Spectral Graph Theory

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The Interplay between:

1) Graph Theory
2) Linear Algebra
3) Numerical Computing

Upside: Faster Algorithms

1) Single Source Shortest Paths
   (possibly negative edge weights)

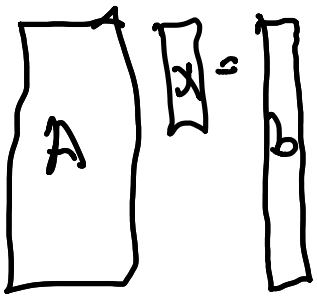
Bellman-Ford: $O(m \cdot n)$ time

m = #edges & n = #vertices

2016: New Method $O(m^{10/7})$

Regression Probs:

Simplest Case: Over Constrained System

e.g. 

Goal $\min_x \|Ax-b\|_2^2$

Standard Answer.

Solve $A^TA = A^Tb$

Prob: Computing A^TA is $O(mn^2)$

New Methods: Find B st $B^TBx' = B^Tb$

then $x \approx x'$

Faster than $O(mn^2)$ time.

Graph Cut Sparsifiers

Input: Graph $G=(V,E)$

Output: Weighted subgraph $H \subseteq G$

s.t. $\forall S \subseteq V$

$$Cut_G(S, \bar{S}) \approx Cut_H(S, \bar{S})$$

Known bounds on number of edges in H.

1) $O(n \log n)$ edges

2) $O(n)$ edges