1 Shur complement

The Shur complement method is an asymptotically better way to calculate the all points shortest path matrix.

1. Partition the graph into 2 sets of nodes $\{S, T\}$

There will be 4 kinds of edges:

$$\begin{array}{lll} From/To & T & S \\ T & A & B \\ S & C & D \end{array} = M$$ As an example, B contains all edges from T to S

1. compute A^* (all internal connections in T)
2. $F = D + CA^*B$
3. F^*

Form the matrix:

$$M^* = \begin{pmatrix} A^* + A^*BF^*CA^* & A^*BF^* \\ F^*CA^* & F^* \end{pmatrix}$$

This matrix is the all shortest path matrix.

Proof: Consider each possible path in terms of combinations of A, B, C, D and think about which entries these will include.

$(CA^*B)_{ij}$ = shortest path form i to j using only vertices in T.

1.1 Timing analyses

$T(n) = 2T(\frac{n}{2}) + cn^3 = cn^3 + 2c(\frac{n}{2})^3 + ... = O(n^3)$ This is better than the all points shortest path calculated by rings method.