Quick Sort & Backwards Analysis

Consider
\[ QS(M) \] (distinct keys)

1) \text{pick random } a \in M \\
2) \text{split } M : 5 < a < 2 \quad (|M|-1 \text{ comparisons}) \\
3) \text{return } QS(5) \times a \times QS(2)

Goal: Expect \# comparisons

Consider dart game:

Init: empty board

| 1 | n |

While \exists \text{ empty square}

pick random empty square

cost = \# empty squares to left & right of dart.

Claim: Expect cost of dart game = Expect cost QS.
Backwards Game:

Init: Full board

Alg: While:exists dart remove a random one.

Cont: #empty sqs to left & right.

Claim: \( \text{Expect cost DG} = \text{Expect cost BW-DG} \)

Analysis of BW-DG

Assume \( i \) darts on board

Consider

\[
S_j^i = \begin{cases} 
1 & \text{if sq}_j \text{ empty & a dart to left or right is removed,} \\
0 & \text{otherwise} 
\end{cases}
\]

Claim: \( \Pr [S_j^i = 1] \leq \frac{2}{i} \)
Consider \( T_i = \sum_{j=1}^{n} S_{ij} \)

Note \( T_i \) is RV = cost of \( i \)th claim removal.

\[
E(T_i) = \sum E(S_{ij}) = \sum \Pr[S_{ij}] \leq \frac{2n}{i}
\]

Consider \( T = \sum_{i=1}^{n} T_i \) total cost

\[
E(T) = \sum E(T_i) \leq 2n \sum \frac{1}{i} = O(n \log n)
\]