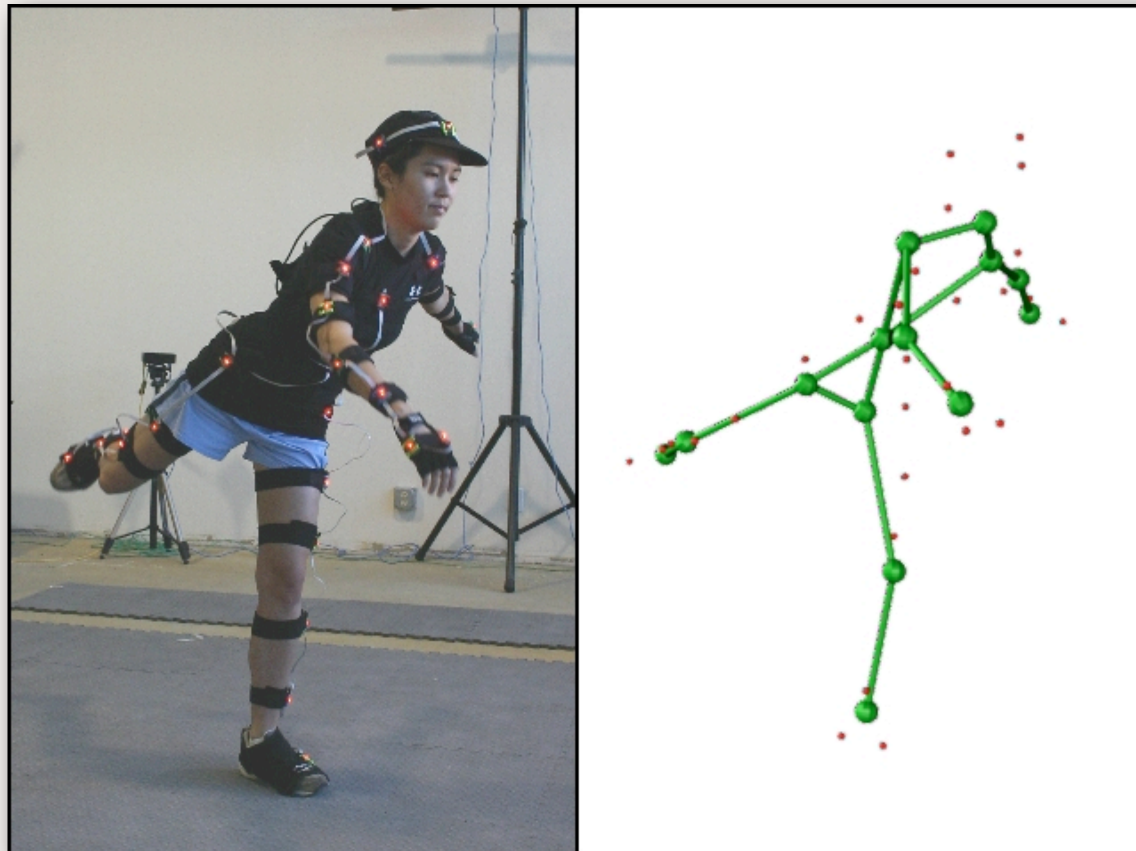
matrix multiplies

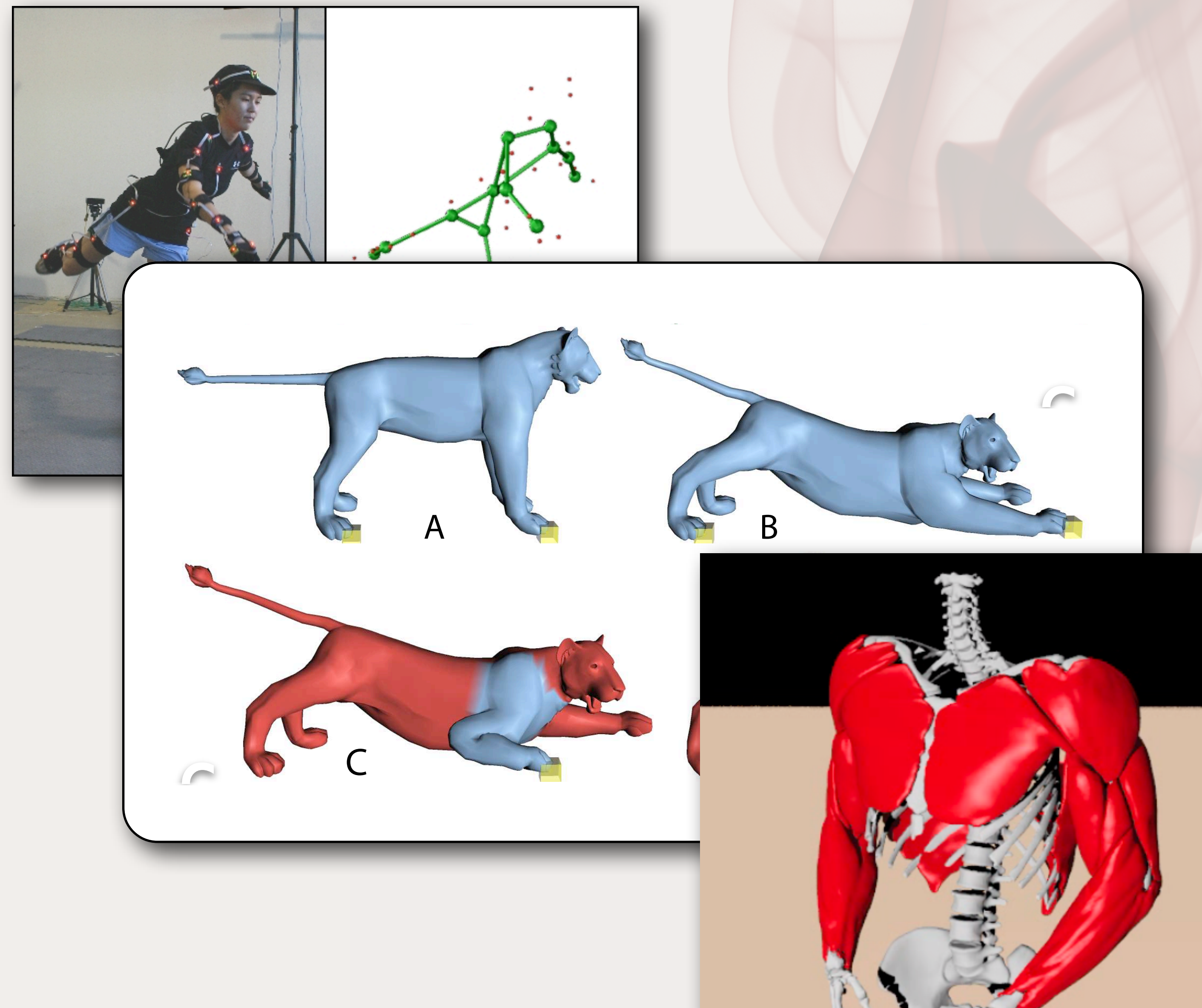
Inverse Kinematics Summary



- **Telescoping composition of functions from root.**
- **Compute derivatives in the *opposite* direction!**

Body Representation

- **Kinematic Skeleton**
- **Anatomical**
- **Pure Mesh**
- **What are the advantages and disadvantages?**



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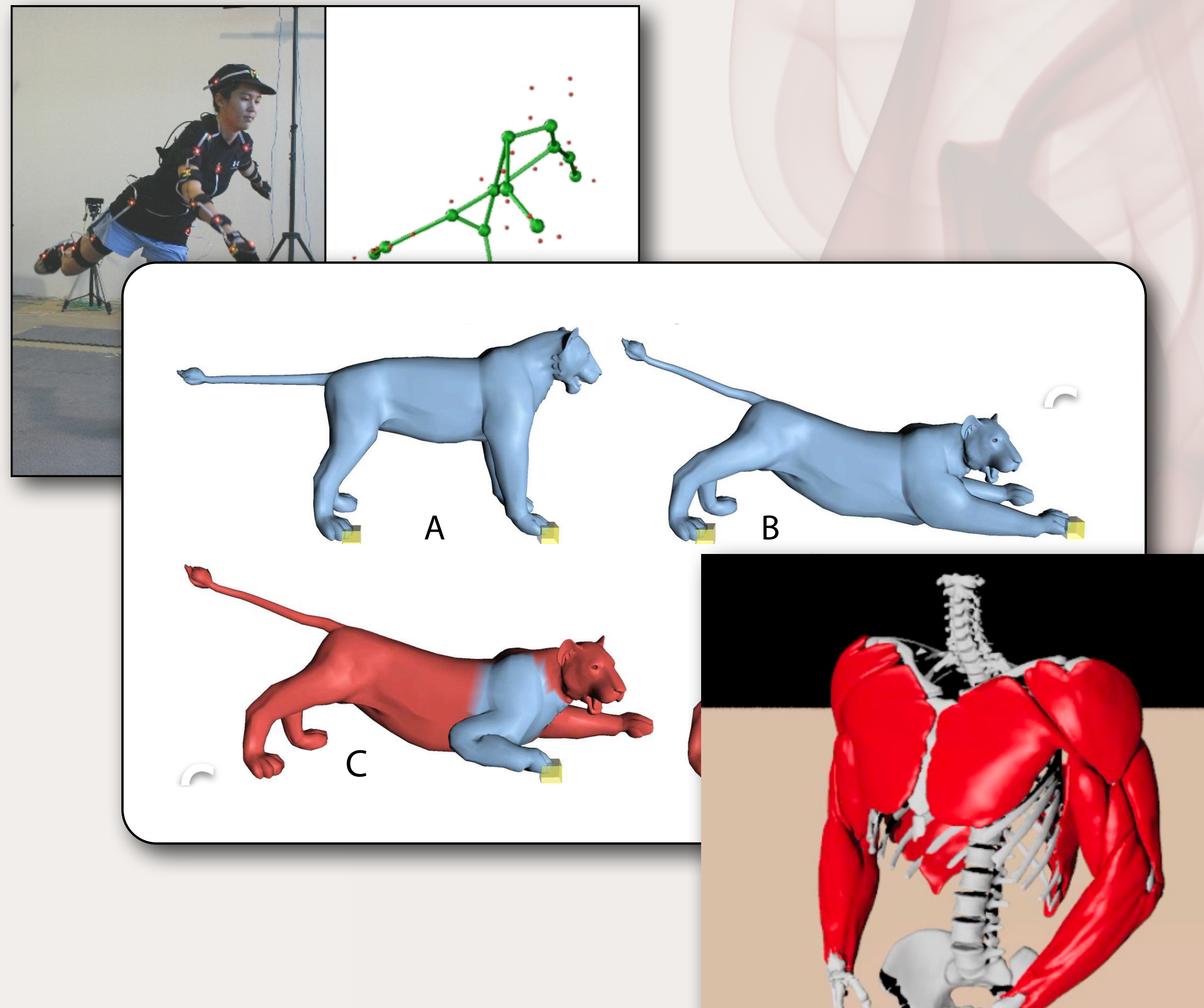
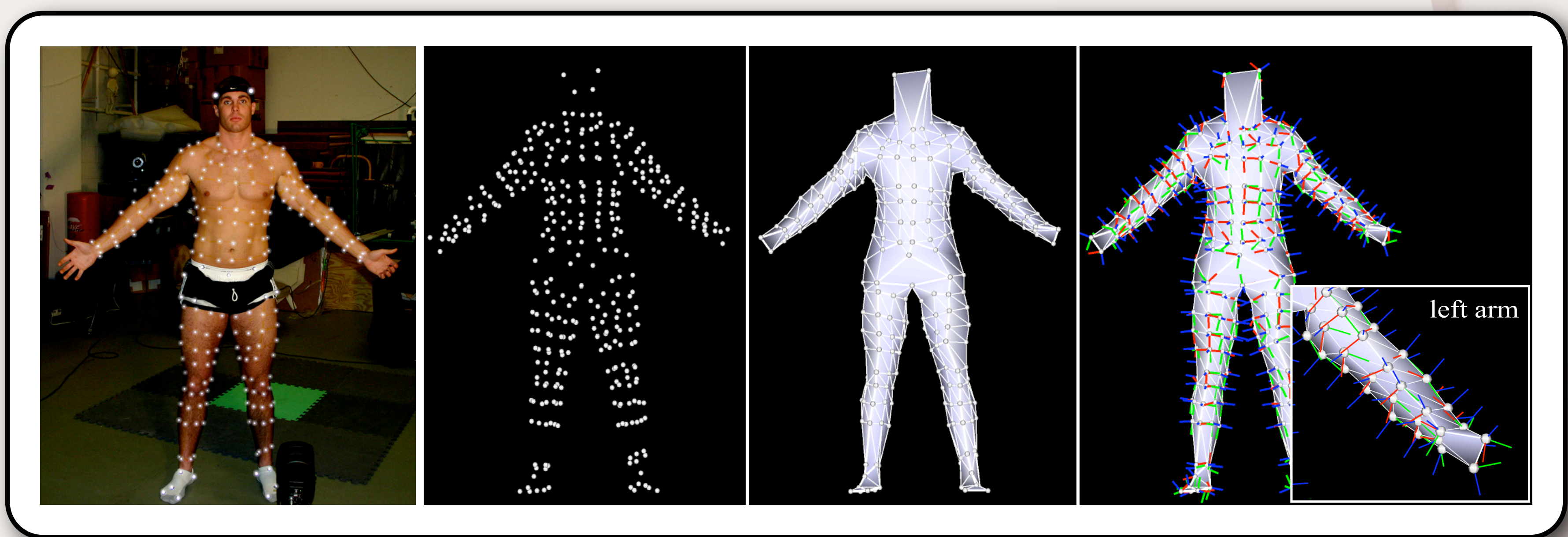


Dense Marker Capture



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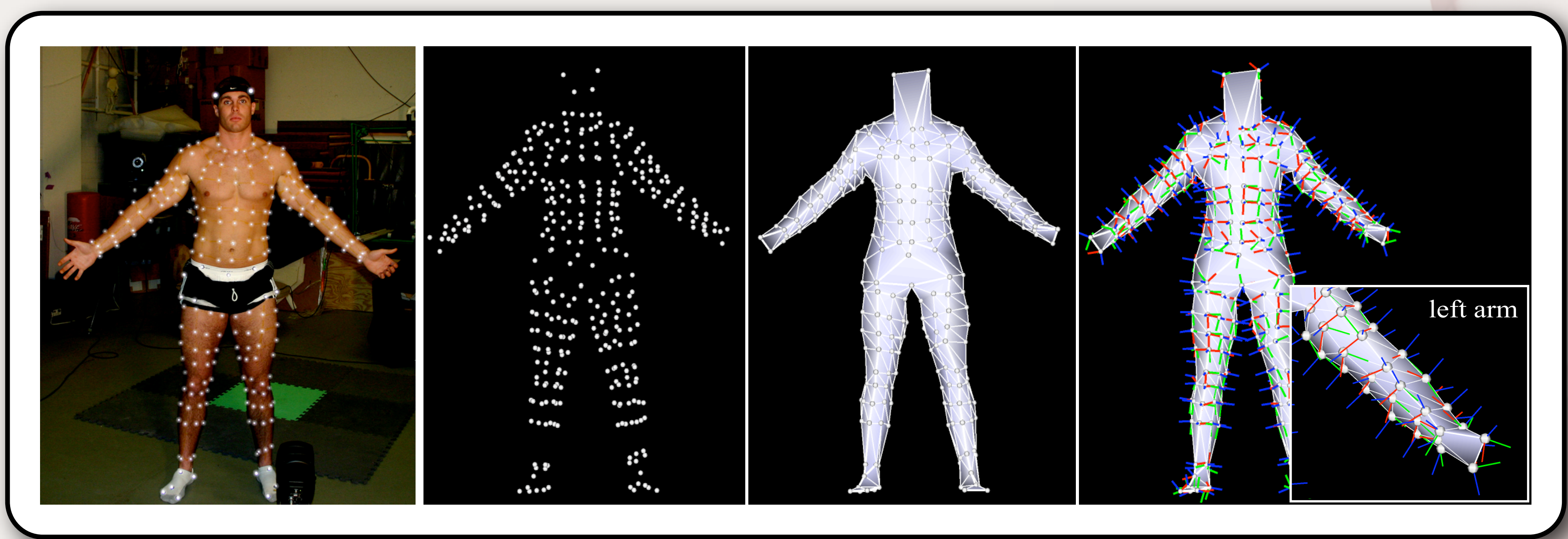
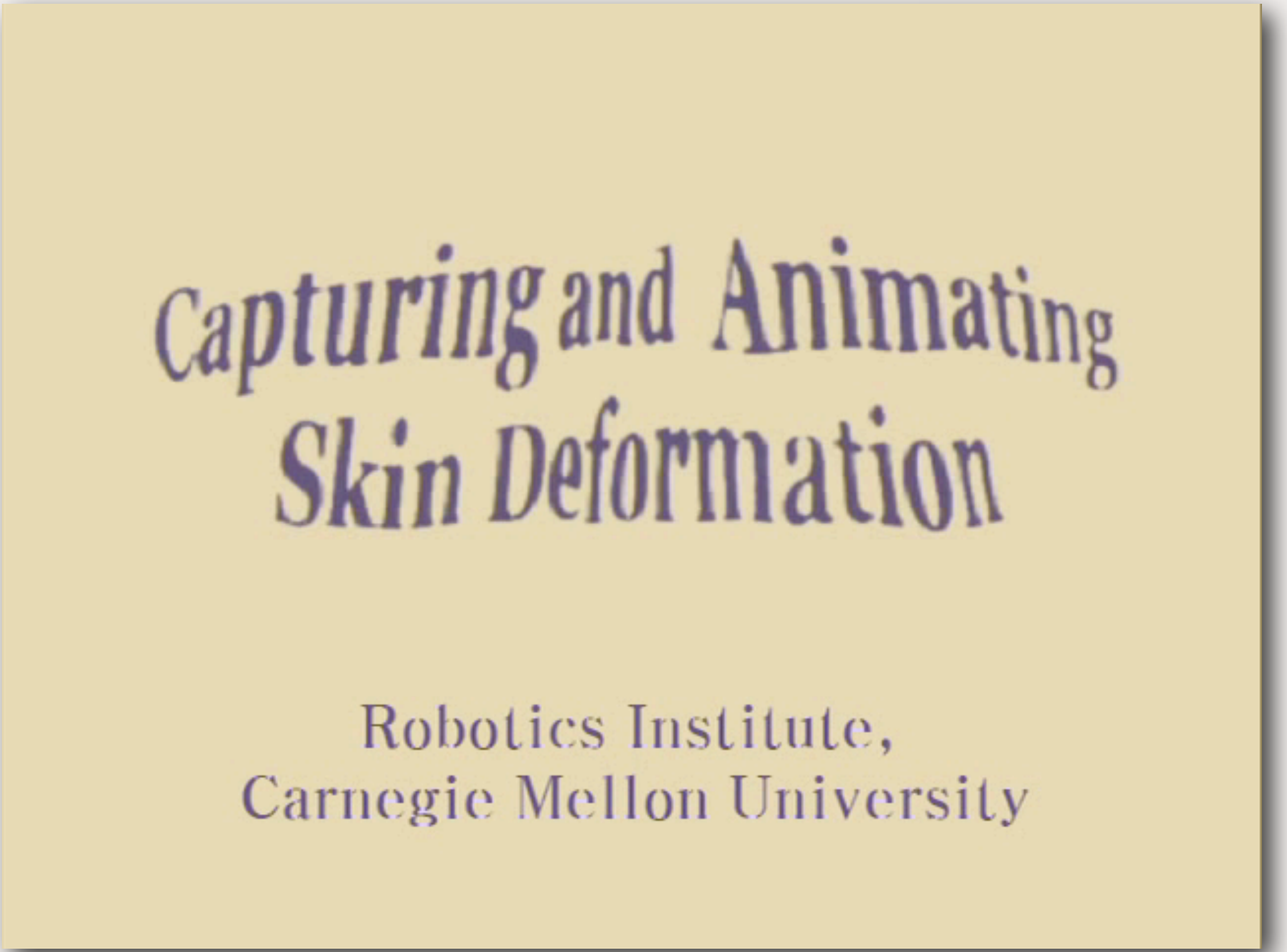
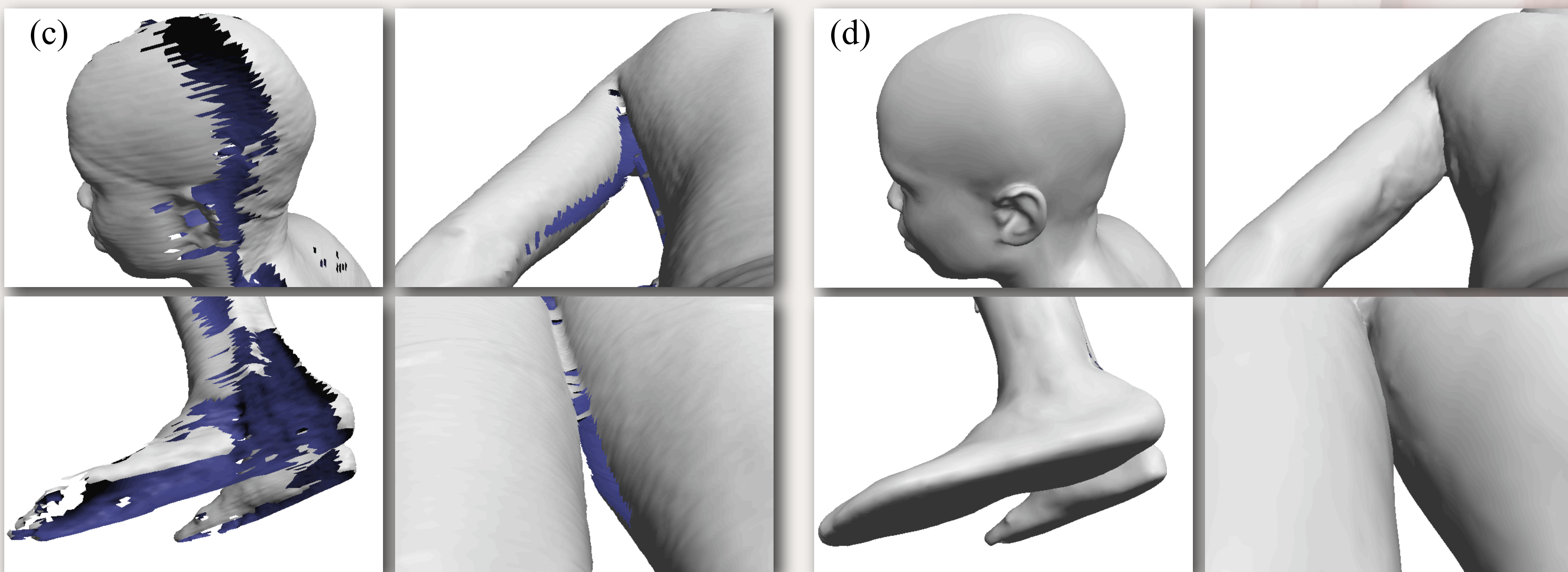




Laser Range Scanning

Filling in Missing Data



source: Allen, Curless, Popović. The space of human body shapes: reconstruction and parameterization from range scans.

How could this be accomplished?

What can you do with a huge set of human meshes in vertex correspondence?

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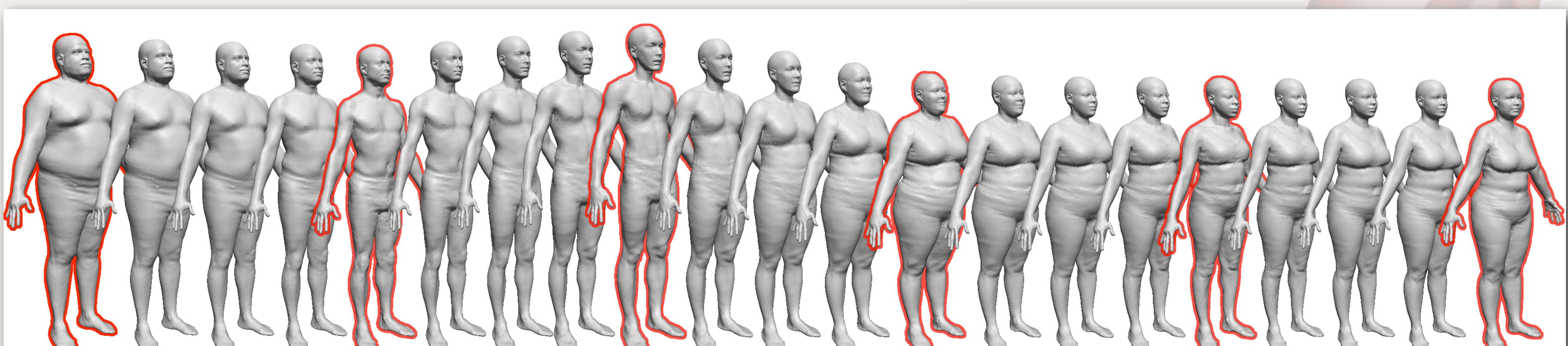

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PCA Shape Analysis

Displacements of Example Poses

PCA Shape Analysis

Displacements of Example Poses = PCA Basis Coefficient Matrix

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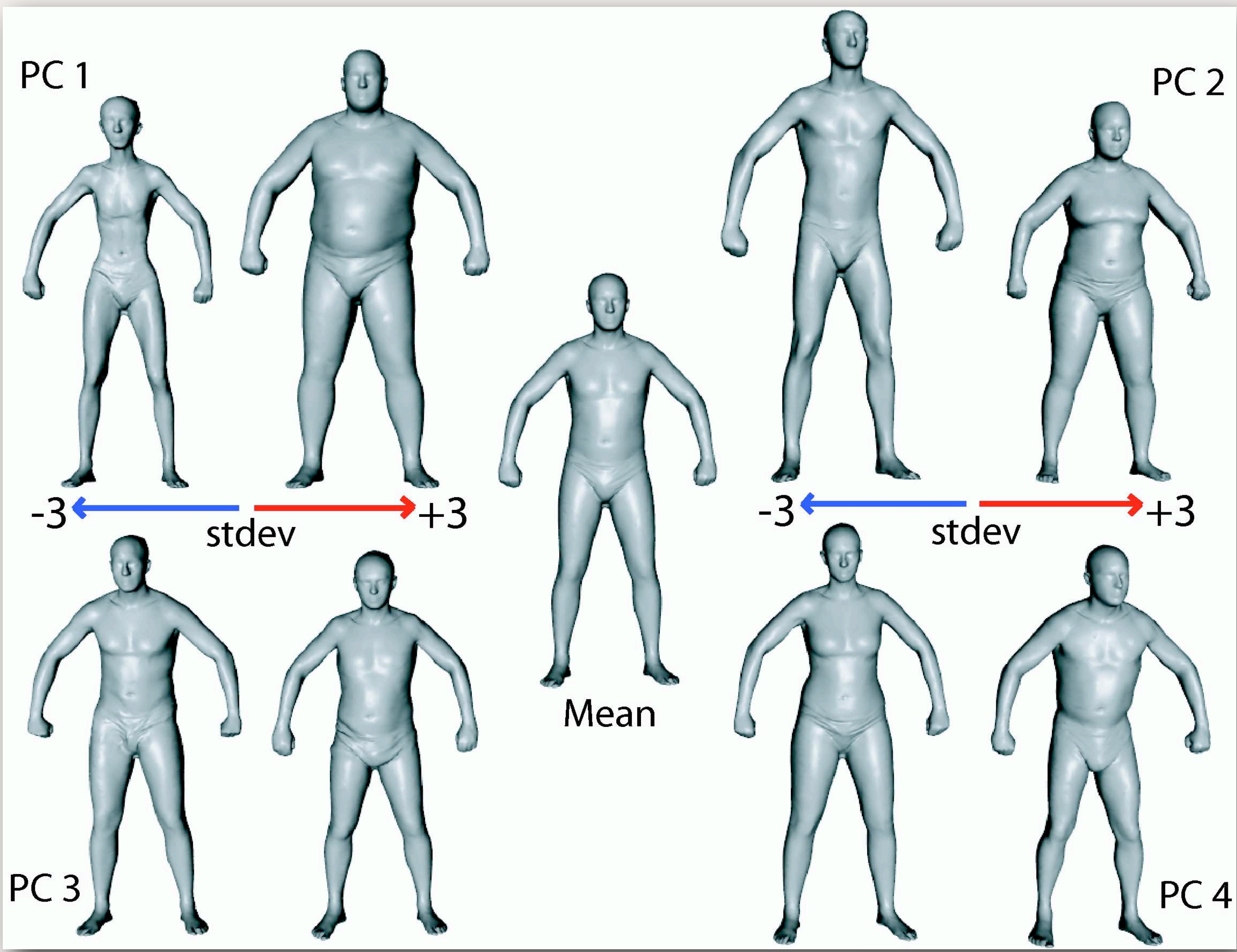
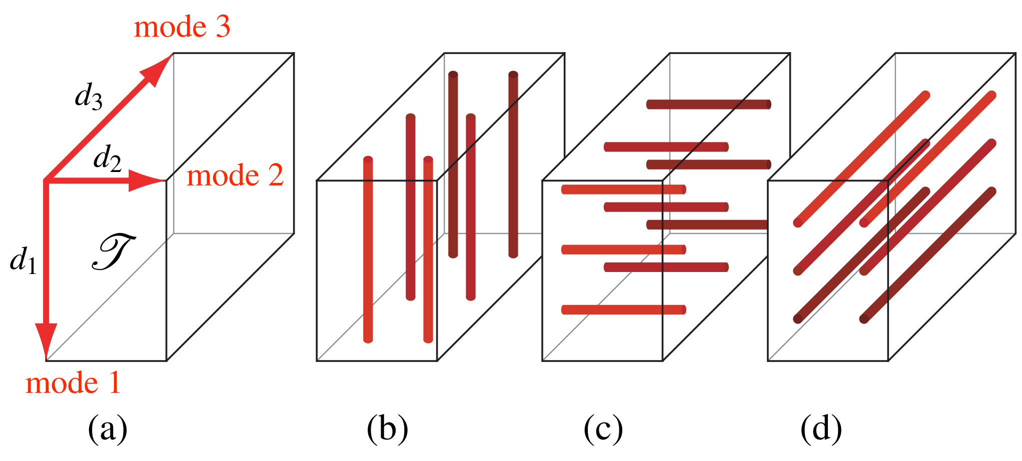
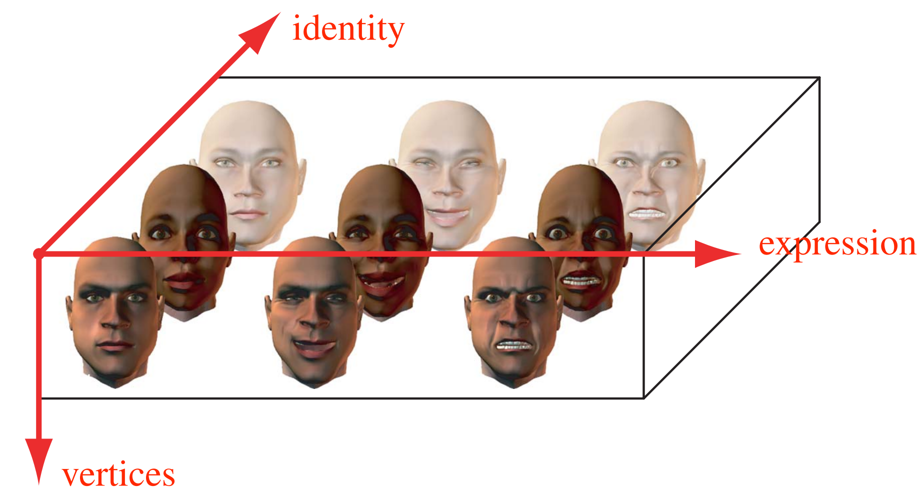


source: Anguelov, Srinivasan, Koller, Thrun, Rodgers, Davis. SCAPE: Shape Completion and Animation of People.

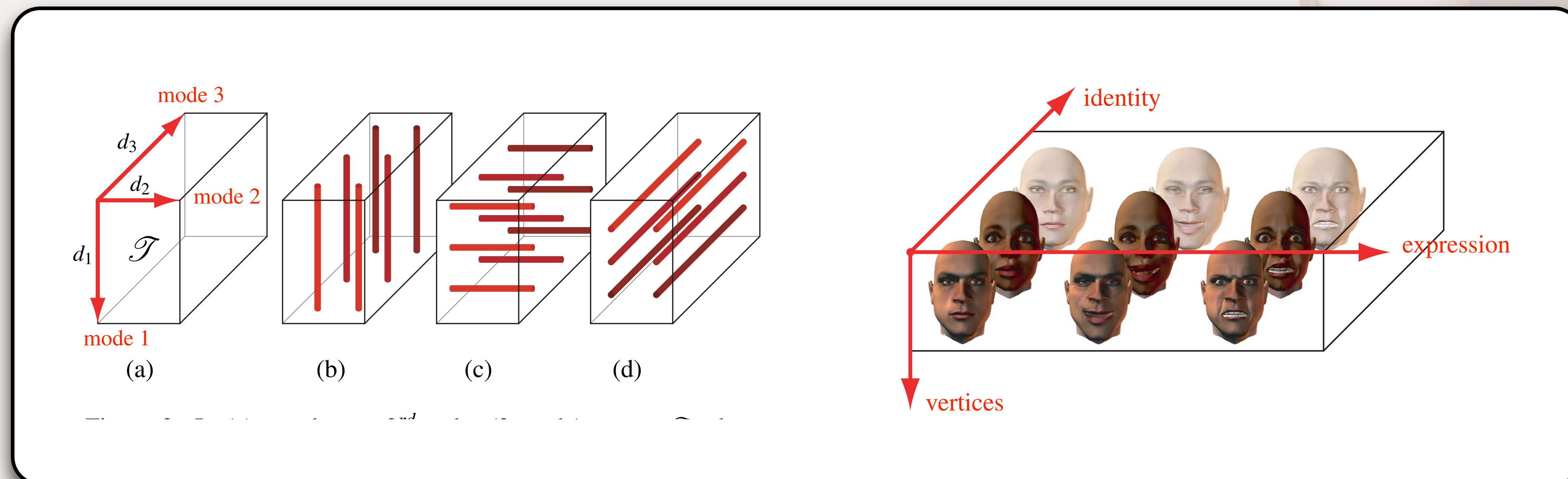
Multilinear Analysis





source: Vlasic, Brand, Pfister, Popović. Face Transfer with Multilinear Models.

Multilinear Analysis



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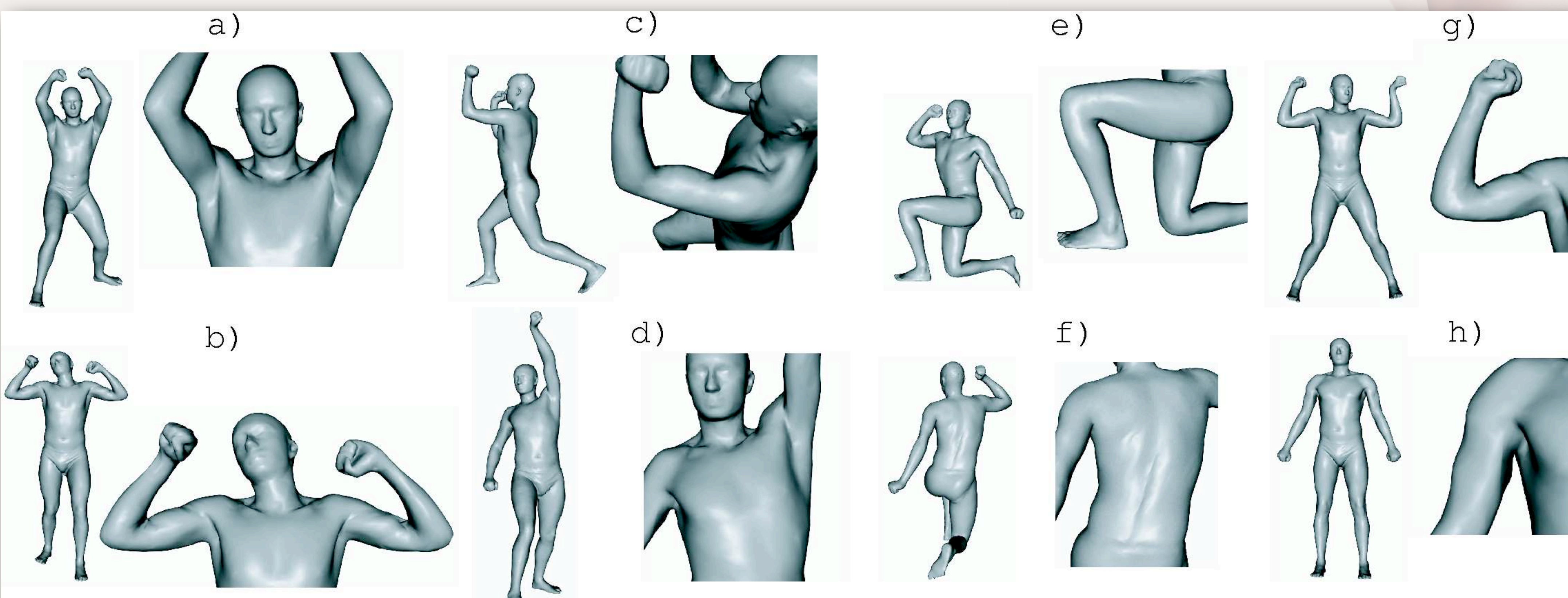
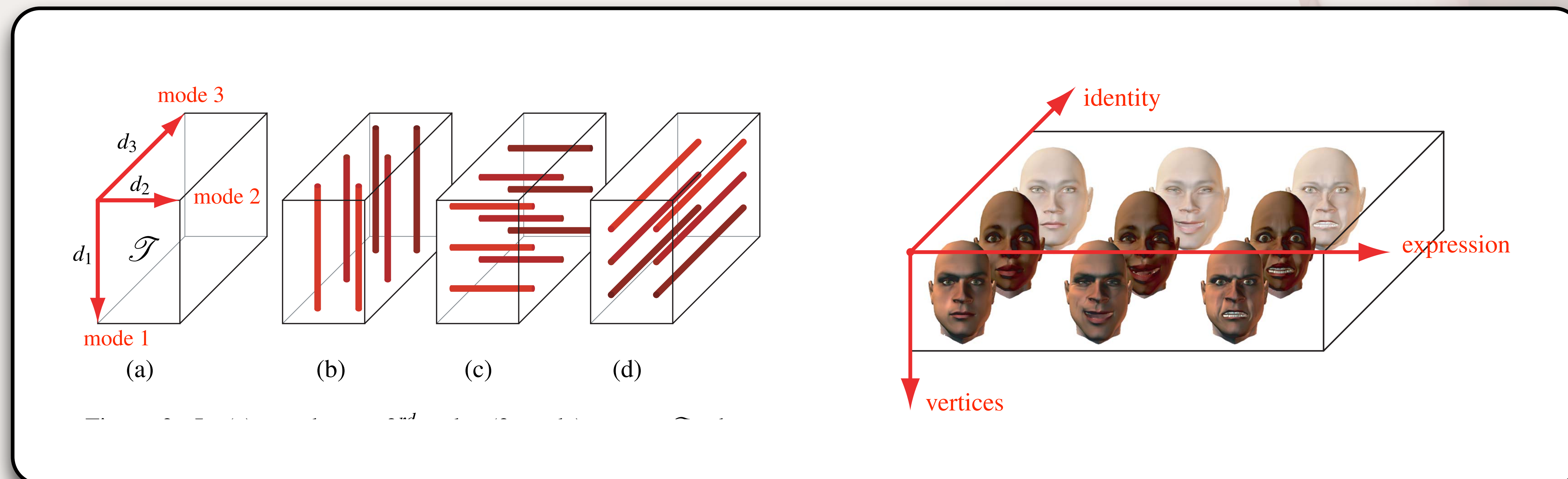


Multilinear Analysis



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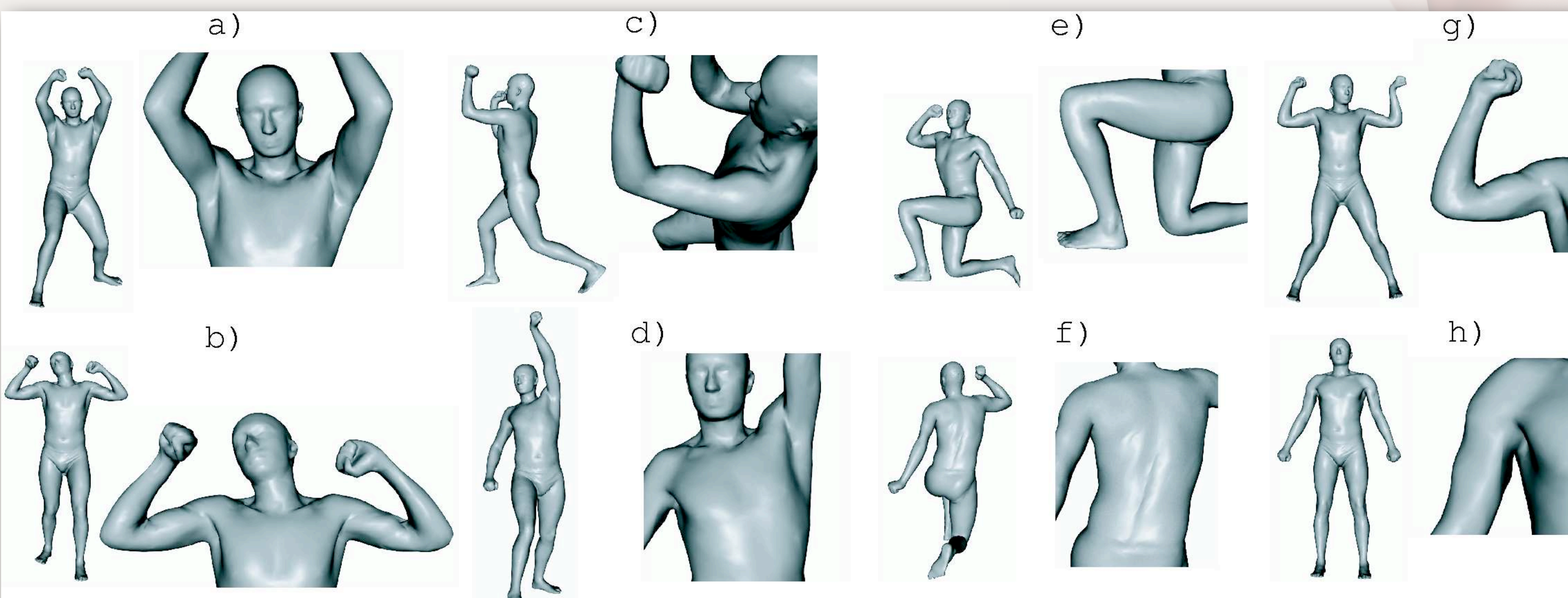

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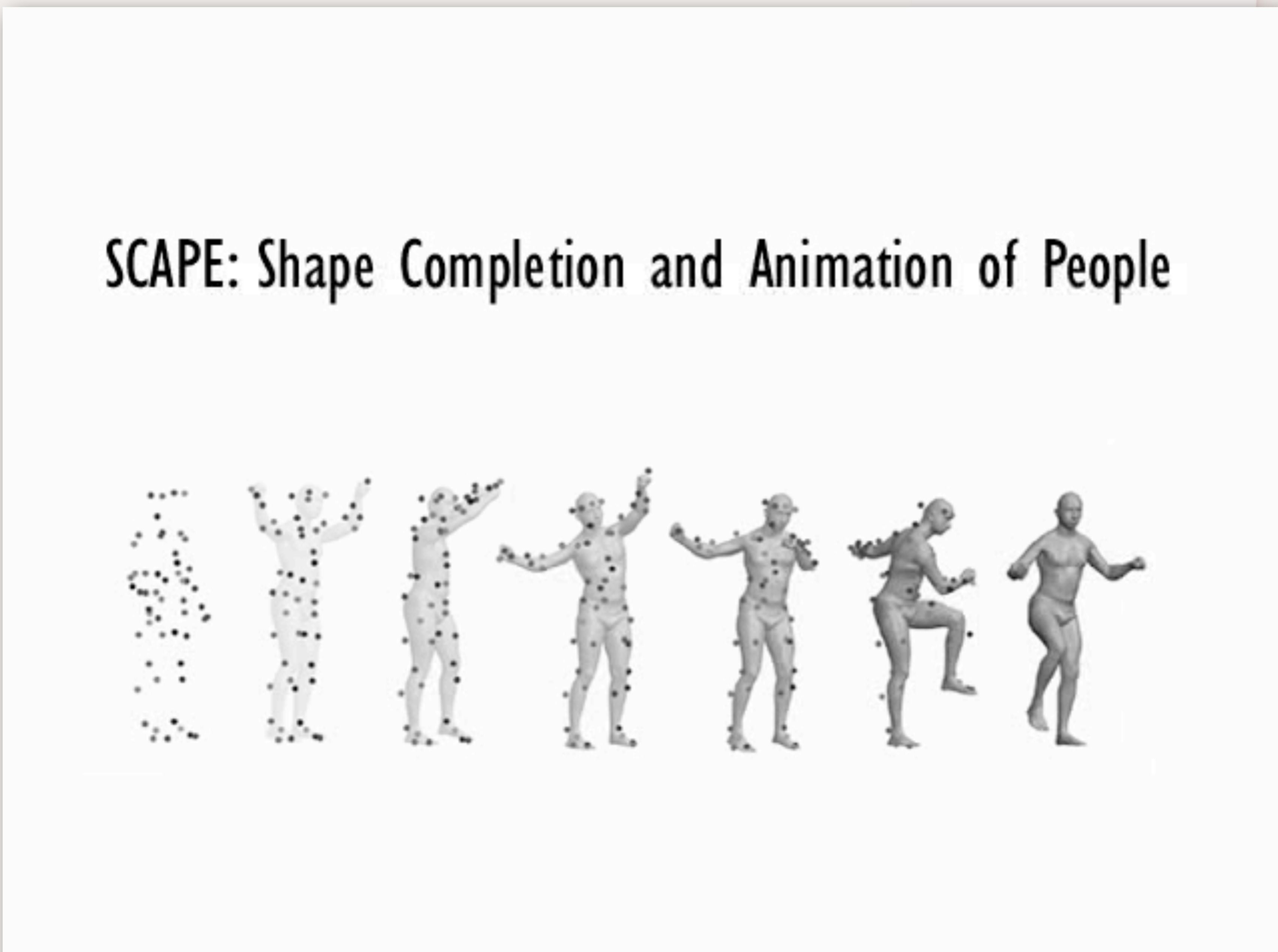
Example



source: Anguelov, Srinivasan, Koller, Thrun, Rodgers, Davis. SCAPE: Shape Completion and Animation of People.

Overview

- State of the art.
- **Body models.**
- Animation
- Questions

source: 3dscience.com

Overview

- State of the art.
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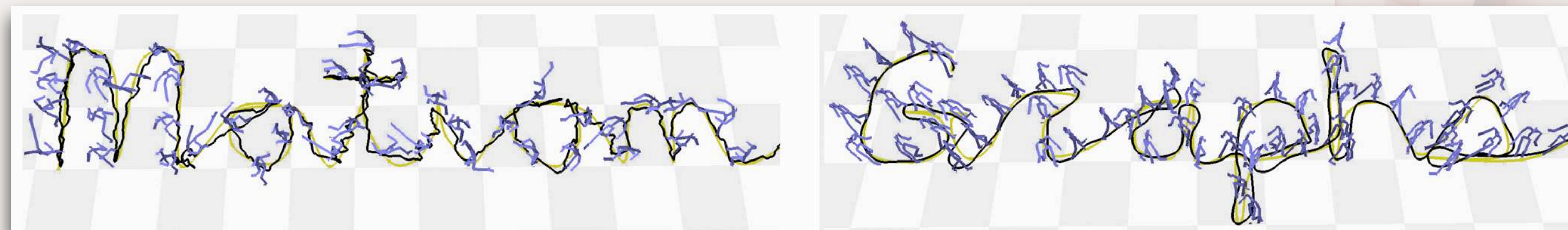
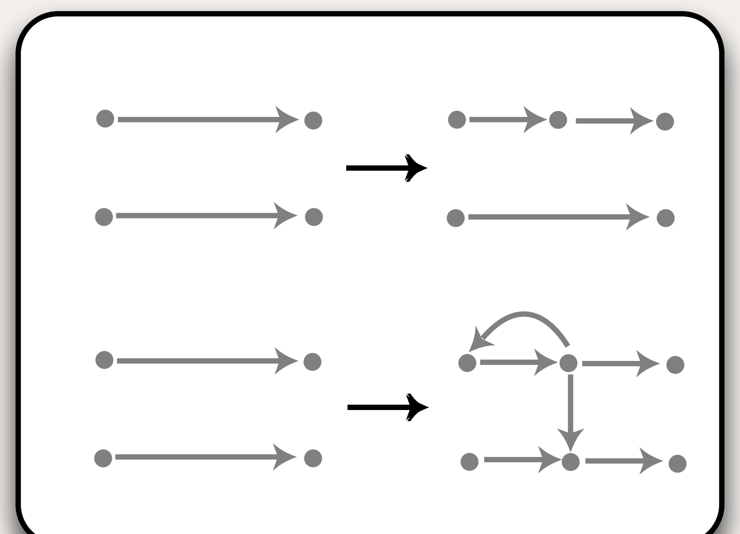
Overview

- **Data-Driven Motion**
- **Physics Based Motion**
- **Motion of other Animals**

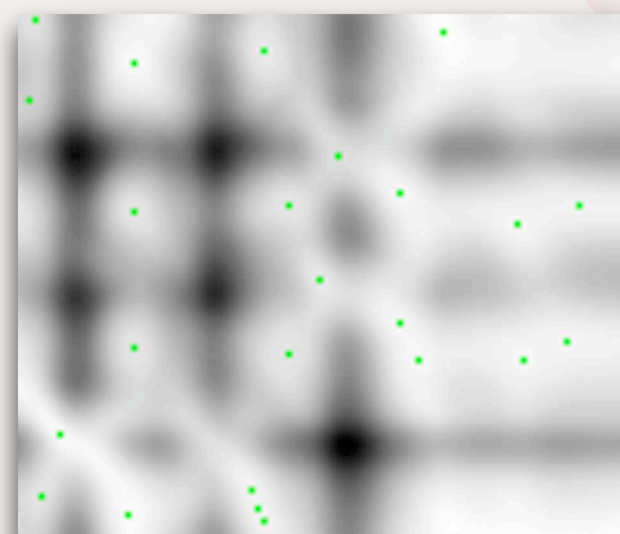
Data Driven Human Animation



source: Kovar, Gleicher, Pighin. Motion Graphs.

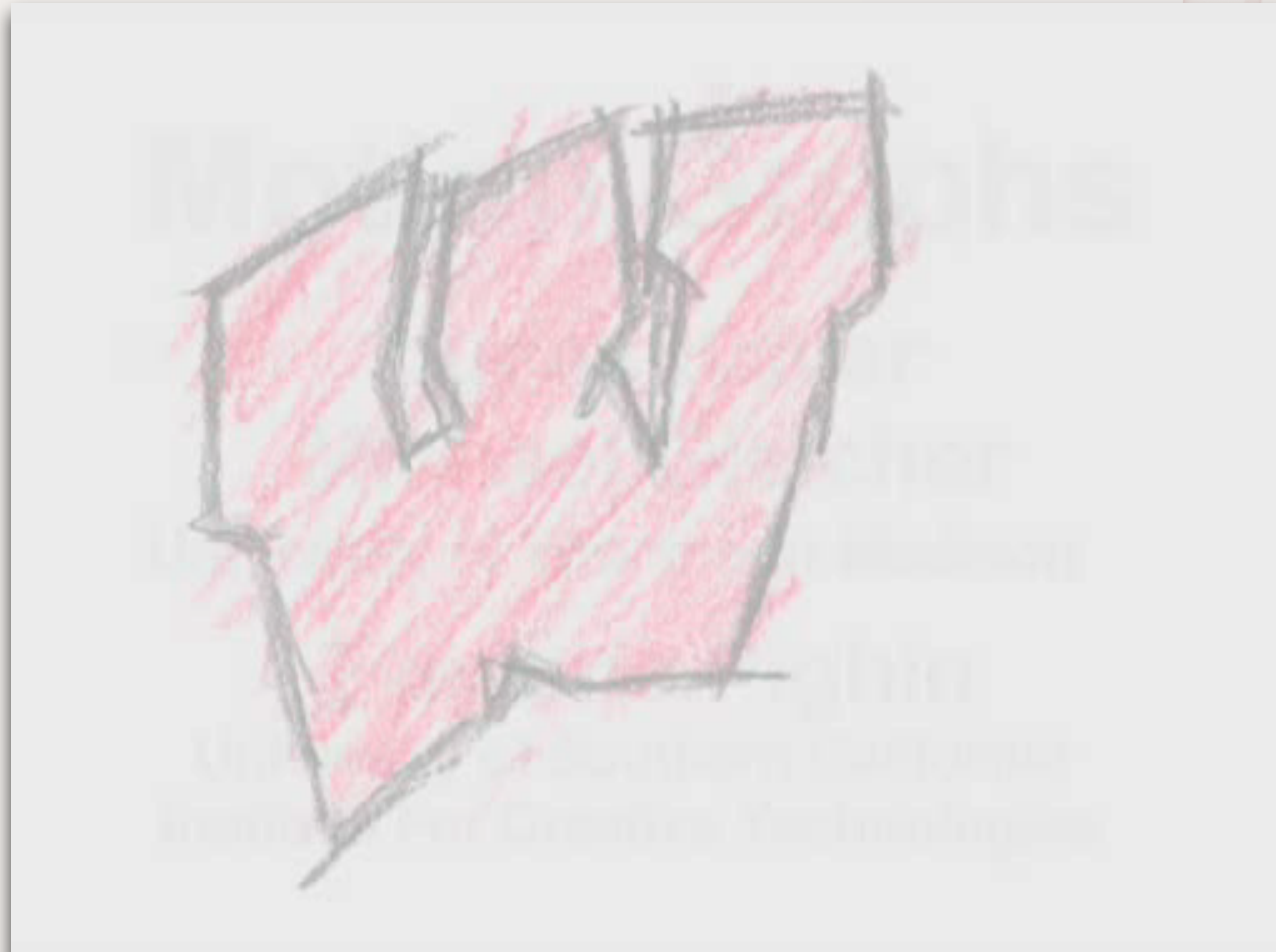


Motion Graph Schematic



Finding Candidate Transitions

Examples

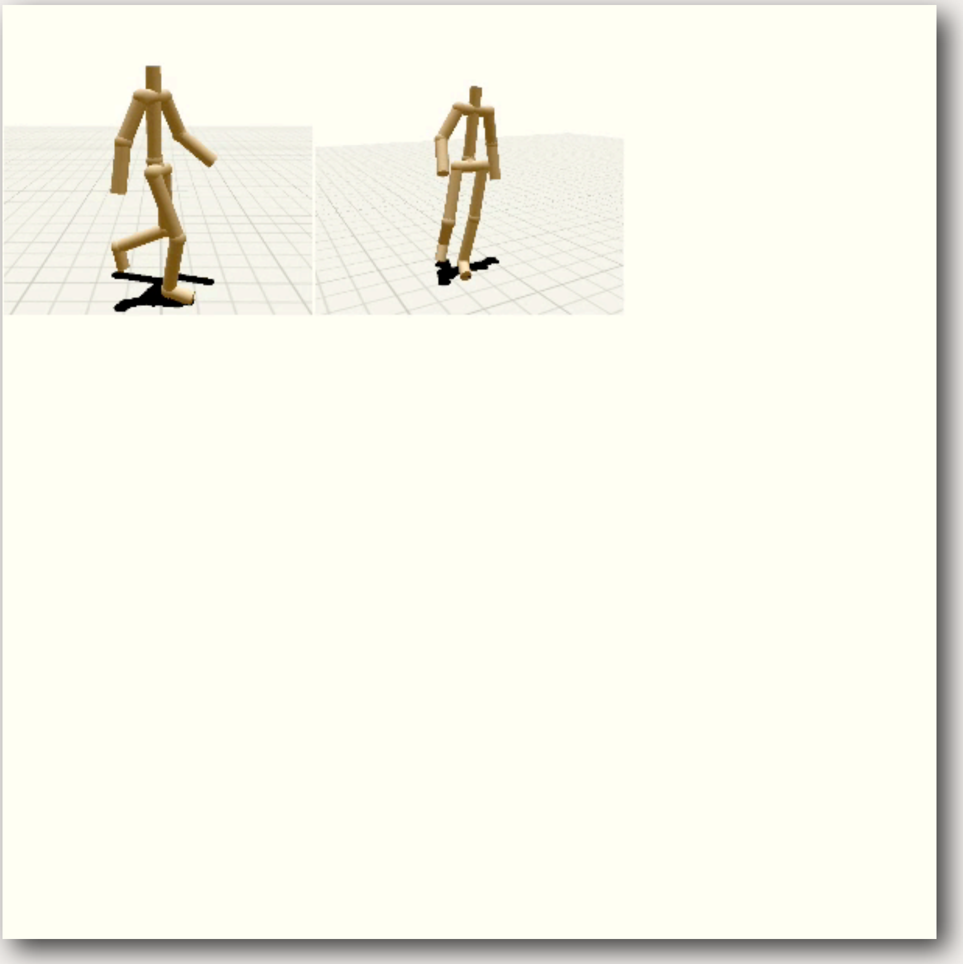


Clips

source: Treuille et al. [2002]

source: Kovar et al. [2002]

Clips



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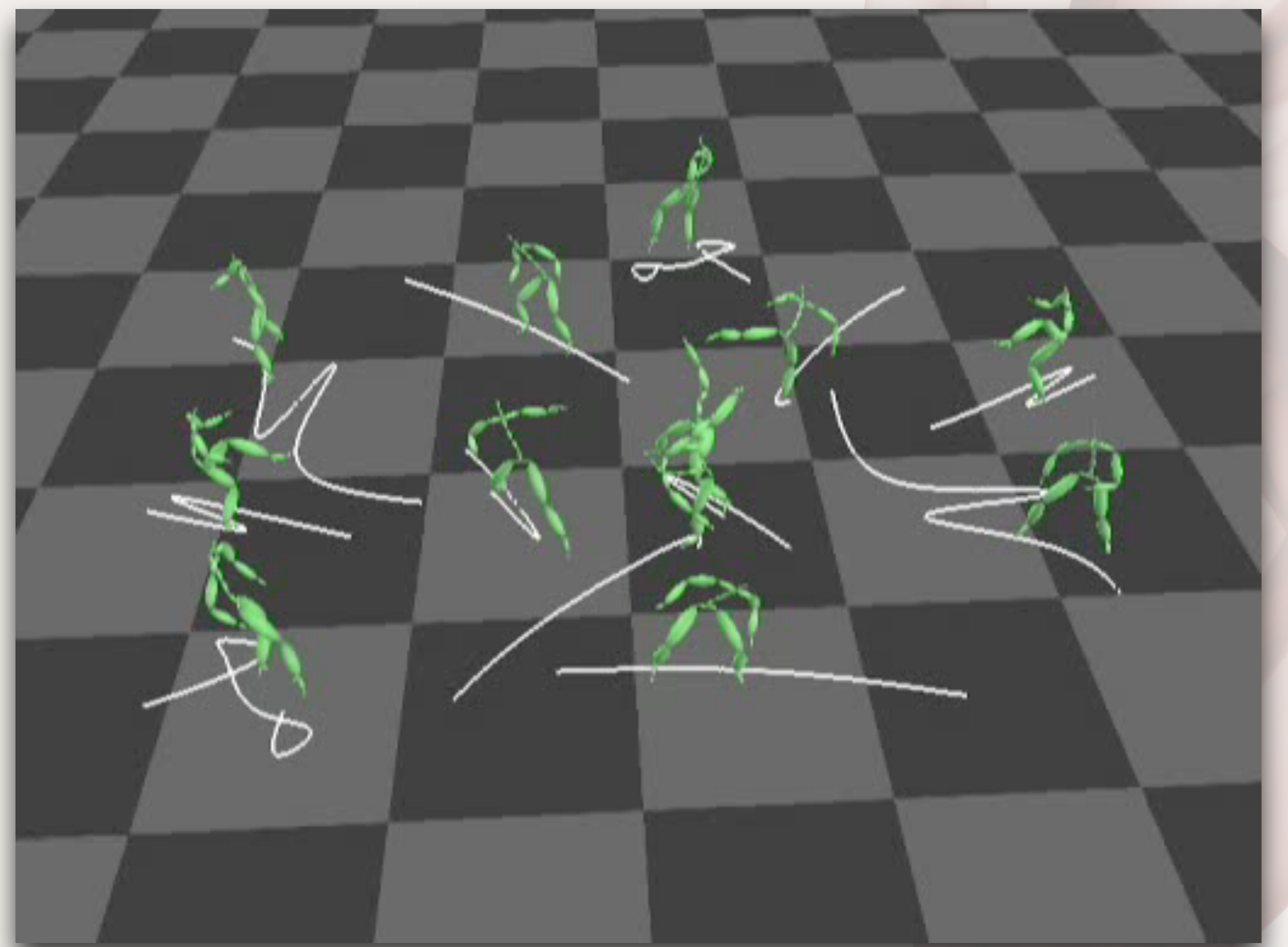

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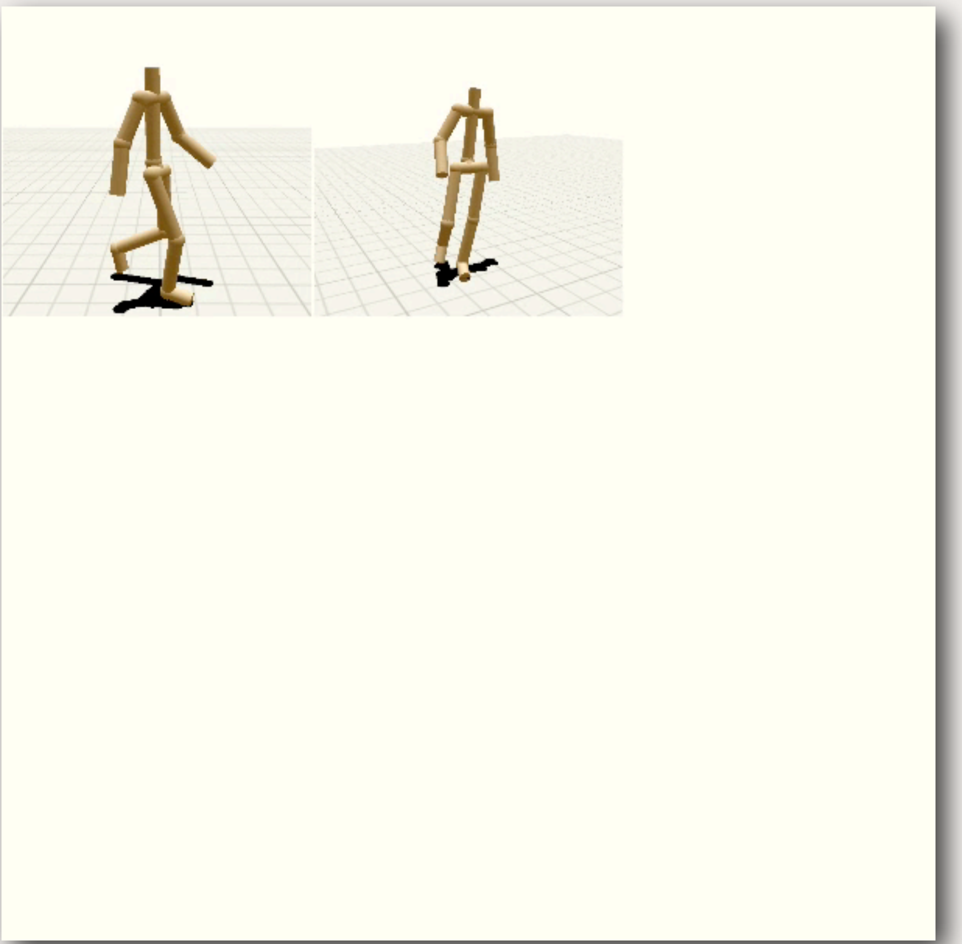
Clips



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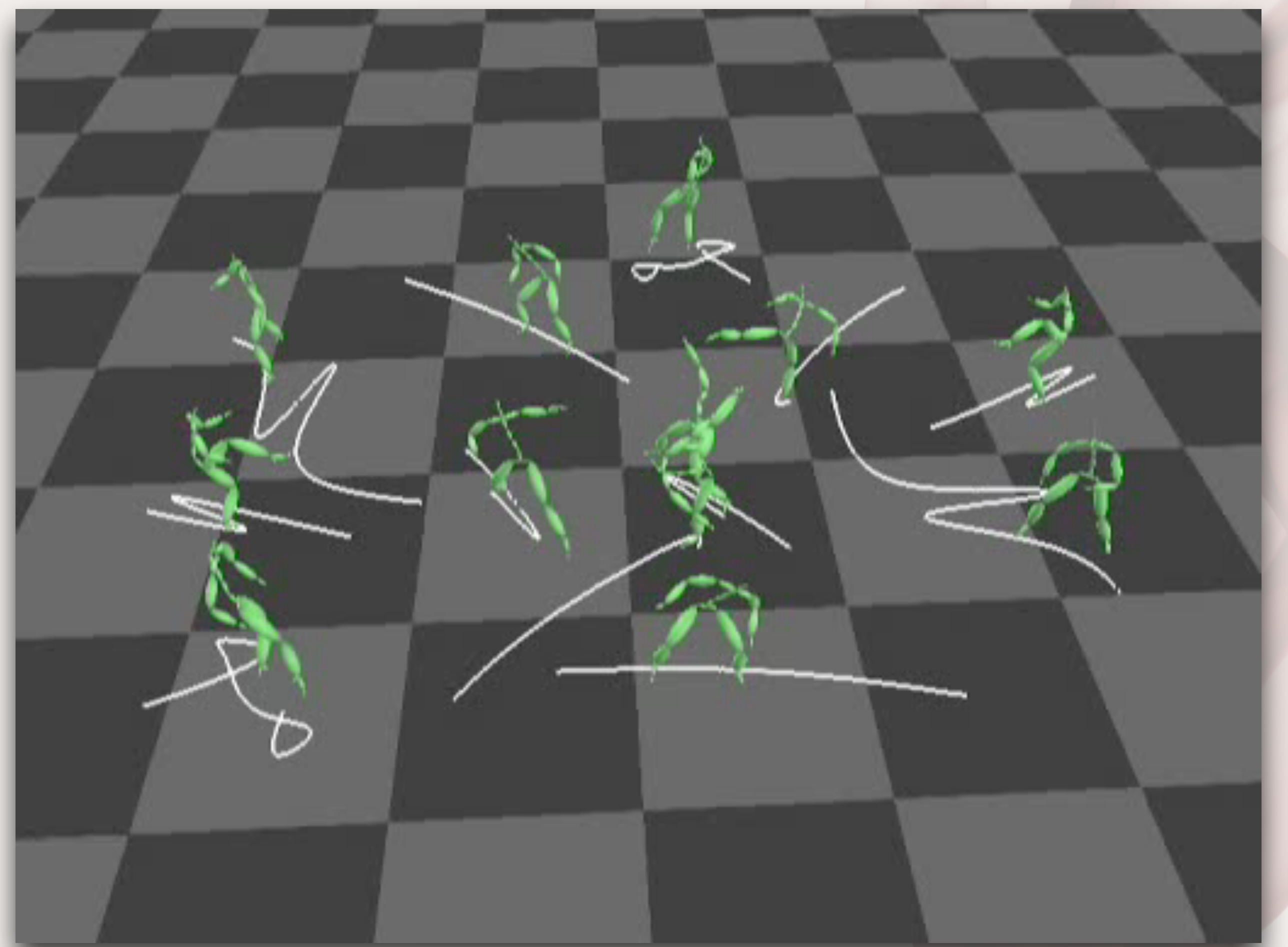

source: Kovar et al. [2002]

Sequences

source: Treuille et al. [2002]

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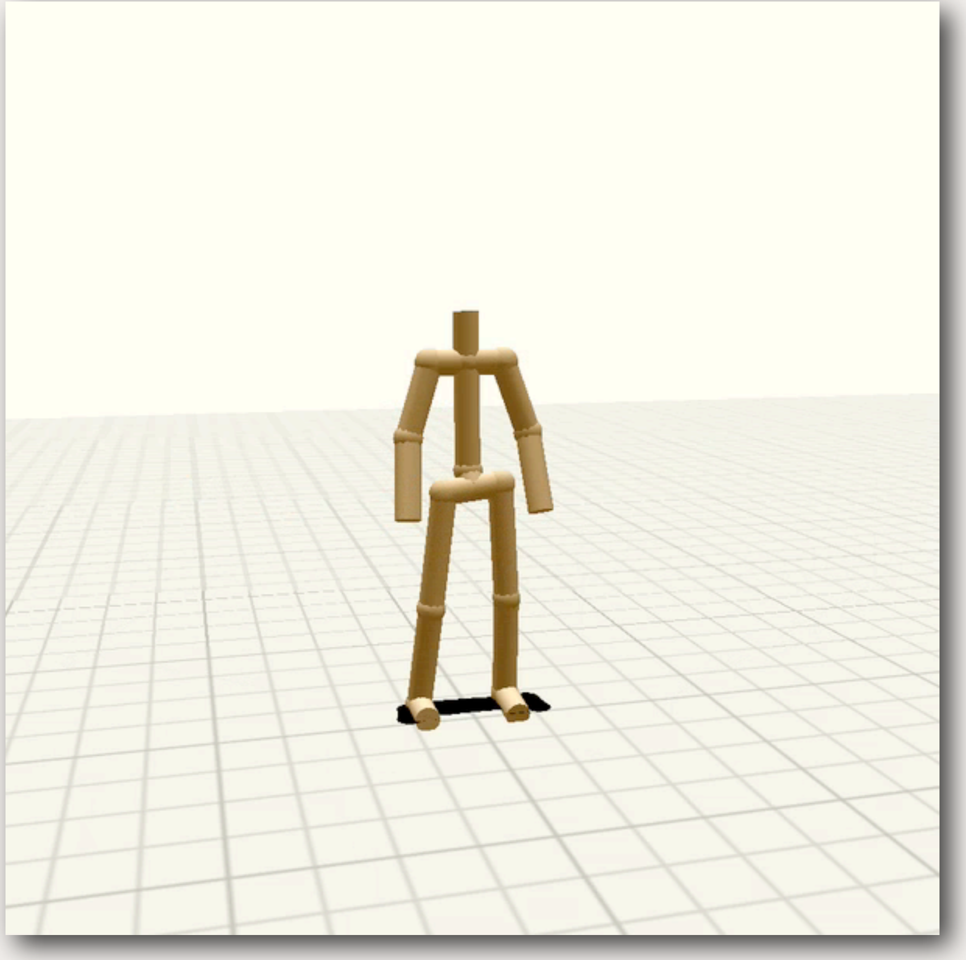
Sequences



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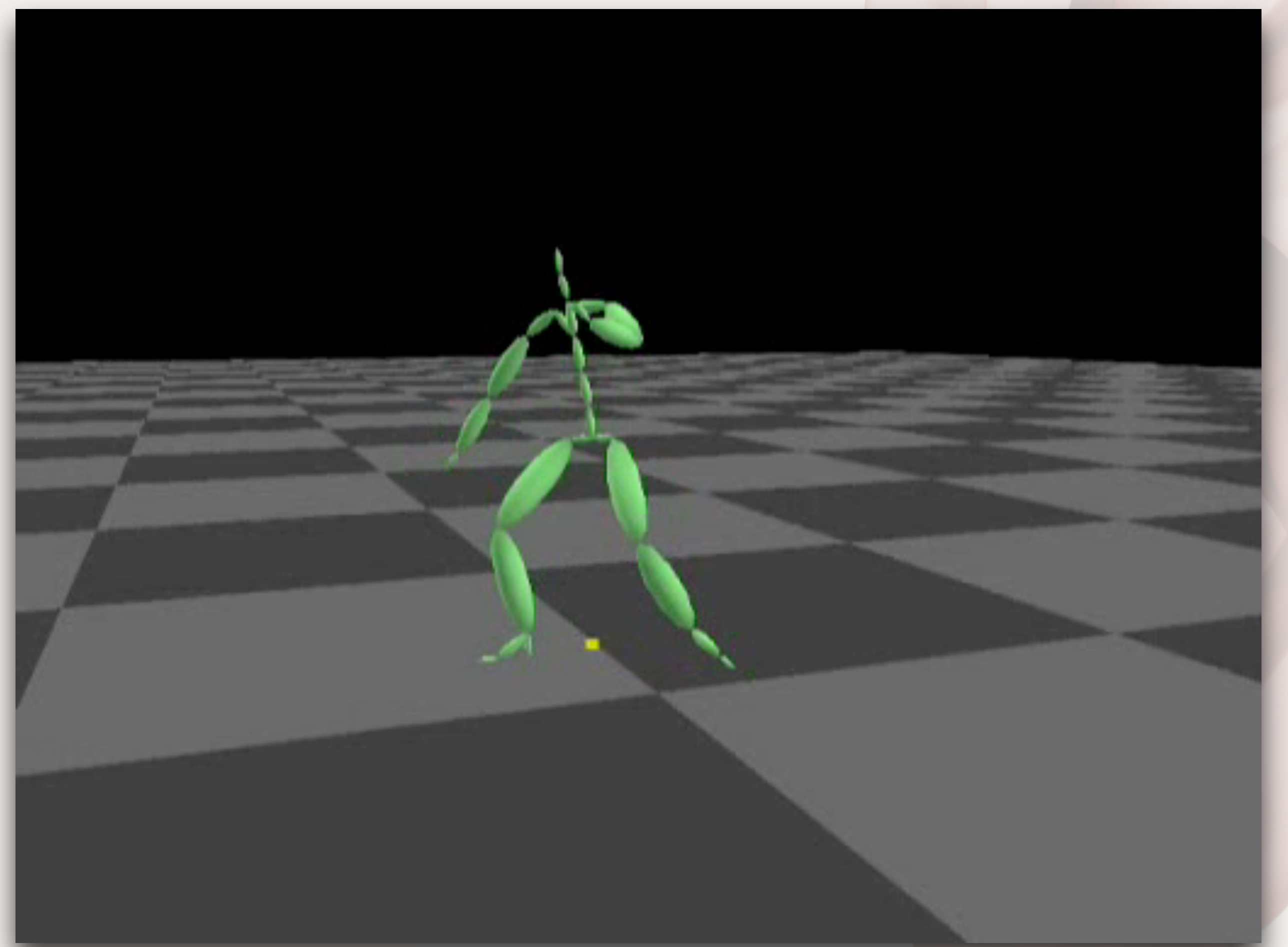

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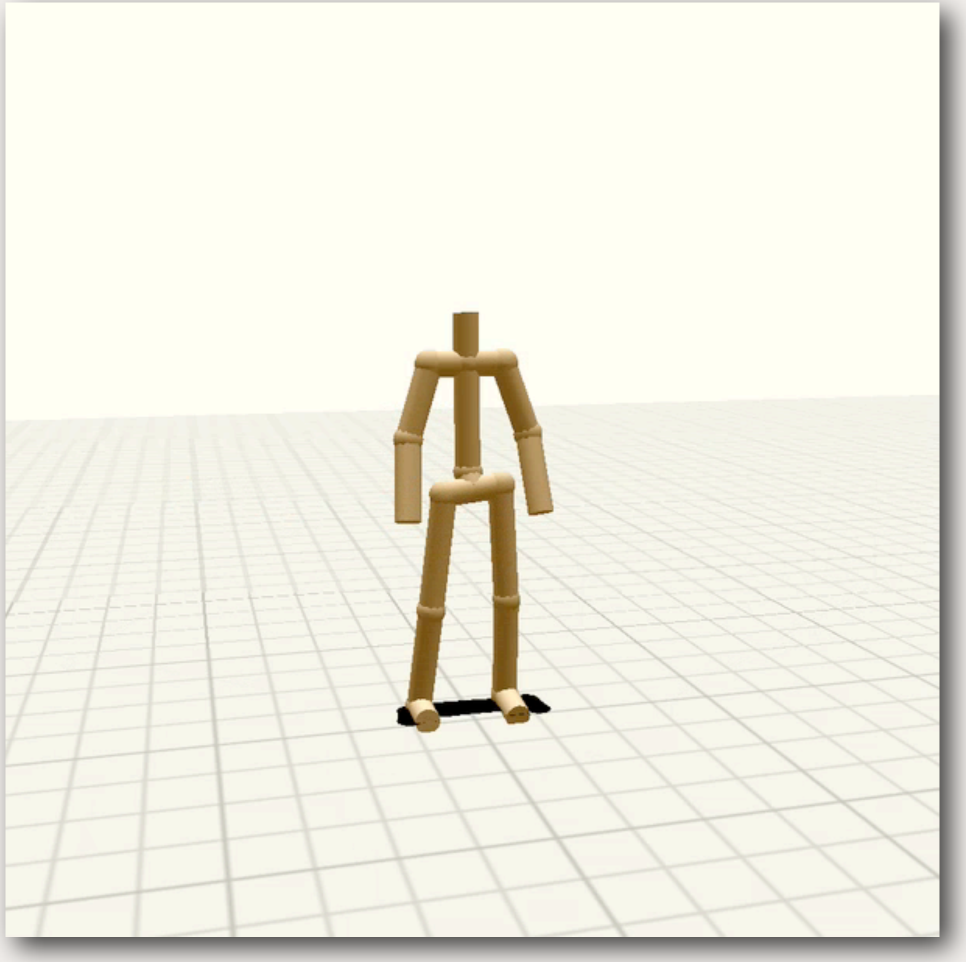
Sequences



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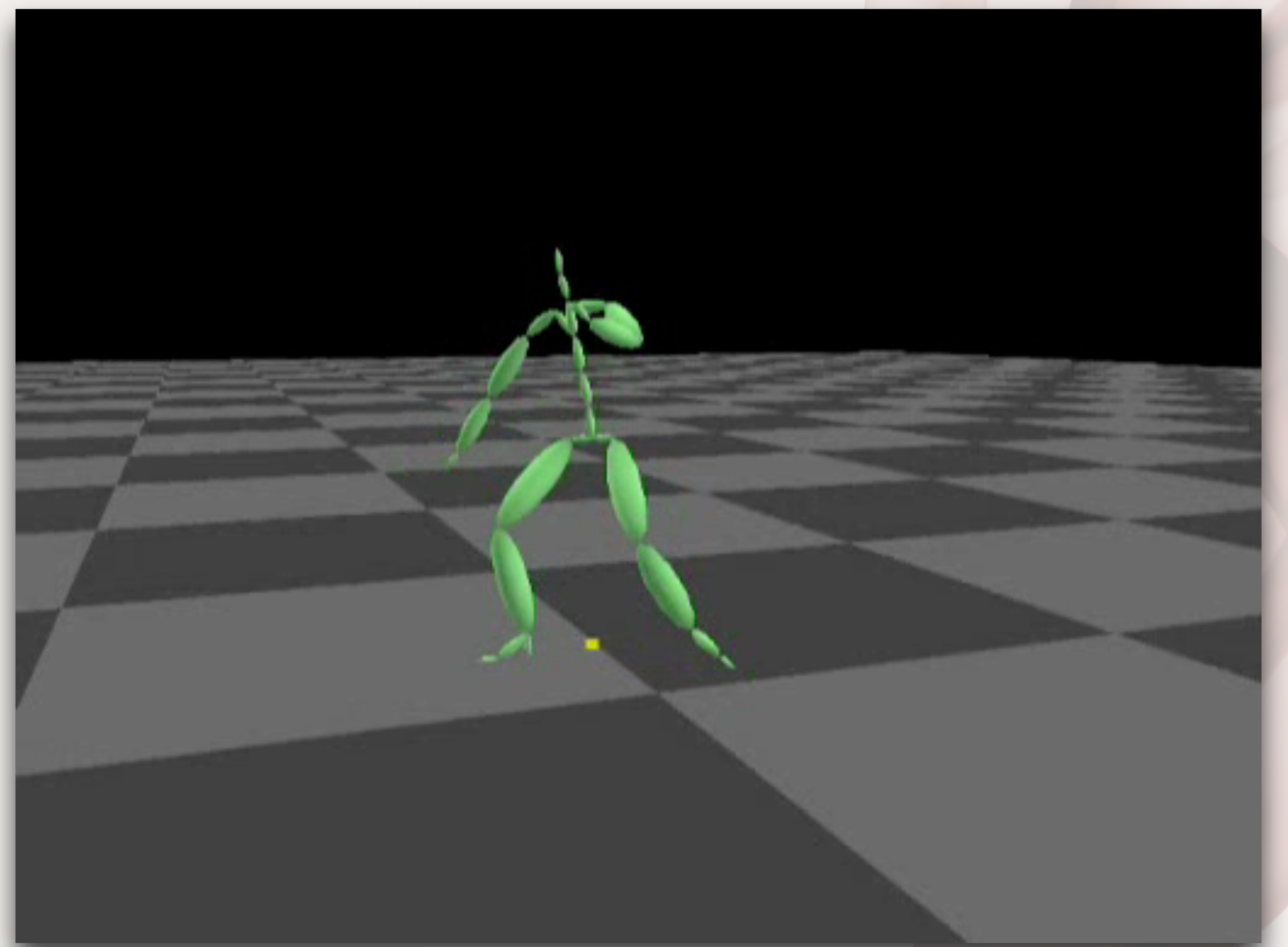

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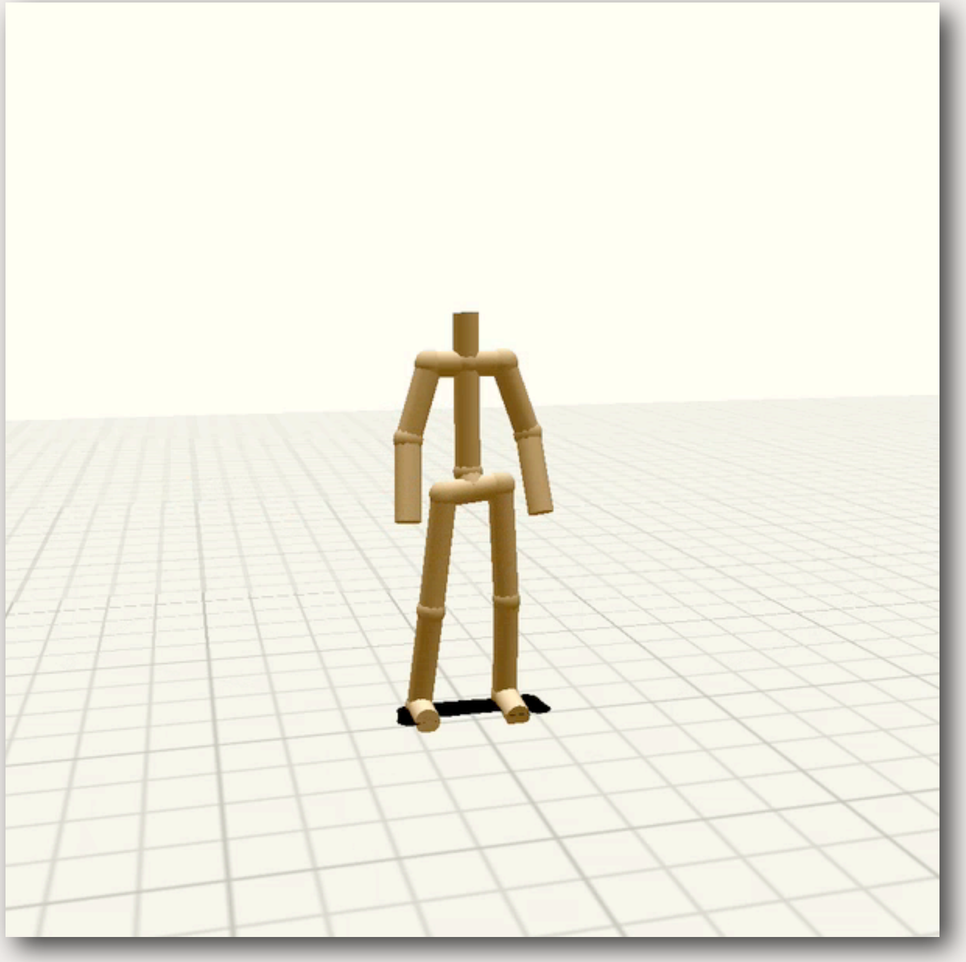

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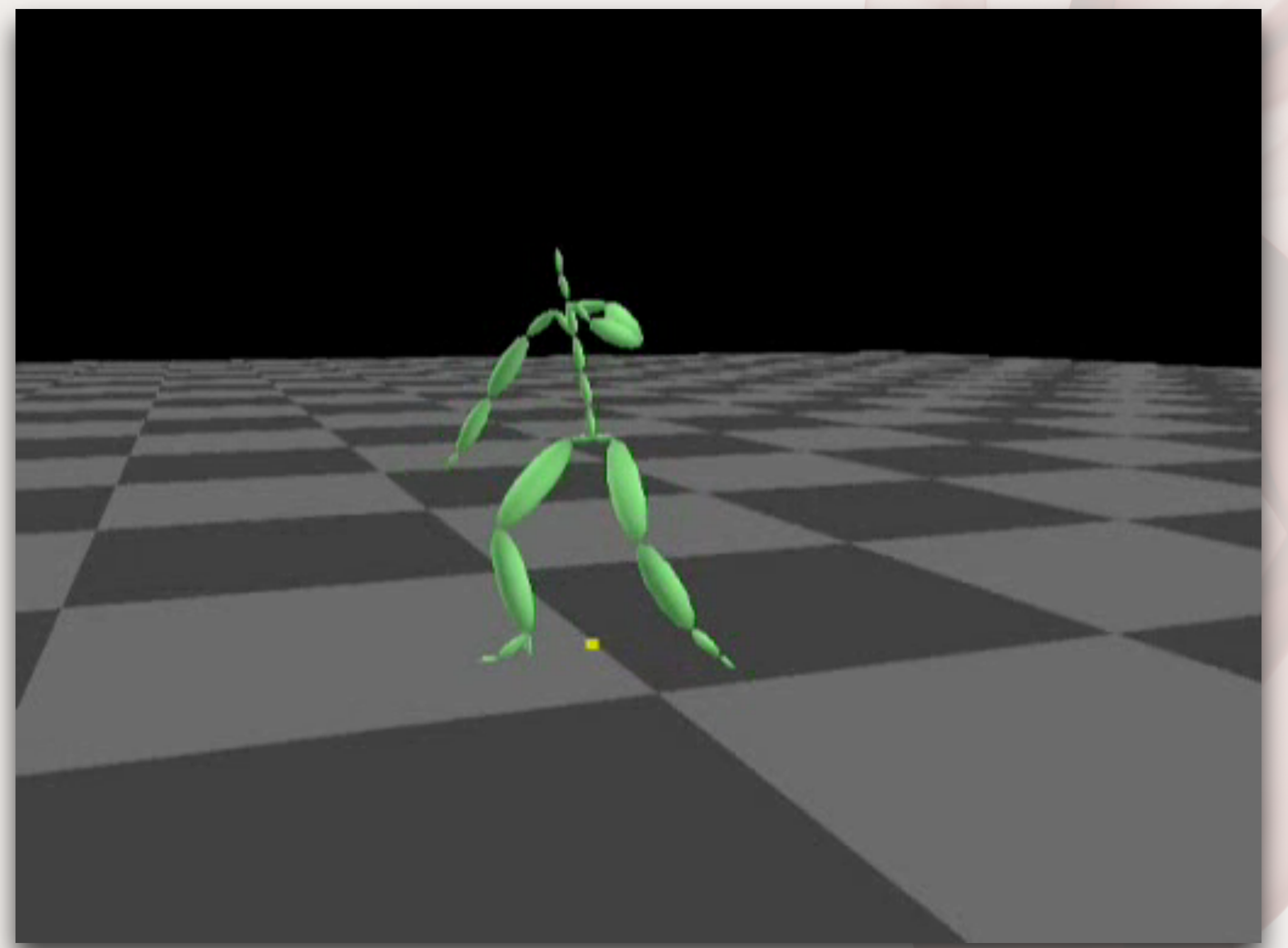
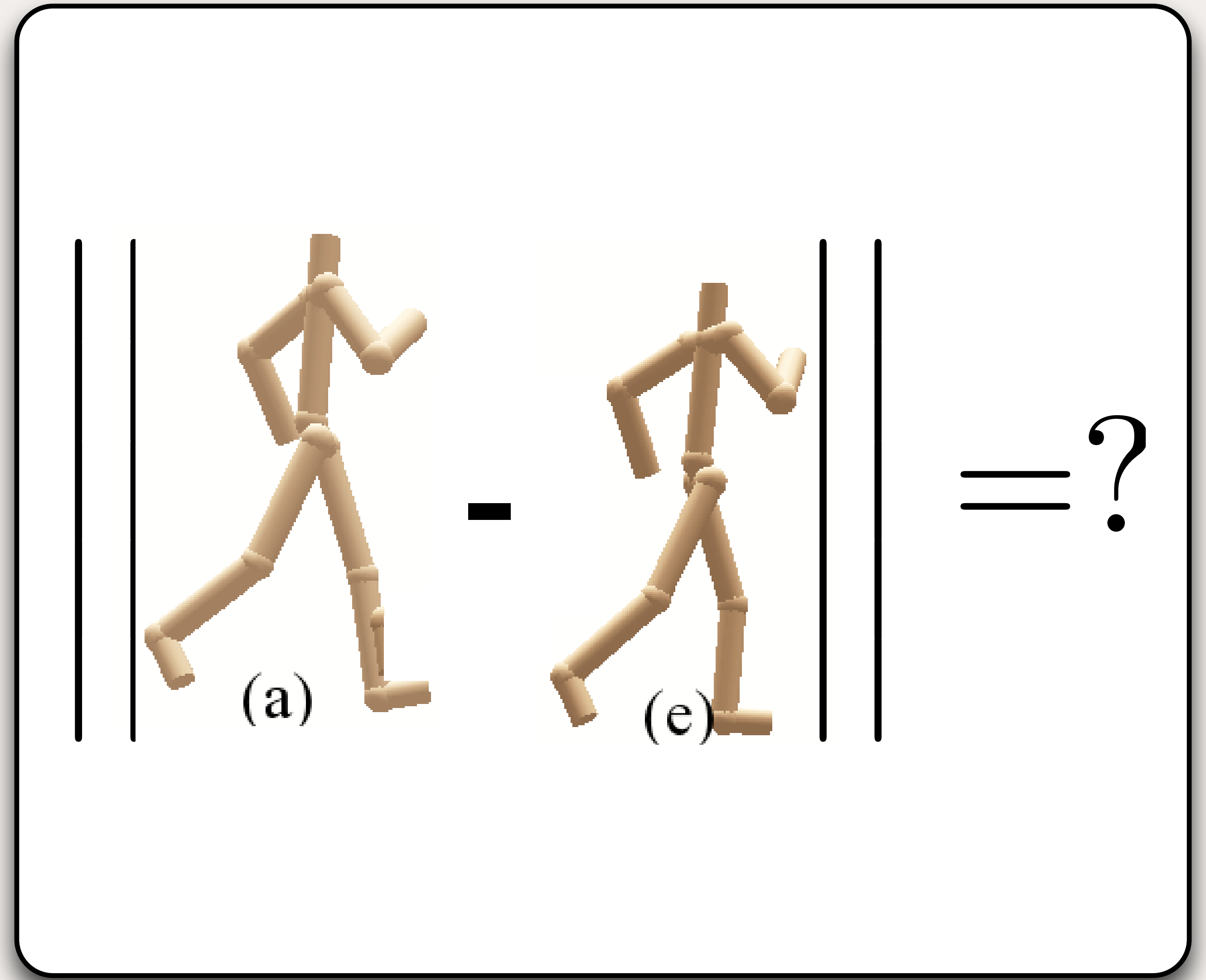


source: Kovar et al. [2002]

How?

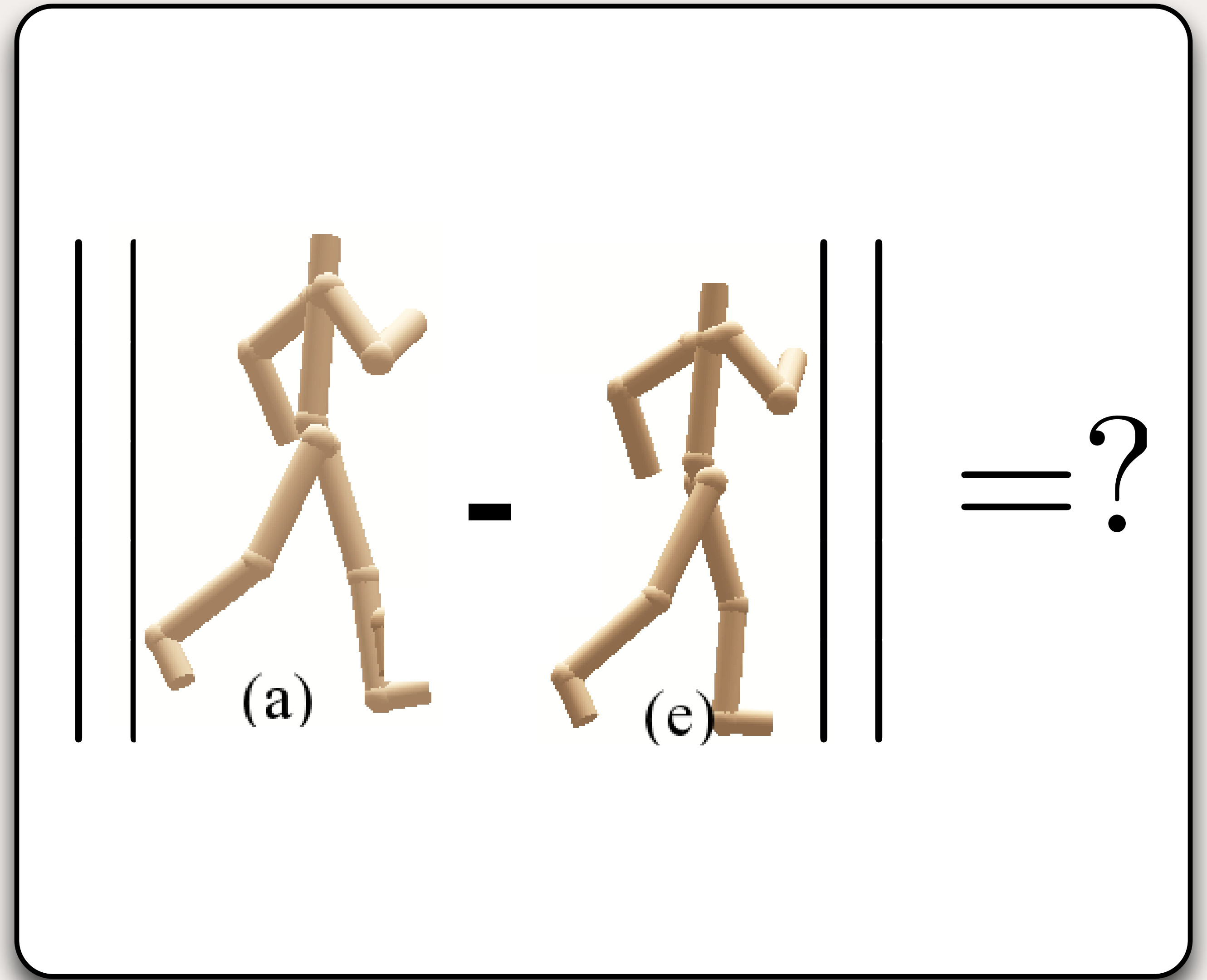
Pose Metrics

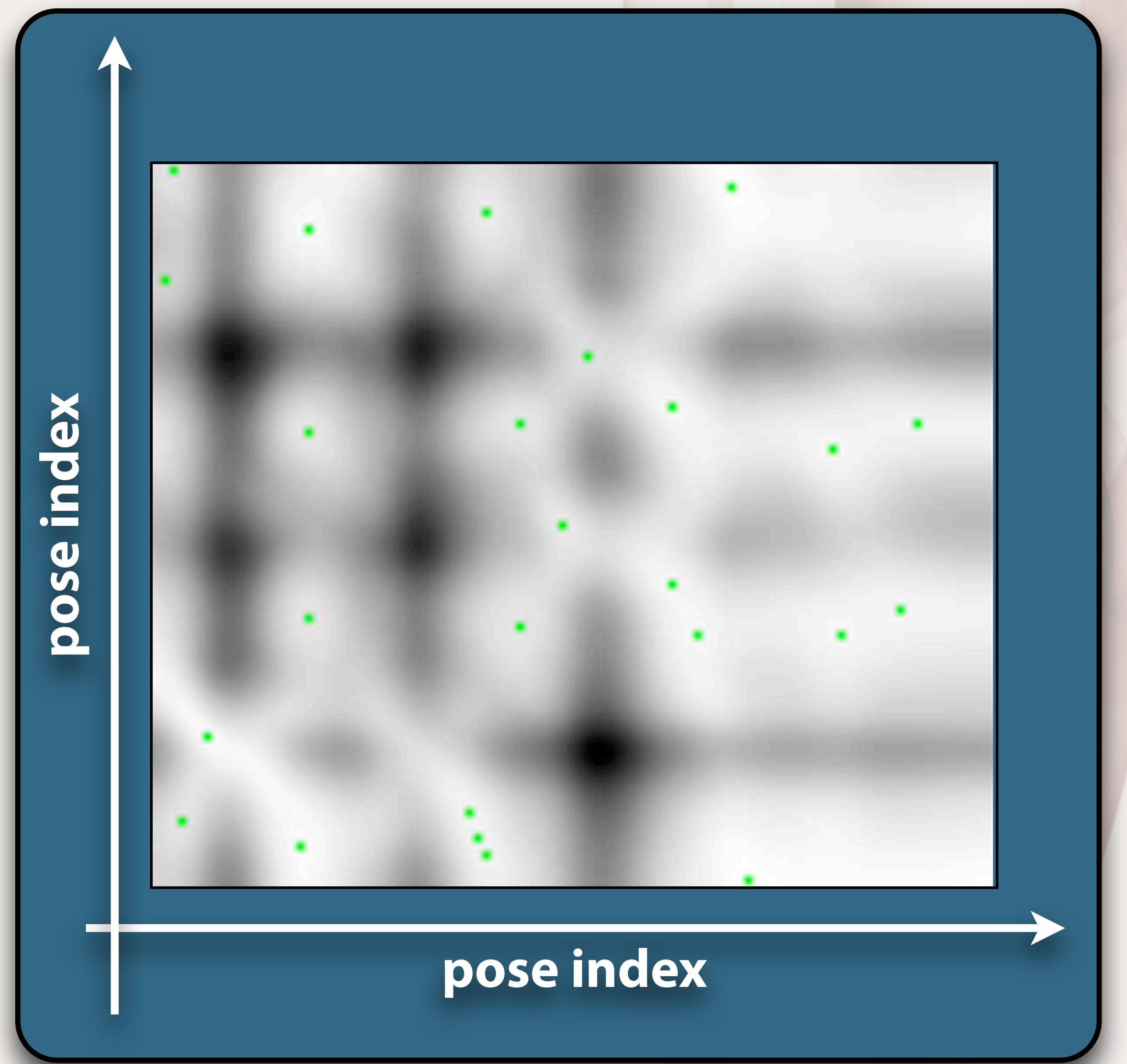
Pose Metrics



How can we define a metric on poses?

Pose Metrics

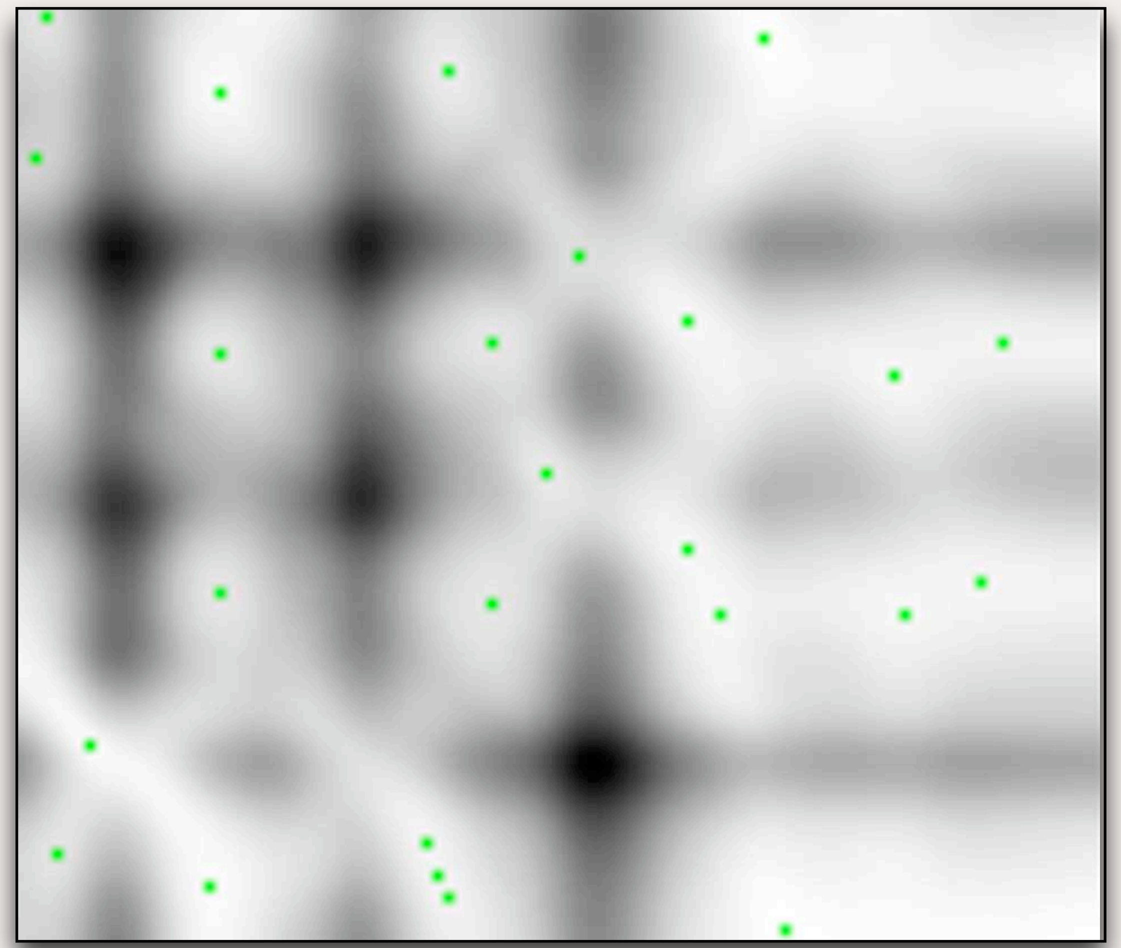




How can we define a metric on poses?

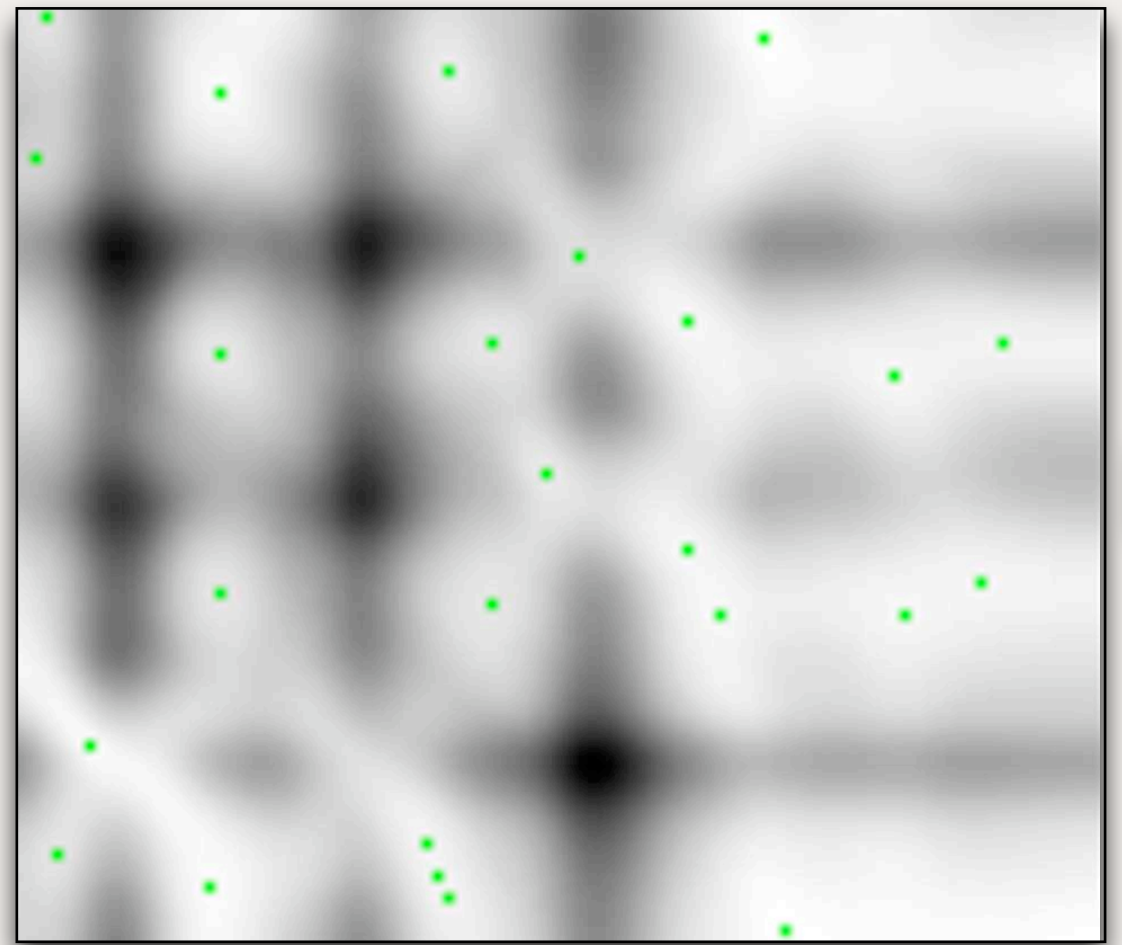
Pairwise pose differences.

Pose Metrics

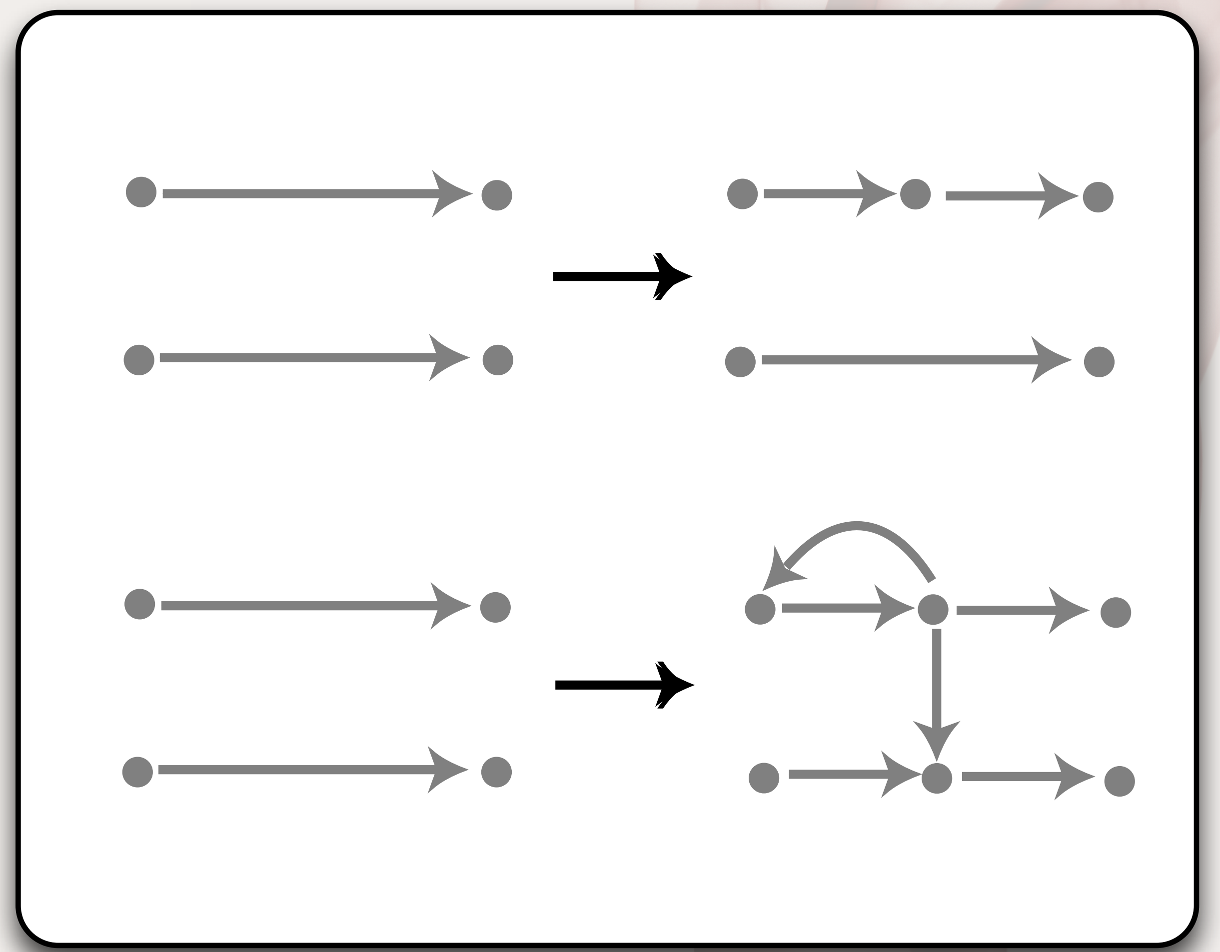


Pairwise Pose Differences

Pose Metrics



Pairwise Pose Differences

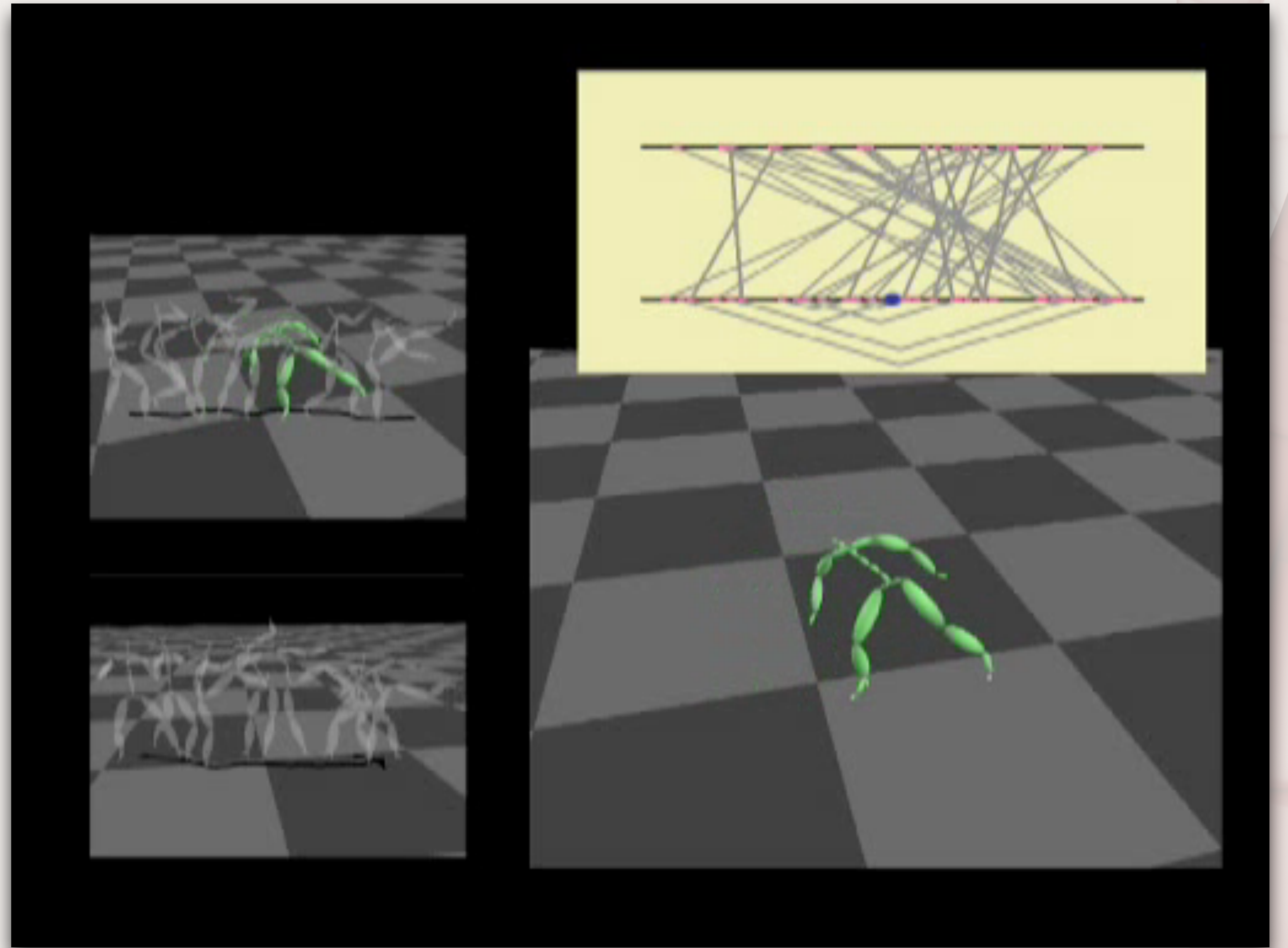


Motion Graph Schematic

Results

source: Kovar et al. [2002]

Results



source: Kovar et al. [2002]

Constraints

- Pose blending may violate physical constriants
  - Linear Momentum Conservation
  - Angular Momentum Conservation
  - Frictional Constraints (“Foot Skate”)

“Foot Skate” Poblem

http://www.youtube.com/watch?v=E_FzgtLVzbI

source: http://www.cs.wisc.edu/graphics/Gallery/kovar.vol/Cleanup/

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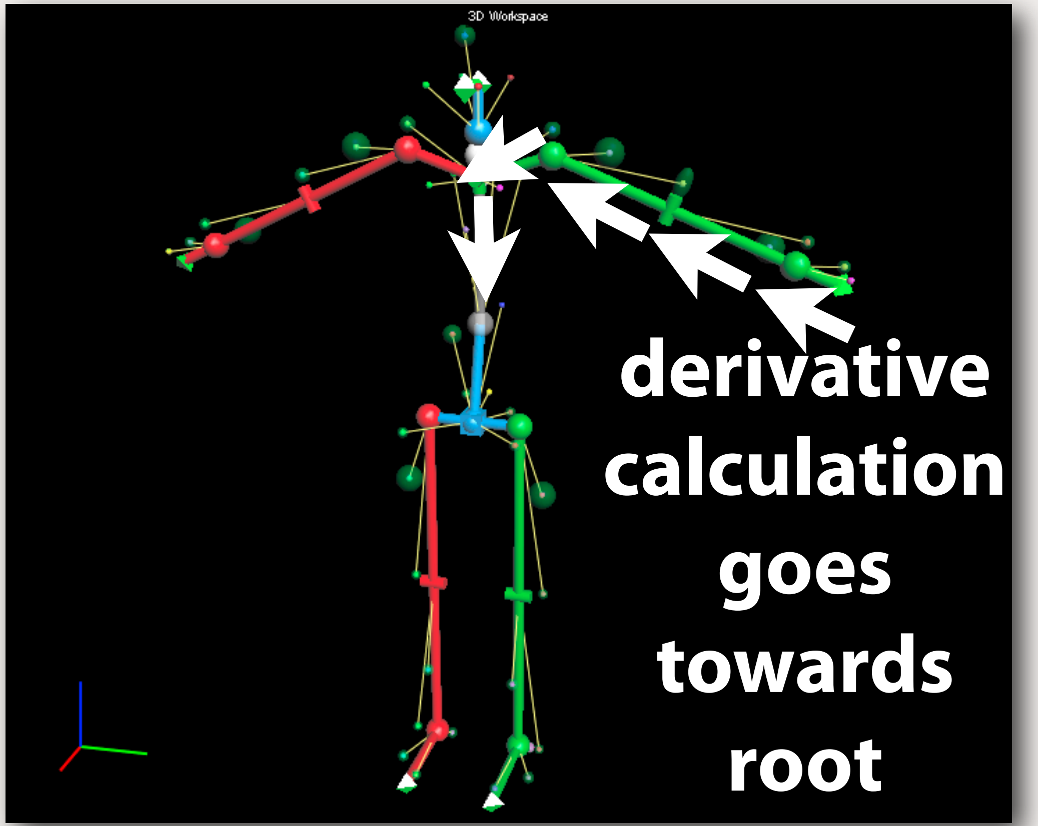
Inverse Kinematic Solution



$$\omega_i = f_{i,\Omega}(\omega_{i-1})$$

$$\frac{dE}{d\Omega} = 2\sum_j (\hat{\mathbf{m}}_j^\star - \hat{\mathbf{m}}_j)^T \frac{d\hat{\mathbf{m}}_j}{d\Omega}$$

$$\frac{d\hat{\mathbf{m}}_j}{d\Omega} = \frac{\partial \hat{\mathbf{m}}_j}{\partial \omega_i}\left(\frac{\partial \omega_i}{\partial \Omega} + \frac{\partial \omega_i}{\partial \omega_{i-1}}\frac{\partial \omega_{i-1}}{\partial \Omega} + \frac{\partial \omega_i}{\partial \omega_{i-1}}\frac{\partial \omega_{i-1}}{\partial \omega_{i-2}}\frac{\partial \omega_{i-2}}{\partial \Omega} + \cdots\right)$$

IK Results

source: http://www.cs.wisc.edu/graphics/Gallery/kovar.vol/Cleanup/

IK Results

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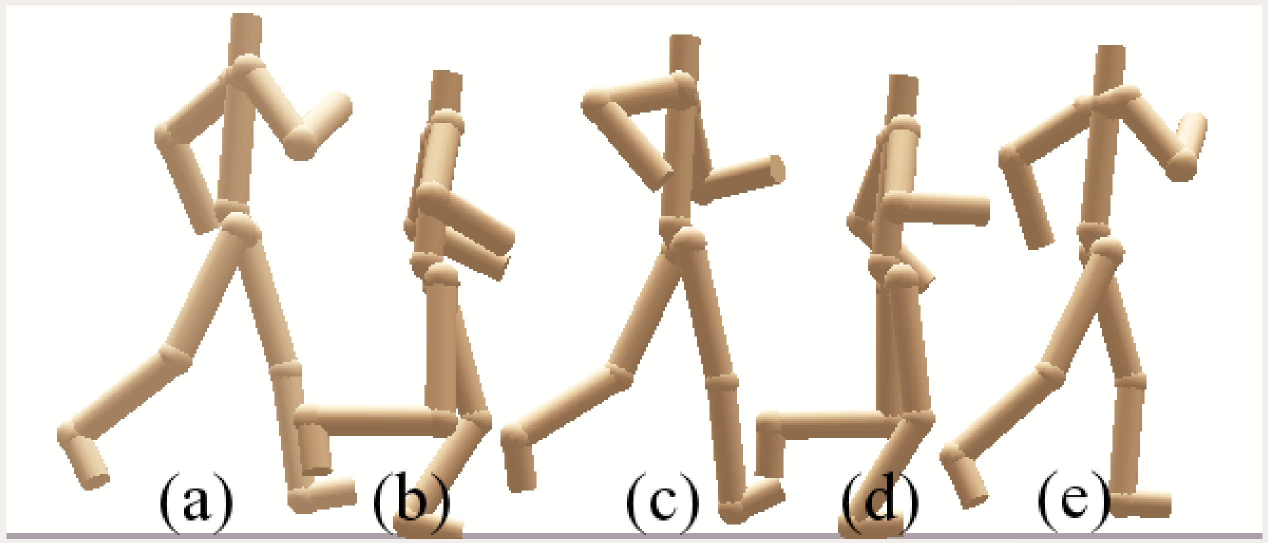
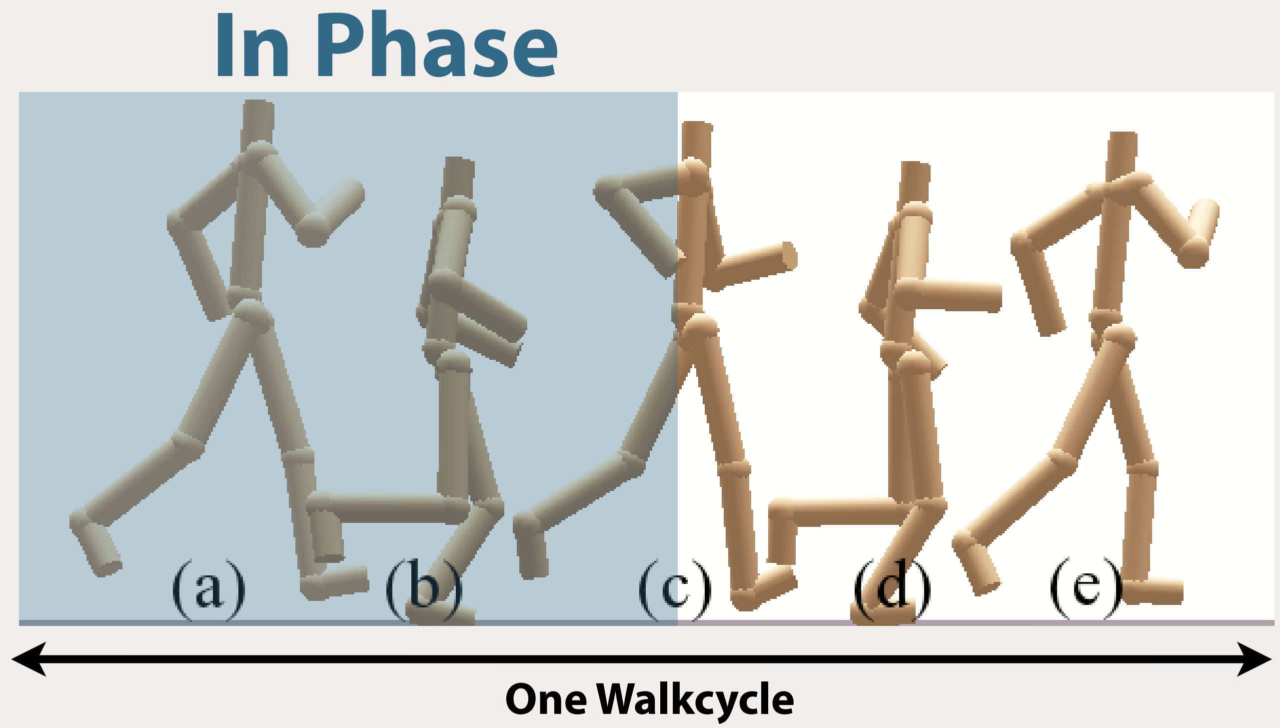
Smart Blending

Smart Blending

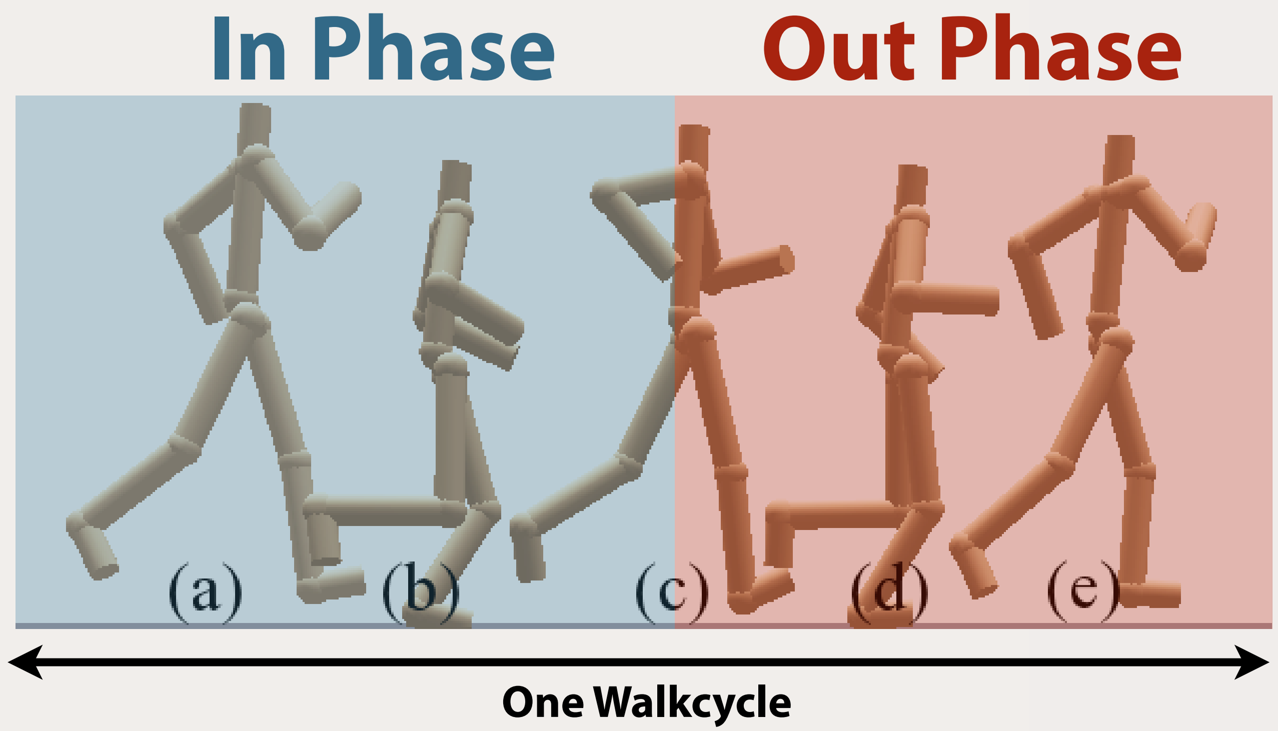


One Walkcycle

Smart Blending



Smart Blending



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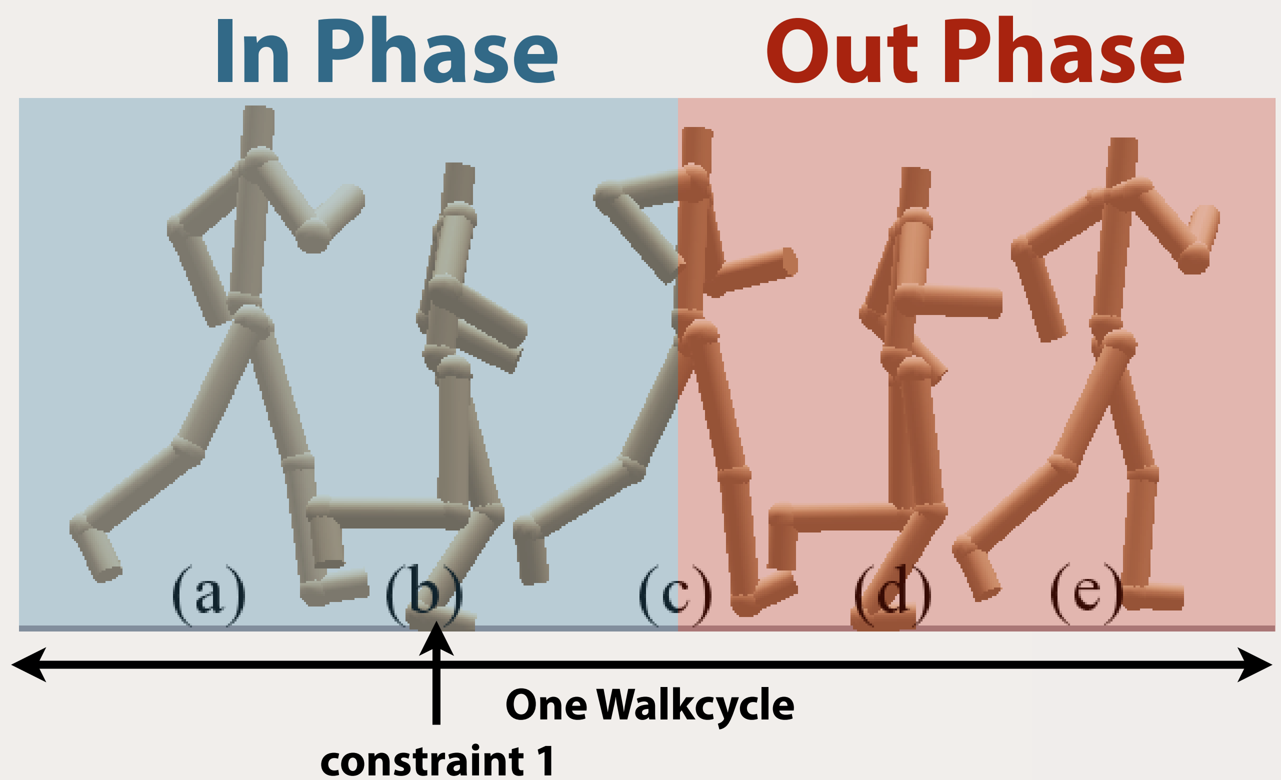

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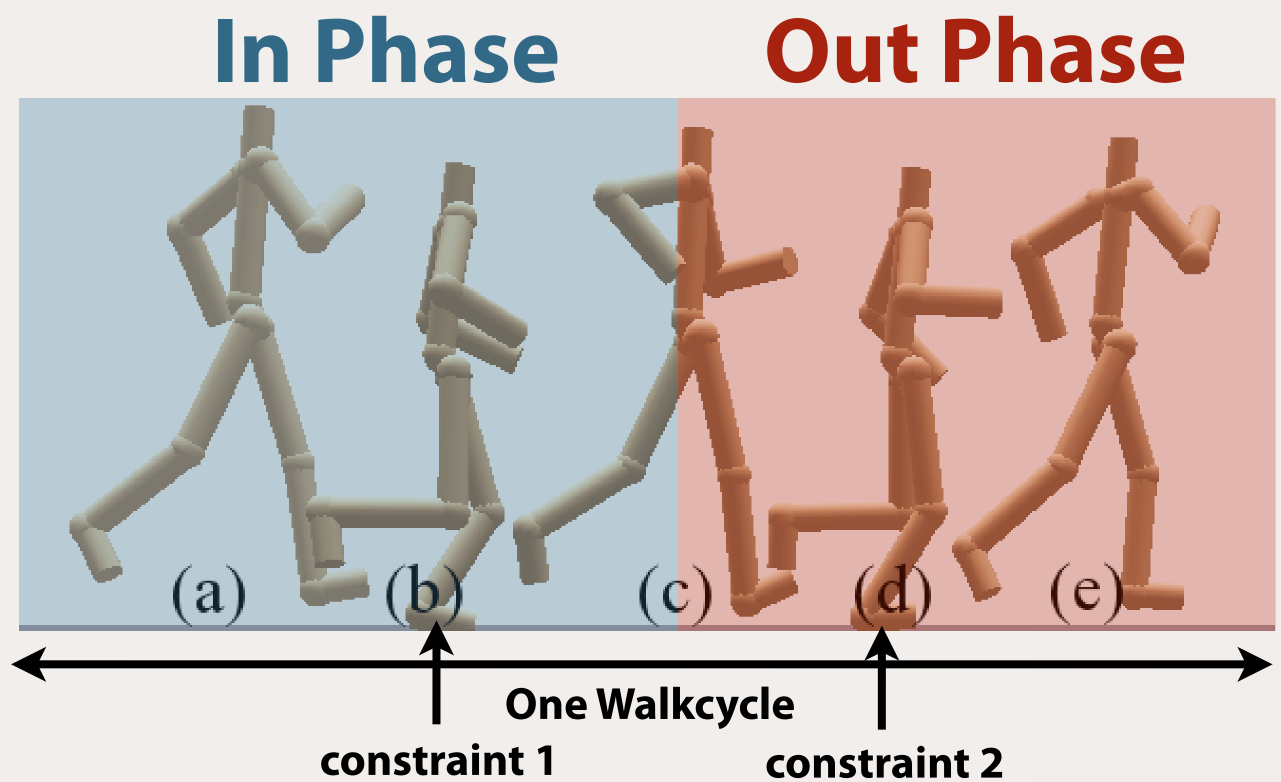

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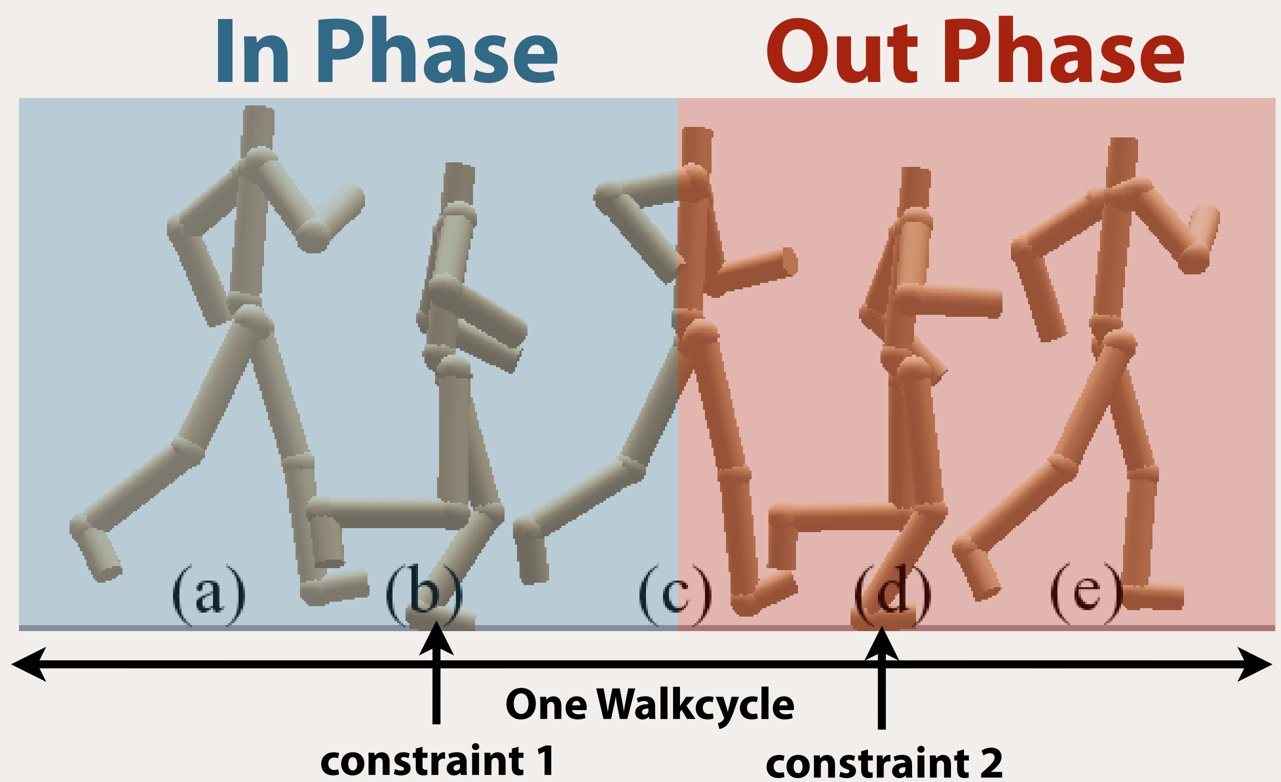
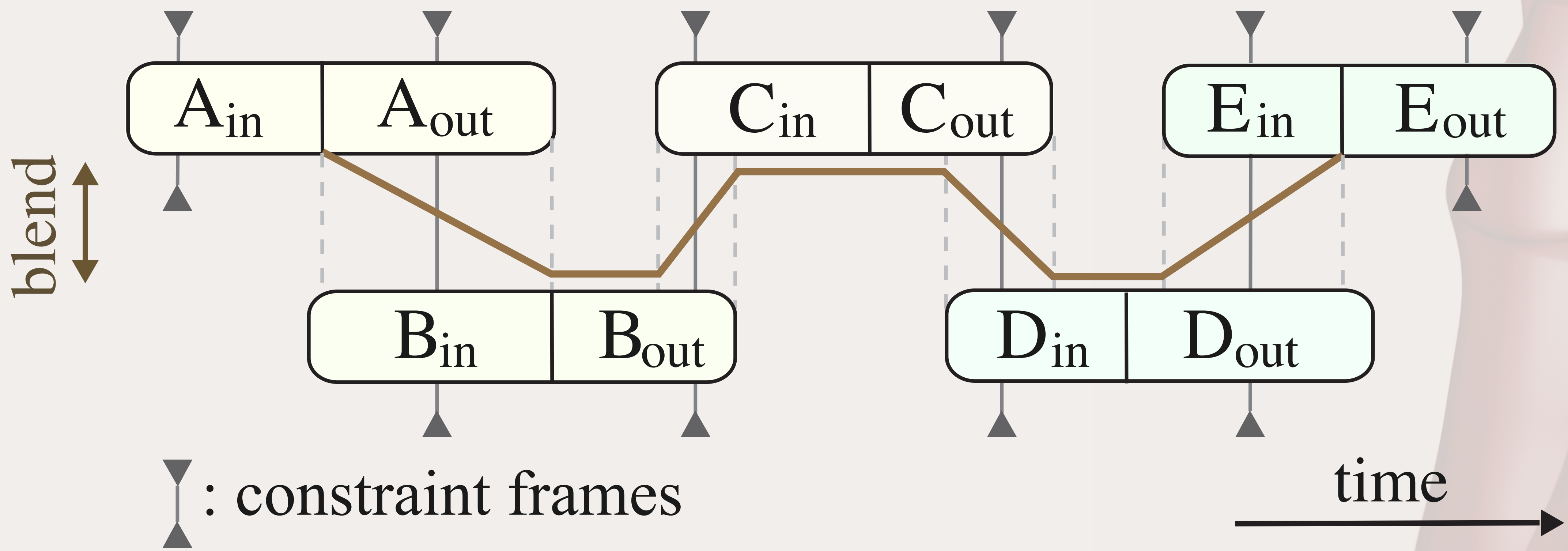
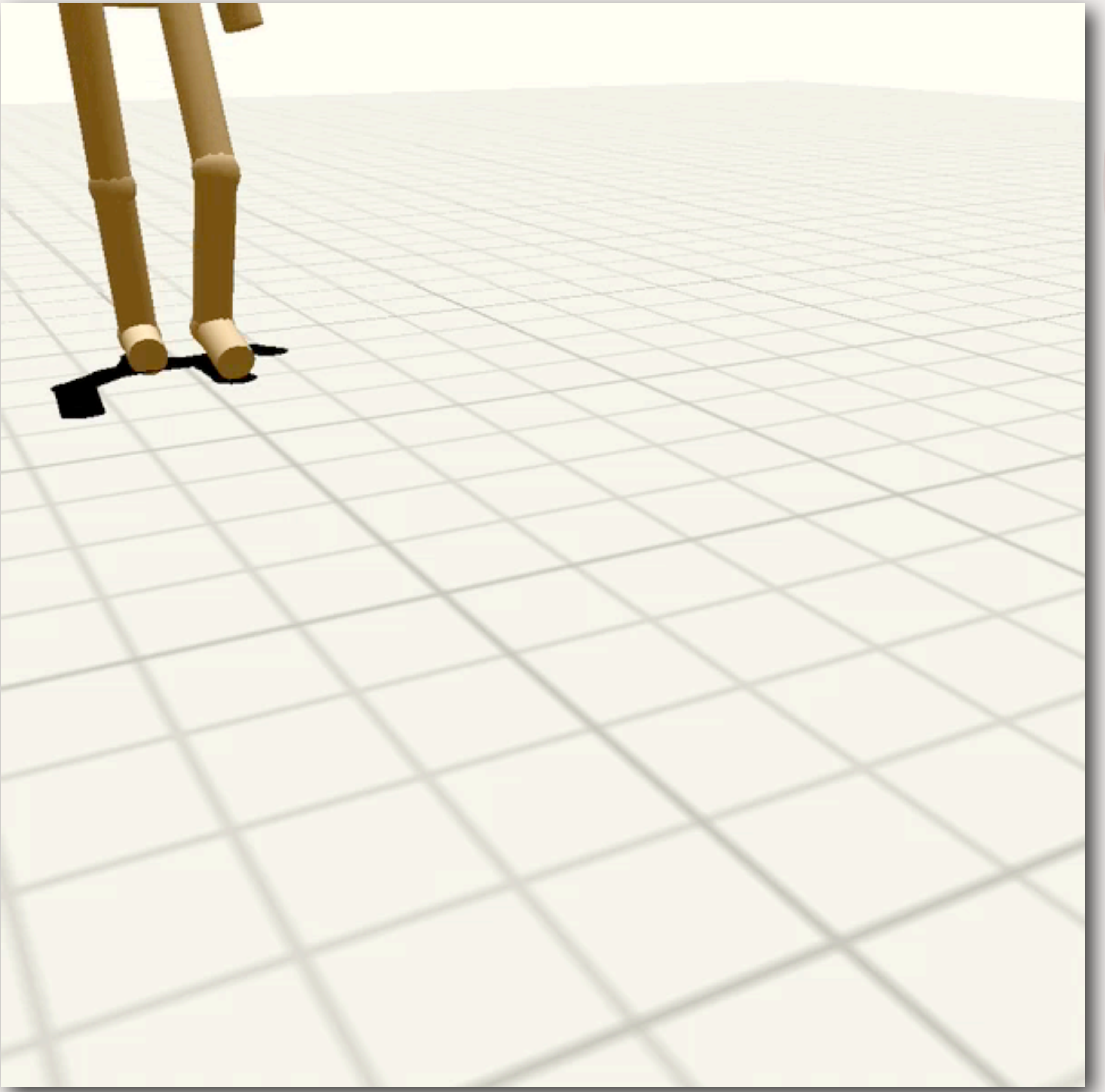




Smart Blending Example

source: Treuille et al. [2002]

Smart Blending Example



source: Treuille et al. [2002]

Open Problems

- How to pick to which clip to transition?
- How to enforce temporal contraints?
- How to generalize beyond the given clips?

Animation Topics

- **Data-Driven Motion**
- Physics Based Motion
- Motion of other Animals

Animation Topics

- Data-Driven Motion
- **Physics Based Motion**
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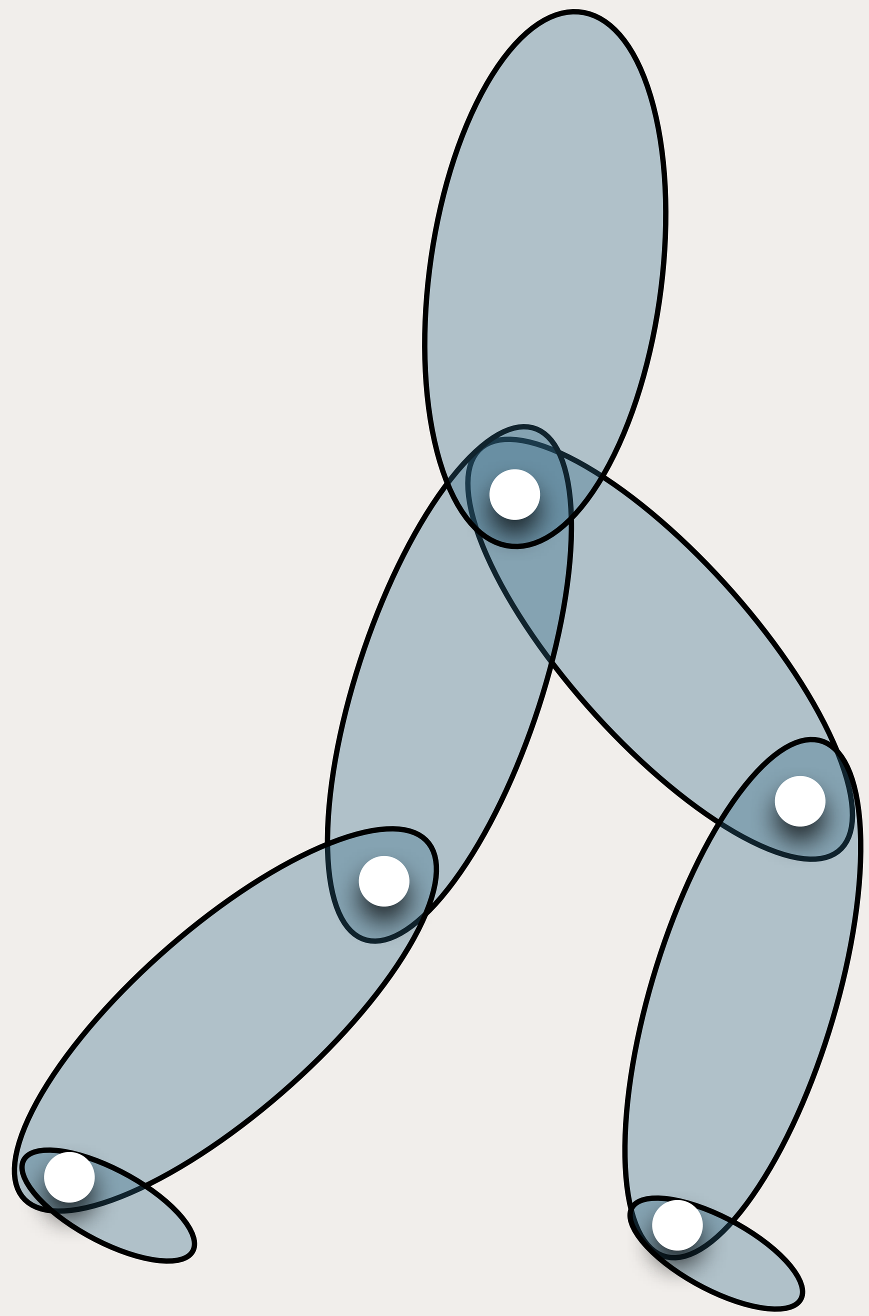
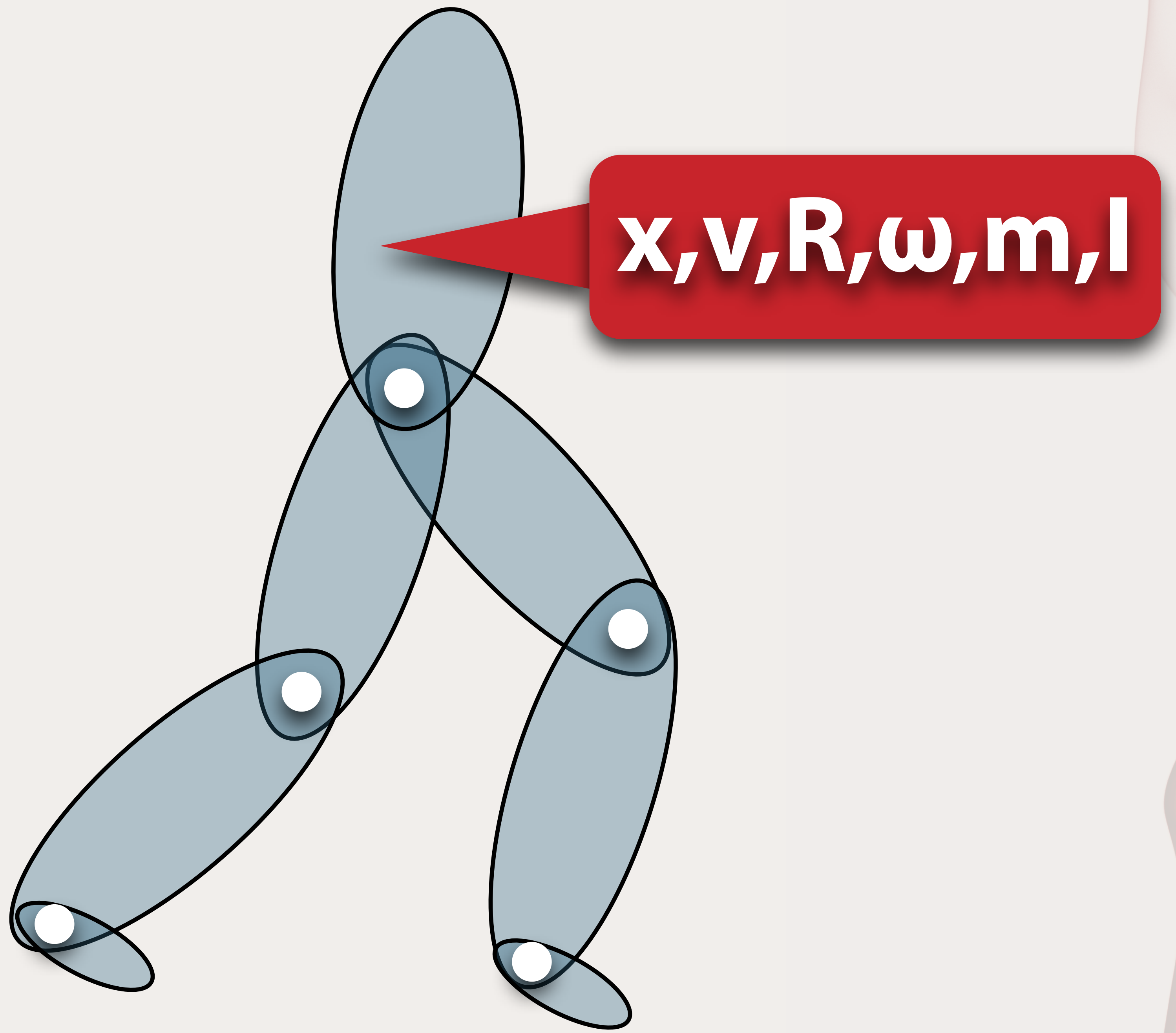
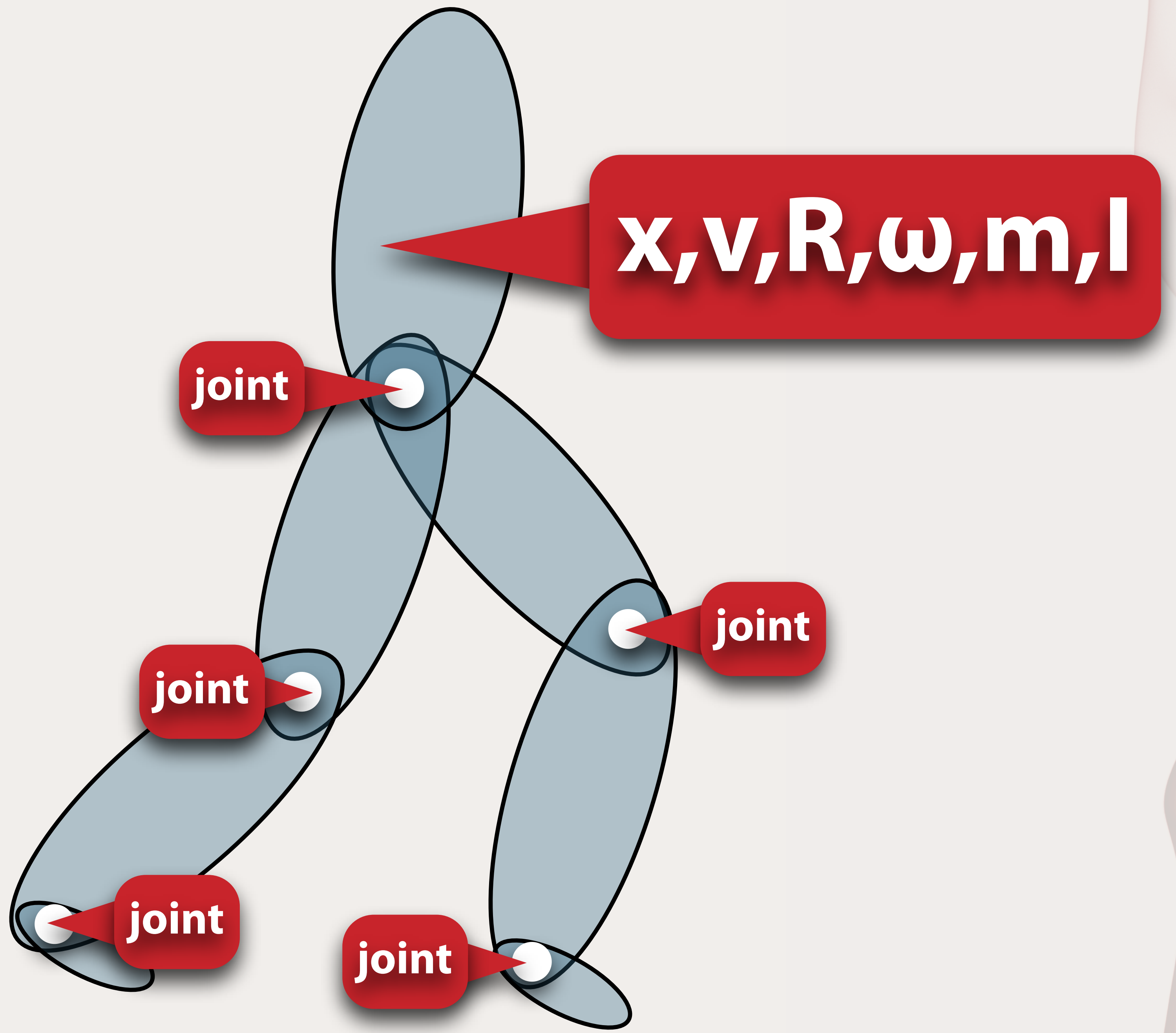
Physical Model



Physical Model



Physical Model



Joint Types

All joints can be written as the composition of...

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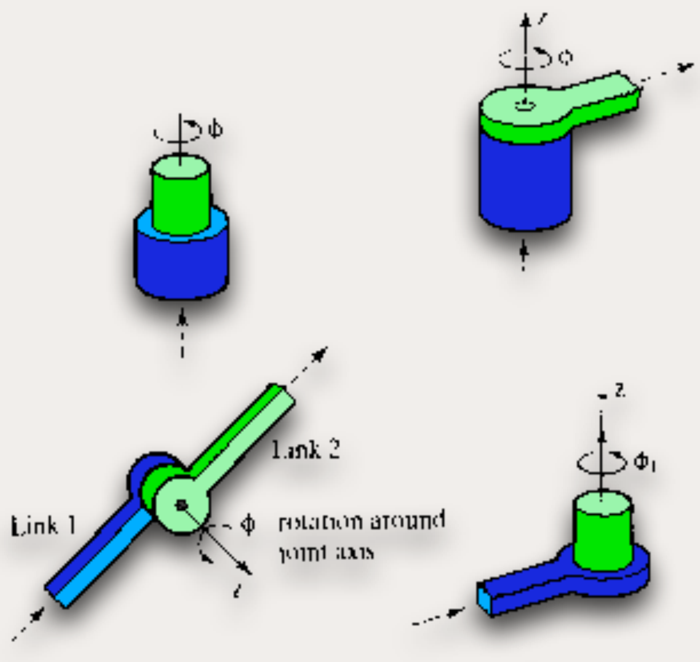
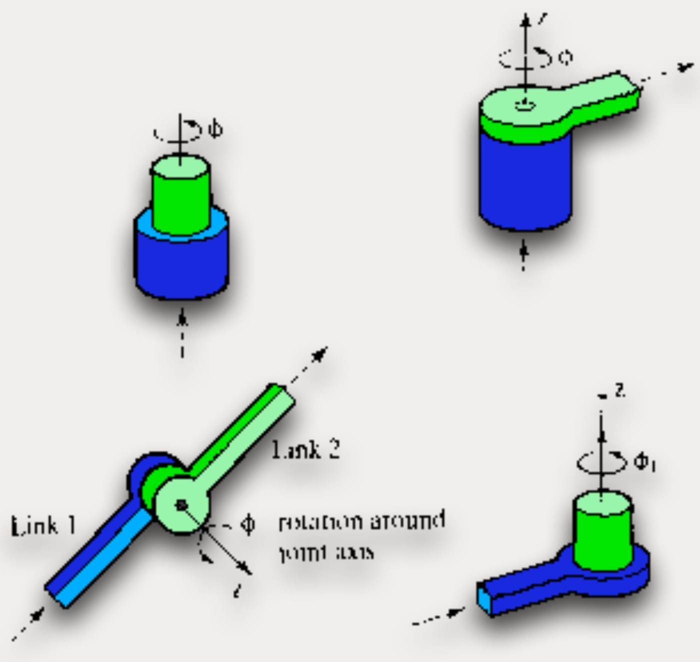
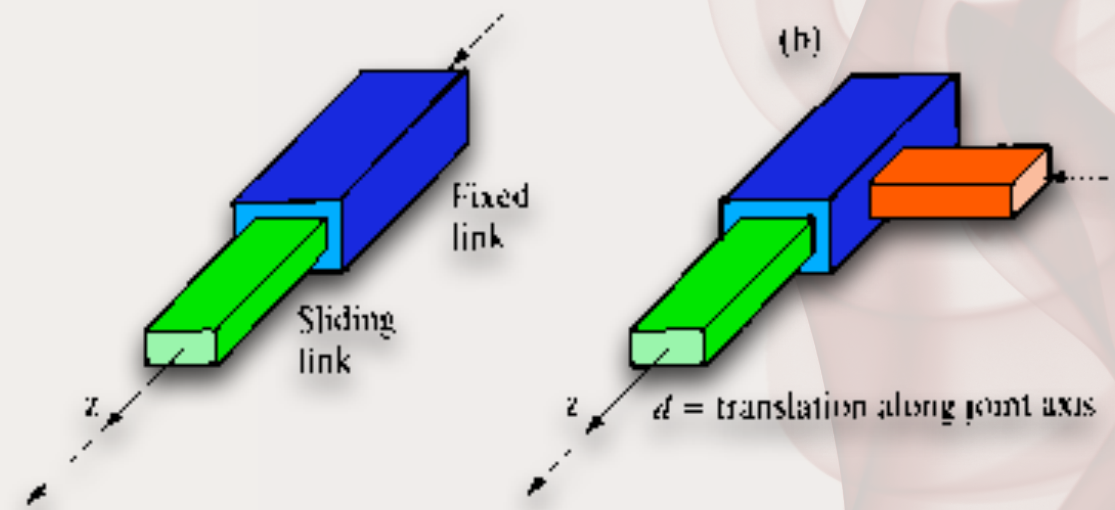


Rotary

Joint Types

All joints can be written as the composition of...

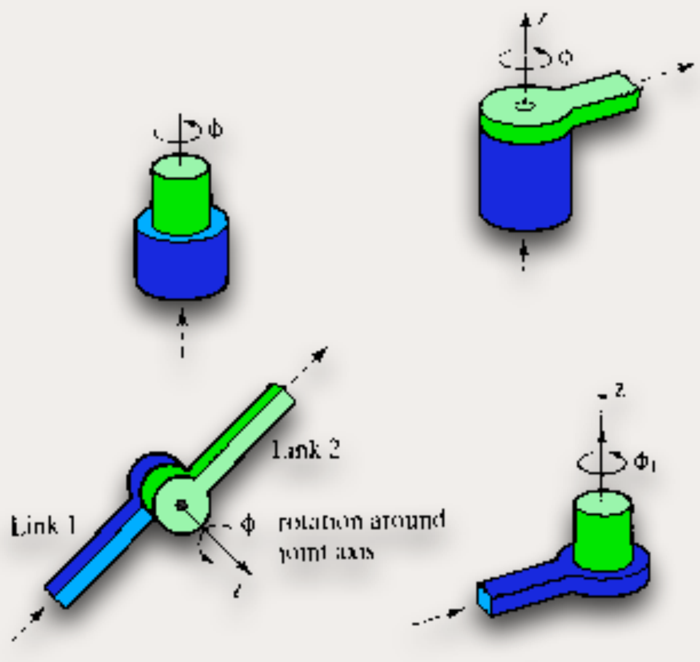




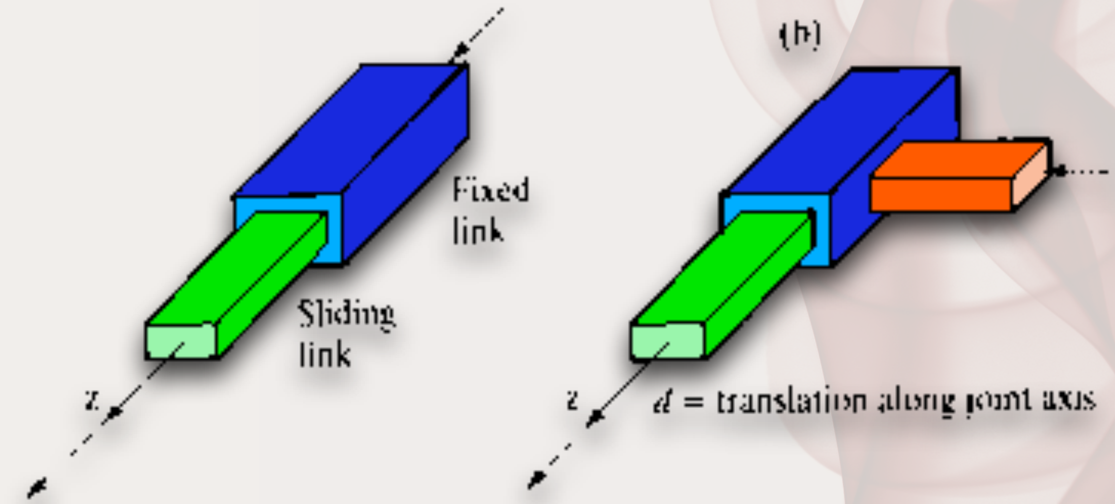
Rotary **Prismatic**

Joint Types

All joints can be written as the composition of...



Rotary

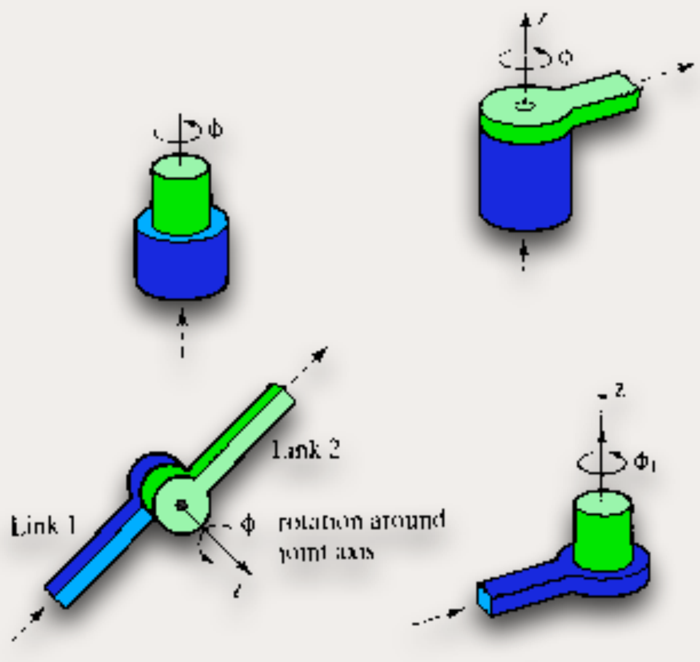


Prismatic

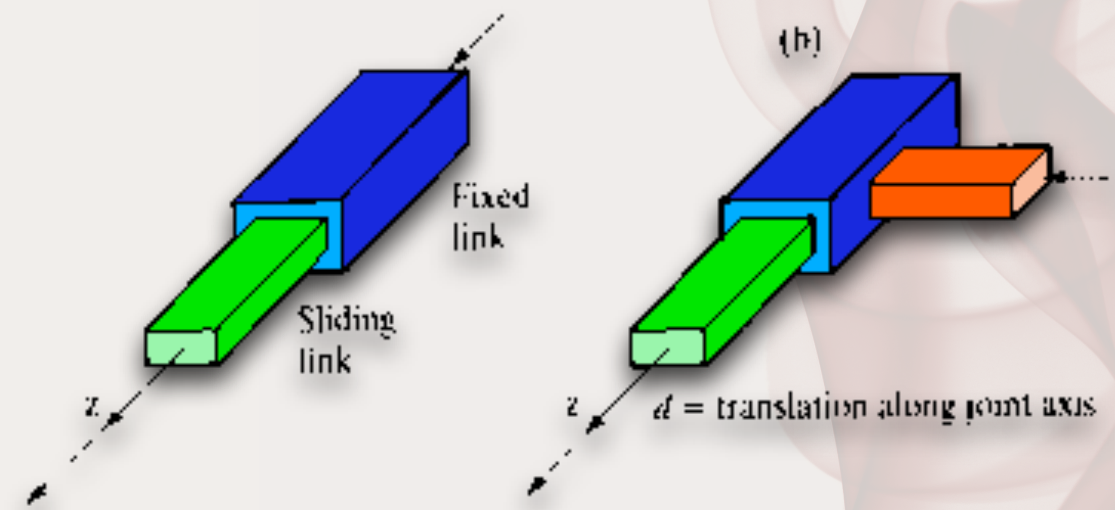
...and have two forms:

Joint Types

All joints can be written as the composition of...



Rotary



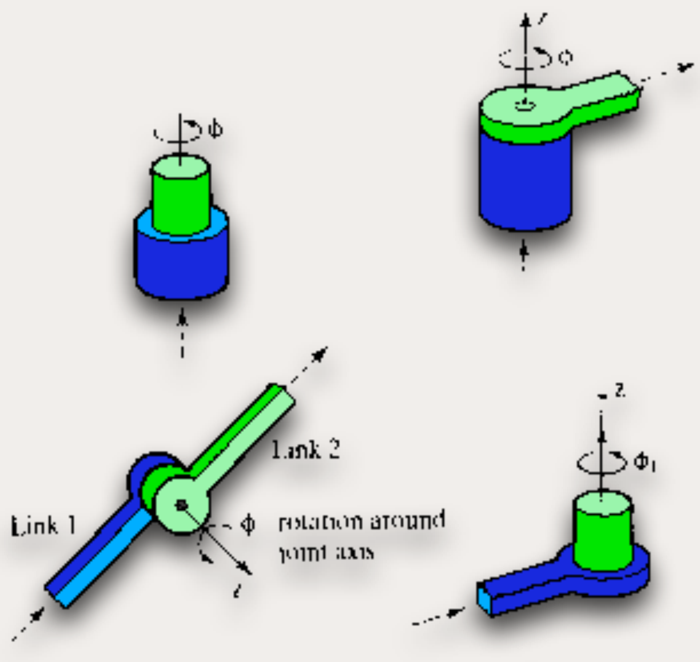
Prismatic

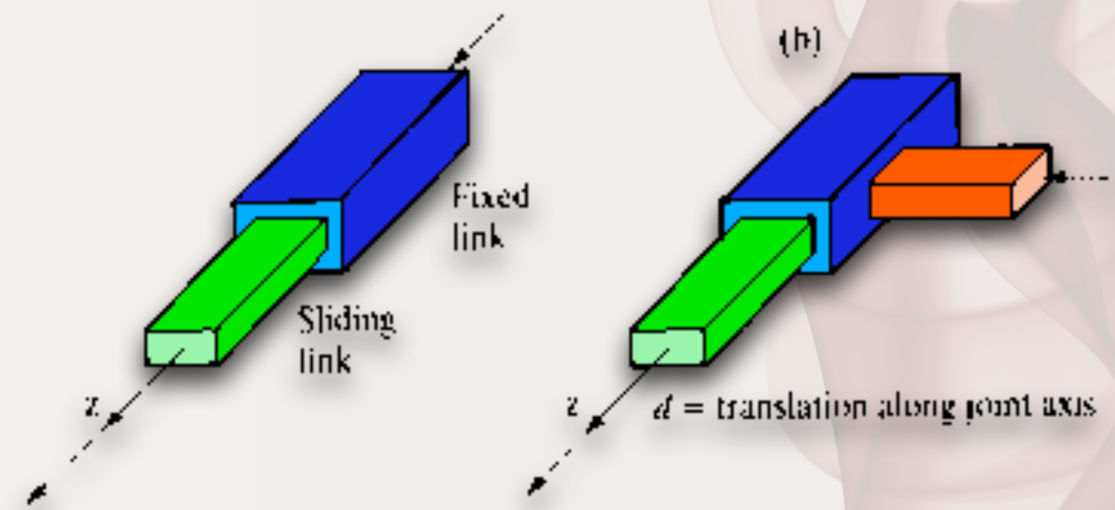
...and have two forms:

1. Constraint Form

Joint Types

All joints can be written as the composition of...



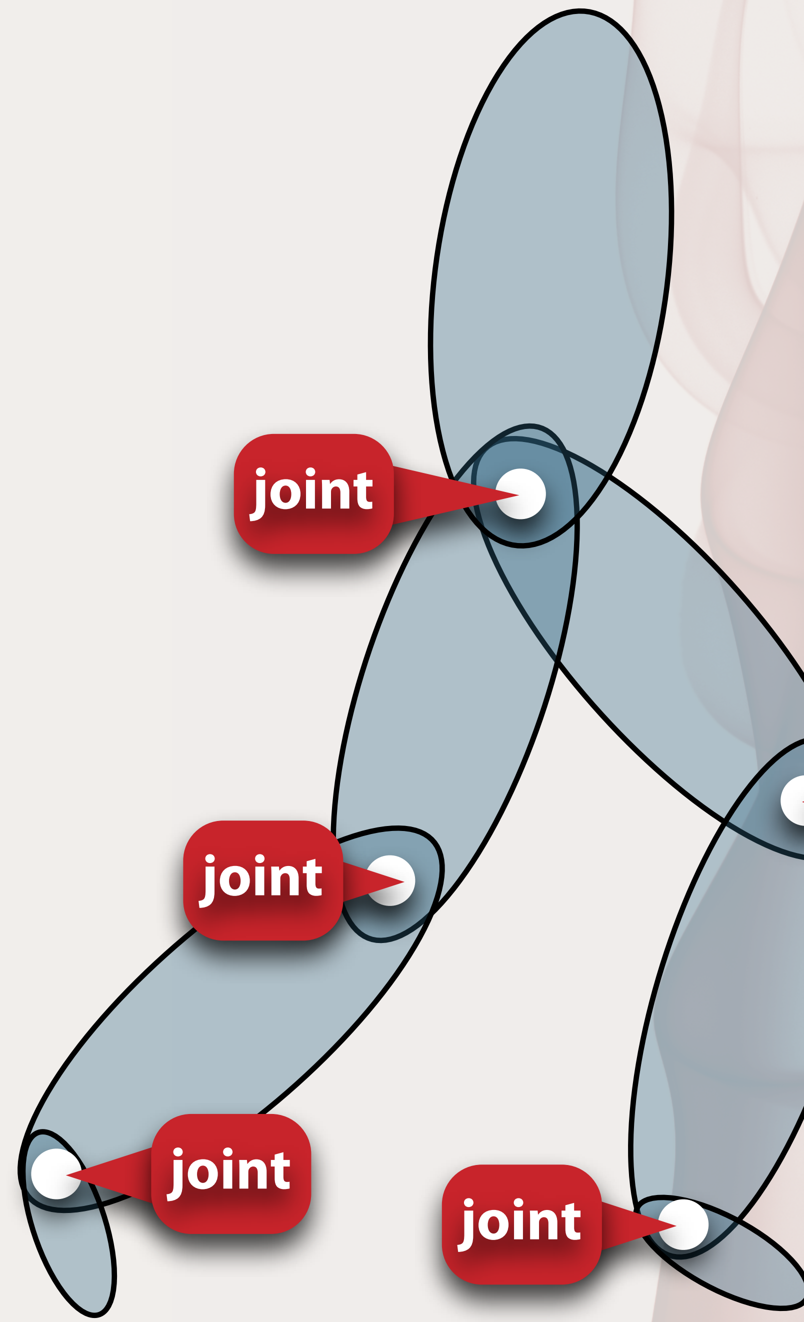


Rotary **Prismatic**

...and have two forms:

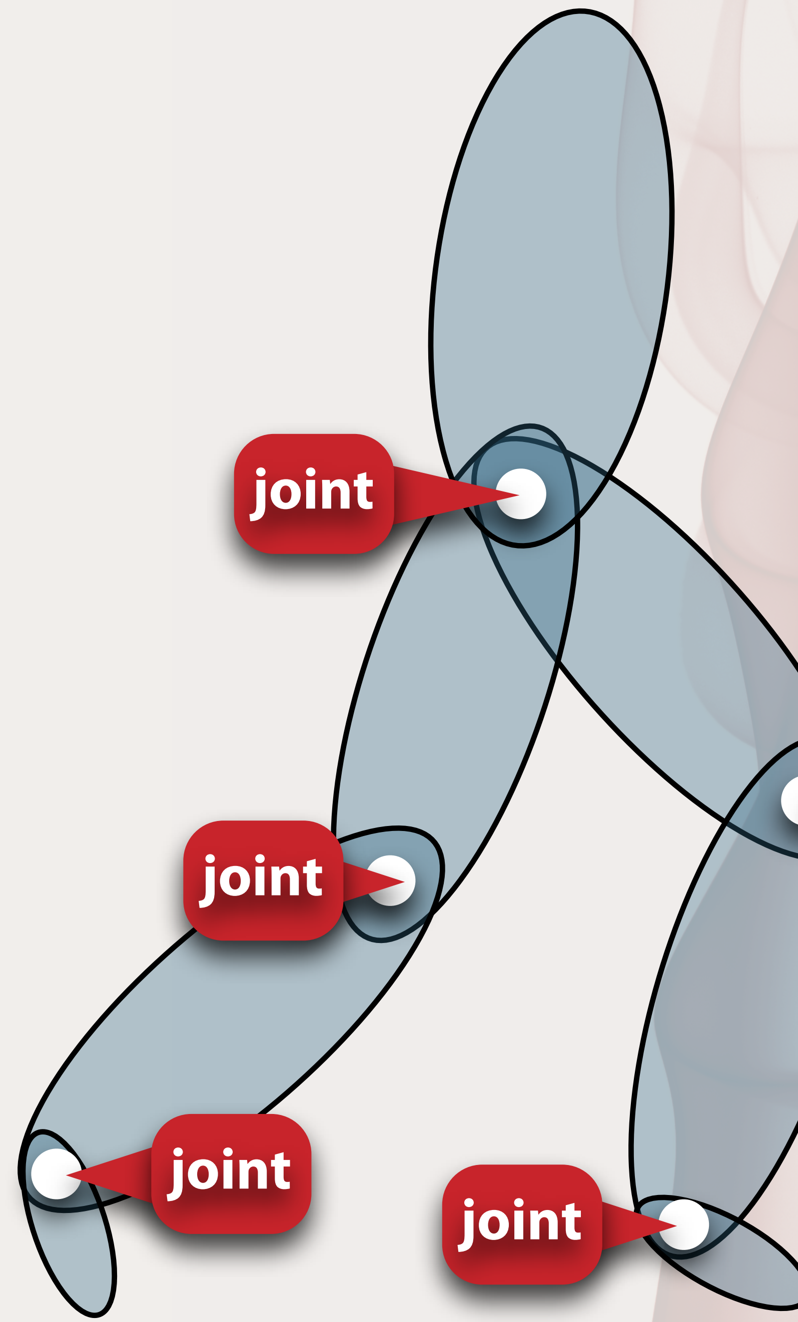
1. Constraint Form
2. Functional Form

Joint Enforcement



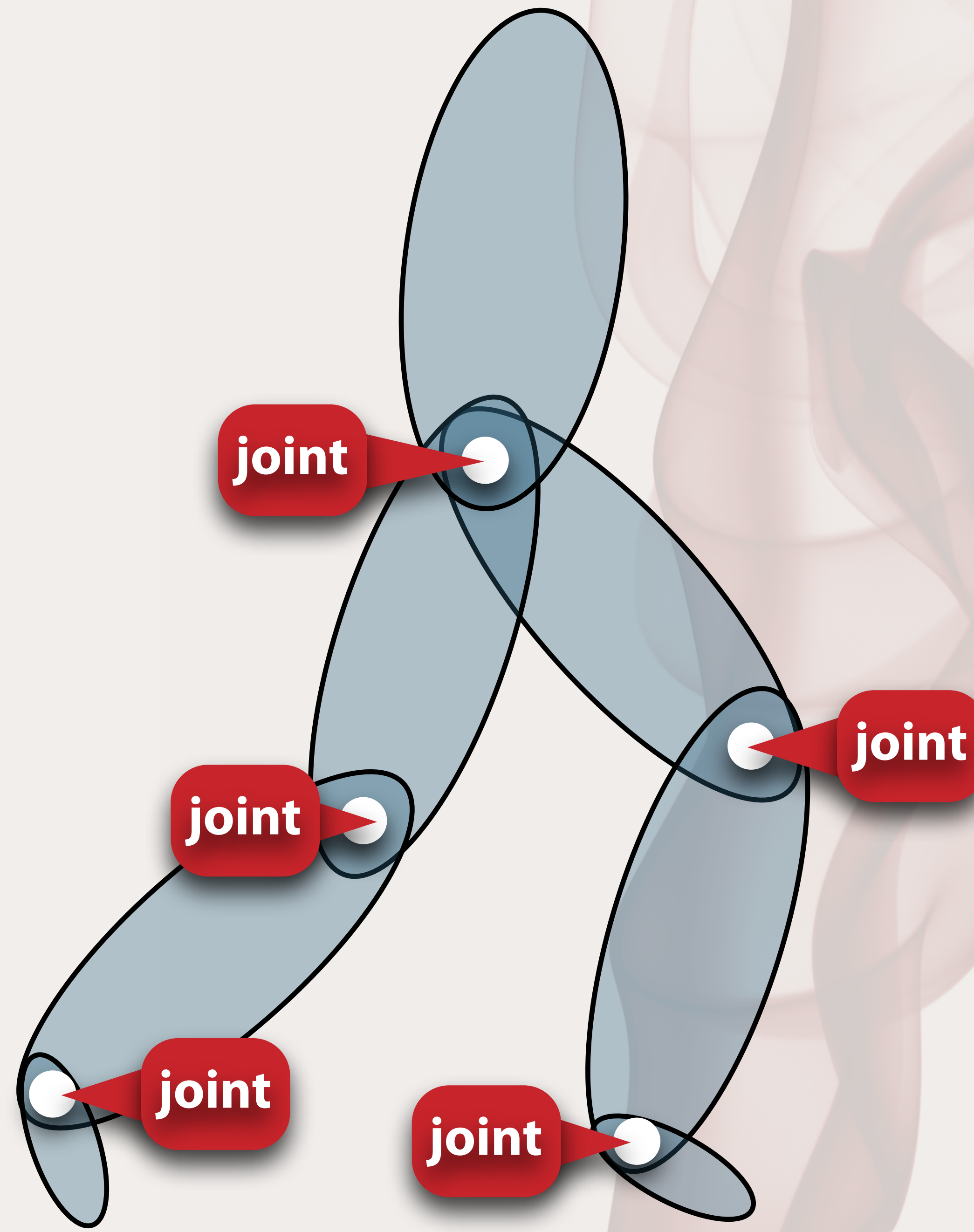
Joint Enforcement

- **Penalty Methods**



Joint Enforcement

- Penalty Methods
- Contraint Methods
  - aka Maximal Coordinate



Joint Enforcement

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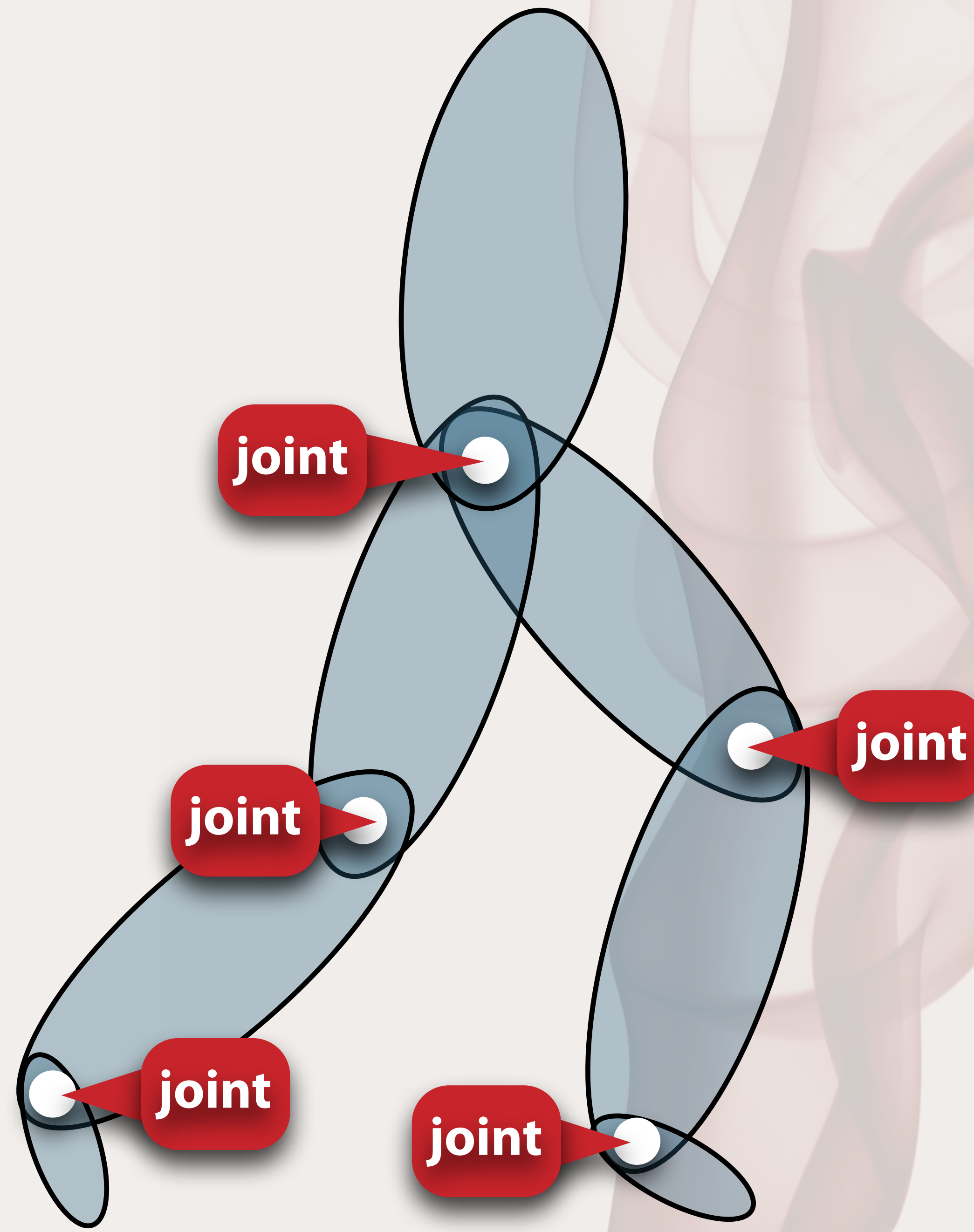
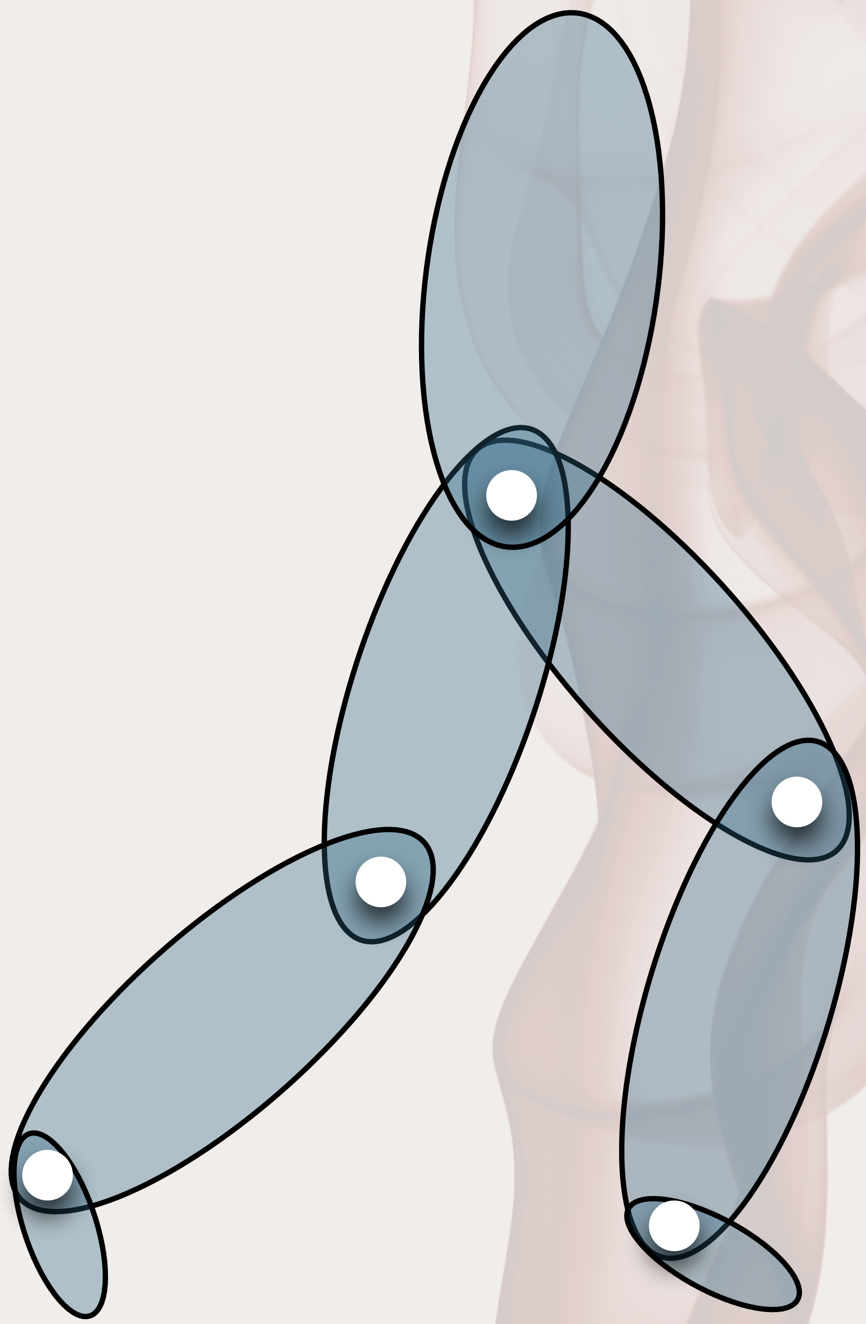
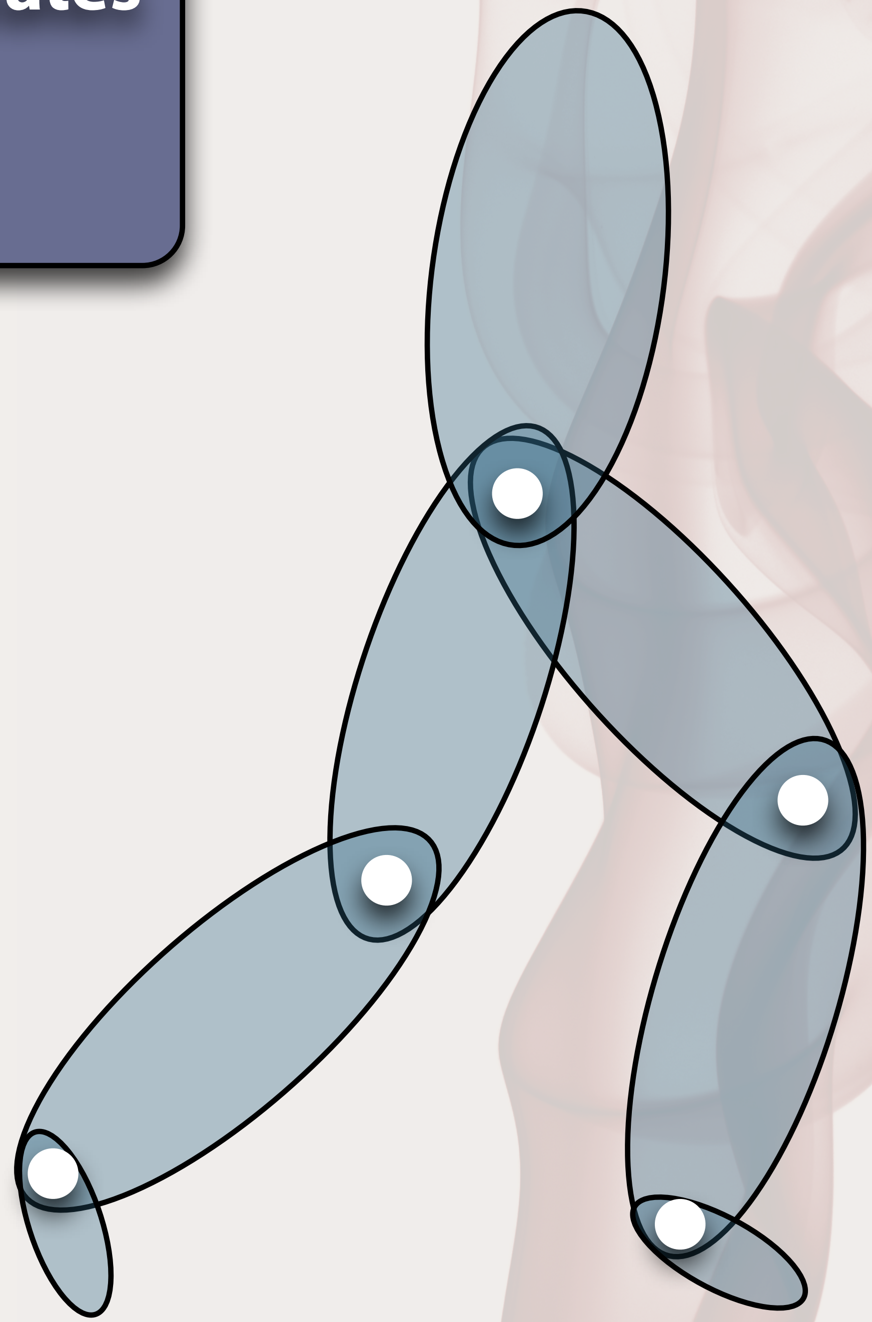


Internal Coordinates



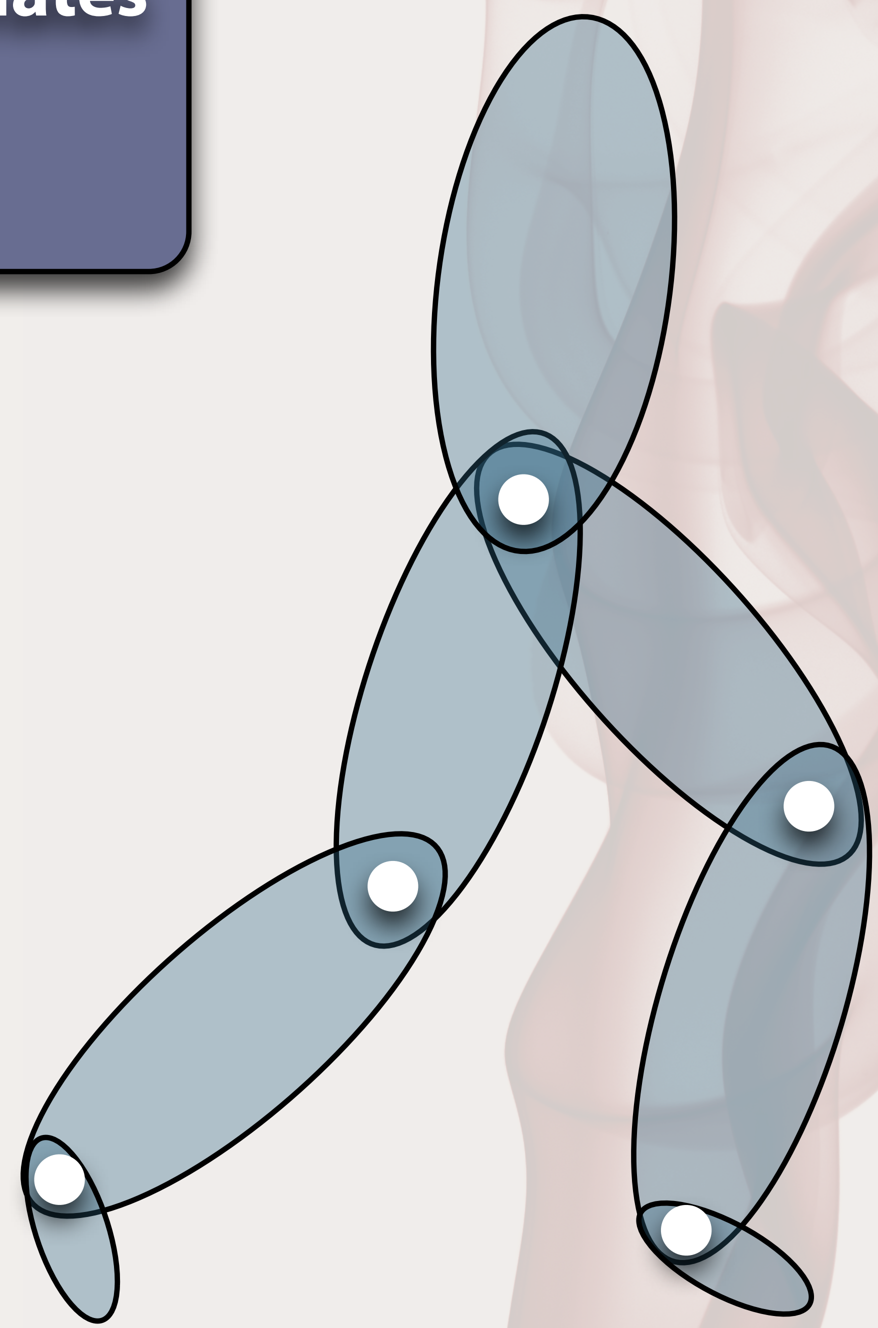
Internal Coordinates

- q: the skeletal coordinates



Internal Coordinates

- q: the skeletal coordinates
- $\dot{q}$: joint velocities



Internal Coordinates

- q: the skeletal coordinates
- $\dot{q}$: joint velocities
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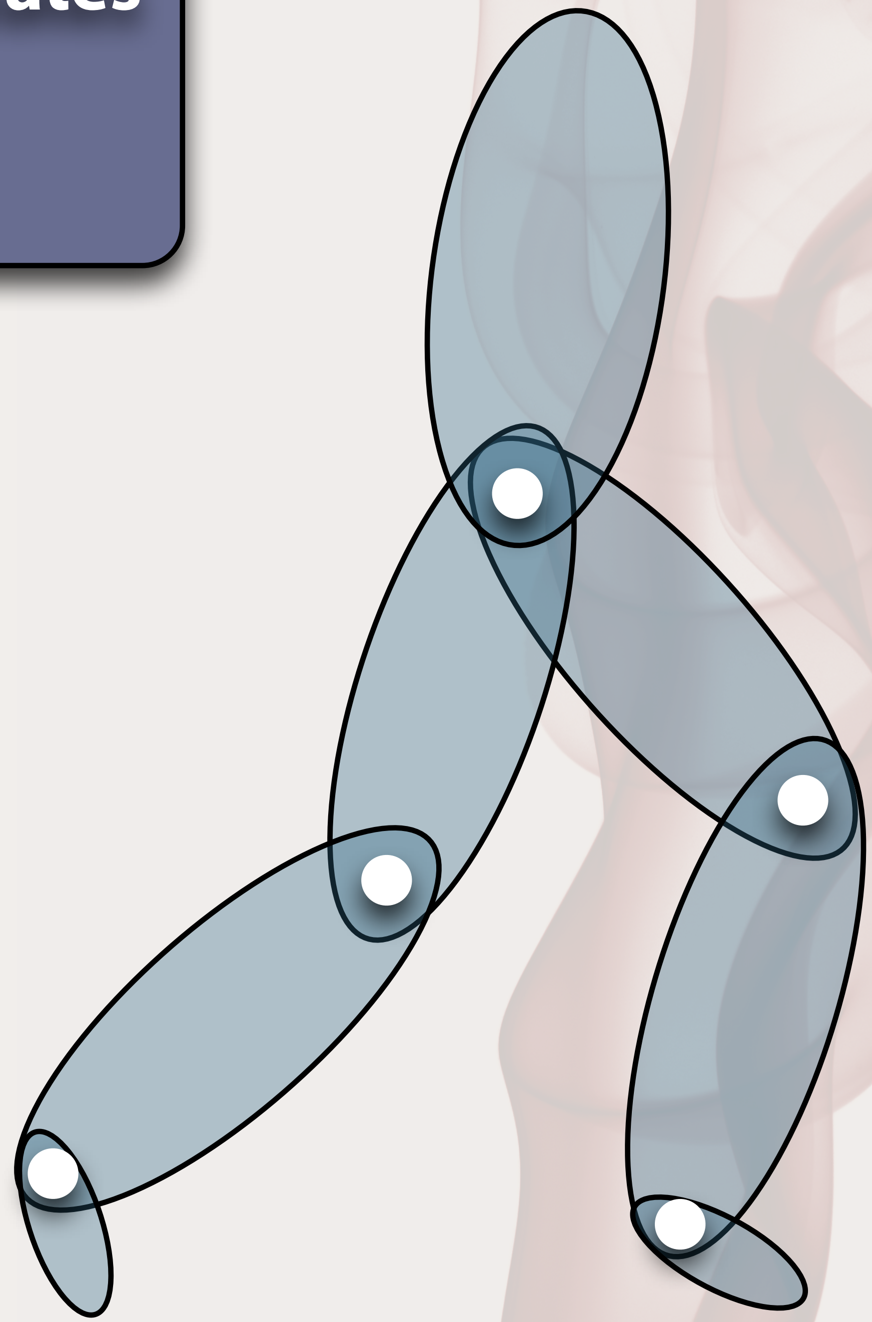

Internal Coordinates

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- $\dot{q}$: joint velocities
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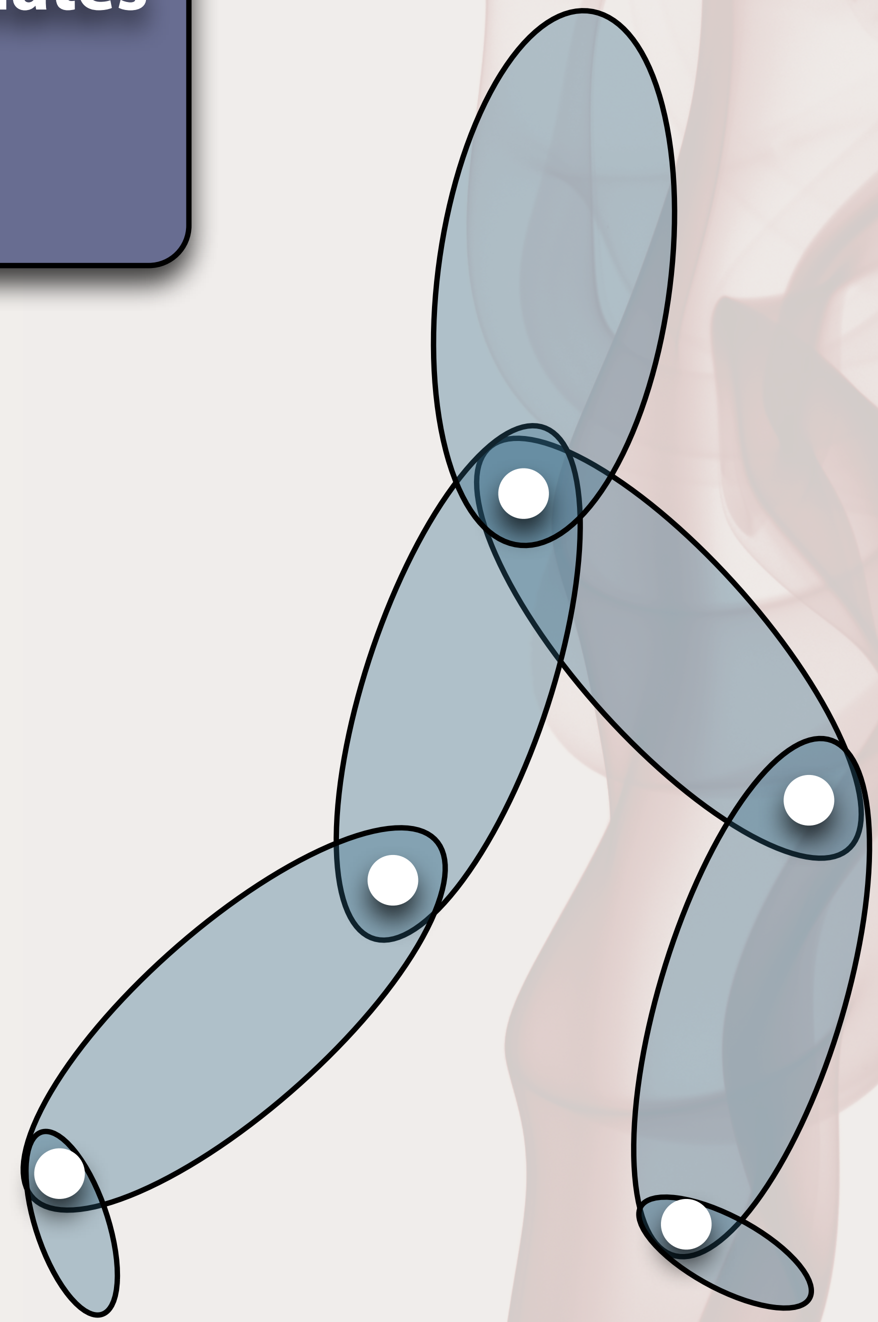
- ***Forward Dynamics Problem***
  - **Compute $\ddot{q} = F(q, \dot{q}, f, \tau)$**



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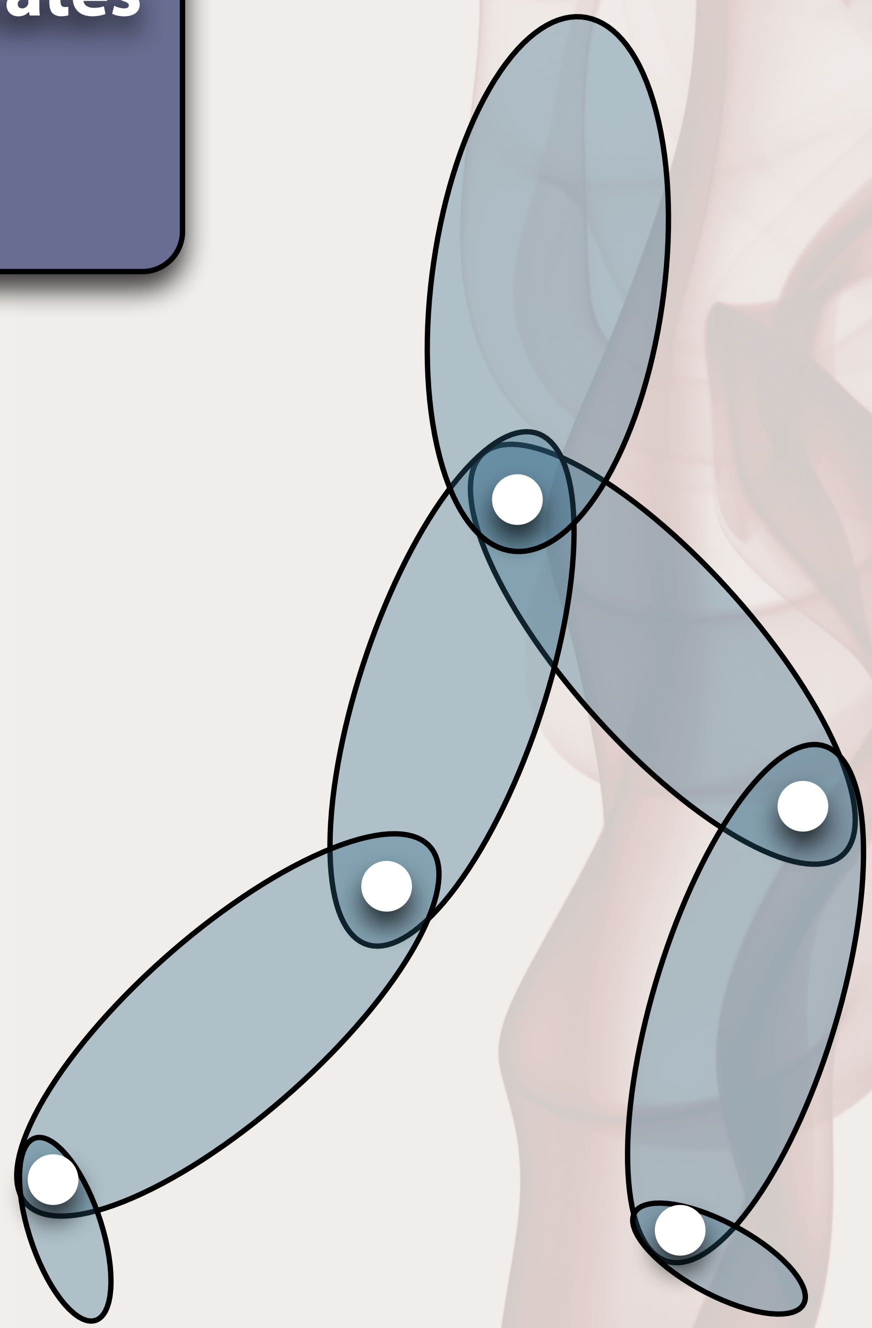

Internal Coordinates

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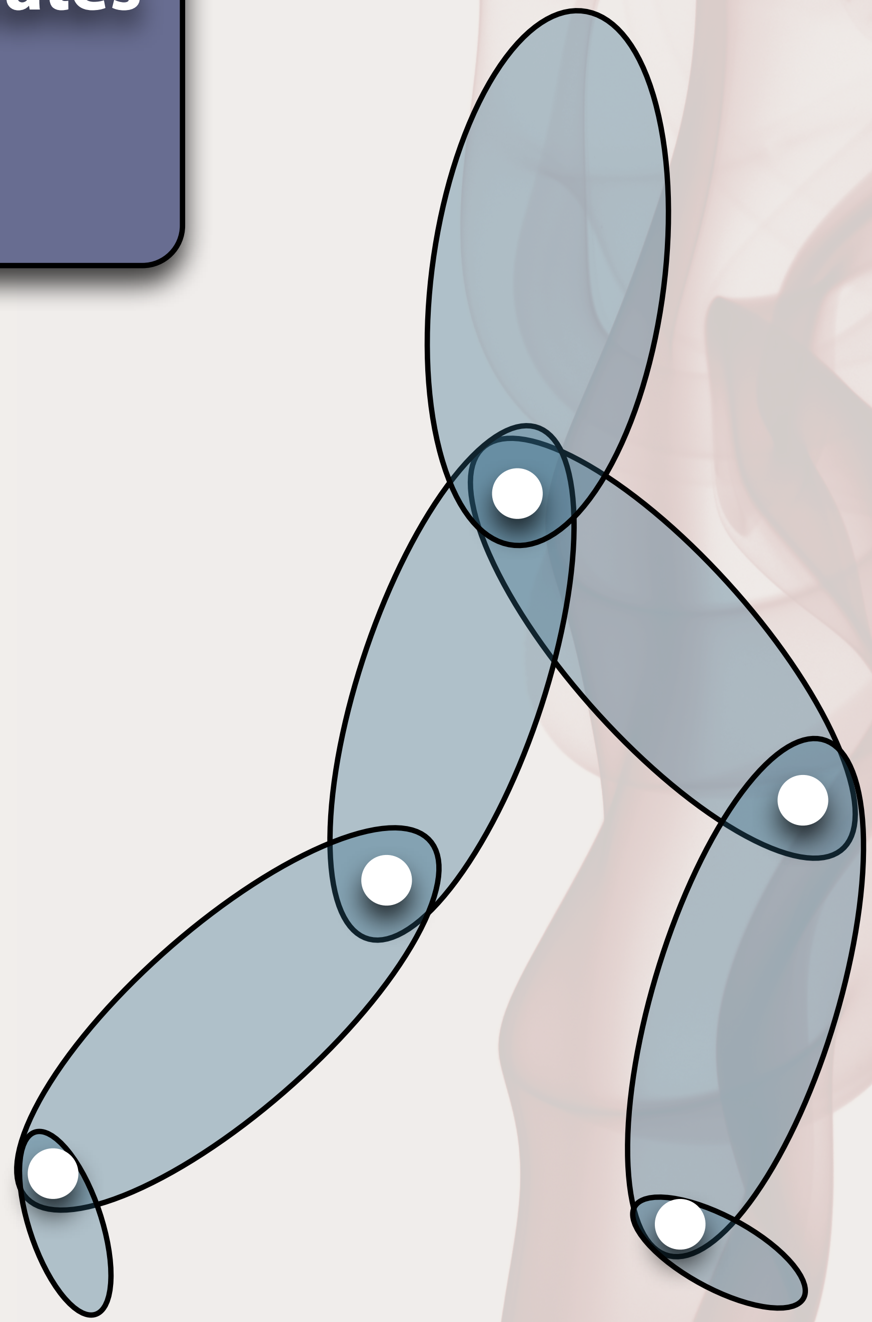
- ***Forward Dynamics Problem***
  - **Compute $\ddot{q} = F(q, \dot{q}, f, \tau)$**
    - **f: external forces**
    - **τ: internal torques**



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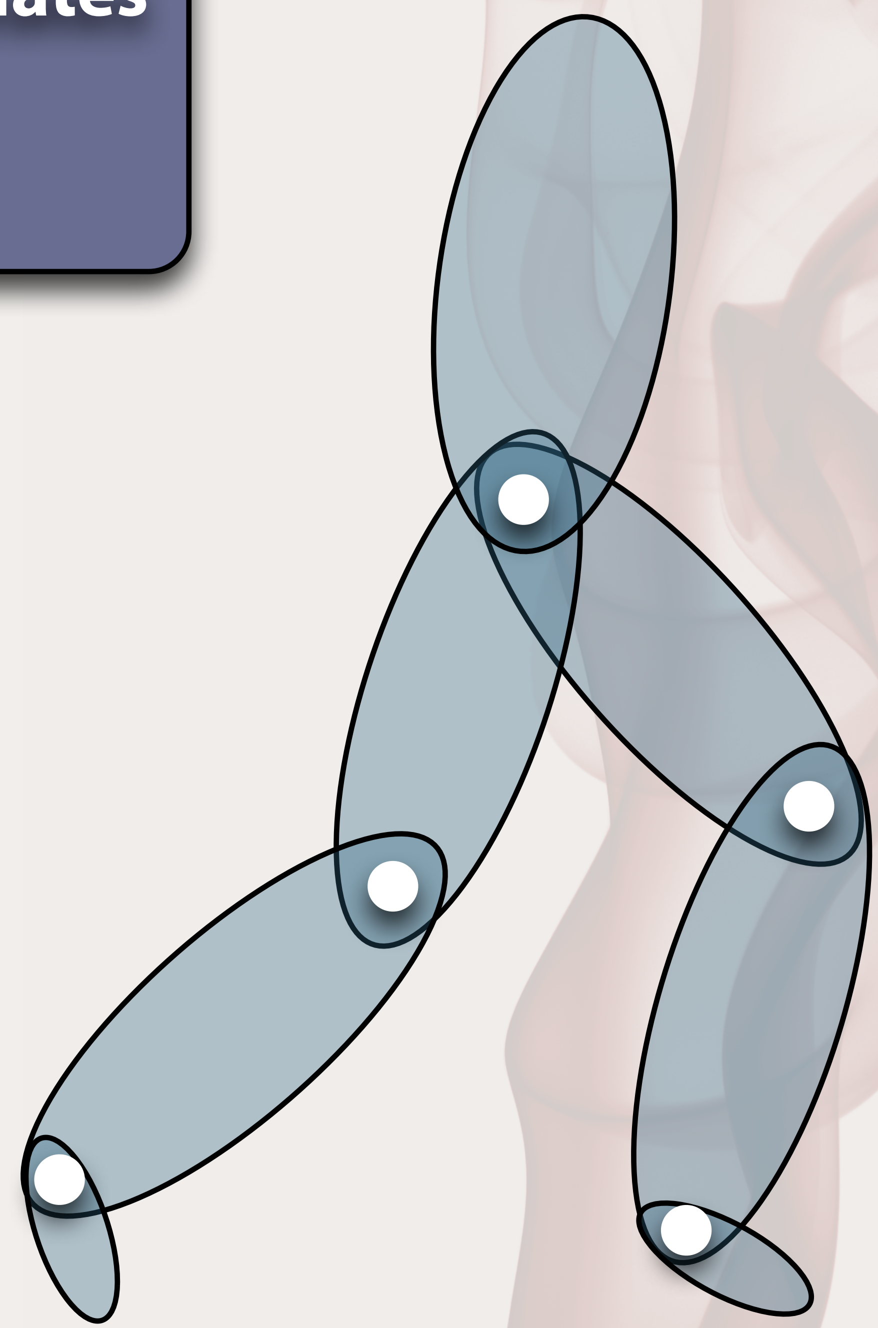
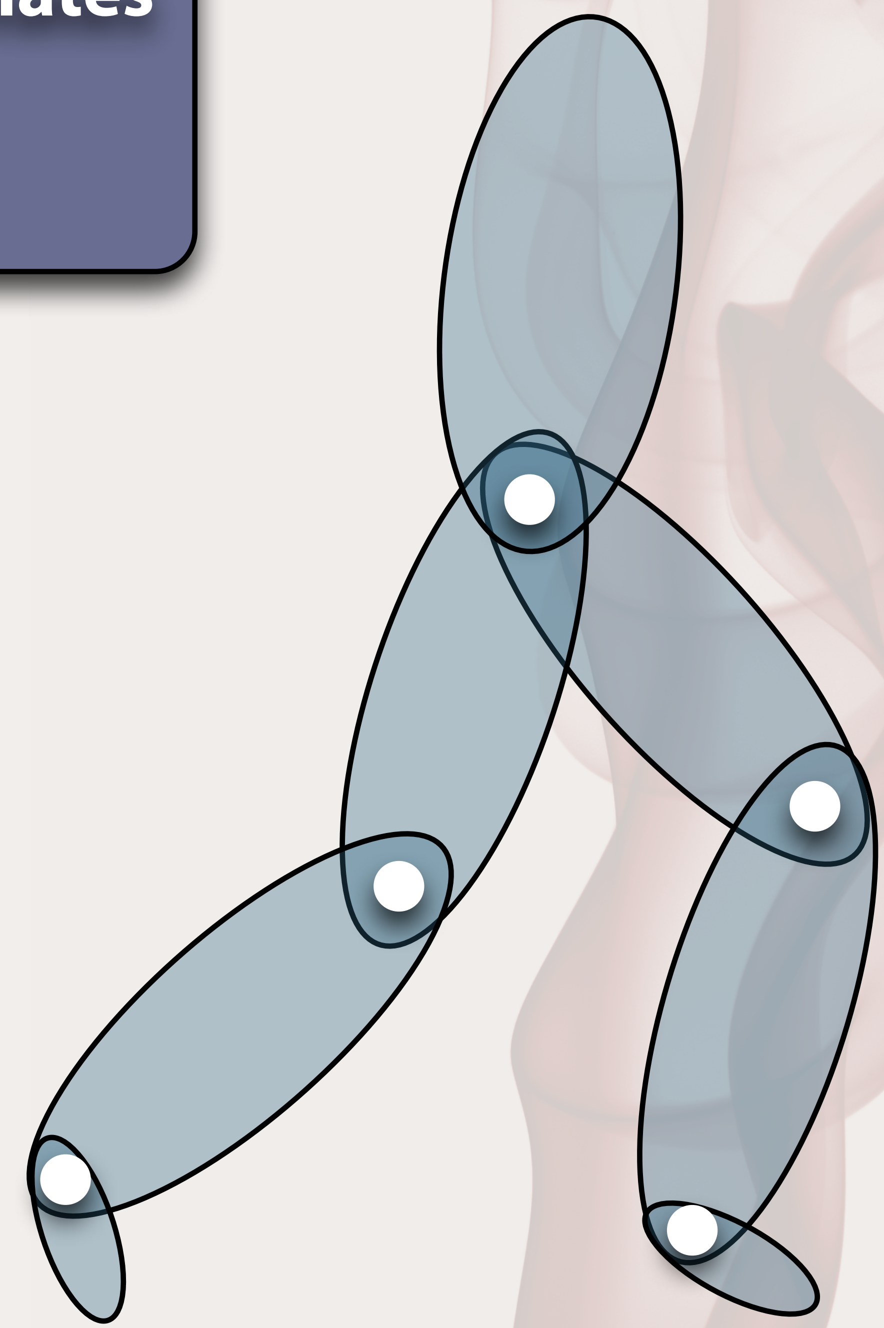
- $\ddot{q}$: joint accelerations

- ***Forward Dynamics Problem***
  - **Compute $\ddot{q} = F(q, \dot{q}, f, \tau)$**
    - **f: external forces**
    - **τ: internal torques**
  - **Then use ODE solver.**



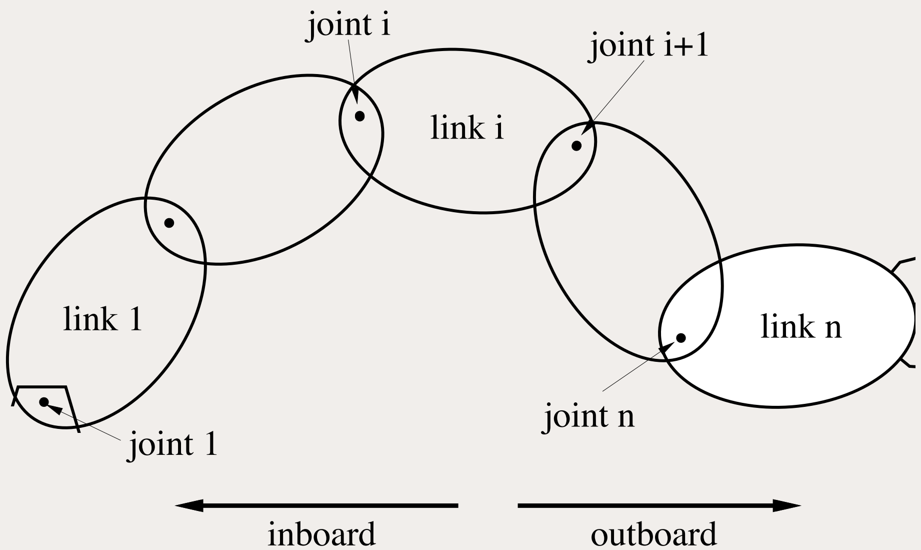
Internal Coordinates

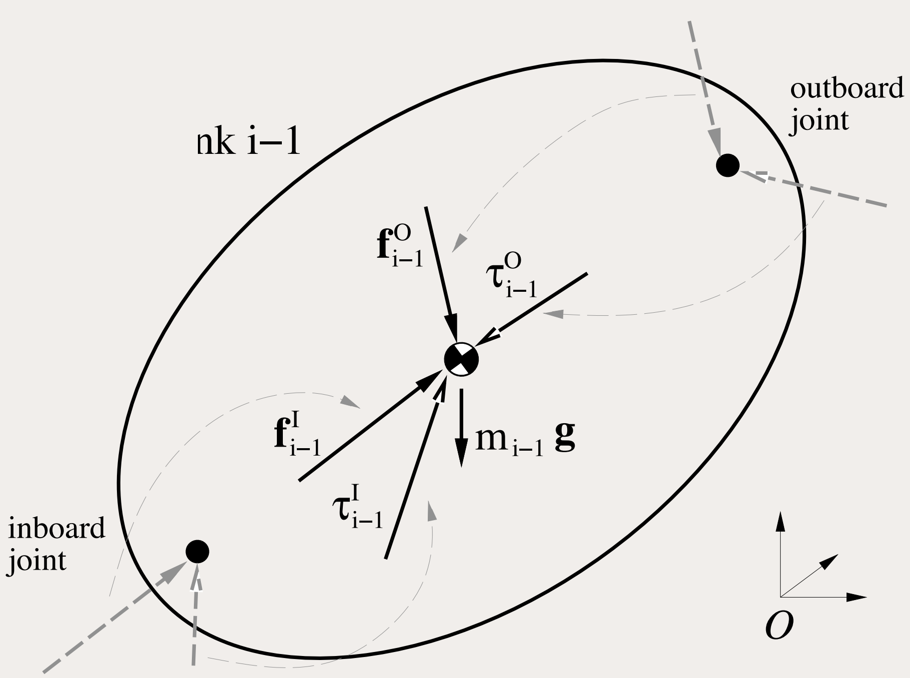
- q: the skeletal coordinates
- $\dot{q}$: joint velocities
- $\ddot{q}$: joint accelerations



- ***Forward Dynamics Problem***
  - **Compute $\ddot{q} = F(q, \dot{q}, f, \tau)$**
    - **f: external forces**
    - **τ: internal torques**
  - **Then use ODE solver.**
- ***Inverse Dynamics Problem***
  - **Compute $\tau = G(q, \dot{q}, \ddot{q}, f)$**

Featherstone Algorithm





Impulse-based Dynamic Simulation of Rigid Body Systems

by

Brian Vincent Mirtich

If joint i is prismatic,

$$\hat{\mathbf{s}}_i'\hat{\mathbf{f}}_i^I = \begin{bmatrix}\mathbf{0}\\ \mathbf{u}_i\end{bmatrix}' \begin{bmatrix}\mathbf{f}\\ \boldsymbol{\tau}-\mathbf{d}_i\times\mathbf{f}\end{bmatrix} = \mathbf{f}\cdot\mathbf{u}_i.$$

The right hand side is the component of the applied force along the joint axis. This force must be supported by the actuator, hence, it is Q_i. If joint i is revolute,

$$\hat{\mathbf{s}}_i'\hat{\mathbf{f}}_i^I = \begin{bmatrix}\mathbf{u}_i\\ \mathbf{u}\times\mathbf{d}_i\end{bmatrix}' \begin{bmatrix}\mathbf{f}\\ \boldsymbol{\tau}-\mathbf{d}_i\times\mathbf{f}\end{bmatrix} = \mathbf{f}\cdot(\mathbf{u}_i\times\mathbf{d}_i) + (\boldsymbol{\tau}-\mathbf{d}_i\times\mathbf{f})\cdot\mathbf{u}_i.$$

The right hand side reduces to $\boldsymbol{\tau}\cdot\mathbf{u}_i$, the component of the applied torque along the joint axis. This torque must be supported by the actuator, hence, it is Q_i. □

Substituting equation (4.23) for link i's spatial acceleration into (4.24) yields

$$\hat{\mathbf{f}}_i^I = \hat{\mathbf{I}}_i^A({}_i\hat{\mathbf{X}}_{i-1}\hat{\mathbf{a}}_{i-1} + \ddot{q}_i\hat{\mathbf{s}}_i + \hat{\mathbf{c}}_i) + \hat{\mathbf{Z}}_i^A.$$

Premultiplying both sides by $\hat{\mathbf{s}}_i'$ and applying Lemma 7 gives

$$Q_i = \hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A({}_i\hat{\mathbf{X}}_{i-1}\hat{\mathbf{a}}_{i-1} + \ddot{q}_i\hat{\mathbf{s}}_i + \hat{\mathbf{c}}_i) + \hat{\mathbf{s}}_i'\hat{\mathbf{Z}}_i^A,$$

from which $\ddot{q}_i$ may be determined:

$$\ddot{q}_i = \frac{Q_i - \hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A\,{}_i\hat{\mathbf{X}}_{i-1}\hat{\mathbf{a}}_{i-1} - \hat{\mathbf{s}}_i'\left(\hat{\mathbf{Z}}_i^A + \hat{\mathbf{I}}_i^A\hat{\mathbf{c}}_i\right)}{\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i}. \qquad (4.27)$$

Substituting this expression for $\ddot{q}_i$ into (4.26) and rearranging gives

$$\begin{aligned}\hat{\mathbf{f}}_{i-1}^I \;=\; & \left[\hat{\mathbf{I}}_{i-1} + {}_{i-1}\hat{\mathbf{X}}_i\left(\hat{\mathbf{I}}_i^A - \frac{\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A}{\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i}\right){}_i\hat{\mathbf{X}}_{i-1}\right]\hat{\mathbf{a}}_{i-1} \\ & + \hat{\mathbf{Z}}_{i-1} + {}_{i-1}\hat{\mathbf{X}}_i\left[\hat{\mathbf{Z}}_i^A + \hat{\mathbf{I}}_i^A\hat{\mathbf{c}}_i + \frac{\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i\left[Q_i - \hat{\mathbf{s}}_i'\left(\hat{\mathbf{Z}}_i^A + \hat{\mathbf{I}}_i^A\hat{\mathbf{c}}_i\right)\right]}{\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i}\right].\end{aligned}$$

Comparing this to the desired form (4.24),

$$\hat{\mathbf{I}}_{i-1}^A = \hat{\mathbf{I}}_{i-1} + {}_{i-1}\hat{\mathbf{X}}_i\left(\hat{\mathbf{I}}_i^A - \frac{\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A}{\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i}\right){}_i\hat{\mathbf{X}}_{i-1} \qquad (4.28)$$

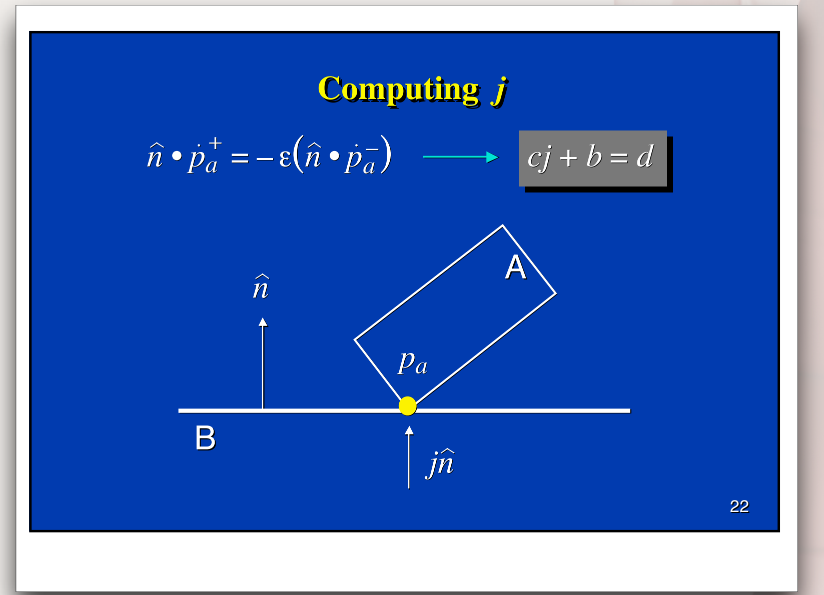
$$\hat{\mathbf{Z}}_{i-1}^A = \hat{\mathbf{Z}}_{i-1} + {}_{i-1}\hat{\mathbf{X}}_i\left[\hat{\mathbf{Z}}_i^A + \hat{\mathbf{I}}_i^A\hat{\mathbf{c}}_i + \frac{\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i\left[Q_i - \hat{\mathbf{s}}_i'\left(\hat{\mathbf{Z}}_i^A + \hat{\mathbf{I}}_i^A\hat{\mathbf{c}}_i\right)\right]}{\hat{\mathbf{s}}_i'\hat{\mathbf{I}}_i^A\hat{\mathbf{s}}_i}\right]. \qquad (4.29)$$

Constraints

Constraints

- **Accelerations are *linear* in applied torques and forces.**

$$a_f = kf + a_0$$



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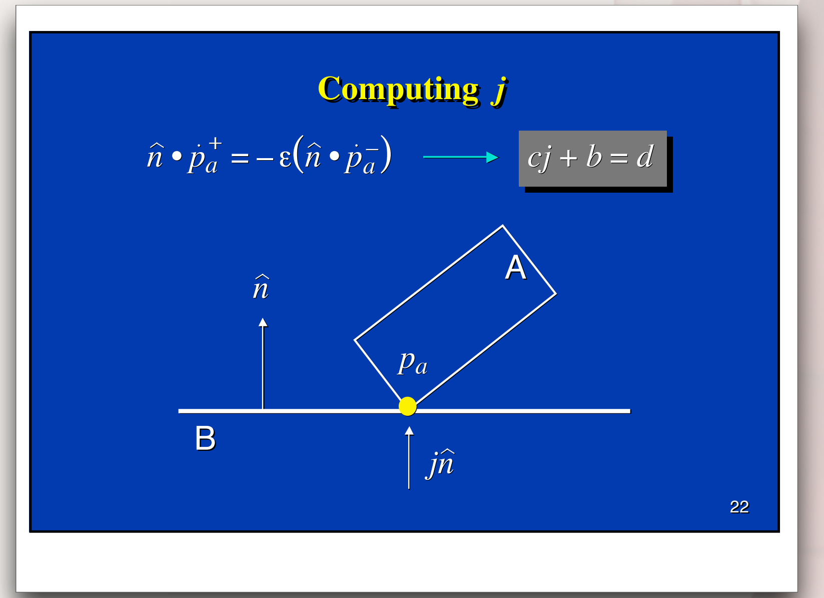
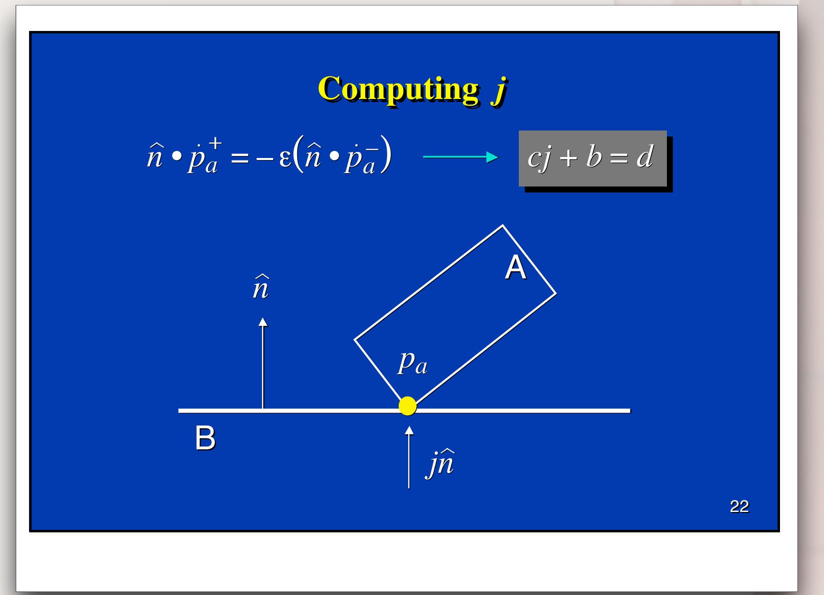


- **Use of "test forces"**

Constraints

- **Accelerations are *linear* in applied torques and forces.**

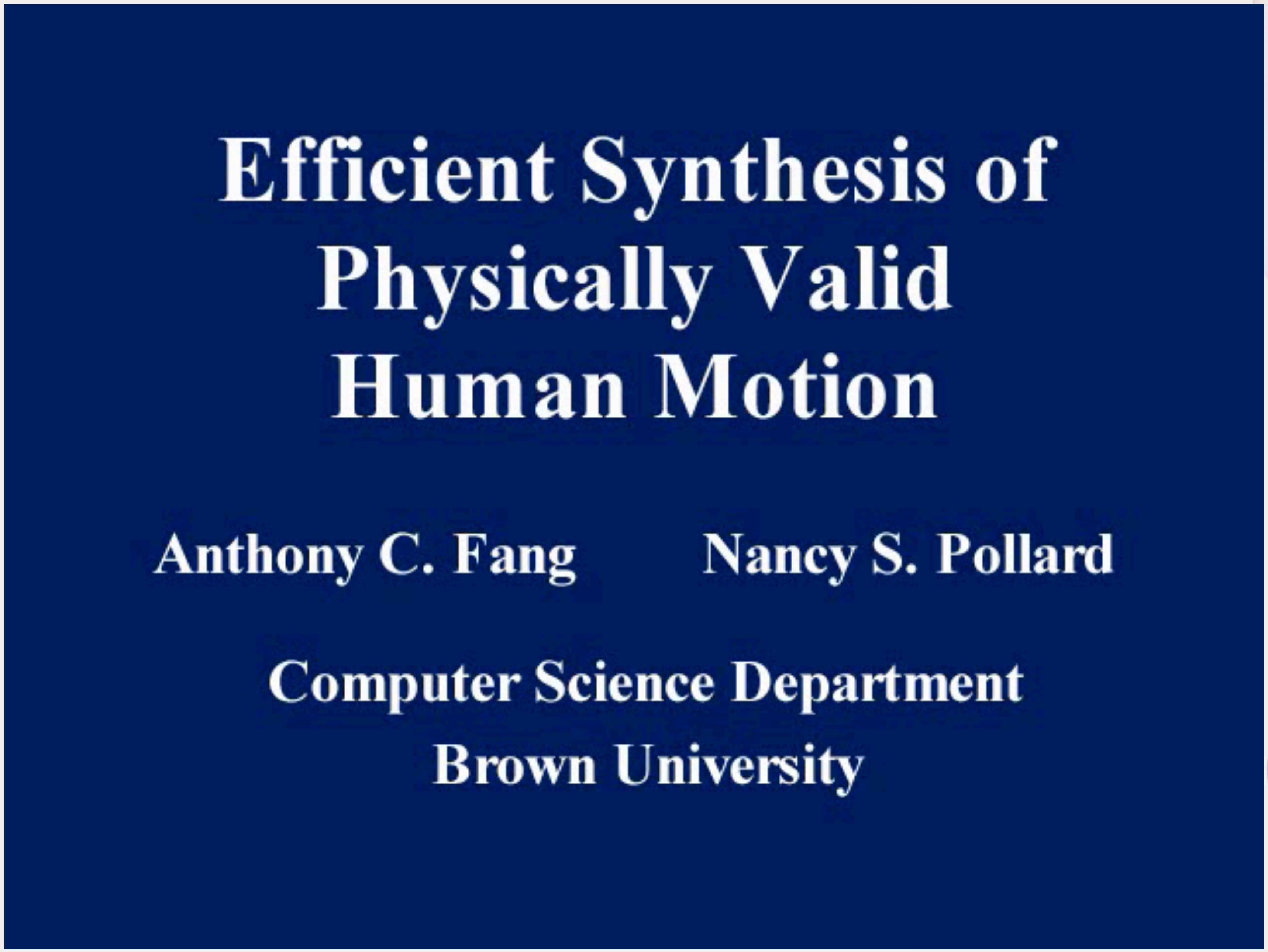
$$a_f = kf + a_0$$



- **Use of "test forces"**
- **Multiple test forces**

$$\mathbf{a}_f = K\mathbf{f} + \mathbf{a}_0$$

Examples


Efficient Synthesis of
Physically Valid
Human Motion
Anthony C. Fang
Nancy S. Pollard
Computer Science Department
Brown University

Animation Topics

- Data-Driven Motion
- **Physics Based Motion**
- Motion of other Animals

Animation Topics

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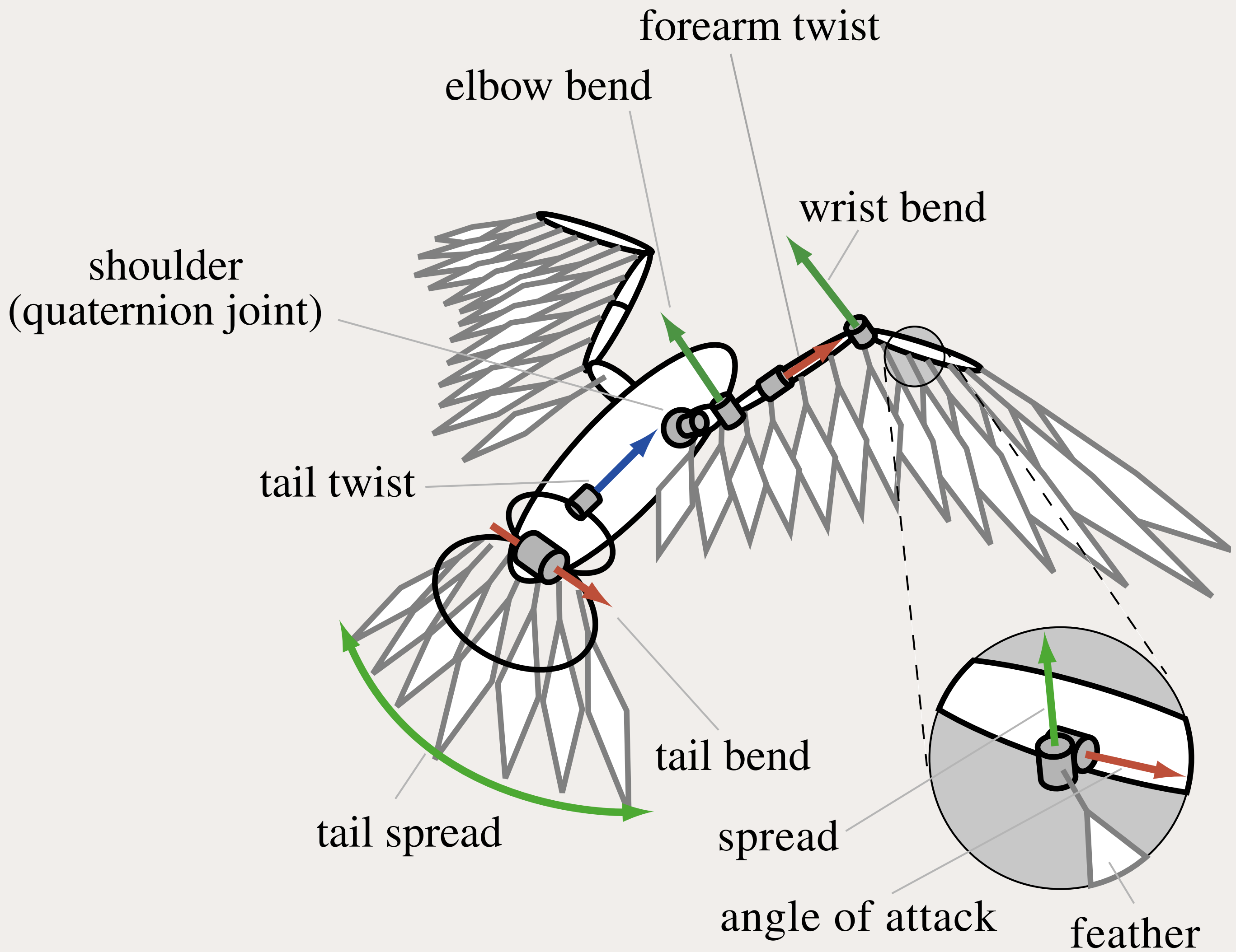
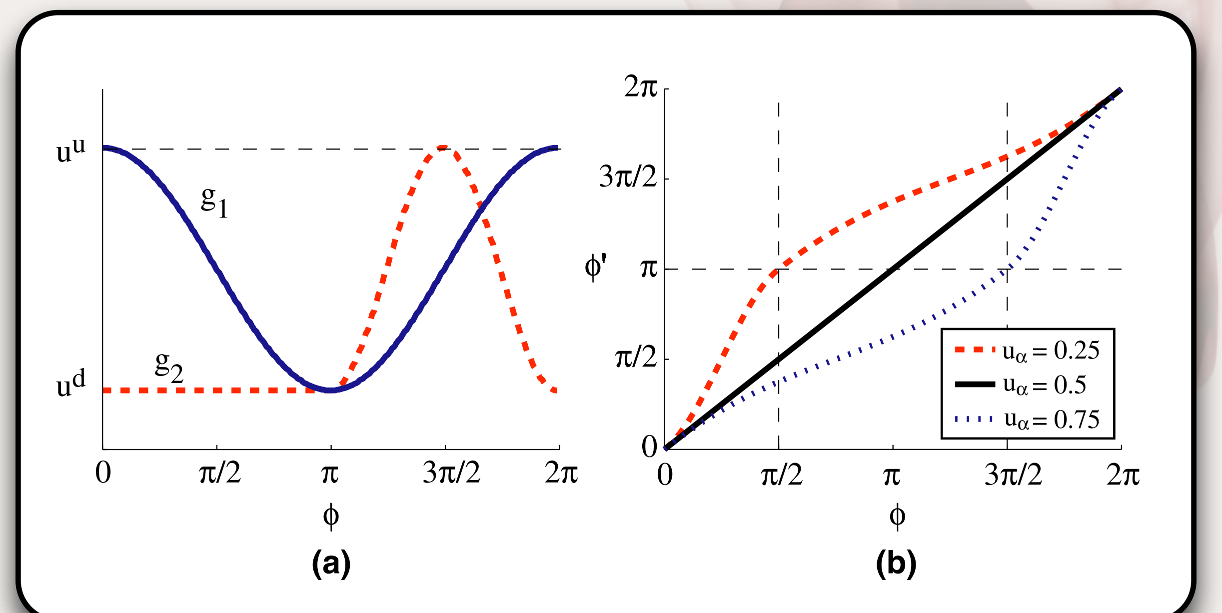
Bird Flight

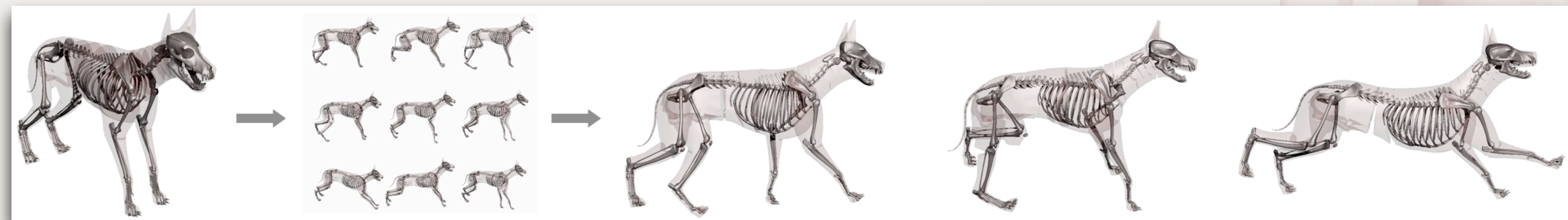




source: Wu and Popović [2003]

Bird Flight Examples

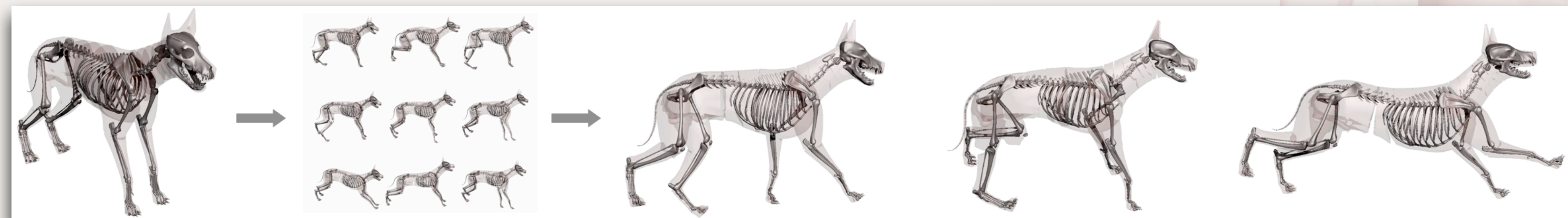
Dogs



Model Vibration Modes Trot = 3 + 14 + 20 Amble = 4 + 14 + 20 Gallop = 0 + 1 + 3

source: Kry et al [2007]

Dogs



Model Vibration Modes Trot = 3 + 14 + 20 Amble = 4 + 14 + 20 Gallop = 0 + 1 + 3

source: Kry et al [2007]

Animation Topics

- Data-Driven Motion
- Physics Based Motion
- **Motion of other Animals**

Overview

- State of the art.
- Body models.
- **Animation**
- Questions

source: 3dscience.com

Overview

- State of the art.
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Questions

- What might be the best physics control paradigm from the animator's perspective?
- How can we control physics (in general)?
- What about linear dynamics (like springs), vs. nonlinear dynamics (like fluids)?
- What about constraints? (e.g. joints)

Student Answewers

- What might be the best physics control paradigm from the animator's perspective?
    - Direct control
        - bezier curves
            - including editing "higher level controls" like accelaration
        - directly editing the density field (in a fluid)
    - "Keyframe" paradigm
    - Many worlds browsing
        - simulate a bunch and then see what looks good
- How can we control physics (in general)?
    - Let the user specify initial conditions
        - And make physical constants customizable
    - Interpolate the state between the keyframes
        - kinematically
        - dynaically
            - this becomes an <u>optimization</u>
    - Set up an energy functions
        - some specify "goals"
        - some specify "not goals"
        - and <u>optimize</u> it
    - we can <u>combine</u> these
        - micro and macro controls
- What about linear dynamics (like springs), vs. nonlinear dynamics (like fluids)?
    - perhaps nonlineararity is a benefit!!! (people don't know what's "real")
- What about constraints? (e.g. joints)
    - can we use these as a form of control?

Overview

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Overview

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