Search I

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Read Russell & Norvig Sections 3.1-3.4.
(Also read Chapters 1 and 2 if you haven't already.)

Next time we'll cover topics related to Section 6.

Search I

Goal-based agent (problem solving agent)

Goal formulation (from preferences). Romania example, (Arad → Bucharest)

Problem formulation: deciding what actions & state to consider.
E.g. not “move leg 2 degrees right.”

No map vs. Map

physical search — deliberative search

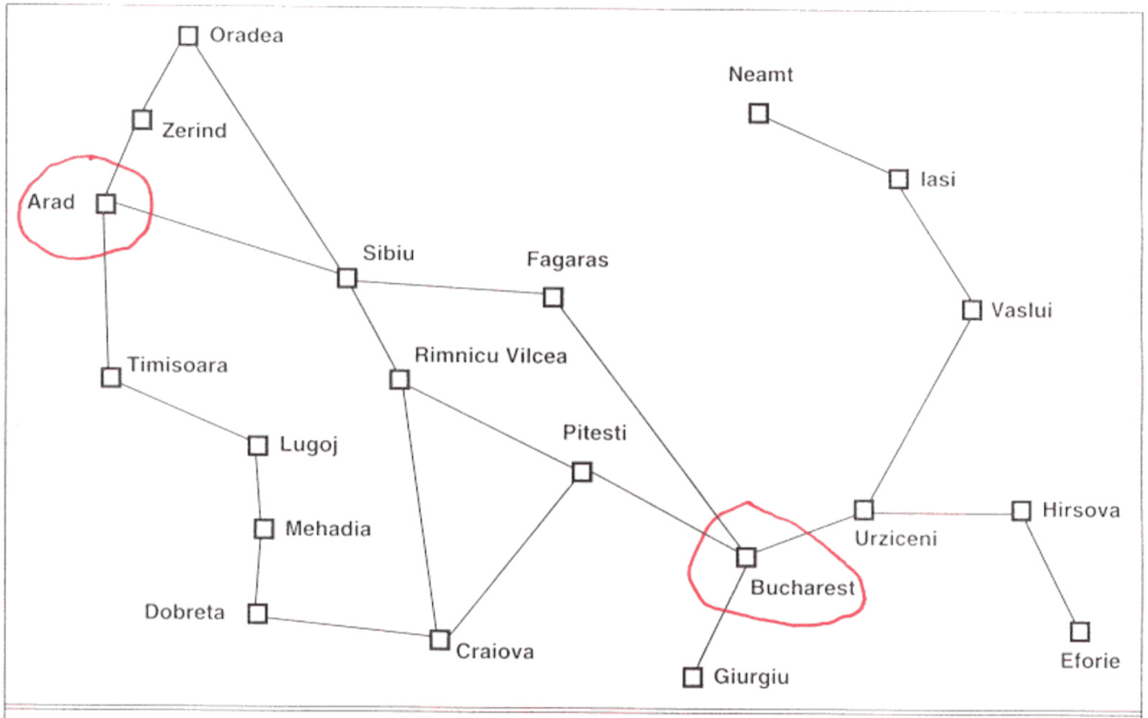


Figure 3.3 A simplified road map of Romania.

Search I

"Formulate, Search, Execute" (sometimes interleave search & execution)

For now we assume

- full observability, i.e., known state
- known effects of actions

Data type *problem*

- Initial state (perhaps an abstract characterization) vs. partial observability (set)
- Operators
- Goal-test (maybe many goals)
- Path-cost-function

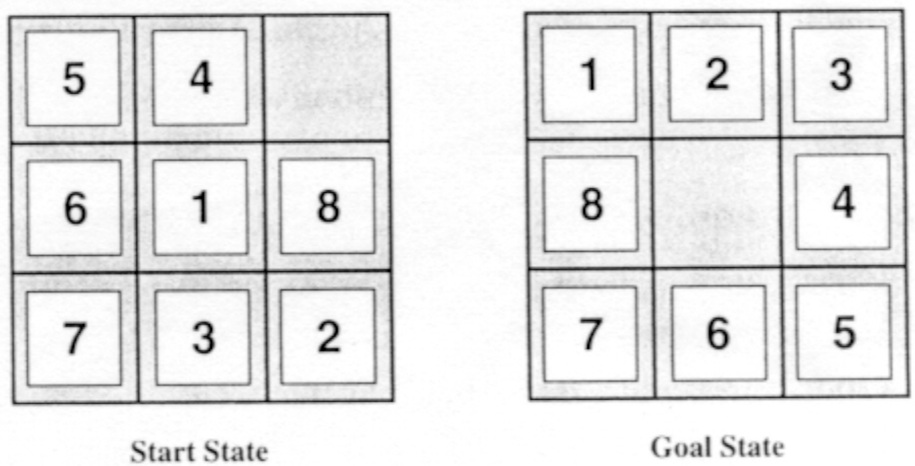
Knowledge representation

- Mutilated chess board example
- Can make huge speed difference in integer programming, e.g., edge versus cycle formulation in kidney exchange

Search I

Example problems demonstrated in terms of the problem definition.

I. 8-puzzle (general class is NP-complete)



A typical instance of the 8-puzzle.

How to model operators? (moving tiles vs. blank)
Path cost = 1

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II. 8-queens (actually, even the general class with n queens happens to have an efficient solution, so search would not be the method of choice)

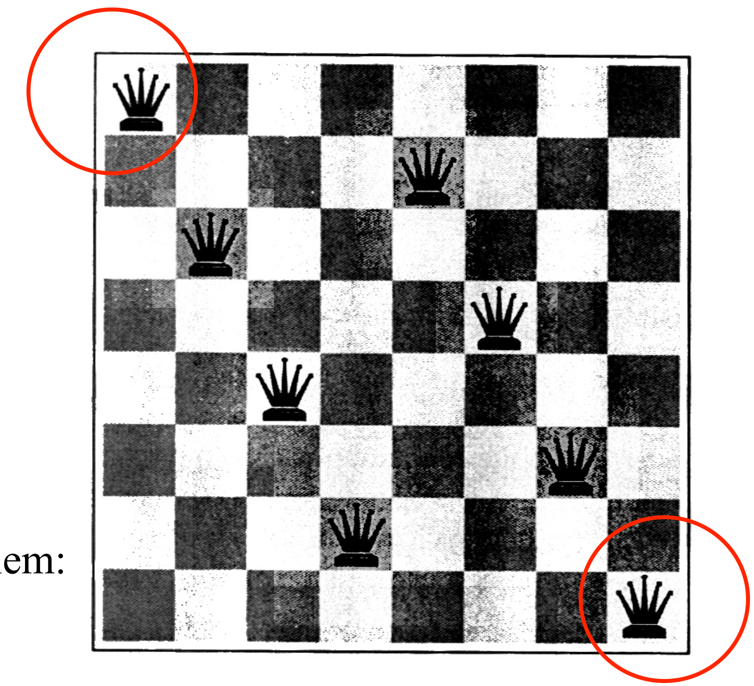
path cost = 0: in this application we are interested in a node, not a path

Incremental formulation:
(constructive search)
States: any arrangement of 0 to 8 queens on board
Ops: add a queen to any square
 # sequences = 64^8

Complete State formulation:
(iterative improvement)
States: arrangement of 8 queens, 1 in each column
Ops: move any attacked queen to another square in the same column

Improved incremental formulation:
States: any arrangement of 0 to 8 queens on board *with none attacked*
Ops: place a queen in the left-most empty column s.t. it is not attacked by any other queen
 # sequences = 2057

Almost a solution to the 8-queen problem:



Search I

III. Rubik' cube ~ 10^{19} states

IV. Crypt arithmetic

	FORTY		29786
+	TEN	+	850
+	TEN	+	850
	SIXTY		31486

V. Real world problems

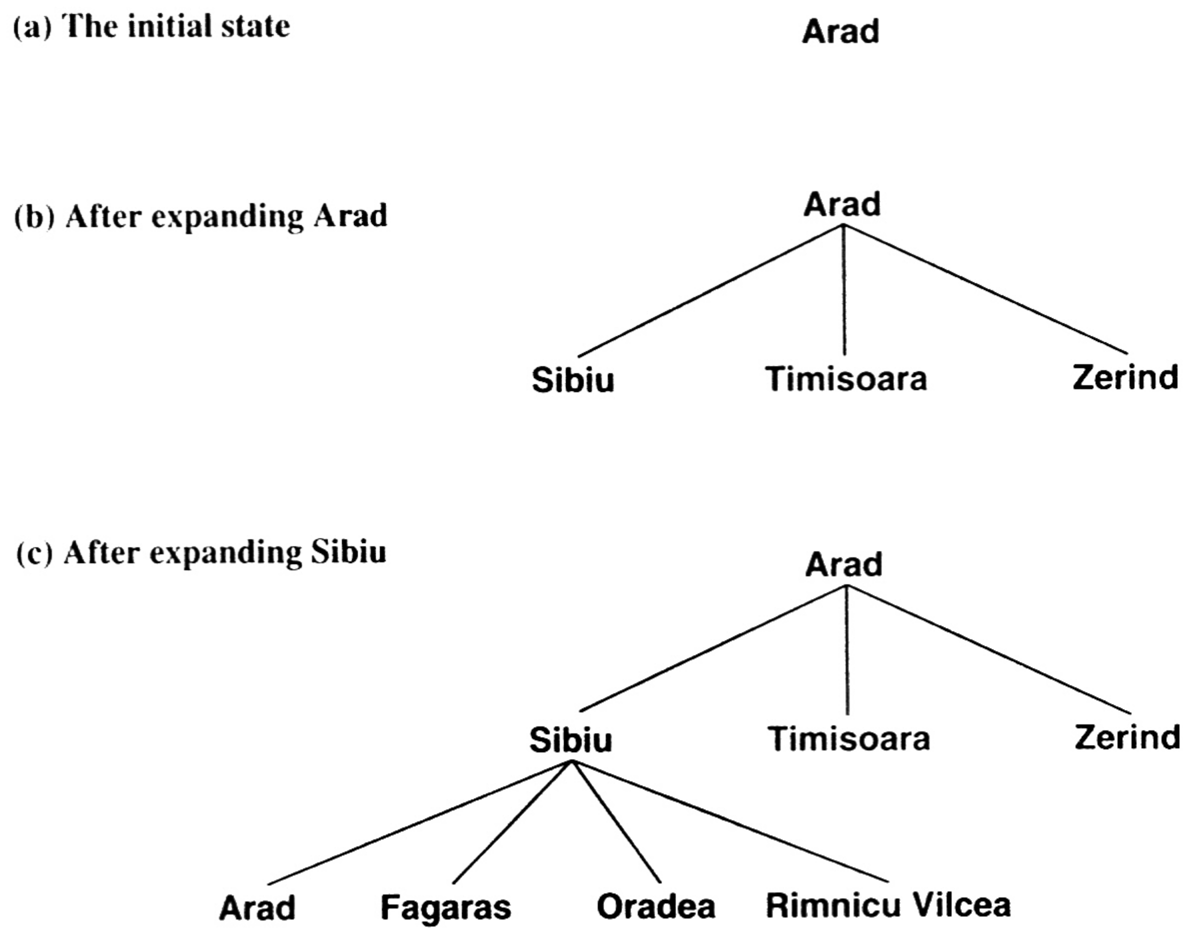
1. Routing (robots, vehicles, salesman)
2. Scheduling & sequencing
3. Layout (VLSI, Advertisement, Mobile phone link stations)
4. Winner determination in combinatorial auctions
5. Which combination of cycles to accept in kidney exchange?

…

Data type *node*

- State
- Parent-node
- Operator
- Depth
- Path-cost

Fringe = frontier = open list (as queue)



Partial search tree for route finding from Arad to Bucharest.

function GENERAL-SEARCH(*problem*, QUEUING-FN) **returns** a solution, or failure

```
nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
loop do
    if nodes is empty then return failure
    node ← REMOVE-FRONT(nodes)
    if GOAL-TEST[problem] applied to STATE(node) succeeds then return node
    nodes ← QUEUING-FN(nodes, EXPAND(node, OPERATORS[problem]))
end
```

The general search algorithm. (Note that QUEUING-FN is a variable whose value will be a function.)

Goodness of a search strategy

- Completeness
- Time complexity
- Space complexity

search cost

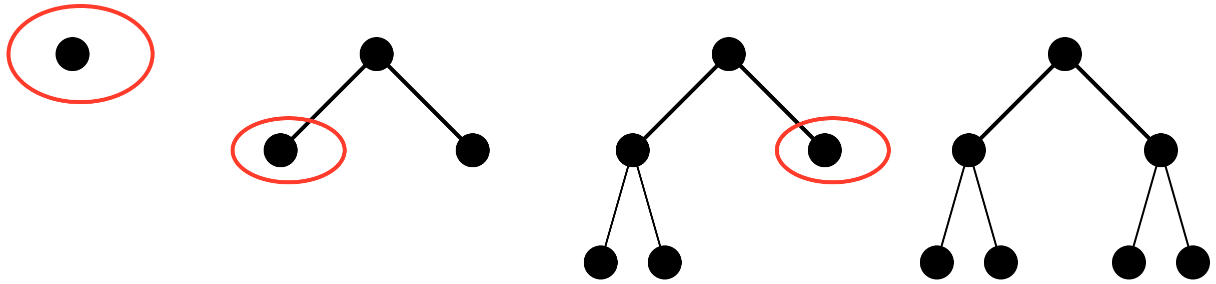
- Optimality of the solution found
  (path cost = domain cost)
- Total cost = domain cost + search cost

Uninformed vs. informed search

Can only distinguish goal states from non-goal state

Breadth-First Search

function BREADTH-FIRST-SEARCH (*problem*) **returns** a solution or failure
  return GENERAL-SEARCH (*problem*, ENQUEUE-AT-END)



Breadth-first search tree after 0,1,2 and 3 node expansions

Breadth-First Search …

Max $1 + b + b^2 + \ldots + b^d$ nodes (d is the depth of the shallowest goal)

- Complete
- Exponential time & memory $O(b^d)$
- Finds optimum if path-cost is a non-decreasing function of the depth of the node.

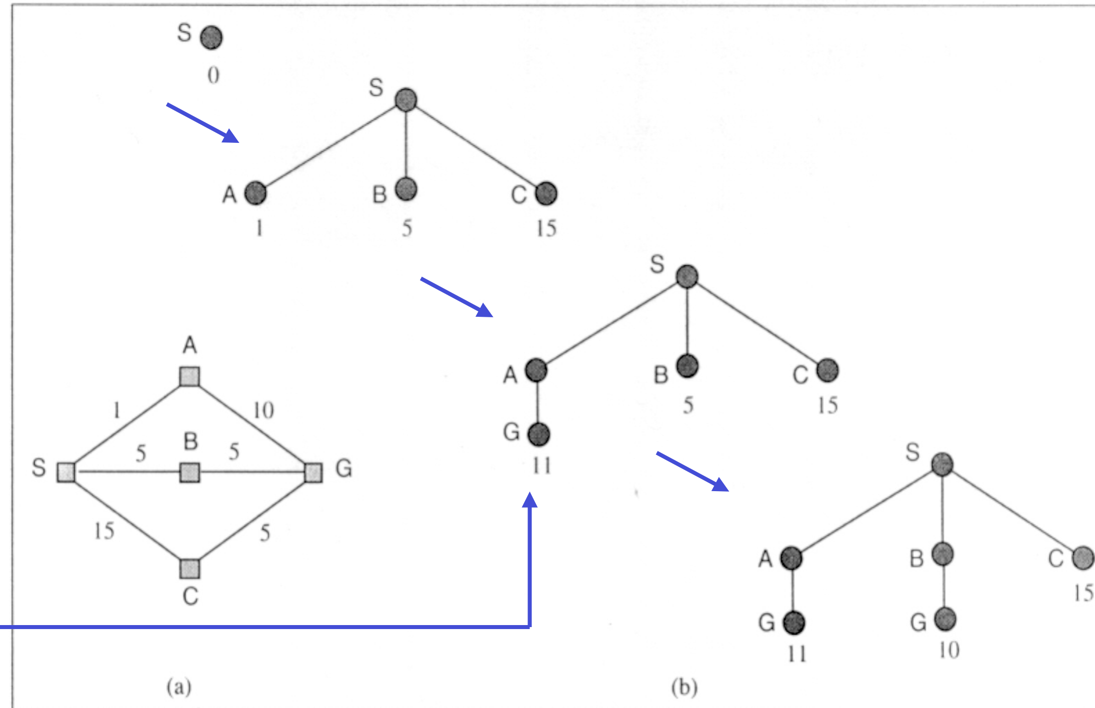
Depth	Nodes	Time	Memory
0	1	1 millisecond	100 bytes
2	111	.1 seconds	11 kilobytes
4	11,111	11 seconds	1 megabyte
6	10^6	18 minutes	111 megabytes
8	10^8	31 hours	11 gigabytes
10	10^{10}	128 days	1 terabyte
12	10^{12}	35 years	111 terabytes
14	10^{14}	3500 years	11,111 terabytes

Time and memory requirements for breadth-first search. The figures shown assume branching factor $b = 10$; 1000 nodes/second; 100 bytes/node.

Uniform-Cost Search

Insert nodes onto open list in ascending order of g(h).

G inserted into open list although it is a goal state. Otherwise cheapest path to a goal may not be found.



A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Finds optimum if the cost of a path never decreases as we go along the path.

$g(\text{SUCCESSORS}(n)) \geq g(n)$

$\Leftarrow$ Operator costs ≥ 0

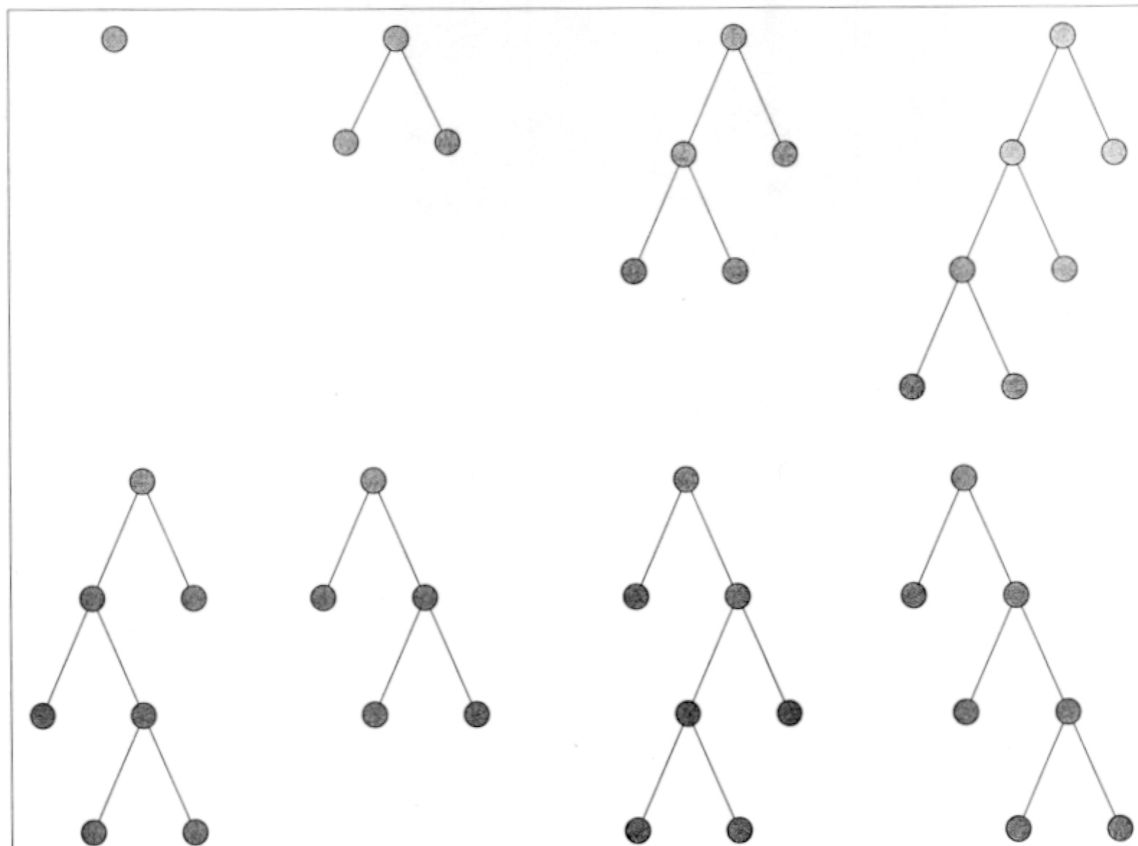
If this does not hold, nothing but an exhaustive search will find the optimal solution.

Depth-First Search

function DEPTH-FIRST-SEARCH (*problem*) **returns** a solution or failure
GENERAL-SEARCH (*problem*, ENQUEUE-AT-FRONT)

Alternatively can use a recursive implementation.

- Time $O(b^m)$ (m is the max depth in the space)
- Space O(bm) !
- Not complete (m may be ∞)
  - E.g. grid search in one direction
- Not optimal



Depth-first search trees for a binary search tree. Nodes at depth 3 are assumed to have no successors.

Depth-Limited Search

- Like depth-first search, except:
  - Depth limit in the algorithm, or
  - Operators that incorporate a depth limit

L = depth limit

Complete if $L \geq d$ (d is the depth of the shallowest goal)

Not optimal (even if one continues the search after the first solution has been found, because an optimal solution may not be within the depth limit L)

$O(b^L)$ time

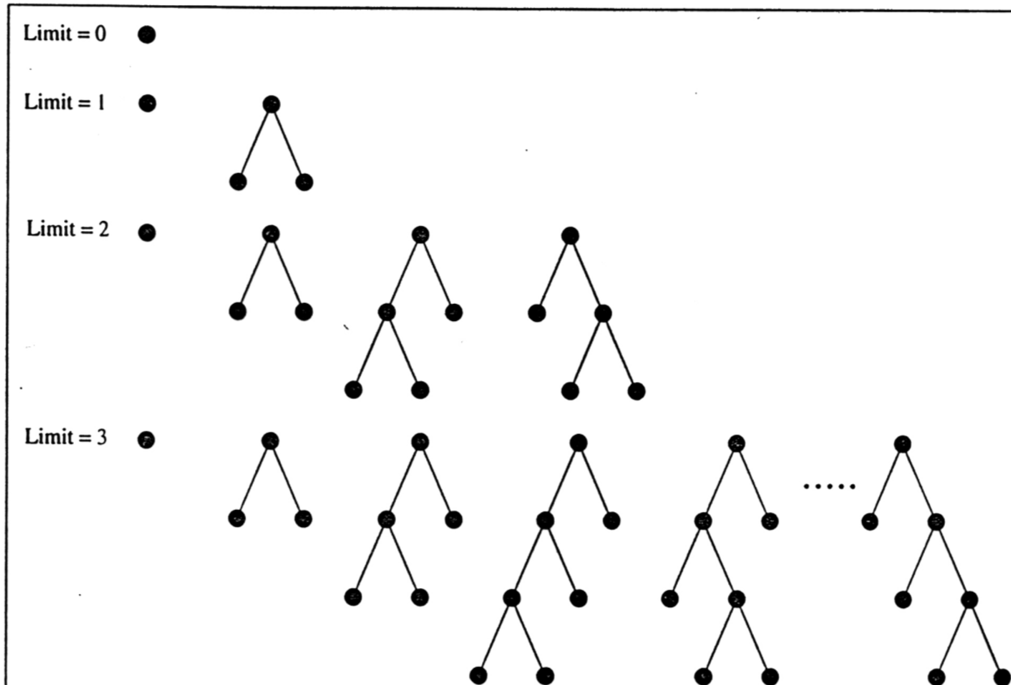
$O(bL)$ space

Diameter of a search space

Iterative Deepening Search

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution sequence
  inputs: *problem*, a problem

  for *depth* ← 0 **to** ∞ **do**
    if DEPTH-LIMITED-SEARCH(*problem*, *depth*) succeeds **then return** its result
  end
  return failure



Four iterations of iterative deepening search on a binary tree.

Iterative Deepening Search

Complete, optimal, $O(bd)$ space

What about run time?
Breadth first search:
$1 + b + b^2 + \ldots + b^{d-1} + b^d$
E.g. b=10, d=5: 1+10+100+1,000+10,000+100,000 = 111,111
Iterative deepening search:
$(d+1)*1 + (d)*b + (d-1)*b^2 + \ldots + 2b^{d-1} + 1b^d$
E.g. 6+50+400+3000+20,000+100,000 = 123,456
In fact, run time is asymptotically optimal: $O(b^d)$. We prove this next…

Next, we examine the running time of DFID on a tree. The nodes at depth d are generated once during the final iteration of the search. The nodes at depth $d-1$ are generated twice, once during the final iteration at depth d, and once during the penultimate iteration at depth $d-1$. Similarly, the nodes at depth $d-2$ are generated three times, during iterations at depths d, $d-1$, and $d-2$, etc. Thus the total number of nodes generated in a depth-first iterative-deepening search to depth d is

$$b^d + 2b^{d-1} + 3b^{d-2} + \cdots + db\,.$$

Factoring out b^d gives

$$b^d(1 + 2b^{-1} + 3b^{-2} + \cdots + db^{1-d})\,.$$

Letting $x = 1/b$ yields

$$b^d(1 + 2x^1 + 3x^2 + \cdots + dx^{d-1})\,.$$

This is less than the infinite series

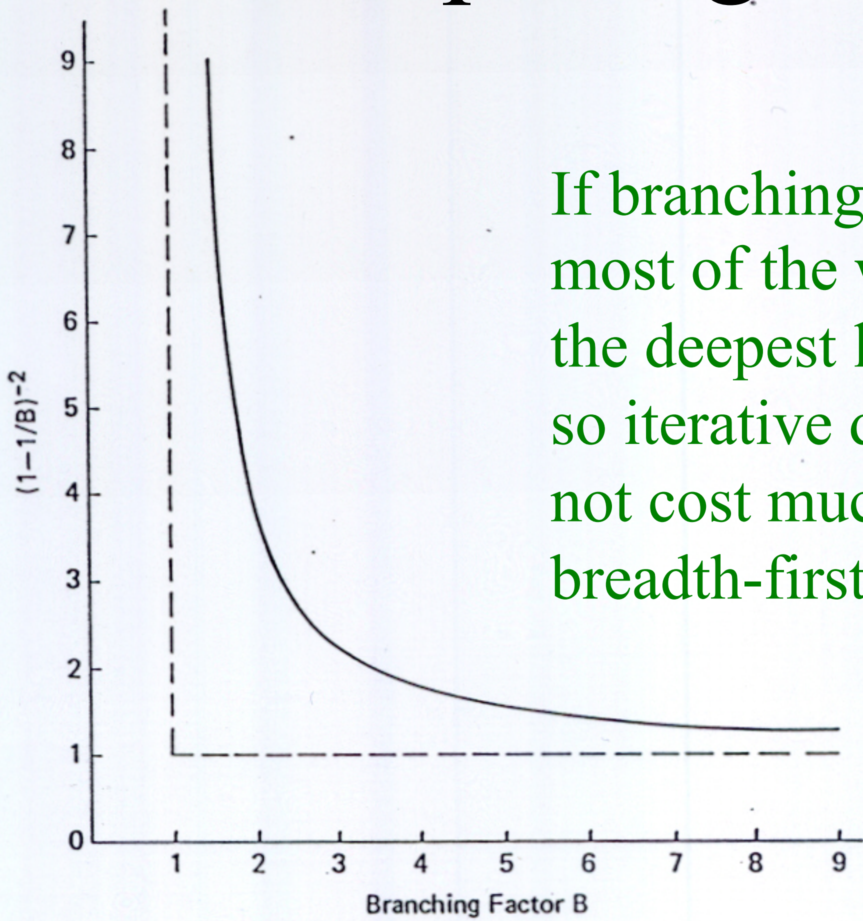
$$b^d(1 + 2x^1 + 3x^2 + 4x^3 + \cdots)\,,$$

which converges to

$$b^d(1-x)^{-2} \quad \text{for abs}(x) < 1\,.$$

Since $(1-x)^{-2}$, or $(1-1/b)^{-2}$, is a constant that is independent of d, if $b > 1$ then the running time of depth-first iterative-deepening is $O(b^d)$.

Iterative Deepening Search…



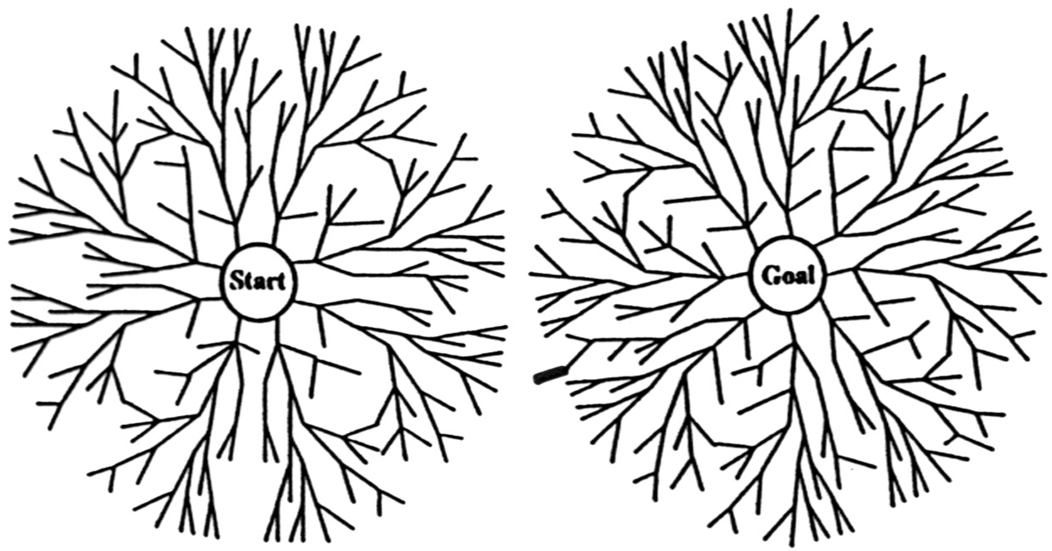
Graph of branching factor vs. constant coefficient as search depth goes to infinity.

If branching factor is *large*, most of the work is done at the deepest level of search, so iterative deepening does not cost much relatively to breadth-first search.

Conclusion:

- Iterative deepening is preferred when search space is large and depth of (optimal) solution is unknown
- Not preferred if branching factor is tiny (near 1)

Bi-Directional Search



A schematic view of a bidirectional breadth-first search that is about to succeed, when a branch from the start node meets a branch from the goal node.

Time $O(b^{d/2})$

Bi-Directional Search …

Need to have operators that calculate predecessors.
What if there are multiple goals?
• If there is an explicit list of goal states, then we can apply a predecessor function to the state set just as we apply the successors function in multiple-state forward search.
• If there is only a description of the goal set, it MAY be possible to figure out the possible descriptions of "sets of states that would generate the goal set"…

Efficient way to check when searches meet: hash table
- 1-2 step issue if only one side stored in the table
Decide what kind of search (e.g. breadth-first) to use in each half.

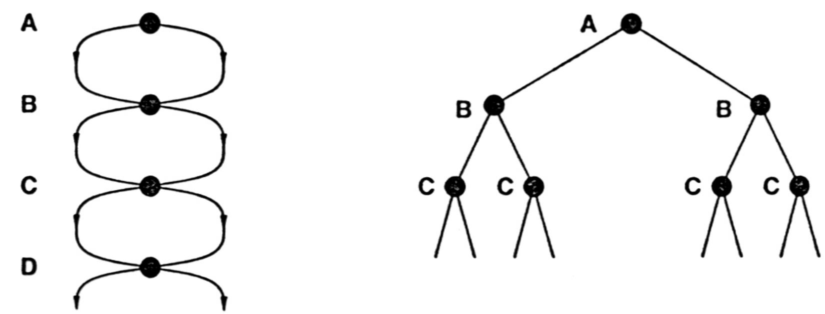
Optimal, complete, $O(b^{d/2})$ time. $O(b^{d/2})$ space (even with iterative deepening) because the nodes of at least one of the searches have to be stored to check matches

Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

b = branching factor
d = depth of shallowest goal state
m = depth of the search space
l = depth limit of the algorithm

Avoiding repeated states



A state space that generates an exponentially larger search tree. The left-hand side shows the state space, in which there are two possible actions leading from A to B, two from B to C, and so on. The right-hand side shows the corresponding search tree.

More effective & more computational overhead

- Do not return to the state you just came from. Have the expand function (or the operator set) refuse to generate any successor that is the same state as the node's parent.
- Do not create paths with cycles in them. Have the expand function (or the operator set) refuse to generate any successor of a node that is the same as any of the node's ancestors.
- Do not generate any state that was ever generated before. This requires every state that is generated to be kept in memory, resulting in a space complexity of $O(b^d)$, potentially. It is better to think of this as $O(s)$, where s is the number of states in the entire state space.

To implement this last option, search algorithms often make use of a hash table that stores all the nodes that are generated. This makes checking for repeated states reasonably efficient. The trade-off between the cost of storing and checking and the cost of extra search depends on the problem: the "loopier" the state space, the more likely it is that checking will pay off.

With loops, the search tree may even become infinite

A detail about Option 3 of the previous slide

- Once a better path to a node is found, all the old descendants (not just children) of that node need to be updated for the new g value
  - This can be done by having both parent and child pointers at each node
  - This can still be a win in overall time because that tree that needs to be updated can still grow later, so we don't want to replicate it