1 Review

- NP = easy to check "yes"
- coNP = easy to check "no"
- NP-hard = can reduce all of NP to this problem
- coNP-hard = can reduce all of coNP to this
- NP-complete = NP and NP-hard
- CoNP-complete = CoNP and CoNP hard

2 3-colorability $\equiv_m^p$ Planar 3-colorability

2.1 3-colorability $\leq_m^p$ planar 3-colorability

There is a 13 node graph ("widget") s.t. the 4 extreme points are constrained so that each pair of opposite corner points must be the same color. Whenever there is a crossing of lines in a planar drawing, insert one of these widgets.

2.2 3-colorability $\geq_m^p$ planar 3-colorability

obvious.

3 Some problems

3.1 knapsack

- S = a set of objects
- $w(a)$ = weight of $a \in S$
- $b(a)$ = benefit of $a \in S$

The knapsack problem is:

$$max \sum_{a \in S'} b(a)$$

subject to:

$$\sum_{a \in S'} w(z) \leq W$$

Phrased as a decision problem this is 'Is there a set S' where $B \geq k$'?

3.2 subset sum

Given a set of integers S is there a subset of S' such that $\sum_{a \in S'} w(a) = B$.

3.3 Partition

Given a set of integers S is there a partition into 2 sets with equal sum?

3.4 Bin packing

Can this list be partitioned into k bins such that each bin sums to $\leq B$.

4 partition $\equiv_m^p$ subset sum

4.1 partition $\leq_m^p$ subset sum

let $M = \sum_{s \in S} s$. Then let $B = \frac{M}{2}$

4.2 partition $\geq_m^p$ subset sum

Add the numbers

- $N - B$
- $N - (M - B)$

For N large. Each of these 2 numbers will be in different partitions and one of these partitions (minus $N - B$) is of the right size.

5 partition $\leq_m^p$ bin packing

Just do 2 bin packing.

6 Linear programming

At one time people didn't think that linear programming was polynomial.

6.1 the problem

pick $\{x_i\}$ to maximize $\sum_i c_i x_i$ subject to $\forall j \sum_i a_{ij} x_j \leq b_j$.

6.2 integer programming

Is there a choice of $\{x_i\}$ that satisfies the inequalities with integers?

This problem is np complete

6.3 subset sum $\leq_m^p$ integer programming

Put in the constraints:

- $\sum_i a_i x_i \leq B$
- $-\sum_i a_i x_i \leq -B$

which implies: $\sum_i a_i x_i = B$.

Also require: $\forall i \; 0 \leq x_i \leq 1$ to reduce subset sum to integer programming.

7 Hamiltonian circuit

Does this graph have a path going through every node exactly once and returning to the start?

7.1 vertex cover $\leq_m^p$ hamiltonian circuit

There is a 4 node directed 'widget' with 2 inputs and 2 outputs s.t. a hamiltonian path can enter through either (or both) input(s) and hit all nodes on a hamiltonian circuit.

A similar widget exists for the undirected case.

Then, you use these widgets in a way which I can't easily describe. For every node in the original graph a loop is created.

8 Travelling salesman problem

Is there a hamiltonian circuit such that:

Is there a hamiltonian circuit such that total cost $\leq k$?

8.1 hamiltonian circuit $\leq_m^p$ TSP

Put weight one on all of the nodes then ask if there is a TSP with $k = \#nodes$.

9 Minimum node deletion bipartite subgraph

Given a graph G, can I make it bipartite by only deleting k nodes?

Can clique $\leq_m^p$ MNDBS?

reduction: Take the original graph, and complement it. Add the number of nodes again, fully interconnecting a new node to the complemented graph. Is this a reduction?

10 New complexity classes

- NC = problems that are parallelizable = solvable in polylog time = $log(n)^{O(1)}$ with a polynomial # of processors.

Does P = NC? Probably not. Example: Does program A finish in time n?
The example is in P-complete.

- BPP = in polytime probably(with high probability) correct. (Bounded error probabilistic problem)
- BQP = same as BPP on a quantum computer.