Day 14 2/25/98 "I'm just trying to confuse you."

1 Planar Graphs

Definition: Planar Graph, G, can be drawn in plane without edges crossing.

Kusatoski's Theorem: G is nonplanar $\Leftrightarrow K_5$ or $K_{3,3}$ is a minor (homeomorphic subgraph). In constructing a homeomorphic subgraph by:

1. removing edges
2. removing isolated vertices.

Claim: if G is planar with no multiple edges then $m \leq 3n - 6$ proof: use euler's formula with a constraint on the maximum number of faces $f \leq \frac{2}{3}m$

definition: $G = (V, E)$ **with** $S \subseteq V$ **is** $f(n)$ **seperator if:**

- $|S| \leq f(n)$
- $\exists$ partition A, B of $V - S$ with $|A|, |B| \leq \frac{2}{3}n$
- there exists no edge from A to B

$\frac{2}{3}n$ is a bit arbitrary.

Square grid's have a $\sqrt{n}$ seperator and tree's have a 1 seperator.

Another definition: $G = (V, E)$ **with** $S \subseteq E$ **is** $f(n)$ **seperator if:**

- $|S| \leq f(n)$
- $\exists$ partition A, B with $|A|, |B| \leq \frac{2}{3}n$
- there exists no edge from A to B

Not every graph has a good edge seperator. For an n-star, there are only $\frac{n}{3}$ edge seperators. If every node has degree at most 3, then you can edge seperate.

2 Graph types

- K_i is a fully connected graph with i nodes
- $K_{n,m}$ is a graph with n vertices connected to another m vertices in every way possible

3 Graph transformations

3.1 geometric dual, G^*

The geometric dual is constructed by drawing an edge crossing each edge in G. Then connect all these new edges together not crossing the old edges in the process.

The geometric dual of K_4 is K_4.

4 combinatorial view of graphs

4.1 permutation view

Convert all edges into directed edges "darts". Number the darts. Cycle through the darts at a particular node, then at each node.

$$\phi = (132)(465)(78)(90)$$

Which means there are 4 connected nodes, the first has darts '1','2', and'3'. The cycling through the darts you go from 1 to 3 to 2 to 1 to 3 to 2 to ....

The reflection permutation moves you along a dart.

$$R = (18)(24)(39)(57)(60)$$

$\phi^* = R\phi = (174)(3058)(269)$

Reasoning: $1 \rightarrow^R 8 \rightarrow^\phi 7 \rightarrow^R 5 \rightarrow^\phi 4 \rightarrow^R 2 \rightarrow^\phi 1$ (continuing forever) Taking every other element, you get: (174). Continue similarly for each cycle. $2 \rightarrow^R 4 \rightarrow^\phi 6 \rightarrow^R 0 \rightarrow^\phi 9 \rightarrow^R 3 \rightarrow^\phi 2$ yields (269).

Each cycle (xy...z) is associated with a face of the graph.

4.2 Edge addition in permutation land

Start with the graph:

$$\phi = (53)(76)(28)(41)$$

$$R = (12)(34)(56)(78)$$

$$\phi^* = R\phi = (1863)(7245)$$

which goes to:

$$\phi = (53)(706)(28)(491)$$

$$R = (12)(34)(56)(78)(90)$$

$$\phi^* = R\phi = (180)(7245)(963)$$

4.3 euler's number

The permutation representation works for more than planar graphs. One interesting question is how many handles the surface which the graph can be embedded on has. In other words: how many holes does the graph have? Examples surfaces: sphere, torus, 2-torus, ...

Use euler's formula: $f - m + n = 2$ where $f = \#faces$, $m = \#edges$, $n = \#vertices$

This can be calculated quickly from the cycle representation.

5 Definitions of tree

- connected graph with $m = n - 1$
- minimal connected graph
- maximally connected graph with no cycles.