1 Motivation

Let $\oplus$ be an associative binary operation:

Definition: All prefix sums

input : $[a_0, ..., a_{n-1}]$

output: $[a_0, a_0 \oplus a_1 \oplus a_3, ..., a_0 \oplus ... \oplus a_{n-1}]$

Prescan output: $[I, a_0, a_0 \oplus a_1, ..., a_0 \oplus ... \oplus a_{n-2}]$

Application: Packing Memory

![Figure 1: Packing memory](image)

2 Prescan

input : $(3,1,7,0,4,1,6,3) \oplus$ addition

Here is the algorithm:

Algorithm:

1. Compute tree of partial sums

2. Set root to zero

3. DOWN!

   (a) Right child $\leftarrow$ Parent $\oplus$ Left child

   (b) Left-child $\leftarrow$ Parent

Here is an example:

For this algorithm

$T(n) = \omega(\log n), W(n) = O(n)$
3 List Ranking

Input: linked list
Output: a mark on each node such that mark = distance from head or mark = distance to tail

Assume:

1. Pointers are in consecutive memory
2. We know location of head and tail
3. Pointers are in arbitrary order
4 Wyllie’s Algorithm

Algorithm 1 Wyllie’s
1: In parallel rank(!) = 1 ; rank(tail) = 0
2: In parallel while succ(head) ≠ nil do
3:   if succ(!) ≠ nil do then
4:      rank(!) = rank(!) ⊕ rank(succ(!))
5:   end if
6: end if

Using n processors and a CREW model for memory, this algorithm does ω(n log n) work in ω(log n) time. Our goal is to reduce it to ω(n) work in ω(log n) time.

5 Random-Mate

Contraction Phase

1. Each live node randomly picks a sex
2. If F → M → X then F → X, M dies
3. Stop when head points to NULL (Only head is alive)

5.0.1 How many rounds needed?

Theorem 5.1. The contraction phase stops in $c \log n$ rounds with high probability.

Proof. Let $P_i$ = Event that node $i$ is still alive after one round.
Note: If node $i$ is some other node besides head, then $\text{Prob}(P_i) = \frac{3}{4}$
Let $P_i^k$ = Event that node $i$ is still alive after $k$ rounds.
Note: $\text{Prob}(P_i^k) = (\frac{3}{4})^k i$ not head
Set $k = c \log (\frac{4}{3})n$

$\text{Prob}(P_i^k) = \left(\frac{1}{4}\right)^k \leq \left(\frac{1}{4}\right)^{c \log (\frac{4}{3}) n} = \frac{1}{n^c}$

Let $P_k$ = Event that some non-head node is still alive. Assume that node$_0$ is the head.

$P_k = P_1^k \cup P_2^k \cup ... \cup P_n^k$

$\text{Prob}(P_k) = \text{Prob}[P_1^k \cup ... \cup P_n^k]$

$\leq \text{Prob}[P_1^k] + ... + \text{Prob}[P_n^k]$

$\leq n \cdot \frac{1}{n^c} = \frac{1}{n^{c-1}}$

If we set $c = 2$ then the contraction phase stops with probability $\leq \frac{1}{n}$.

In the expansion phase we run contraction phase “backwards”.

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Figure 5: