This homework is just a list of fundamental linear algebra facts you should know. You should have an idea how the proofs also go.

1 Orthogonal Projection
Suppose that $Ax = b$ is an over constrained linear system and we would like to find an $x$ to minimize $|Ax - b|^2_2$, the $L^2_2$ distance, where the columns of $A$ are independent.

1. Show that the answer to our minimization problem is:
   1) The system $A^T A \bar{x} = A^T b$ always has a solution.
   2) The $\bar{x}$ is solution to our problem.

2. The projection of $b \in \mathbb{R}^m$ onto the column space of an $m \times n$ matrix $A$ is the linear matrix $A(A^T A)^{-1} A^T$.

2 Spectral Theorem
Suppose that $A$ is a symmetric $n \times n$ real matrix. Show that $A$ has the following properties:

1. The eigenvalues are all real.

2. $A$ has a complete set of eigenvalues and eigenvectors, i.e., Its eigenvectors span a space of dimension $n$.

3. $A = U^T \Lambda U$ where the rows of $U$ are an orthonormal set of eigenvectors for $A$ and $\Lambda$ is a diagonal matrix of eigenvalues for $A$.

4. If $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of $A$ and $x_1, \ldots, x_n$ are the respective orthonormal eigenvectors as column vectors then
   $$ A = \lambda_1 x_1 x_1^T + \cdots + \lambda_n x_n x_n^T $$

3 Matrix Exponential
Assuming that $A$ is real symmetric, use the Spectral Theorem show that $e^A$ is well defined and give a simple expression for it. That is, how do the eigenvalues and eigenvectors of $A$ relate to those of $e^A$?