15-750
4/26/19

All Prefix Sums & Prescan

Let $\oplus$ be an associative binary op.

eg $+, \cdot$

Def All-Prefix-Sums

Input $[a_0, \cdots, a_{n-1}]$

output $[a_0, a_0 \oplus a_1, a_0 \oplus a_1 \oplus a_2, \cdots, a_0 \oplus \cdots \oplus a_{n-1}]$

Def Prescan

output $[I, a_0, a_0 \oplus a_1, \cdots, a_0 \oplus \cdots \oplus a_{n-2}]$

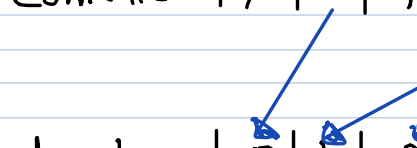
Note Prefix Sum (A) = Prescan $(A) \oplus A$

elementwise addition

Prescan Application

Packing Out Memory

location	0	1	2	3	4	5	6	7
contents	∅	*	∅	∅	*	*	∅	*



location	0	1	2	3	4	5	6	7
contents	*	*	*	*	∅	∅	∅	∅

Use Prescan

Input (0,1,0,0,1,1,0,1)

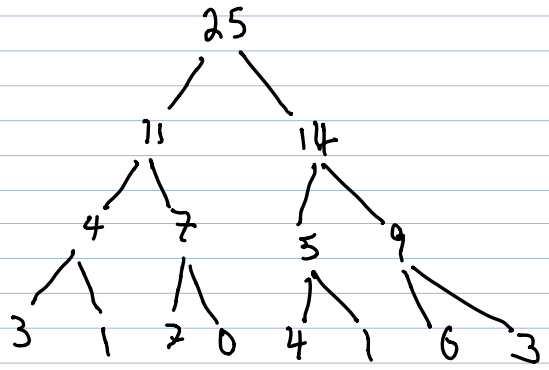
Prescan (0,0,1,1,1,2,3,3)

Note Google: MapReduce not defined for arrays!

Parallel Prefix By Example

Input $(3,1,7,0,4,1,6,3)$ $\oplus$ addition

Prefix Alg 1) Compute tree of partial sums.
2) Set root to zero
3) Run Down!



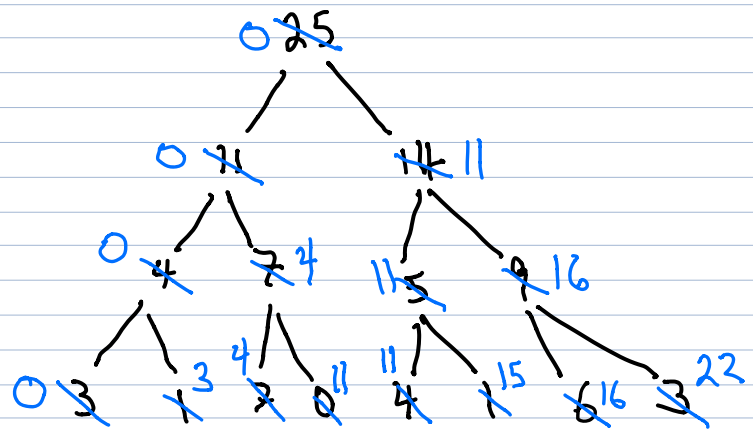
Proc Down: In ll do

a) Right-Child $\Leftarrow$ Parent $\oplus$ Left-child

b) Left-Child $\Leftarrow$ Parent

c) Repeat on children

Down



Precan (0, 3, 4, 11, 11, 15, 16, 22)

Input (3, 1, 7, 0, 4, 1, 6, 3)

$T(n) = O(\log n)$

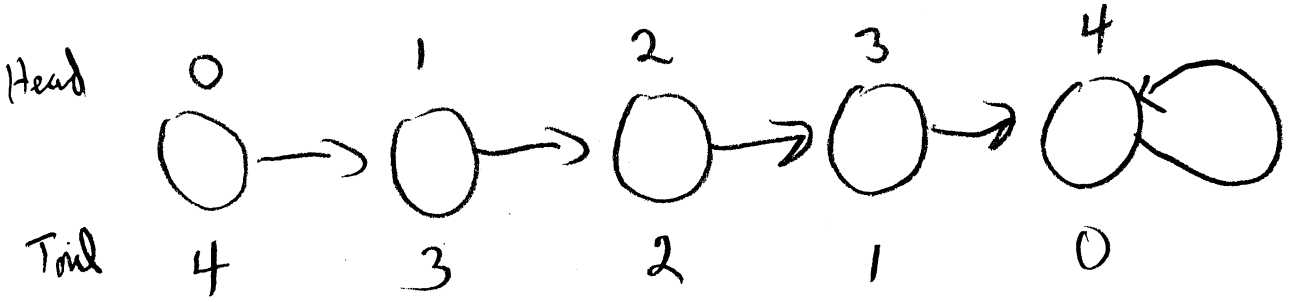
$P(n) = O(n)$ $\Rightarrow P.T = O(n \cdot \log n)$

List Ranking

input: linked-list

output: a mark on each node s.t.

mark = distance from head or
= distance to tail



Assume:

1) pointers are in consecutive memory
2) We know location of head & tail
3) pointers in arbitrary order.

Wyllie's Alg

In|| rank(!) := 1 ; rank(tail) = 0

In|| while succ(head) ≠ nil do

if succ(!) ≠ nil do

rank(!) := rank(!) ⊕ rank(succ(!))

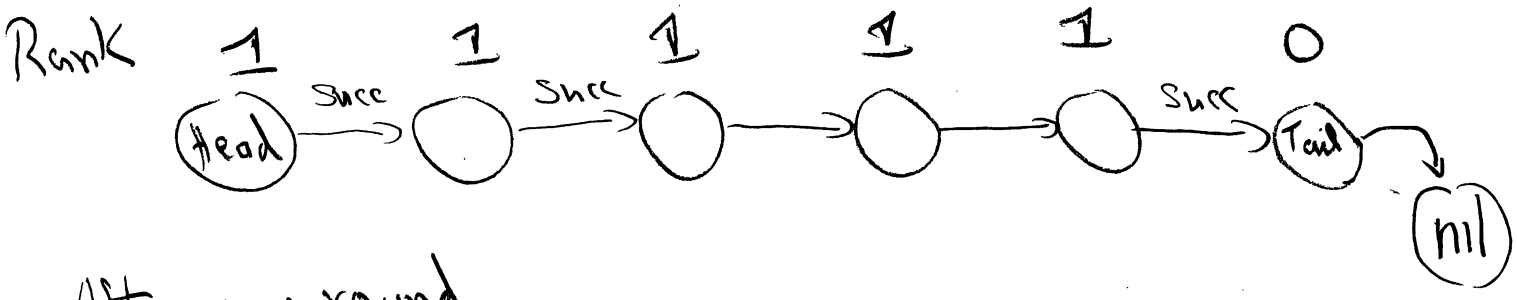
succ(!) := succ(succ(!))

fi

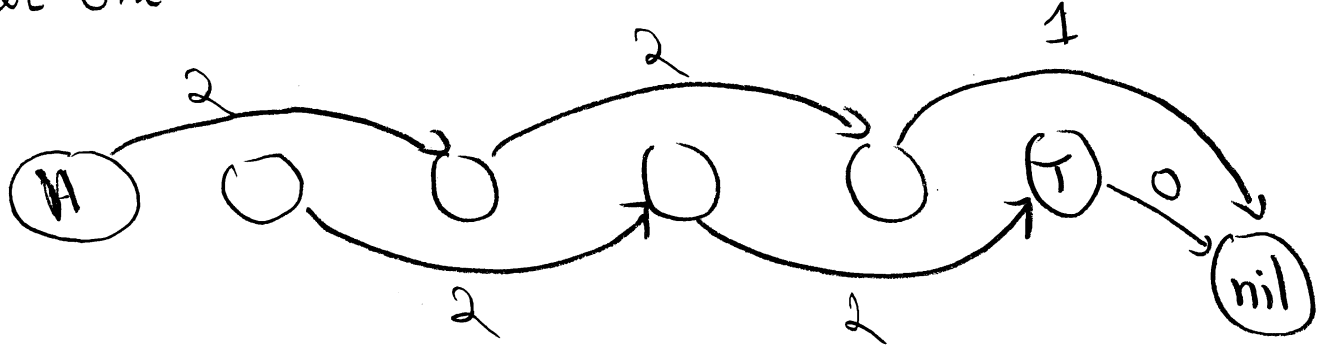
	Goal
# processors = n	
Time = $O(\log n)$	$O(\log n)$
Work = $O(n \log n)$	$O(n)$
CREW model	

Wyllie's Alg

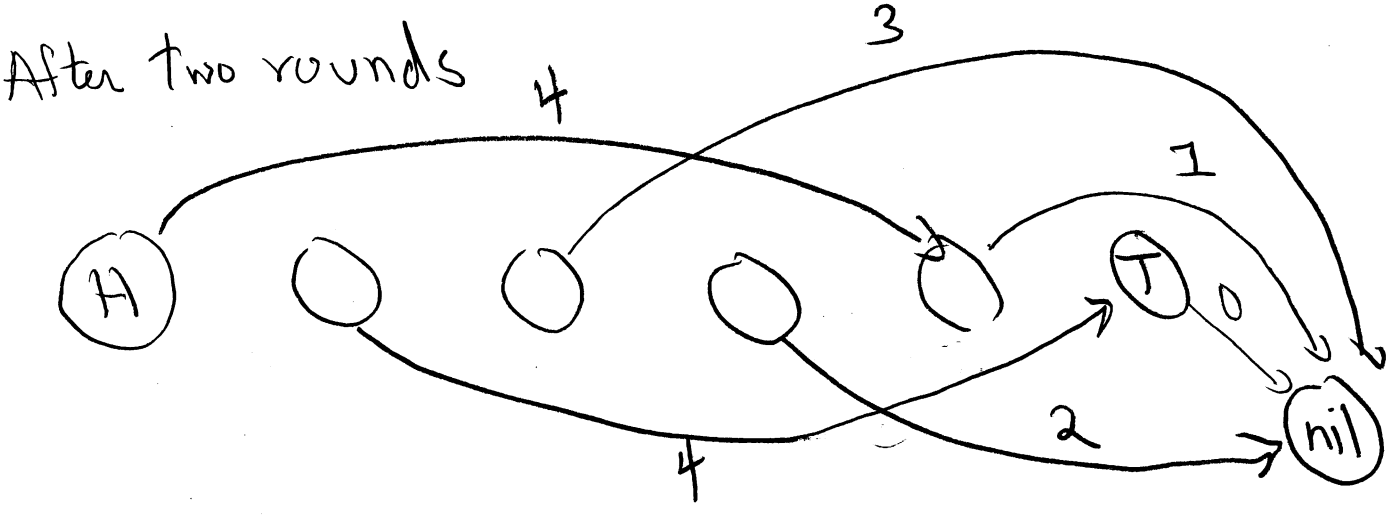
Initial



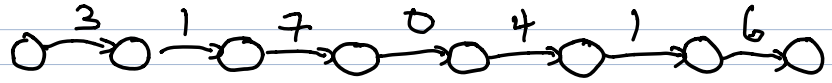
After one round



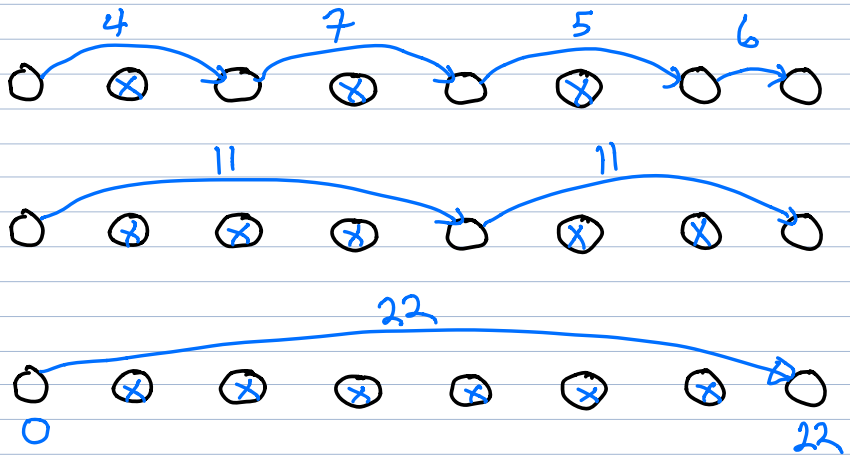
After two rounds



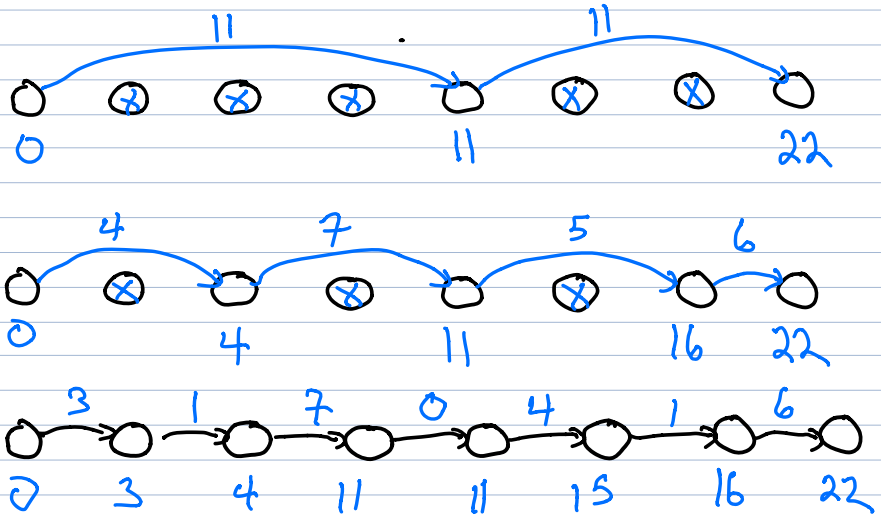
Prescan on a linked list



Recursively spliceout every other element!



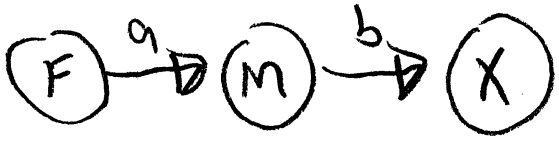
Fill in missing node values.



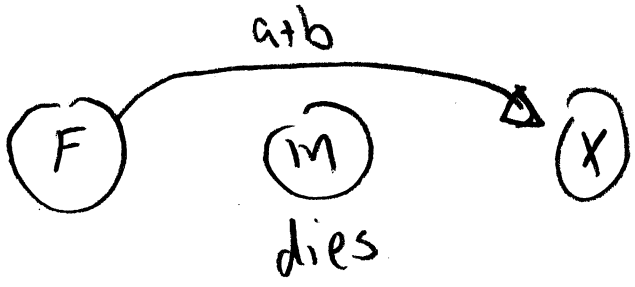
Random-Mate

1) Contraction Phase

   1) Each live node randomly picks a sex

   2) If 

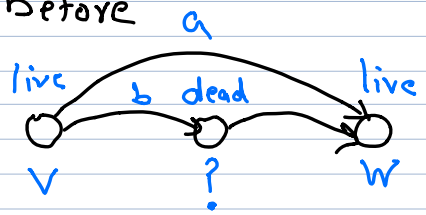
then

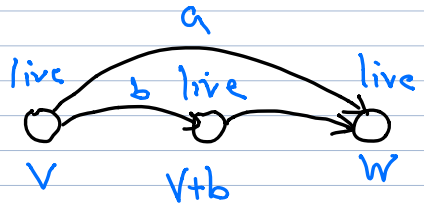


   3) Stop when head points to nil.
      (only head is live)

In the expansion phase we run contraction phase "backward".

eg before





Random Mate Timing Analysis

Thm The contraction phase for Random Mate terminates with high prob.

Pf Let $P_i^k \equiv$ Event that node i is still live after k rounds.

Note If node i not head then $\text{Prob}[P_i^1] = 3/4$

Since each round is independent

$\text{Prob}[P_i^k] = (3/4)^k$ for node i not head.

Set $k = \lceil c \log_{4/3} n \rceil$

$$\text{Prob}[P_i^k] \approx 1/(4/3)^k \leq \frac{1}{(4/3)^{c \log_{4/3} n}} = \frac{1}{n^c}$$

Let $P^k \equiv$ Event that some non-head node is still live.

Assume that node$_0$ is the head.

$$P^k = P_1^k \cup \cdots \cup P_n^k$$

$$\text{Prob}[P^k] = \text{Prob}[P_1^k \cup \cdots \cup P_n^k]$$

$$\leq \text{Prob}[P_1^k] + \cdots + \text{Prob}[P_n^k] \leq \frac{n}{n^c} = \frac{1}{n^{c-1}}$$

□

Euler Tours

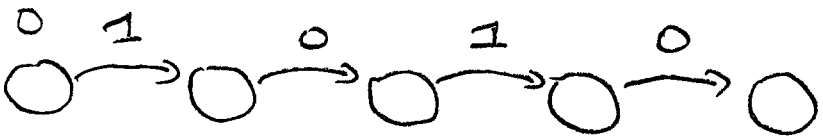
Prob: Preorder number a tree

Input: A tree stored using pointers

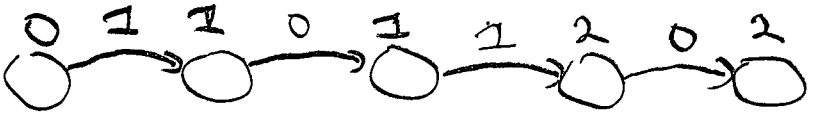
Output: Preorder numbering

Weighted List Ranking

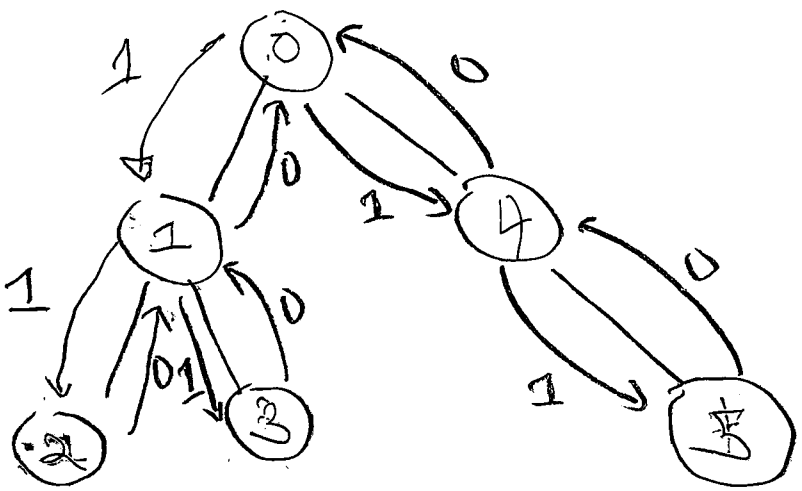
Input:



Output:



Preorder



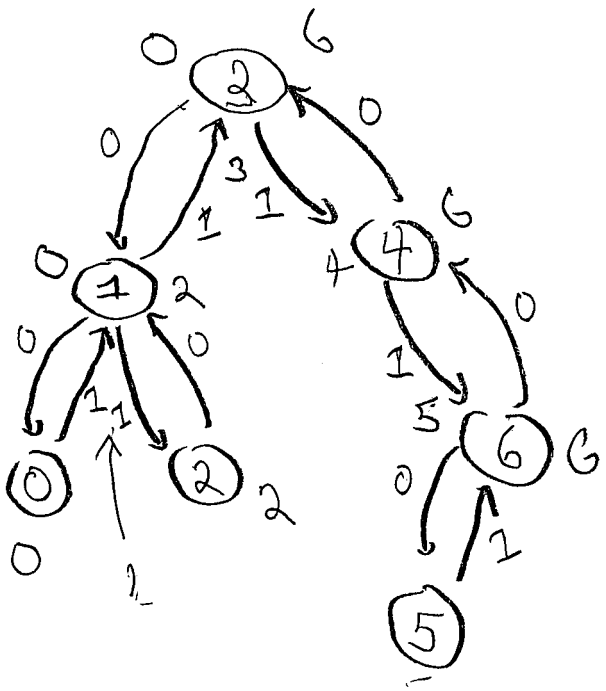
Use First-Visit value.

Preorder

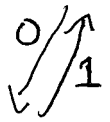
1 ≡ weight of down edge

0 ≡ weight of up edge

Inorder?



left-edge



right-edge

Use "Middle-Visit" Value

Postorder?

Size of Subtree?