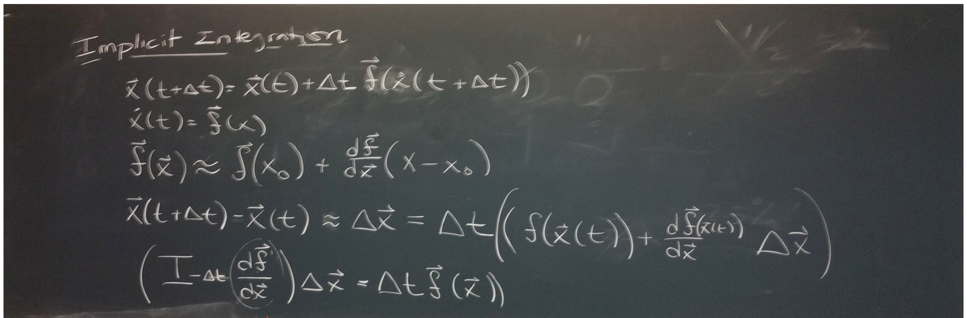
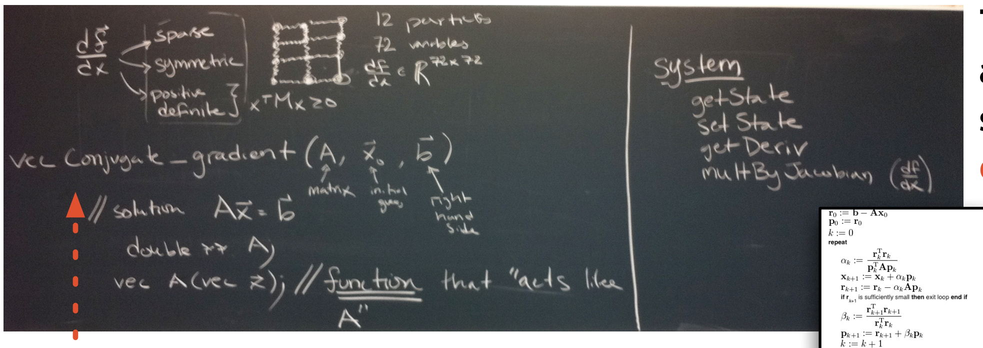
Implicit Integration + Large Linear Systems

15-867: The Animation of Natural Phenomena



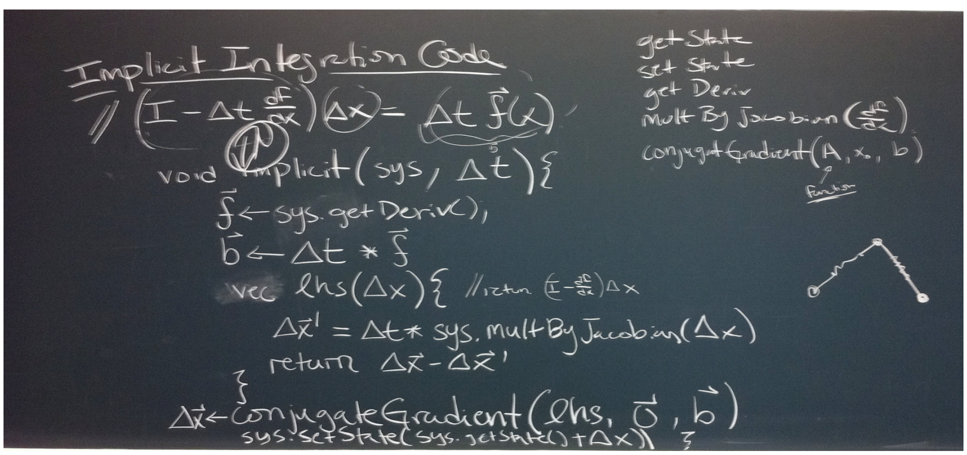
We can approximate implicit integration of a nonlinear system as follows. The integrator turns into a solution to a linear system. The key is the large sparse **Jacobian** matrix, df/dx.



$\mathbf{r}_0 := \mathbf{b} - \mathbf{A}\mathbf{x}_0$
$\mathbf{p}_0 := \mathbf{r}_0$
$k := 0$
repeat

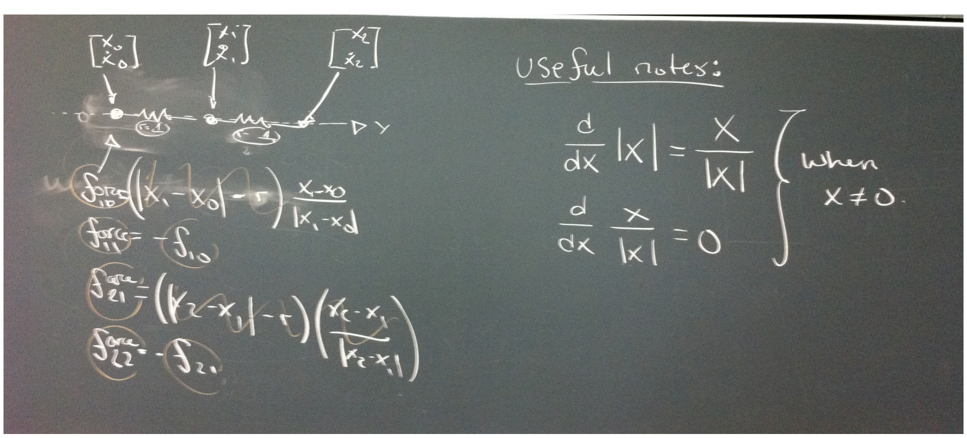
$\alpha_k := \dfrac{\mathbf{r}_k^{\mathrm{T}}\mathbf{r}_k}{\mathbf{p}_k^{\mathrm{T}}\mathbf{A}\mathbf{p}_k}$
$\mathbf{x}_{k+1} := \mathbf{x}_k + \alpha_k\mathbf{p}_k$
$\mathbf{r}_{k+1} := \mathbf{r}_k - \alpha_k\mathbf{A}\mathbf{p}_k$
if $\mathbf{r}_{k+1}$ is sufficiently small **then** exit loop **end if**
$\beta_k := \dfrac{\mathbf{r}_{k+1}^{\mathrm{T}}\mathbf{r}_{k+1}}{\mathbf{r}_k^{\mathrm{T}}\mathbf{r}_k}$
$\mathbf{p}_{k+1} := \mathbf{r}_{k+1} + \beta_k\mathbf{p}_k$
$k := k + 1$

The Jacobian is large, sparse, symmetric and positive definite. This allows us to solve this system with the **conjugate gradient** method.



Using the conjugate gradient method, we can now write down **pseudocode for implicit integration**. We only need:

- getState
- setState
- getDeriv
- multByJacobian
- conjugateGradient



Question: What is the Jacobian for three "beads on a wire" connected by undamped linear springs?