Algorithms Core 15-750 Final Spring 1998

DATE: 1-4PM Monday 5 May 1998, in Wean 5409

NAME: ______________________ EMAIL: ______________________

1	2	3	4	5	Total	6
32	15	15	15	15	92	15 extra

You may use the three books from the course (CLR, AHU, Kozen), your personal notes, and class handouts.

Answer each question on separate sheets of paper for grading purposes. Make sure that each page has your name and email address. You will get full credit for the first 5 questions. The sixth question is for extra credit

Make sure you include with each algorithm you design the following three bullets.

DESCRIPTION: A high level description of the algorithm.

CORRECTNESS: At least a high level outline of how the full proof would go. Possibly citing lemmas from class.

TIMING: A timing analysis.

The algorithms will be graded based on what you put in each bullet.

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1 Short Answer Questions. [4Pts per question. Total 32Pts]

For each statement answer whether it is true our false include at most 2 short sentences for explanation.

1) Strassen Algorithm for matrix multiplication can be applied to any semi-ring provided we have additive and multiplicative identity elements.

2) Treaps are efficient search trees for performing inserts, deletes and search, but the analysis in class and in Kozen does not extend to join.

3) Using Fibonacci heaps we can delete elements in constant worst case time.

4) Tarjan's Union-Find data structure is very fast, almost linear time, but it also uses more than linear space.

5) The following matching problem is NP-hard. Given a graph, not necessarily bipartite, find a maximum number of vertex disjoint edges.

6) The graph 4-color problem is NP-complete, i.e., the set of all 4-colorable graphs is NP-complete.

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7) We know that the expected number of comparisons for sorting a randomly list of n elements bounded below by $\Omega(n \log n)$. Therefore the expect number of comparisons needed to compute the convex hull of n random points in, say, a the unit square in the plane will also be bounded below by $\Omega(n \log n)$.

8) The $s-t$ maximum flow problem can be expressed as a linear programming problem but the number of constraints in general is exponential in the size of the graph.

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2 Finding a Square in the Plane[15Pts]

Give an $O(n^2 \log n)$ time algorithm for the following problem: Given n points in the plane determine if any four of the points form the vertices of a square. The square need not be aligned with the x or y axis.

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3 Boolean Matrix Multiplication[15Pts]

Suppose that A and B are two $n \times n$ Boolean matrices. Further assume each element in A and B is chosen randomly and independently from the set $\{0,1\}$. Give an algorithm which computes the product $A \cdot B$ over the Boolean semi-ring $(\{0,1\}, \vee, \wedge, 0, 1)$ in expected time $O(n^2)$.

e.g.,

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

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4 IP for Maximum Independent Set[15Pts]

Let $G = (V, E)$ be an undirected graph. Show how to express the maximum independent set problem for G as a zero-one integer programming problem.

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5 Solving a Recurrence Relation by Induction[15Pts]

Consider the following recurrence relation

$$T(1) = 0$$

$$T(n) \leq \left[(\frac{2}{n-1}) \sum_{i=1}^{n-1} T(i) \right] + c \quad \text{for} \quad n \geq 2.$$

By induction show that $T(n) = O(n)$. The c is a positive constant.

Hint: You may want to assume by induction that $T(n) \leq an + b$ for some constants a and b, and sufficiently large n.

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6 Covering Half the Edges[15Pts]

Let G be an undirected graph. A "half cover" of size k is a set of k vertices that includes at least one endpoint of at least half the edges of G. Consider the languages HALF-COVER consisting of all pairs G, k such that the graph G has a half-cover of size at most k. Show that the language HALF-COVER is NP-complete.

(Hint: Consider Vertex-Cover Problem.)