Quick Sort & Backwards Analysis

15-750
2/17/16

Consider

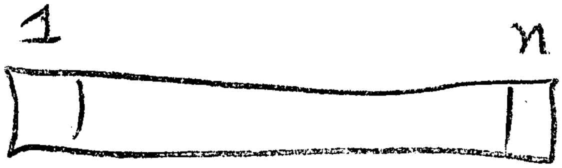
$QS(M)$ (distinct keys)

1) pick random $a \in M$
2) Split M: $S < a < L$ ($|M|-1$ comparisons)
3) return $QS(S) * a * QS(L)$

Goal: Expect # comparisons

Consider dart game:

Init: empty board

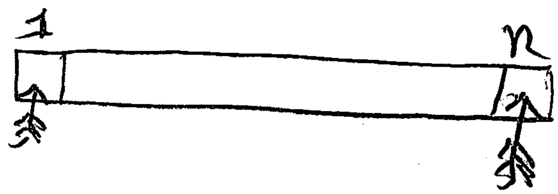


While $\exists$ empty square
pick random empty sq
cost = # empty sqs to left & right of dart.

Claim Expect cost of dart game = Expect cost QS.

Backwards Game:

Init: Full board



Alg While: ∃ dart remove a random one.

Cost: # empty Sqs to left & right.

Claim Expect cost DG = Expect cost BW-DG

Analysis of BW-DG

Assume i darts on board

Consider

$$S_j^i = \begin{cases} 1 & \text{if } Sq_j \text{ empty \& a dart to left or right is removed.} \\ 0 & \text{O.W} \end{cases}$$

Claim $\Pr[S_j^i = 1] \leq \frac{2}{i}$

Consider $T_i = \sum_{j=1}^{n} S_j^i$

<u>Note</u> T_i is RV $\equiv$ cost of ith dart removal.

$$E(T_i) = \sum E(S_j^i) = \sum \Pr[S_j^i] \leq \frac{2n}{i}$$

Consider $T = \sum_{i=1}^{n} T_i$ total cost

$$E(T) = \sum E(T_i) \leq 2n \sum \frac{1}{i} = O(n \log n)$$