Particle Fluids

Adrien Treuille
Overview

- Smoothed Particle Hydrodynamics (SPH) Basics
- Fluid Behavior
- The Navier-Stokes Equations
- SPH Simulation
- Rendering
- Results
- Questions
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Particle Model

Properties

• Position - $x$
• Velocity - $v$
• Mass - $m$
• Density - $\rho$
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Fluid Behavior

- Gravity
- Resistance to Compression
- Viscosity
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Fluid Forces

\[ m \ddot{v} = f \]
Fluid Forces

\[ \rho \dot{v} = f \]
Fluid Forces

\( \rho \ddot{v} = f \)

Gravity

Resistance to Compression

Viscosity
Fluid Forces

\[ \rho \dot{v} = f_{\text{gravity}} + f_{\text{pressure}} + f_{\text{viscosity}} \]

Gravity

Resistance to Compression

Viscosity
Fluid Forces

\[ \rho \dot{v} = \mathbf{f}_{\text{gravity}} + \mathbf{f}_{\text{pressure}} + \mathbf{f}_{\text{viscosity}} \]

Gravity

Resistance to Compression

Viscosity
Gravity

\[ f_{\text{gravity}} = \rho g \]
\[ \rho \dot{v} = f_{\text{gravity}} + f_{\text{pressure}} + f_{\text{viscosity}} \]
\[ \dot{\rho v} = \rho g + f_{\text{pressure}} + f_{\text{viscosity}} \]
\[ \rho \dot{v} = \rho g + f_{\text{pressure}} + f_{\text{viscosity}} \]

- **Gravity**
- **Resistance to Compression**
- **Viscosity**
Pressure

\[ f_{\text{pressure}} = -\nabla p \]
\[ p \propto \rho \]
\[ \rho \dot{v} = \rho g + f_{\text{pressure}} + f_{\text{viscosity}} \]
\[ \dot{\rho} \mathbf{v} = \rho \mathbf{g} - \nabla p + \mathbf{f}_{\text{viscosity}} \]

**Gravity**

**Resistance to Compression**

**Viscosity**
\[ \dot{\rho} \vec{v} = \rho g - \nabla p + \mathbf{f}_{\text{viscosity}} \]

**Fluid Forces**

- **Gravity**
- **Resistance to Compression**
- **Viscosity**

Tuesday, October 25, 11
\[ \rho \dot{v} = \rho g - \nabla p + f_{\text{viscosity}} \]

- Gravity
- Resistance to Compression
- Viscosity
Viscosity

• How do we drive the fluid velocity to be like its neighbors?

• Use a spring!

\[ f_{\text{viscosity}} \propto \mathbf{v}_{\text{neighborhood}} - \mathbf{v} \]

• What is this?
Viscosity

\[ \mathbf{v}(x - \Delta x) \rightarrow \mathbf{v}(x) \rightarrow \mathbf{v}(x + \Delta x) \]

\[ \mathbf{v}^{(1D)}_{\text{neighborhood}} \approx \frac{1}{2} \mathbf{v}(x - \Delta x) + \frac{1}{2} \mathbf{v}(x + \Delta x) \]

\[ \mathbf{v}^{(2D)}_{\text{neighborhood}} \approx \frac{1}{4} \mathbf{v}(x - \Delta x) + \frac{1}{4} \mathbf{v}(x + \Delta x) + \frac{1}{4} \mathbf{v}(x - \Delta y) + \frac{1}{4} \mathbf{v}(x + \Delta y) \]

\[ \mathbf{f}_{\text{viscosity}} \propto \mathbf{v}_{\text{neighborhood}} - \mathbf{v} \]

**What is this?**
**Viscosity**

By Substitution...

\[ f_{\text{viscosity}} \propto \frac{1}{2}v(x - \Delta x) + \frac{1}{2}v(x + \Delta x) - v \]

\[ f_{\text{viscosity}} \propto v(x - \Delta x) + v(x + \Delta x) - 2v \]

Taking Limits...

\[ f_{\text{viscosity}} = \mu \nabla^2 v \]
\[ \rho \dot{v} = \rho g - \nabla p + f_{\text{viscosity}} \]

- **Gravity**
- **Resistance to Compression**
- **Viscosity**
\[ \rho \dot{\mathbf{v}} = \rho g - \nabla p + \mu \nabla^2 \mathbf{v} \]

- **Gravity**
- **Resistance to Compression**
- **Viscosity**
\[ \rho \ddot{\mathbf{v}} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v} \]

- **Gravity**
- **Resistance to Compression**
- **Viscosity**
Navier-Stokes

\( \rho \dot{v} = \rho g - \nabla p + \mu \nabla^2 v \)

How do we evaluate these quantities using a particle system?
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Kernel Functions

\[ W(r) \]

or

\[ W(r, h) \]

- Properties:
  - Symmetric: \[ W(x) = W(-x) \]
  - Finite Support: \[ W(x) = 0 \quad \forall \|x\| > h \]
  - Flat at center: \[ \nabla W(0) = 0 \]
  - Normalized: \[ \int W(x) \, dx = 1 \]
A simple kernel is that and use it in all but two cases. An important feature of this divergence of yields the surface normal field pointing into the fluid and the everywhere else. This field is called surface. For the force density acting near the surface we get following kernel free to design kernels for special purposes. We designed the conducive to stability. Apart from those constraints, one is all even and normalized (see Fig. 2). In addition, kernels depend on the choice of the smoothing kernels. The kernels Stability, accuracy and speed of the SPH method highly de-

3.5. Smoothing Kernels

To distribute the surface traction among particles near the surface and to get a force density we multiply by a normal-
Interpolation

For a physical quantity $A$:

$$A_S(r) = \sum_j m_j \frac{A_j}{\rho_j} W(r - r_j, h),$$

where $W$ is the smoothing kernel, $m$ is the mass, $A$ is the quantity, $\rho$ is the density, and $h$ is the kernel width.

Example, density:

$$\rho_S(r) = \sum_j m_j \frac{\rho_j}{\rho_j} W(r - r_j, h)$$
What About

Function:

\[ A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r} - \mathbf{r}_j, h), \]

Gradient:

\[ \nabla A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\mathbf{r} - \mathbf{r}_j, h) \]

Laplacian:

\[ \nabla^2 A_S(\mathbf{r}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla^2 W(\mathbf{r} - \mathbf{r}_j, h). \]
For One Particle:

\[ p_j = \kappa (\rho_j - \bar{\rho}) \]

pressure force = \(-\nabla p(r)\)

Spatial Pressure Evaluation:

\[ \nabla p(r) = \sum_j m_j \frac{p_j}{\rho_j} \nabla W(r - r_j, h) \]

But wait... is this symmetric?
\[-\nabla p(r) = \sum_{j} p_j \nabla W(r - r_j)\]
Pressure

\[-\nabla p(r) = \sum_{j} p_j \nabla W(r - r_j)\]

\[f_A = \]
\[-\nabla p(r) = \sum_j p_j \nabla W(r - r_j)\]

\[f_A = -p_A \nabla W(0)\]
\[ -\nabla p(r) = \sum_{j} p_{j} \nabla W(r - r_{j}) \]

\[ f_{A} = -p_{A} \nabla W(0) - p_{B} \nabla W(-d_{1}) \]
\[-\nabla p(r) = \sum_j p_j \nabla W(r - r_j)\]

\[f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1 - d_2)\]
\[-\nabla p(r) = \sum_{j} p_j \nabla W(r - r_j)\]

\[f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1-d_2)\]

\[f_B = -p_A \nabla W(d_1) - p_B \nabla W(0) - p_C \nabla W(-d_2)\]

\[f_C = -p_A \nabla W(d_1+d_2) - p_B \nabla W(d_2) - p_C \nabla W(0)\]
\[-\nabla p(r) = \sum_{j} p_j \nabla W(r - r_j)\]

\[f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1-d_2)\]

\[f_B = -p_A \nabla W(d_1) - p_B \nabla W(0) - p_C \nabla W(-d_2)\]

\[f_C = -p_A \nabla W(d_1+d_2) - p_B \nabla W(d_2) - p_C \nabla W(0)\]


\[
-\nabla p(r) = \sum_j p_j \nabla W(r - r_j)
\]

\[
f_A = -p_A \nabla W(0) - p_B \nabla W(-d_1) - p_C \nabla W(-d_1 - d_2)
\]

\[
f_B = -p_A \nabla W(d_1) - p_B \nabla W(0) - p_C \nabla W(-d_2)
\]

\[
f_C = -p_A \nabla W(d_1 + d_2) - p_B \nabla W(d_2) - p_C \nabla W(0)
\]

\[
g_1 = -\nabla W(-d_1) \quad g_2 = -\nabla W(-d_2)
\]
\[-\nabla p(r) = \sum_j p_j \nabla W(r - r_j)\]

\[
\begin{align*}
\mathbf{f}_A &= \mathbf{p}_B \mathbf{g}_1 \\
\mathbf{f}_B &= -\mathbf{p}_A \mathbf{g}_1 + \mathbf{p}_C \mathbf{g}_2 \\
\mathbf{f}_C &= -\mathbf{p}_B \mathbf{g}_2
\end{align*}
\]

\[
\begin{align*}
\mathbf{g}_1 &= -\nabla W(-\mathbf{d}_1) \\
\mathbf{g}_2 &= -\nabla W(-\mathbf{d}_2)
\end{align*}
\]
Pressure

\[ -\nabla p(r_i) = \sum_j \frac{p_i + p_j}{2} \nabla W(r_i - r_j) \]

\[
\begin{align*}
\mathbf{f}_A &= \frac{1}{2}(p_A + p_B) \mathbf{g}_1 \\
\mathbf{f}_B &= -\frac{1}{2}(p_A + p_B) \mathbf{g}_1 + \frac{1}{2}(p_B + p_C) \mathbf{g}_2 \\
\mathbf{f}_C &= -\frac{1}{2}(p_B + p_C) \mathbf{g}_2
\end{align*}
\]

\[
\mathbf{g}_1 = -\nabla W(-d_1) \quad \mathbf{g}_2 = -\nabla W(-d_2)
\]
Pressure

\[ f_{i}^{\text{viscosity}} = \mu \nabla^2 v(r_a) = \mu \sum_j m_j \frac{v_j}{\rho_j} \nabla^2 W(r_i - r_j, h). \]

Symmetrization:

\[ f_{i}^{\text{viscosity}} = \mu \sum_j m_j \frac{v_j - v_i}{\rho_j} \nabla^2 W(r_i - r_j, h). \]

Spring that pulls the particle towards the velocity of it's neighbors.
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Fast Marching Algorithm
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Example

Predictive–Corrective Incompressible SPH

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Comparison

SPH

Grid-based
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Questions

- Which phenomena does the grid-based method capture better? Why?
- Which phenomena does SPH capture better? Why?
- Which do neither capture? (How could we?)
- How could we turn our SPH simulator into a grid-based simulator?
- **SPH simulation**
  - seems to represent turbulent flows a little better \((g3)\)
    - small groups of particles \((g2)\)
    - “rough” flows \((g2)\)
    - sprays, splashes \((g1) (g4)\)
  - better for splashing \((g2)\)
  - better for water \((g2)\)
  - flow is kind of bumpy
    - does not settle to an attractive rest state \((g1)\)

- **grid-based simulation**
  - seems to be much smoother \((g3)\)
  - fewer “damping” artifacts \((g3)\)
    - the surface seems to “settle down” more slowly \((g3)\)
  - the grid based method “looses less detail” \((g3)\)
  - worse for splashing \((g2)\)
  - good for “smoother fluids” \((g2) (g1) (g4)\)
    - easier to render \((?) (g4)\)
  - allow for larger timesteps \((g2)\)
  - must be confined to a boundary \((g2)\)

- **neither**
  - bubbles \((g2) (g1)\)

- **possible**
  - how could we combine the two?! \((g1) (g4)\)
    - use grid based methods to get a continuous surface, but use SPH to get
      small particles, etc.