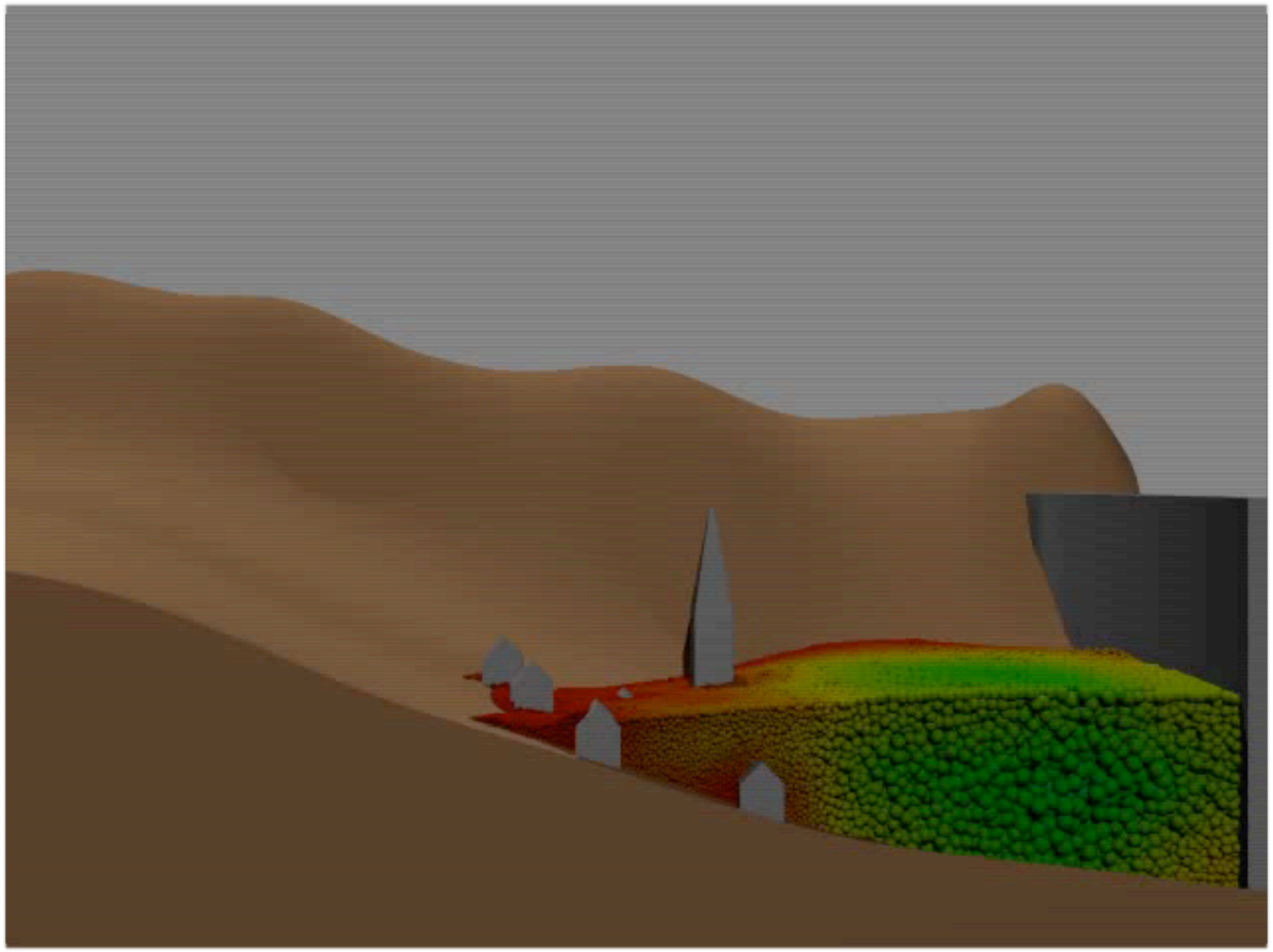
Particle Fluids



Adrien Treuille

Overview

- Smoothed Particle Hydrodynamics (SPH) Basics
- Fluid Behavior
- The Navier-Stokes Equations
- SPH Simulation
- Rendering
- Results
- Questions

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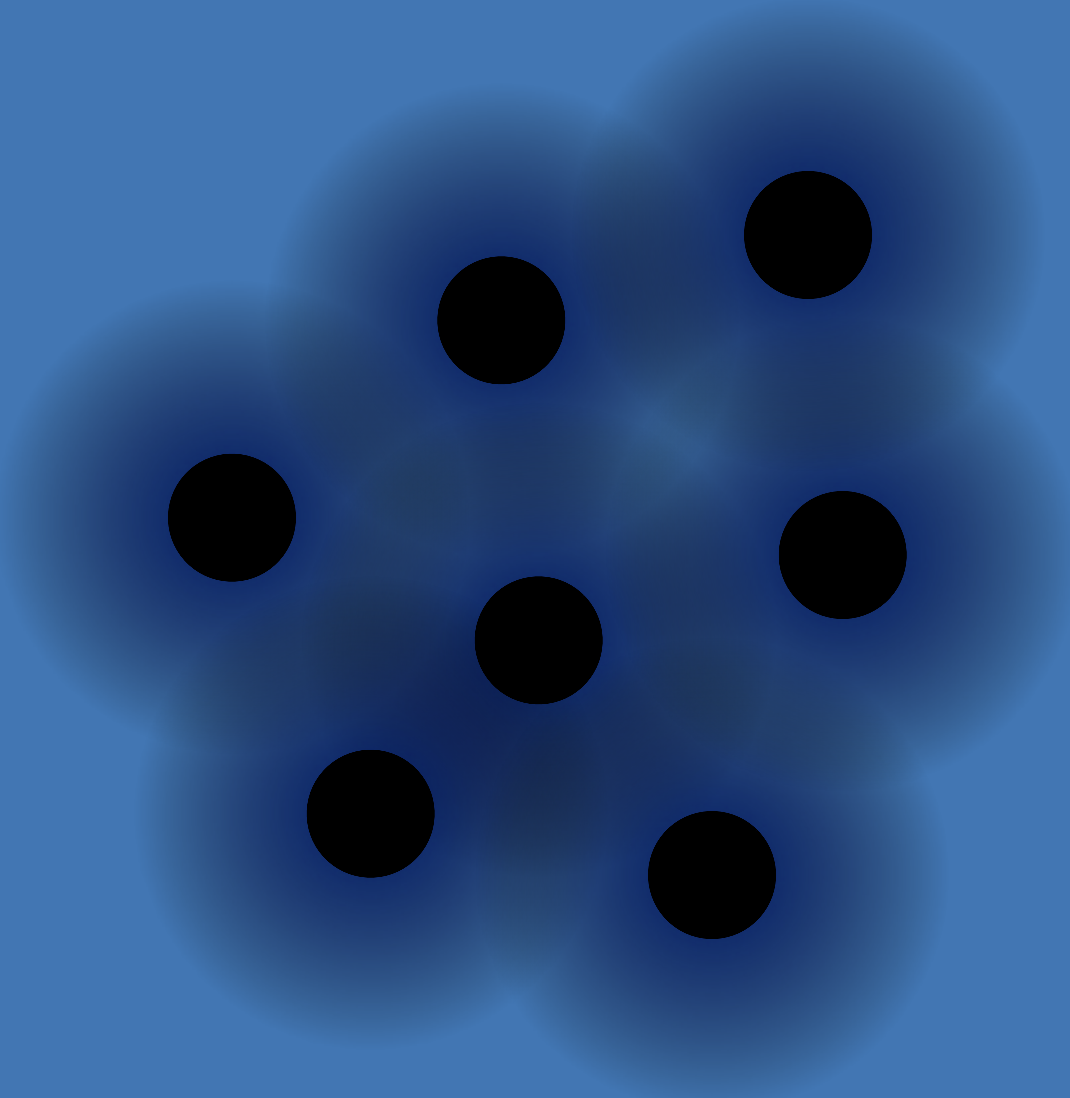
Particle Model



Properties

- Position - $\boldsymbol{x}$
- Velocity - $\boldsymbol{v}$
- Mass - m
  - Density - ρ

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Fluid Behavior

Gravity

Resistance to Compression

Viscosity

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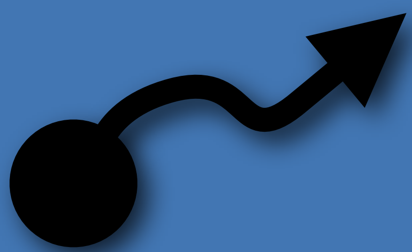
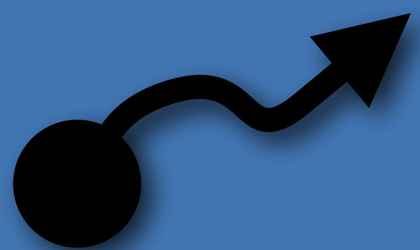
Fluid Forces



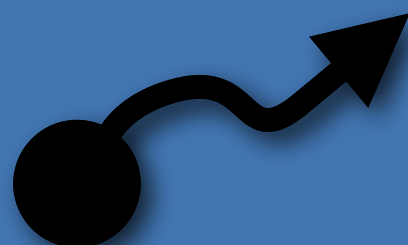
$$m\dot{\mathbf{v}} = \mathbf{f}$$

Fluid Forces

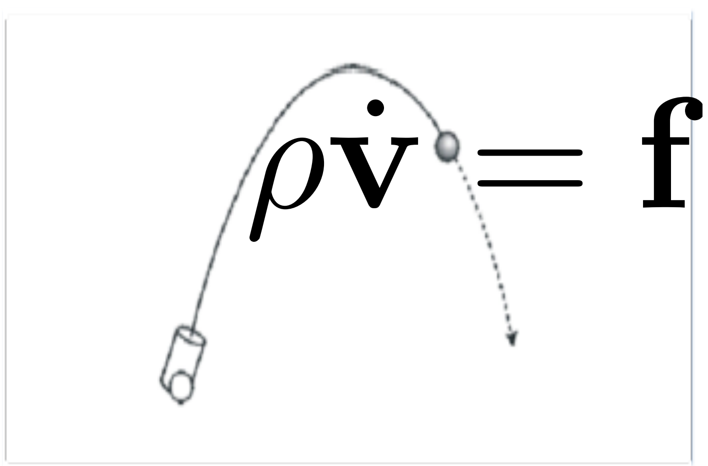


$$\rho \dot{\mathbf{v}} = \mathbf{f}$$

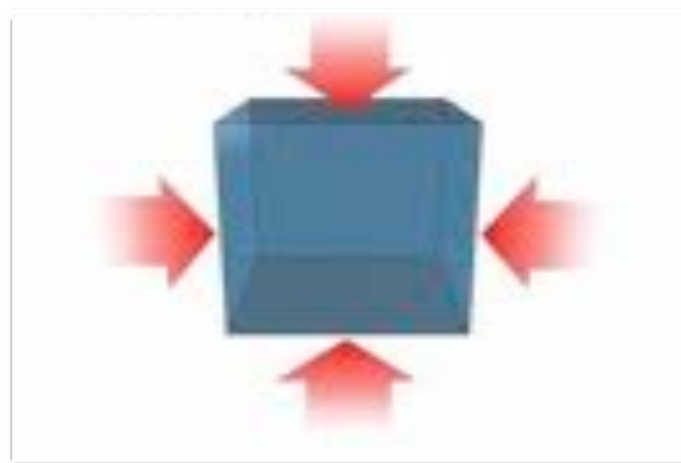
Fluid Forces



$$\rho \dot{\mathbf{v}} = \mathbf{f}$$



Gravity

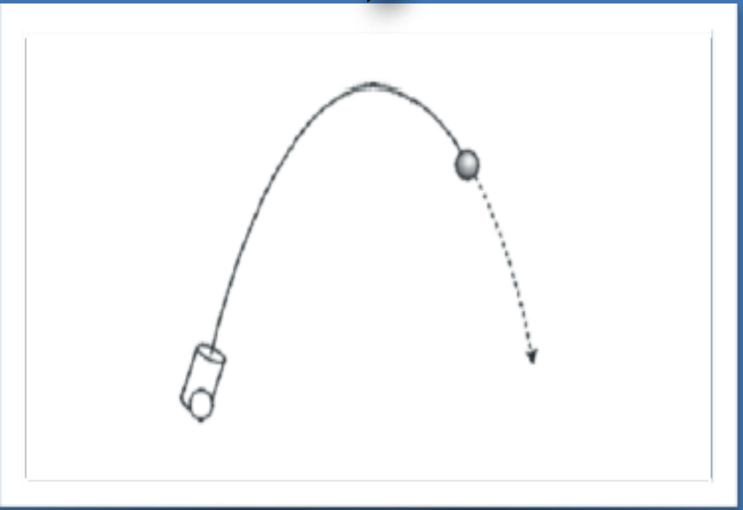


Resistance to Compression

Viscosity

Fluid Forces

$$\rho\dot{\mathbf{v}} = \mathbf{f}_{\text{gravity}} + \mathbf{f}_{\text{pressure}} + \mathbf{f}_{\text{viscosity}}$$



Gravity

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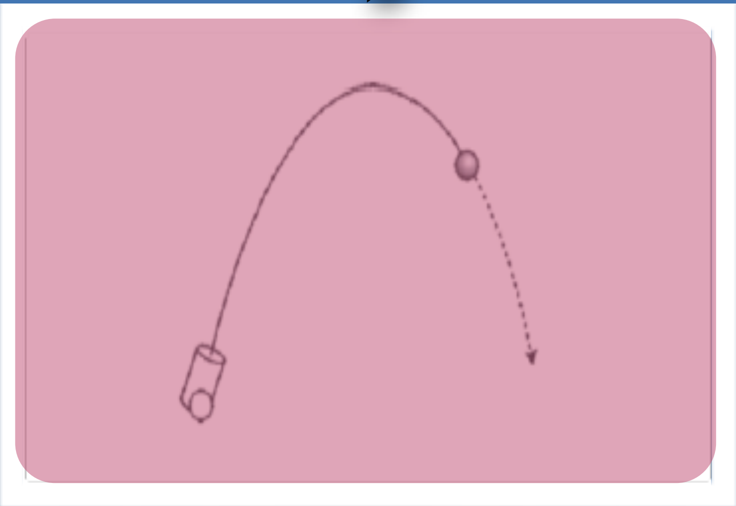
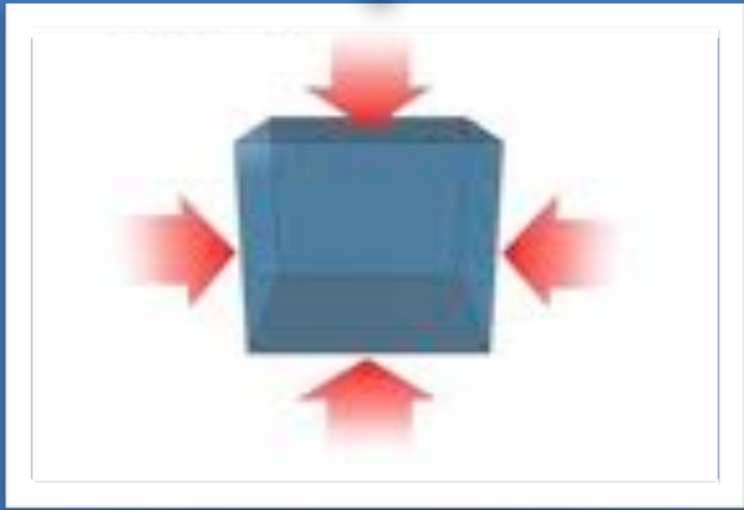
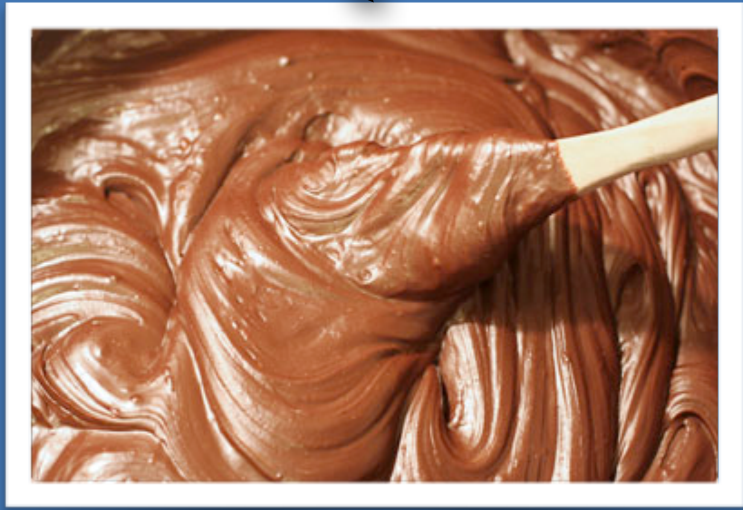


Gravity



Resistance to Compression



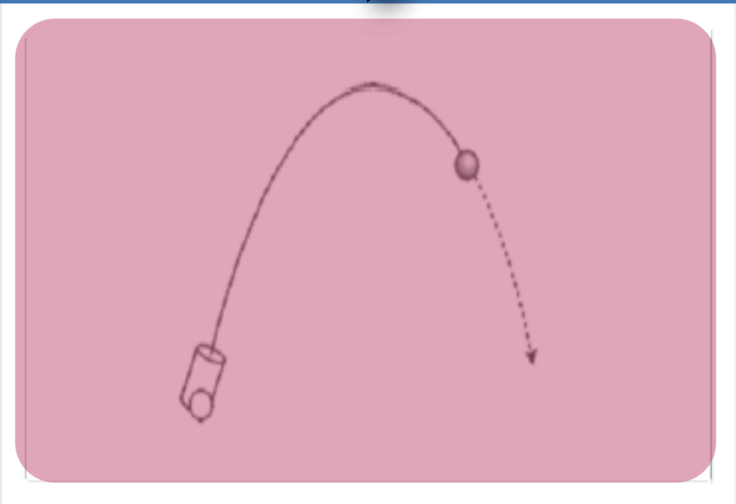
Viscosity

Gravity

$$\mathbf{f}_{\text{gravity}} = \rho \mathbf{g}$$

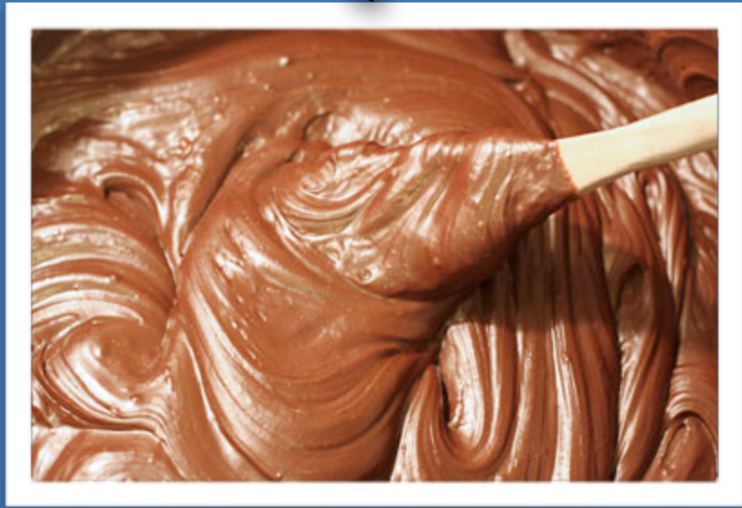
Fluid Forces

$$\rho \dot{\mathbf{v}} = \mathbf{f}_{\text{gravity}} + \mathbf{f}_{\text{pressure}} + \mathbf{f}_{\text{viscosity}}$$



Gravity

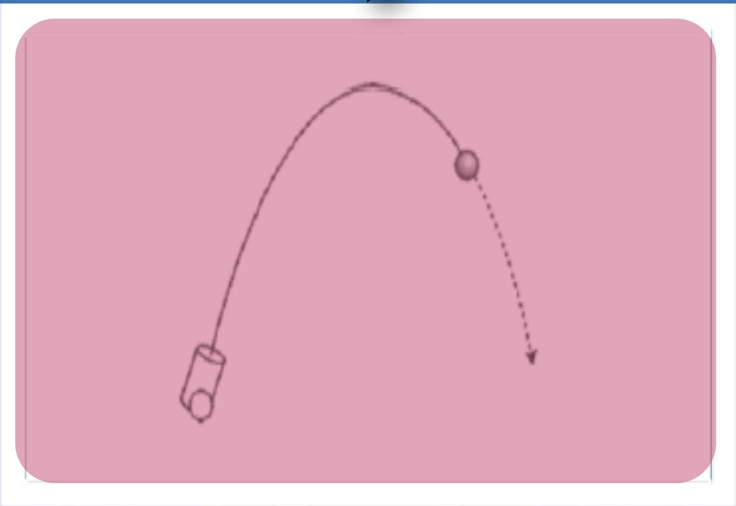
Resistance to Compression



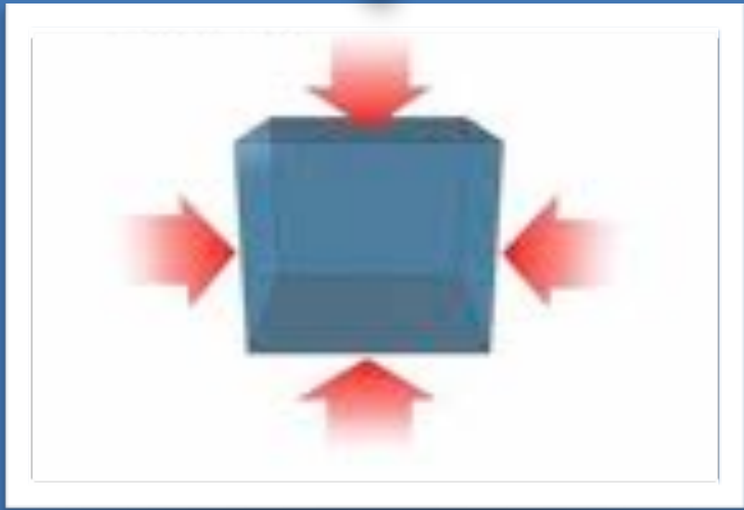
Viscosity

Fluid Forces

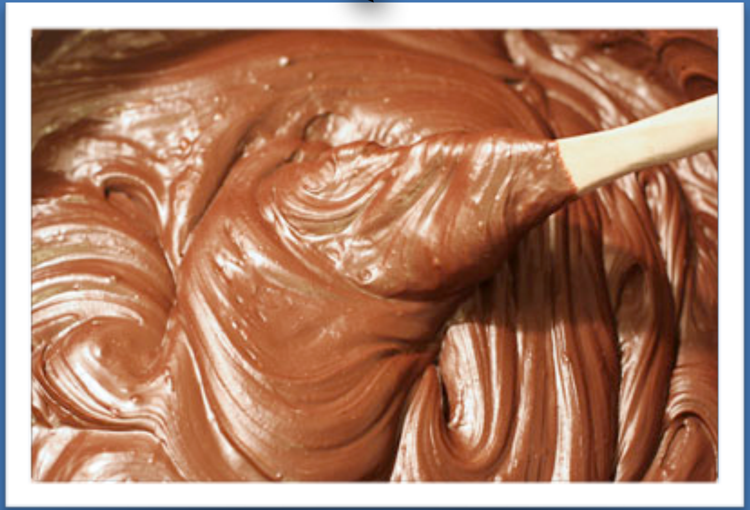
$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} + \mathbf{f}_{\text{pressure}} + \mathbf{f}_{\text{viscosity}}$$



Gravity



Resistance to Compression

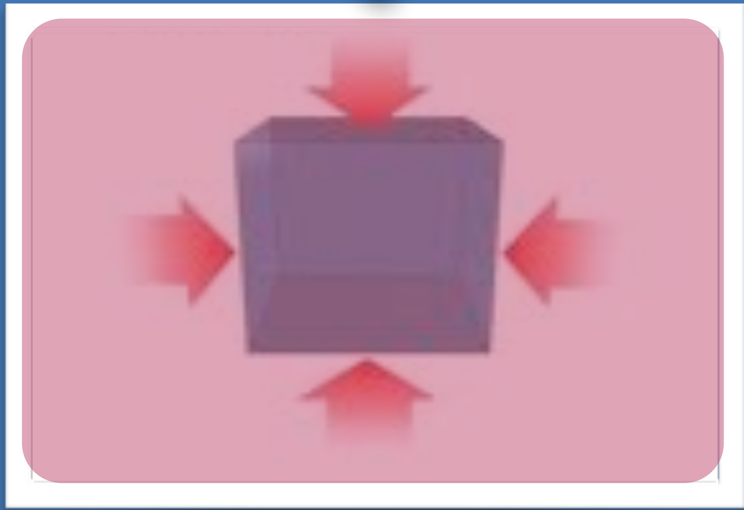


Viscosity

Fluid Forces

$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} + \mathbf{f}_{\text{pressure}} + \mathbf{f}_{\text{viscosity}}$$

Gravity



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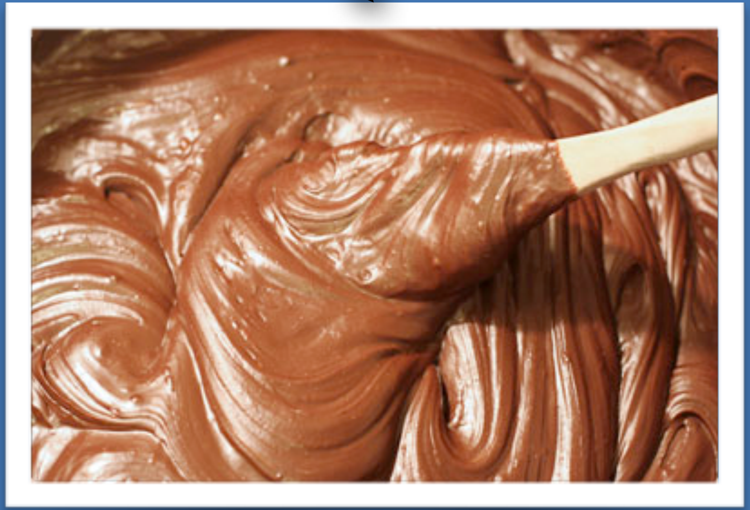
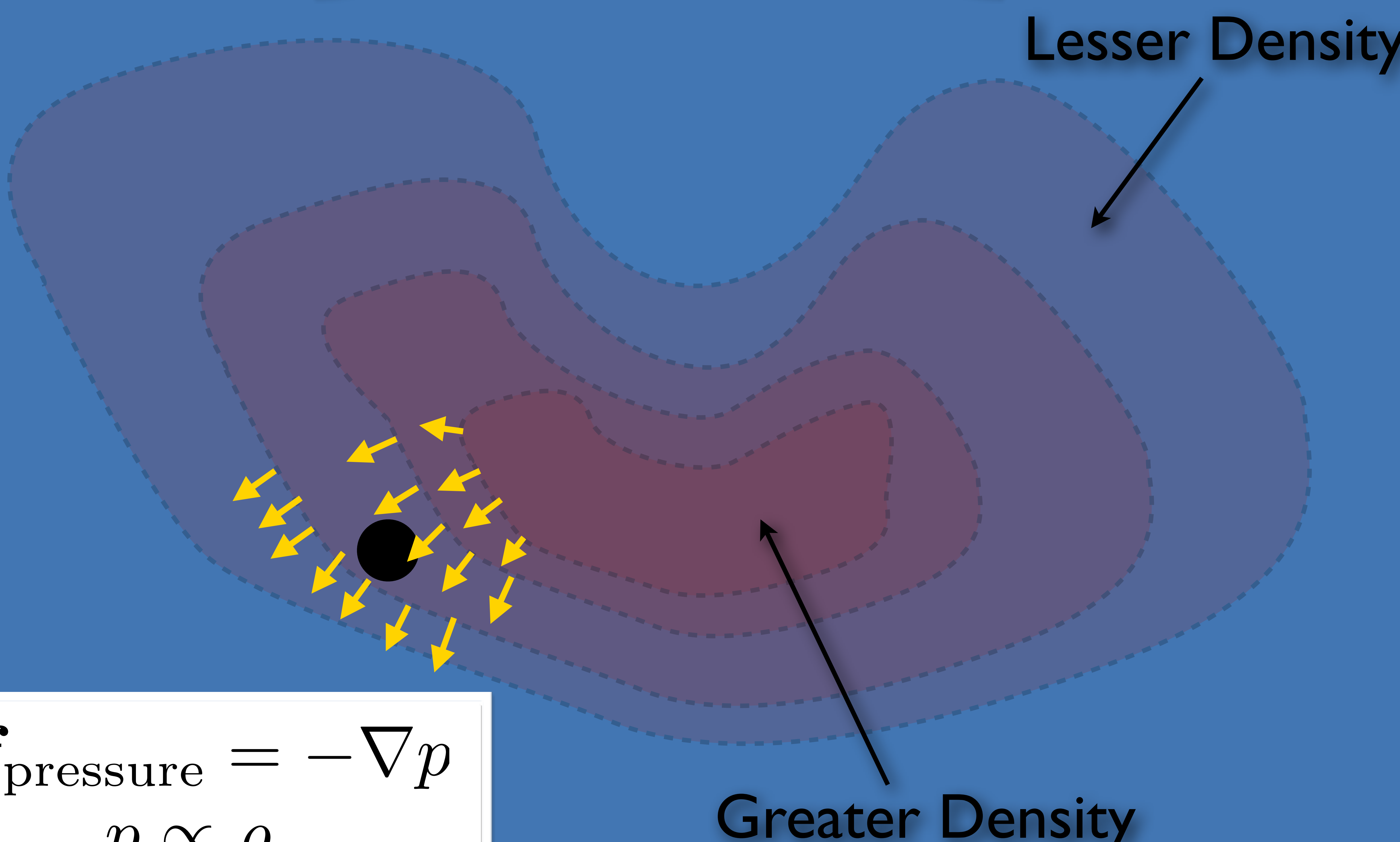


Viscosity

Pressure



$$\mathbf{f}_{\text{pressure}} = -\nabla p$$
$$p \propto \rho$$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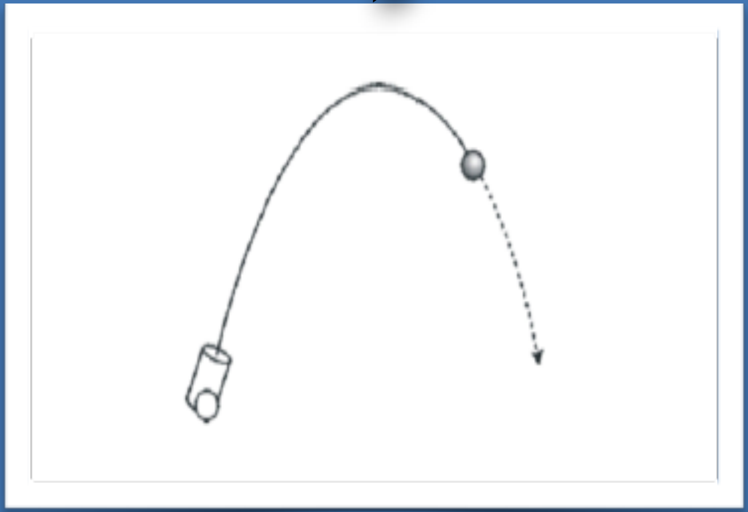
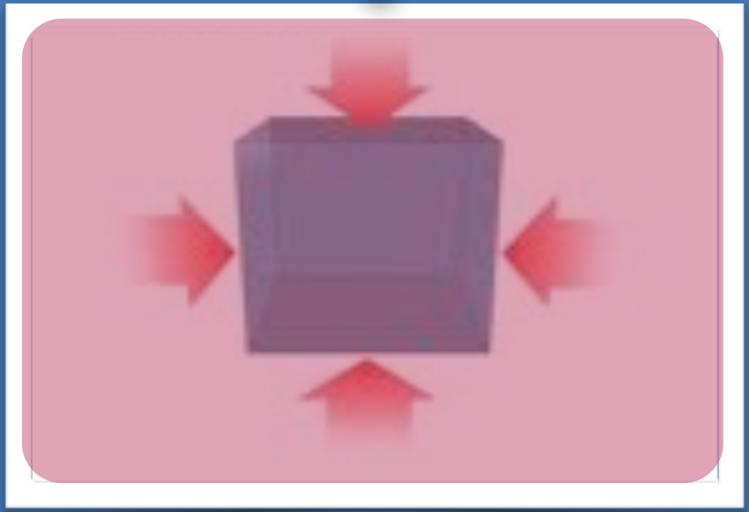
Fluid Forces

$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} + \mathbf{f}_{\text{pressure}} + \mathbf{f}_{\text{viscosity}}$$



Gravity

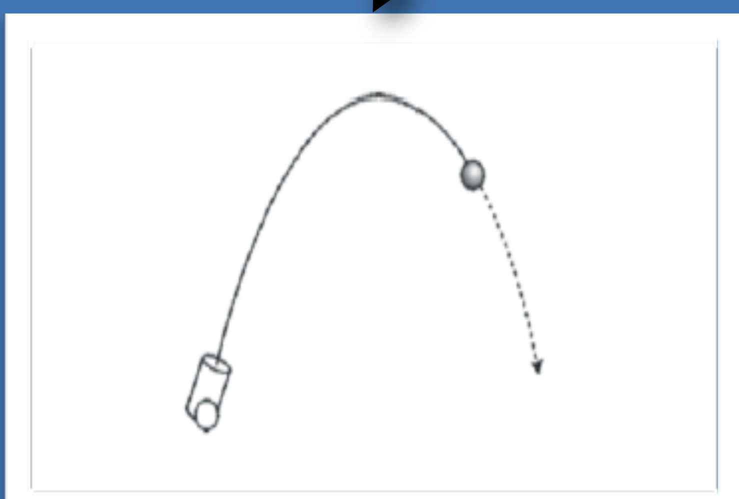


Resistance to Compression

Viscosity

Fluid Forces

$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} \quad -\nabla p \quad + \mathbf{f}_{\text{viscosity}}$$



Gravity

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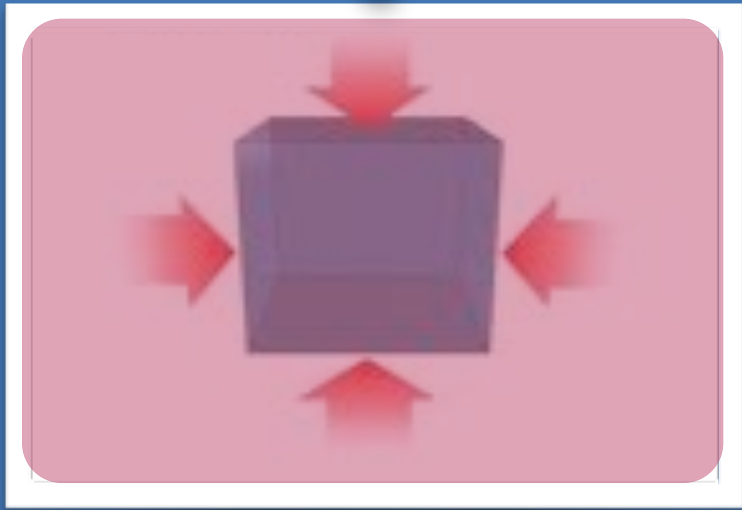
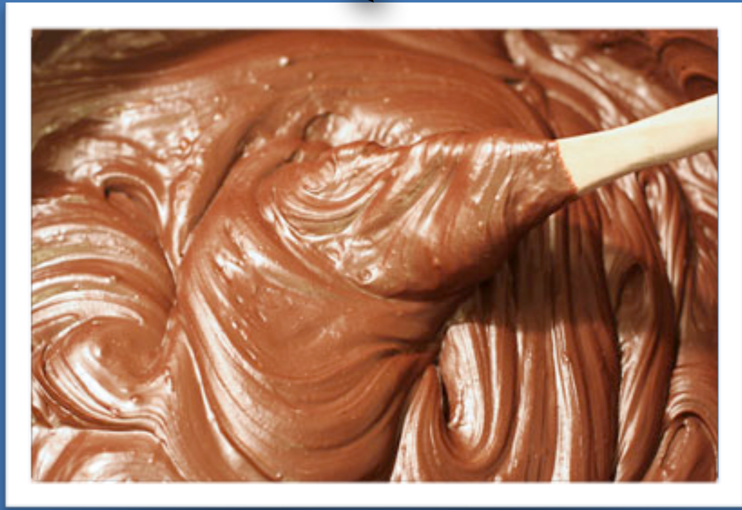




Gravity

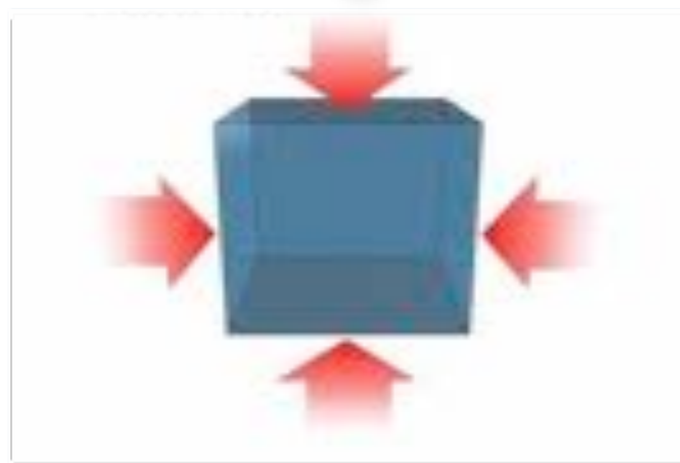
Resistance to Compression

Viscosity

Fluid Forces

$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} - \nabla p + \mathbf{f}_{\text{viscosity}}$$

Gravity



Resistance to Compression

Viscosity

Viscosity

- How do we drive the fluid velocity to be like its neighbors?

- Use a spring!

$$\mathbf{f}_{\text{viscosity}} \propto \underbrace{\mathbf{v}_{\text{neighborhood}}}_{\text{What is this?}} - \mathbf{v}$$

- What is this?

Viscosity

$$\mathbf{v}(x-\Delta x) \longleftrightarrow \mathbf{v}(x) \longleftrightarrow \mathbf{v}(x+\Delta x)$$

$$\mathbf{v}^{(1D)}_{\text{neighborhood}} \approx \tfrac{1}{2}\mathbf{v}(x-\Delta x)+\tfrac{1}{2}\mathbf{v}(x+\Delta x)$$

$$\mathbf{v}^{(2D)}_{\text{neighborhood}} \approx \tfrac{1}{4}\mathbf{v}(x-\Delta x)+\tfrac{1}{4}\mathbf{v}(x+\Delta x)+ \tfrac{1}{4}\mathbf{v}(x-\Delta y)+\tfrac{1}{4}\mathbf{v}(x+\Delta y)$$

$$\mathbf{f}_{\text{viscosity}} \propto \underbrace{\mathbf{v}_{\text{neighborhood}}}_{\text{What is this?}} - \mathbf{v}$$

- What is this?

Viscosity

By Substitution...

$$f_{\text{viscosity}} \propto \frac{1}{2}\mathbf{v}(x - \Delta x) + \frac{1}{2}\mathbf{v}(x + \Delta x) - \mathbf{v}$$

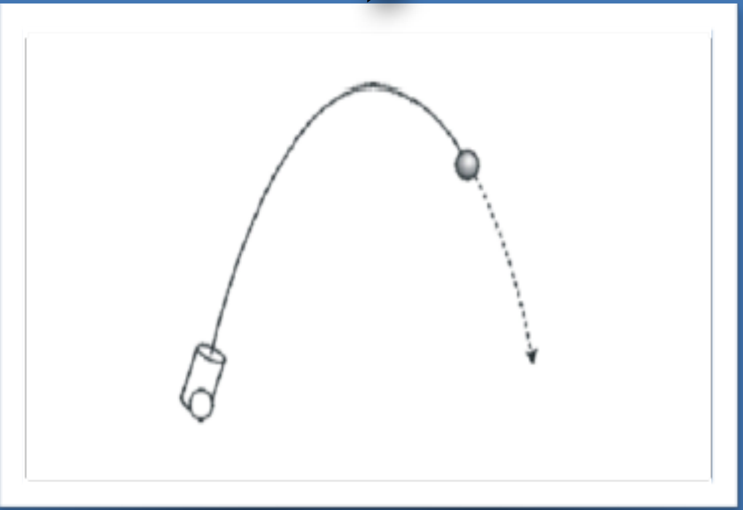
$$f_{\text{viscosity}} \propto \mathbf{v}(x - \Delta x) + \mathbf{v}(x + \Delta x) - 2\mathbf{v}$$

Taking Limits...

$$f_{\text{viscosity}} = \mu \nabla^2 \mathbf{v}$$

Fluid Forces

$$\rho \dot{\mathbf{v}} = \rho \mathbf{g} - \nabla p + \mathbf{f}_{\text{viscosity}}$$



Gravity

Resistance to Compression

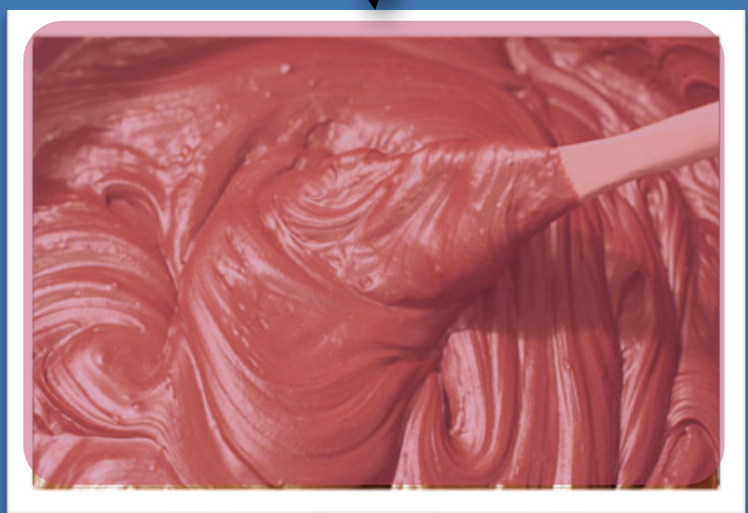
Viscosity

Fluid Forces

$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} \quad -\nabla p \quad + \mu\nabla^2\mathbf{v}$$

Gravity

Resistance to Compression

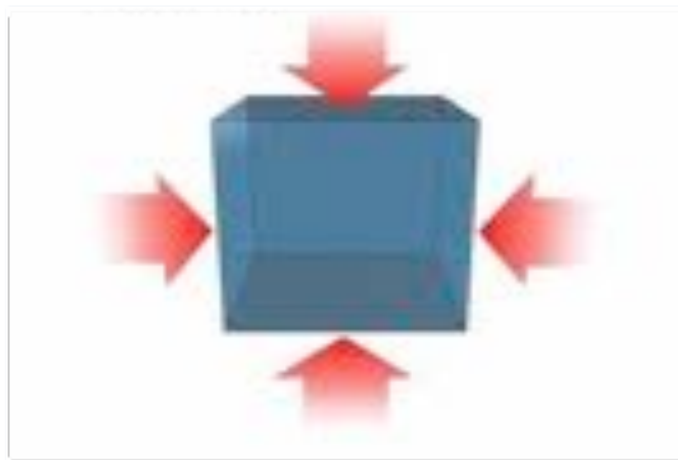


Viscosity

Navier-Stokes

$$\rho \dot{\mathbf{v}} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v}$$

Gravity



Resistance to Compression

Viscosity

Navier-Stokes

$$\rho\dot{\mathbf{v}} = \rho\mathbf{g} - \nabla p + \mu\nabla^2\mathbf{v}$$

How do we evaluate these quantities using a particle system?

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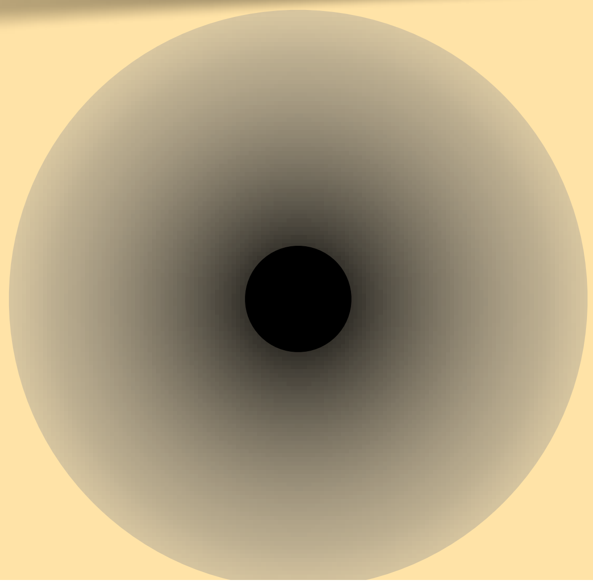
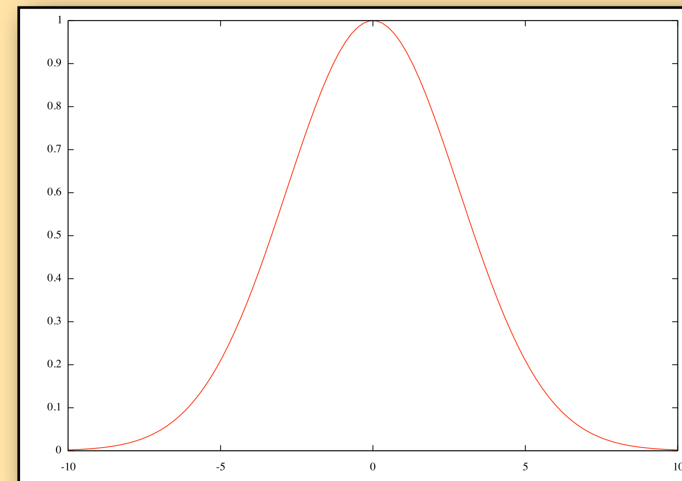
Kernel Functions



$$W(\mathbf{r})$$

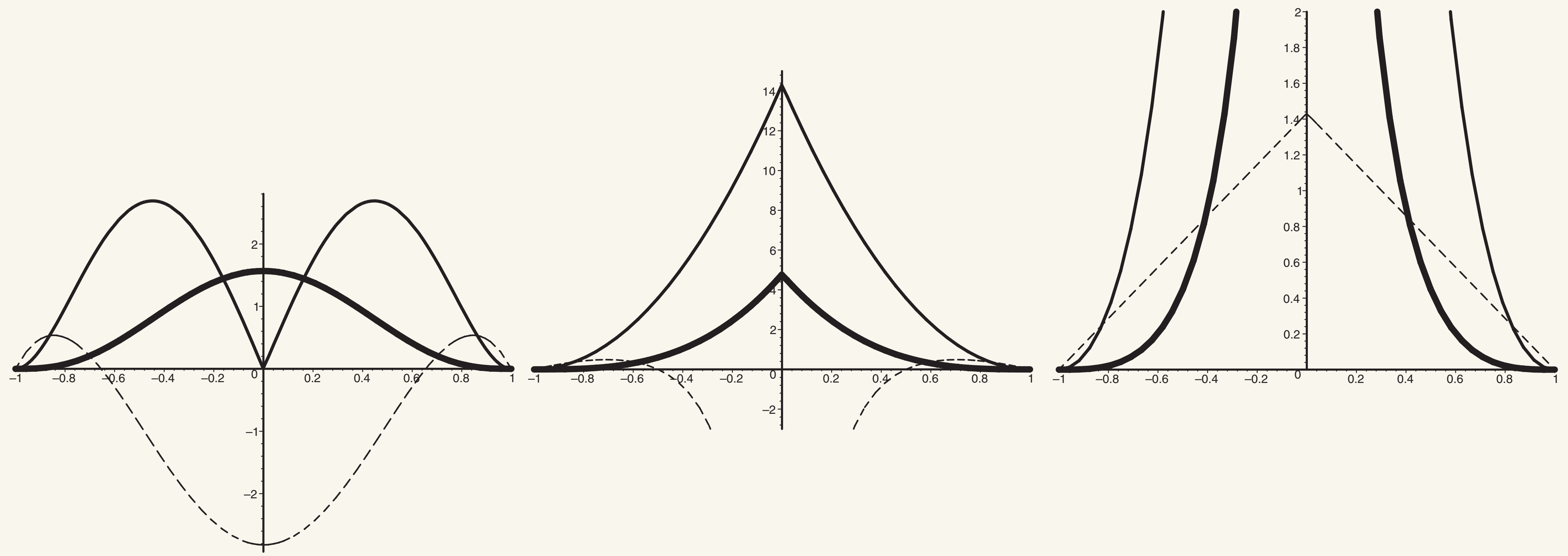
or

$$W(\mathbf{r}, h)$$



- Properties:
  - Symmetric: $W(\mathbf{x}) = W(-\mathbf{x})$
  - Finite Support: $W(\mathbf{x}) = 0 \ \forall ||x|| > h$
  - Flat at center: $\nabla W(0) = 0$
  - Normalized: $\int W(\mathbf{x}) d\mathbf{x} = 1$

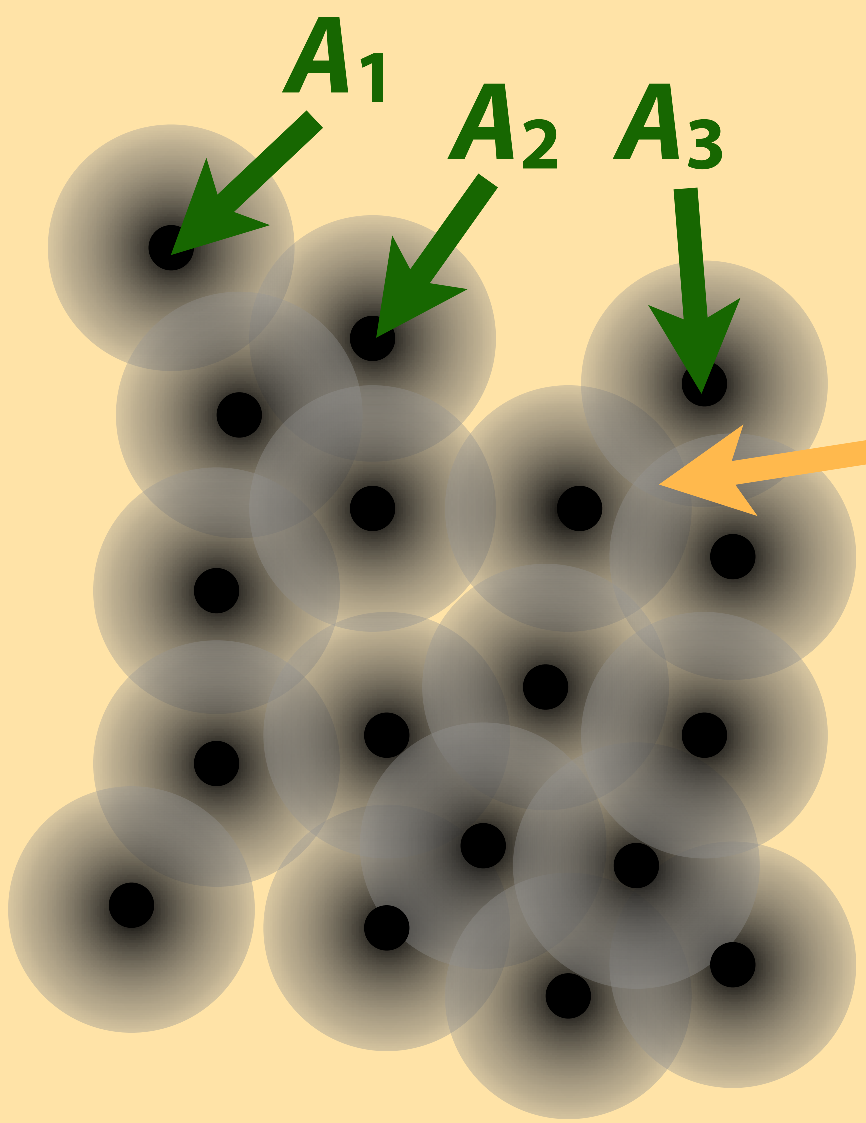
Kernel



(a) (b) (c)

Kernel
Gradient Length
Laplacian

Interpolation



For a physical quantity A:

quantity

mass

position

$$A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h),$$

kernel

density

kernel width

Example, density:

$$\rho_S(\mathbf{r}) = \sum_j m_j \frac{\rho_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h)$$

What About

Function:

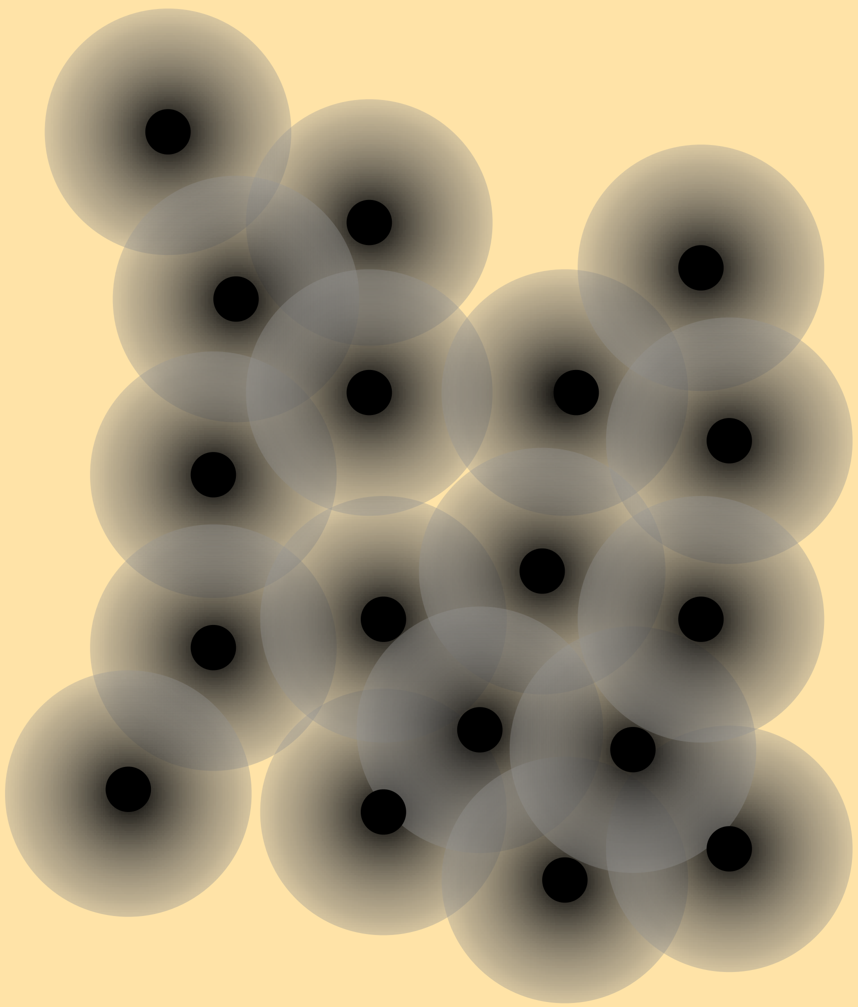
$$A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r}-\mathbf{r}_j, h),$$

Gradient:

$$\nabla A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\mathbf{r}-\mathbf{r}_j, h)$$

Laplacian:

$$\nabla^2 A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla^2 W(\mathbf{r}-\mathbf{r}_j, h).$$



But we're going to play tricks. ;-)

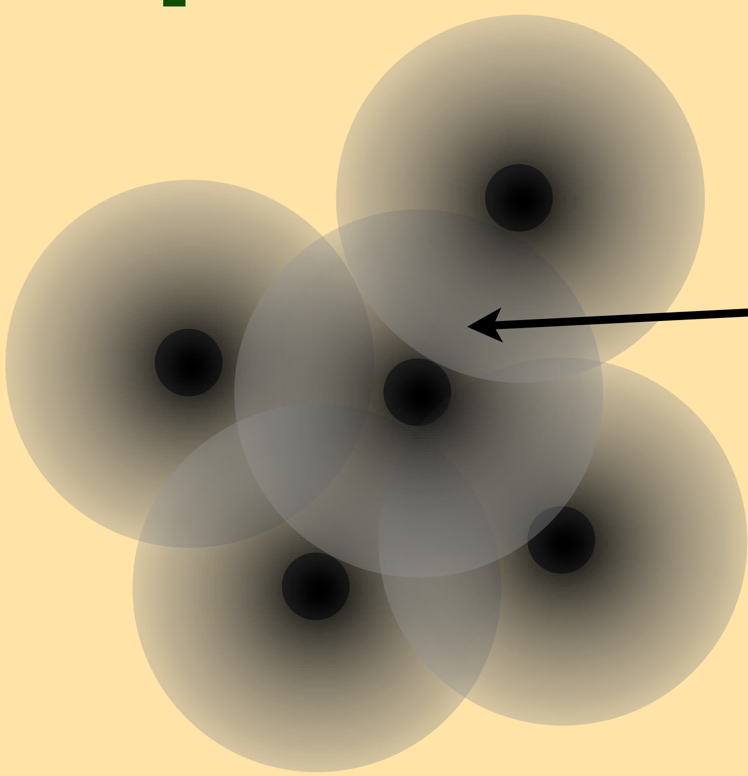
Pressure

For One Particle:

$$p_j = \kappa(\rho_j - \overline{\rho})$$

$$\text{pressure force} = -\nabla p(\mathbf{r})$$

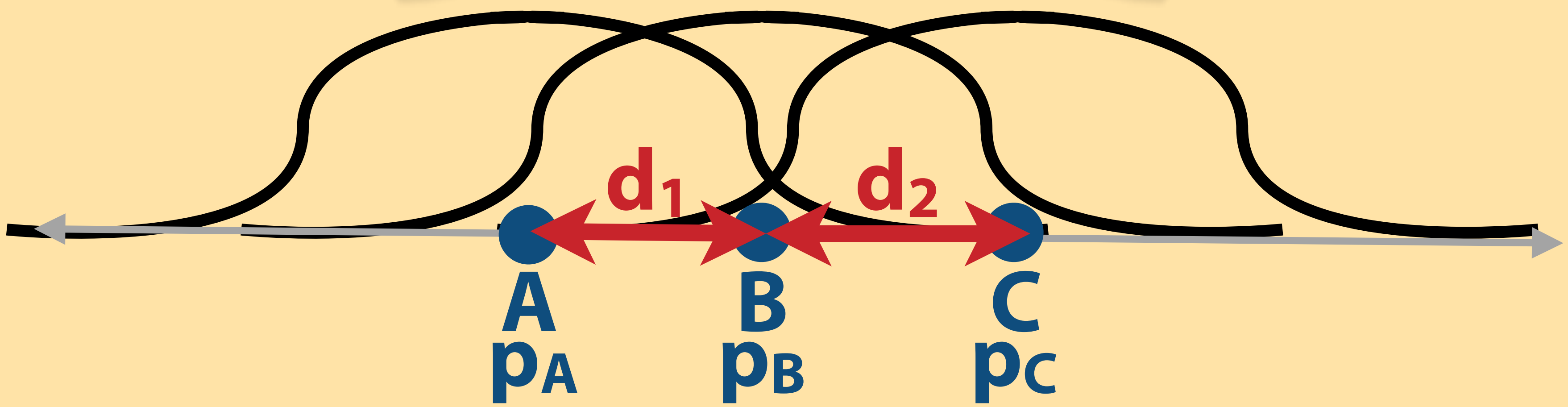
Spatial Pressure Evaluation:



$$\nabla p(\mathbf{r}) = \sum_j m_j \frac{p_j}{\rho_j} \nabla W(\mathbf{r} - \mathbf{r}_j, h)$$

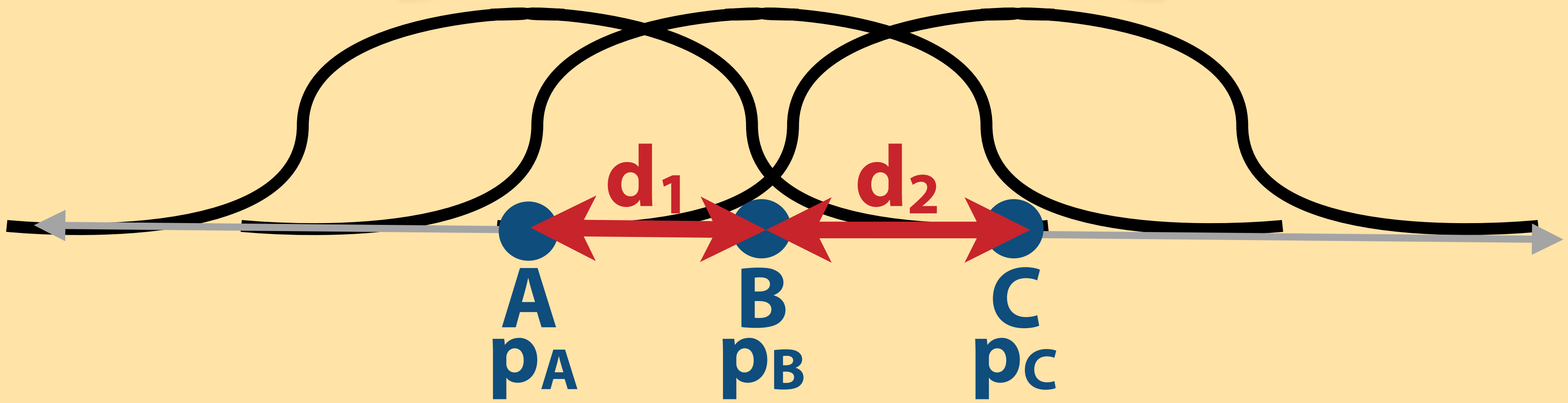
But wait... is this symmetric?

Pressure



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

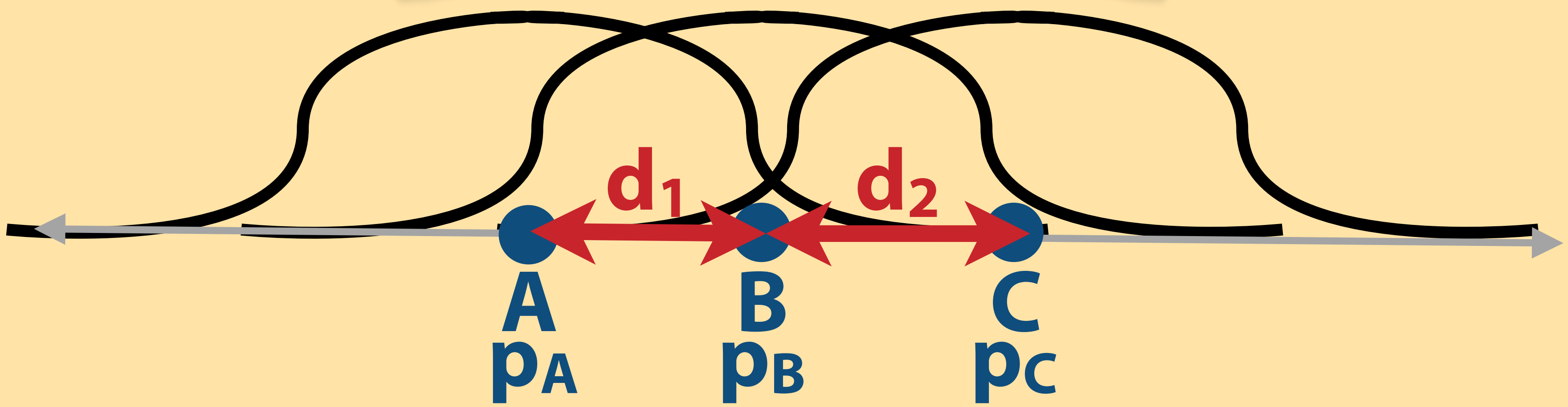
Pressure



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$\mathbf{f_A} =$

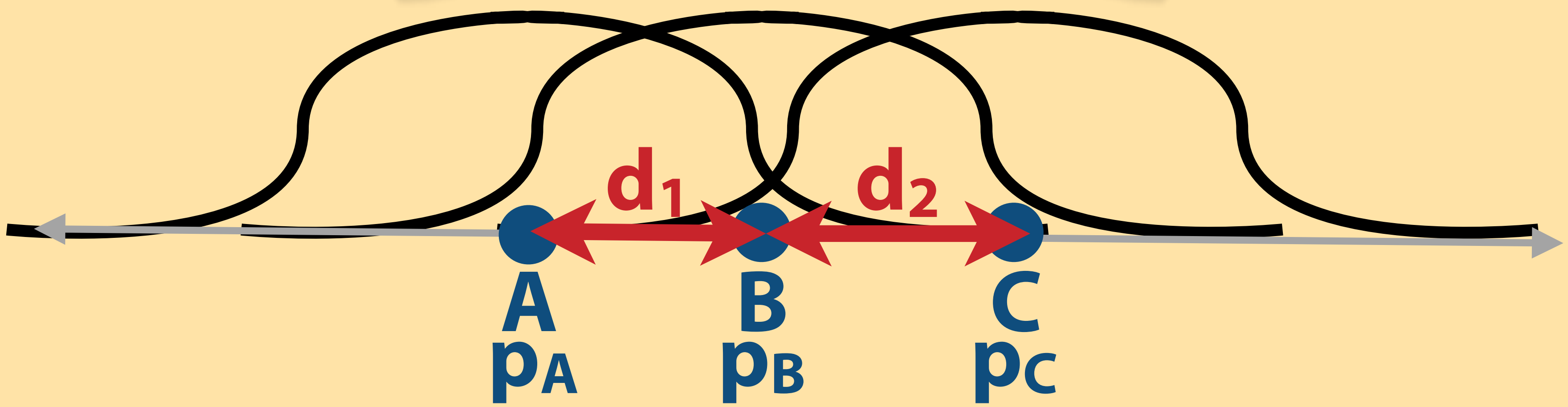
Pressure



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f_A = -p_A \nabla W(0)}$$

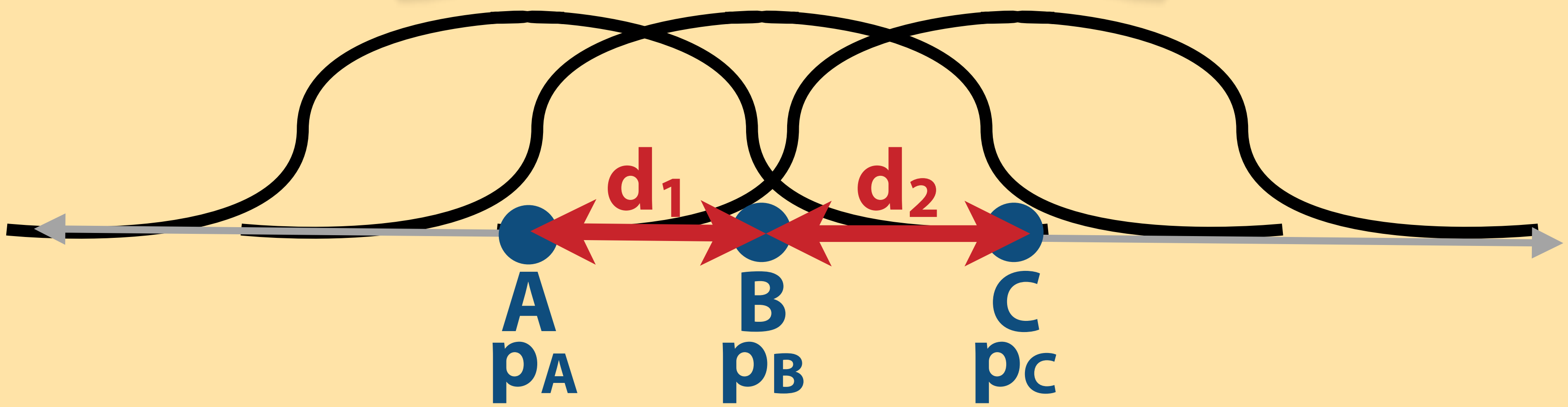
Pressure



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1)}$$

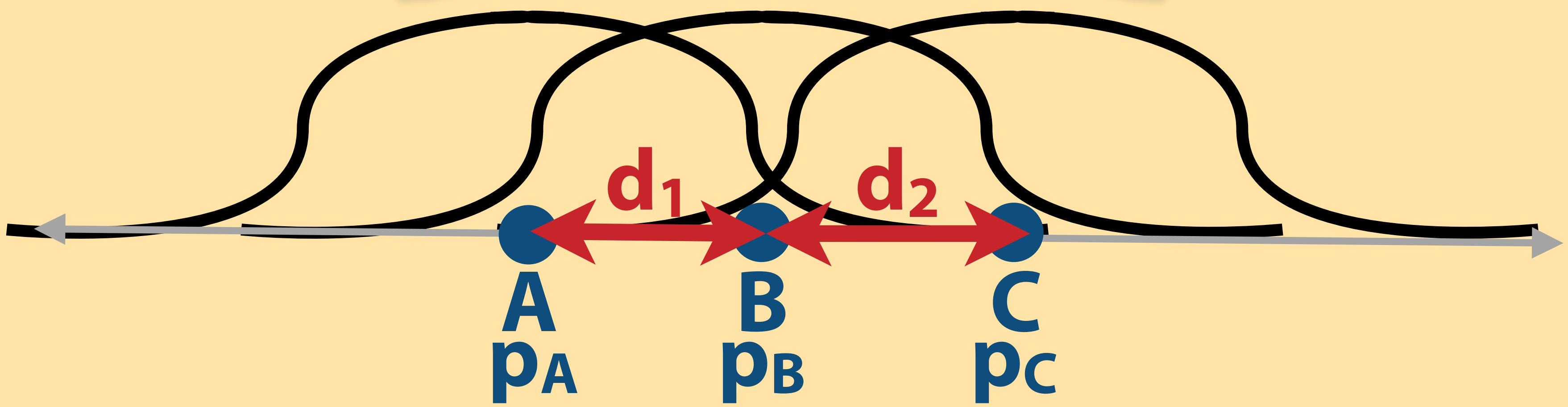
Pressure



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1 - d_2)}$$

Pressure



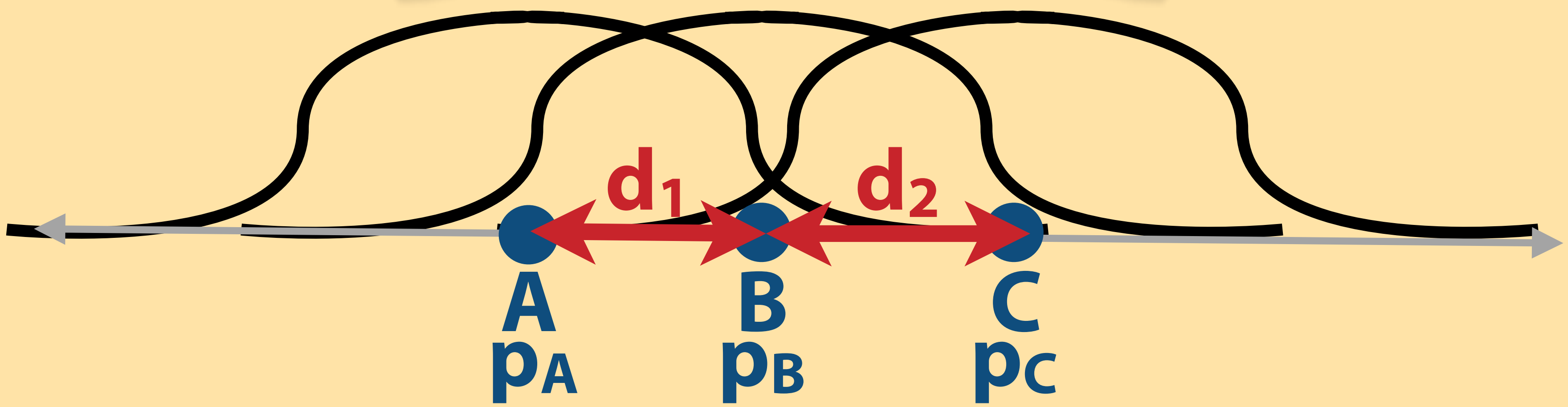
$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\mathbf{f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1-d_2)}$$

$$\mathbf{f_B = -p_A \nabla W(d_1) - p_B \nabla W(0) - p_C \nabla W(-d_2)}$$

$$\mathbf{f_C = -p_A \nabla W(d_1+d_2) - p_B \nabla W(d_2) - p_C \nabla W(0)}$$

Pressure



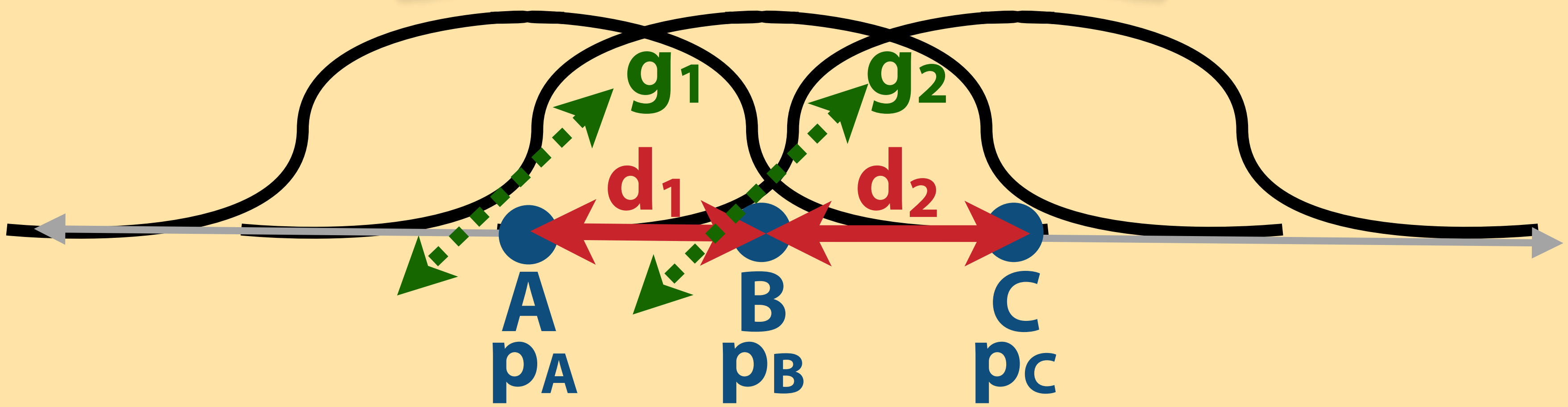
$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$f_A = \cancel{-p_A \nabla W(0)} - p_B \nabla W(-d_1) - \cancel{p_C \nabla W(-d_1 - d_2)}$$

$$f_B = -p_A \nabla W(d_1) - \cancel{p_B \nabla W(0)} - p_C \nabla W(-d_2)$$

$$f_C = \cancel{-p_A \nabla W(d_1 + d_2)} - p_B \nabla W(d_2) - \cancel{p_C \nabla W(0)}$$

Pressure



$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

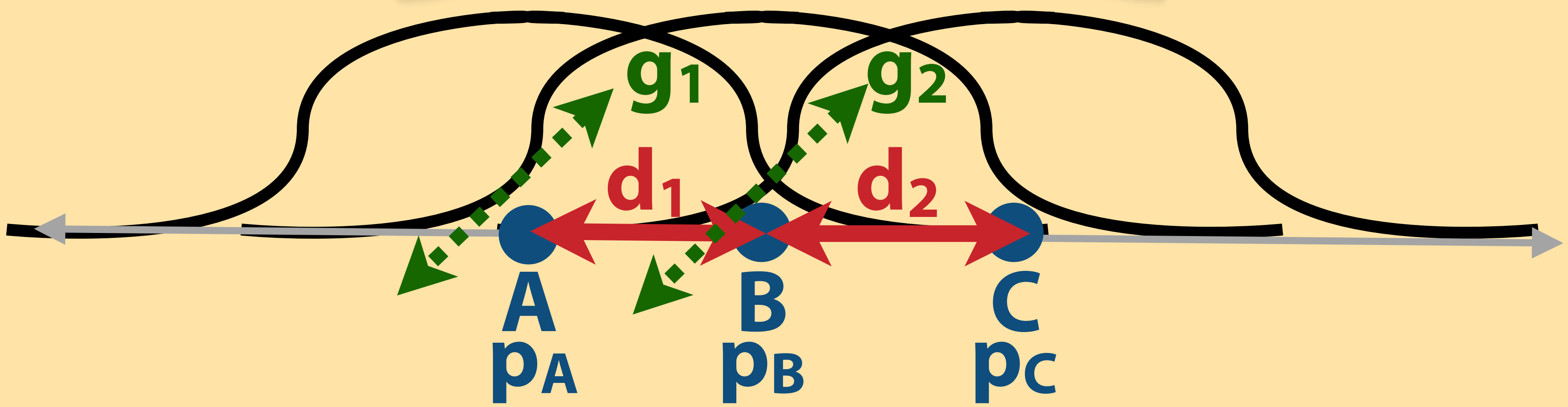
$$f_A = -\cancel{p_A \nabla W(0)} - p_B \nabla W(-d_1) - \cancel{p_C \nabla W(-d_1-d_2)}$$

$$f_B = -p_A \nabla W(d_1) - \cancel{p_B \nabla W(0)} - p_C \nabla W(-d_2)$$

$$f_C = -\cancel{p_A \nabla W(d_1+d_2)} - p_B \nabla W(d_2) - \cancel{p_C \nabla W(0)}$$

$$g_1 = -\nabla W(-d_1) \quad g_2 = -\nabla W(-d_2)$$

Pressure

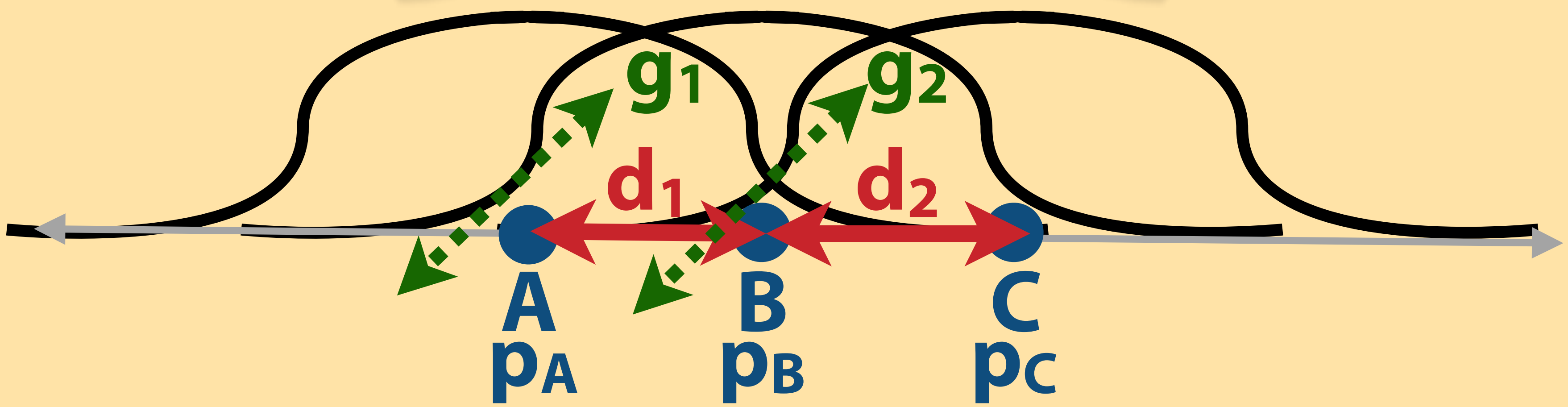


$$-\nabla p(\mathbf{r}) = \sum_j p_j \nabla W(\mathbf{r} - \mathbf{r}_j)$$

$$\left.\begin{aligned} f_A &= p_B\, g_1 \\ f_B &= -p_A\, g_1 + p_C\, g_2 \\ f_C &= -\, p_B\, g_2 \end{aligned}\right\} \neq 0$$

$$g_1 = -\,\nabla W(-d_1) \quad g_2 = -\,\nabla W(-d_2)$$

Pressure



$$-\nabla p(\mathbf{r}_i) = \sum_j \frac{p_i + p_j}{2} \nabla W(\mathbf{r}_i - \mathbf{r}_j)$$

$$\left.\begin{aligned} f_A &= \tfrac{1}{2}(p_A + p_B)g_1 \\ f_B &= -\tfrac{1}{2}(p_A + p_B)g_1 + \tfrac{1}{2}(p_B + p_C)g_2 \\ f_C &= -\tfrac{1}{2}(p_B + p_C)g_2 \end{aligned}\right\} = 0$$

$$g_1 = -\nabla W(-d_1) \quad g_2 = -\nabla W(-d_2)$$

Pressure

$$\mathbf{f}_i^{\text{viscosity}} = \mu \nabla^2 \mathbf{v}(\mathbf{r}_a) = \mu \sum_j m_j \frac{\mathbf{v}_j}{\rho_j} \nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h).$$

Symmetrization:

$$\mathbf{f}_i^{\text{viscosity}} = \mu \sum_j m_j \frac{\mathbf{v}_j - \mathbf{v}_i}{\rho_j} \nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h).$$

Spring that pulls the particle towards the velocity of it's neighbors.

Overview

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- The Navier-Stokes Equations
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- Rendering
- Results
- Questions

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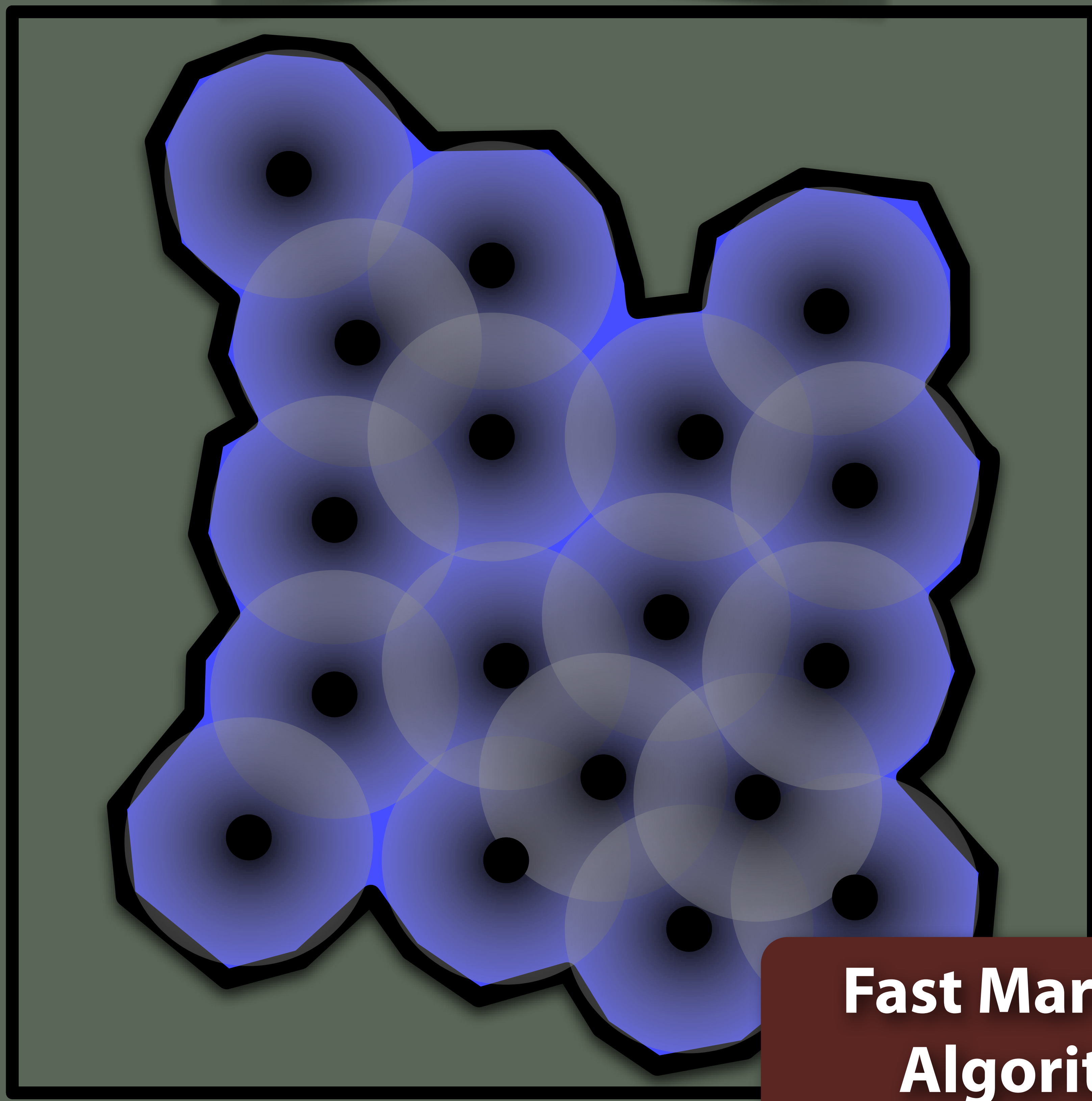
Rendering



Fast Marching Algorithm

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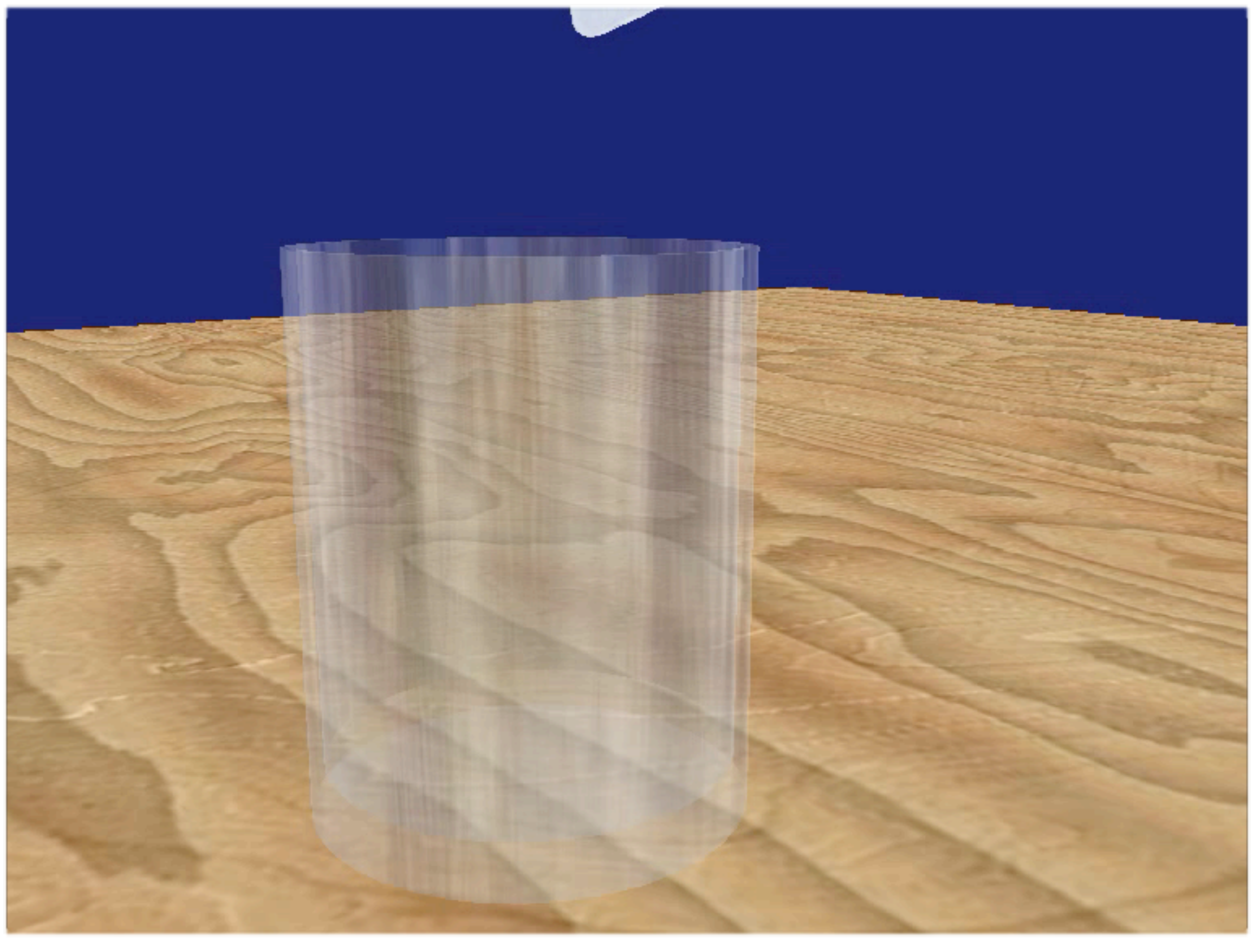
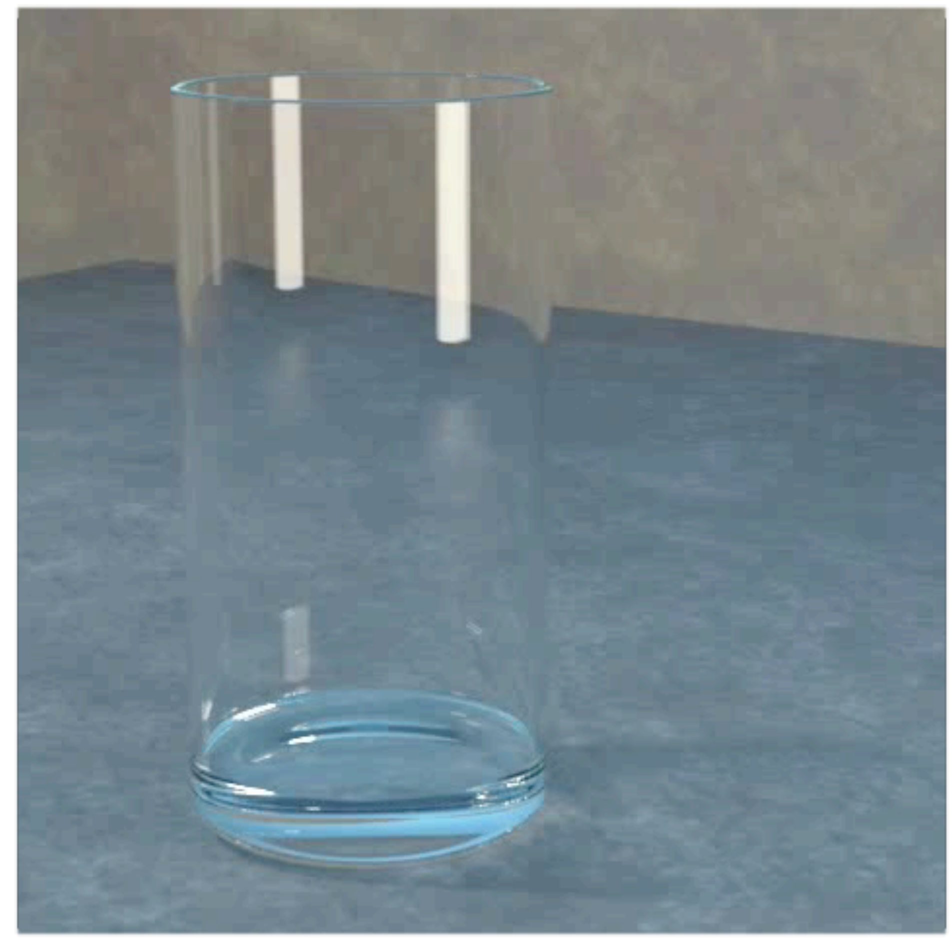
Example


Predictive-Corrective
Incompressible SPH
Barbara Solenthaler, Renato Pajarola
University of Zurich
VMML
visualization and multimedia lab
university of zürich

Comparison



SPH



Grid-based

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Questions

- Which phenomena does the grid-based method capture better? Why?
- Which phenomena does SPH capture better? Why?
- Which do neither capture? (How could we?)
- How could we turn our SPH simulator into a grid-based simulator?

- **SPH simulation**
  - seems to represent turbulent flows a little better (**g3**)
    - small groups of particles (**g2**)
    - “rough” flows (**g2**)
    - sprays, splashes (**g1**) (**g4**)
  - better for splashing (**g2**)
  - better for water (**g2**)
  - flow is kind of bumpy
    - does not settle to an attractive rest state (**g1**)

- **grid-based simulation**
  - seems to be much smoother (**g3**)
  - fewer “damping” artifacts (**g3**)
    - the surface seems to “settle down” more slowly (**g3**)
  - the grid based method “looses less detail” (**g3**)
  - worse for splashing (**g2**)
  - good for “smoother fluids” (**g2**) (**g1**) (**g4**)
    - **easier to render** (?) (**g4**)
  - allow for larger timesteps (**g2**)
  - must be confined to a boundary (**g2**)

- **neither**
  - bubbles (**g2**) (**g1**)

- **possible**
  - **how could we combine the two?!** (**g1**) (**g4**)
    - use grid based methods to get a continuous surface, but use SPH to get small particles, etc.