Parallel Algorithms II

15-750
3/26/10

Packing out memory

Input: Array of used & unused memory

Ouput: Array of used followed by unused memory

	1	0	1	1	1	1	0	0	1	0	1	1
Input:	A	Ø	A	B	D	C	Ø	Ø	D	Ø	A	B

Output:	A	A	B	D	C	D	A	B	Ø	Ø	Ø	Ø
	0	1	2									

Solution: Use Prescan on array

$$M_i = \begin{cases} 1 & \text{if ith memory used} \\ 0 & \text{o.w} \end{cases}$$

Euler Tours

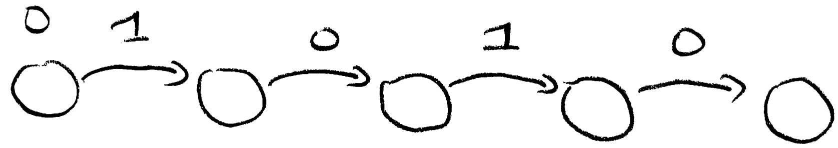
Prob: Preorder number a tree

Input: A tree stored using pointers

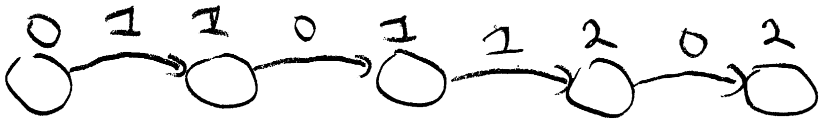
Output: Preorder numbering

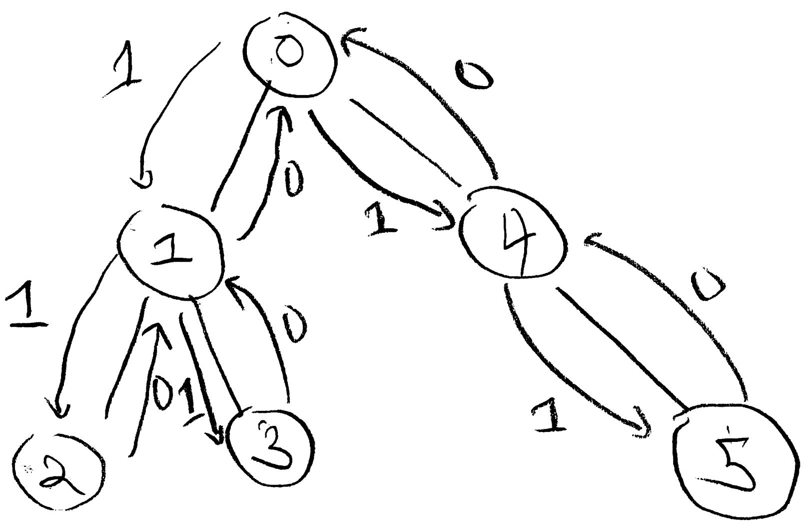
Weighted List Ranking

Input:



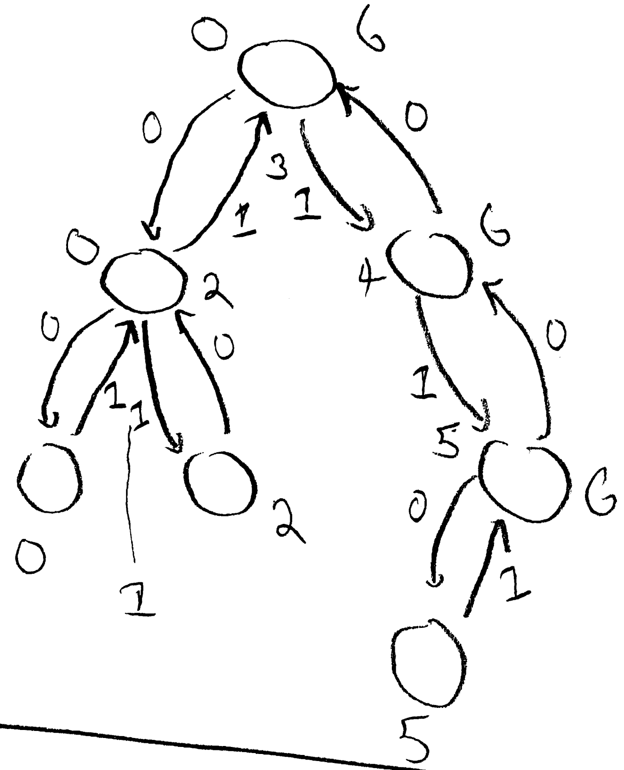
Output:





1 ≡ weight of down edge
0 ≡ weight of up edge

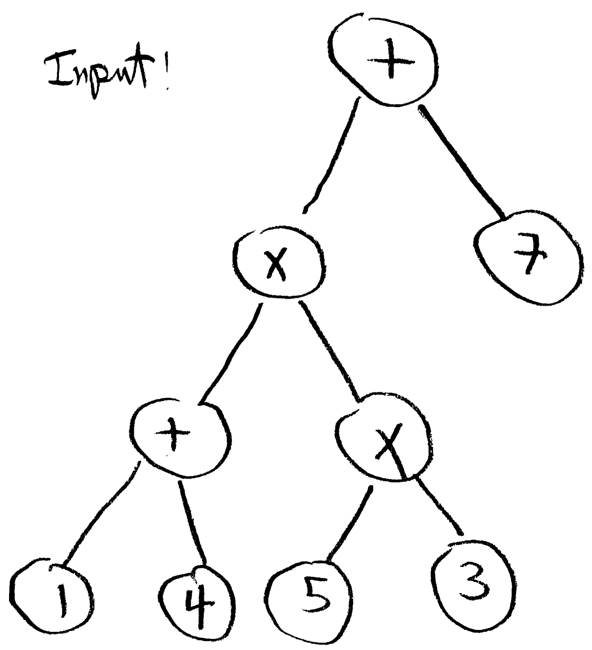
Inorder:



Postorder?

Parallel Expression Evaluation

Eg Input!



Output! Value, all subvalues

Goal! Parallel Alg

Simple Alg

1) Assign a processor to each node.

While tree non empty **do**

2) if Leaf "send" value to parent
delete node [RAKE]

3) if node has 2 values then evalute.

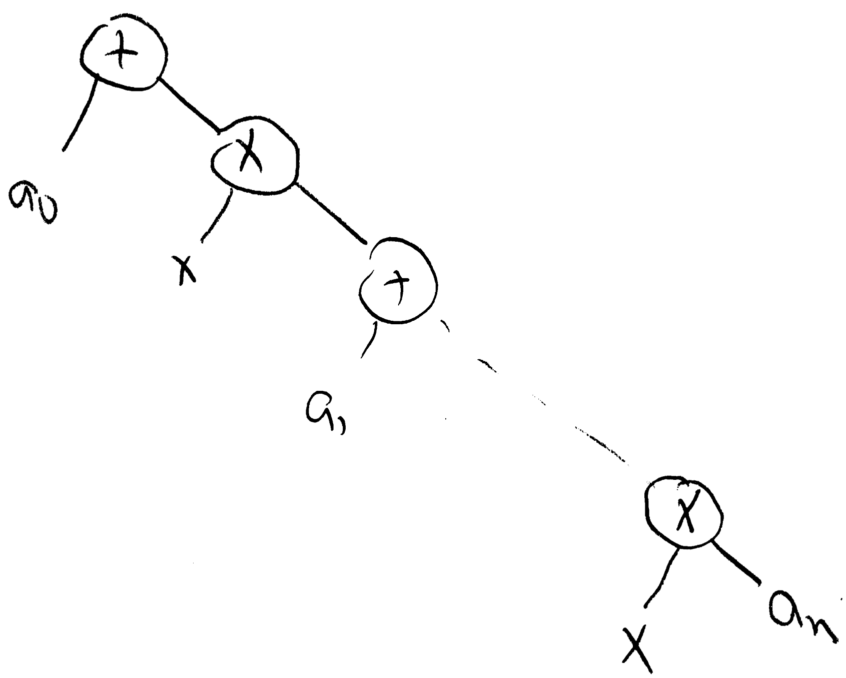
Worst Case for simple Alg

Recall: Horner's Rule

Input: polynomial $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

Alg: $a_0 + x(a_1 + x(\cdots + x(a_{n-1} + x(a_n)\cdots)$

As a tree



Simple Alg

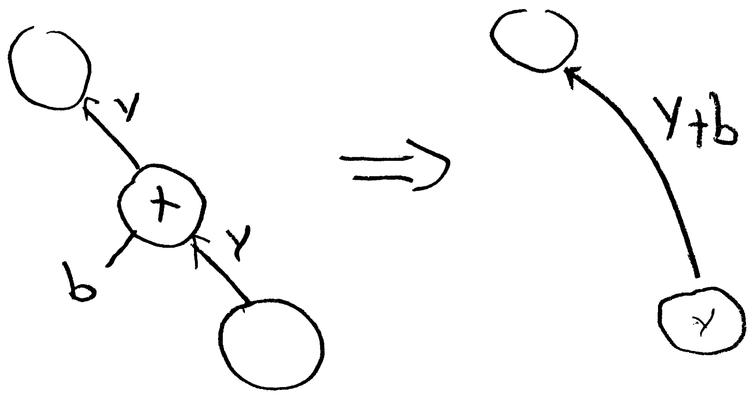
$O(n)$ Time

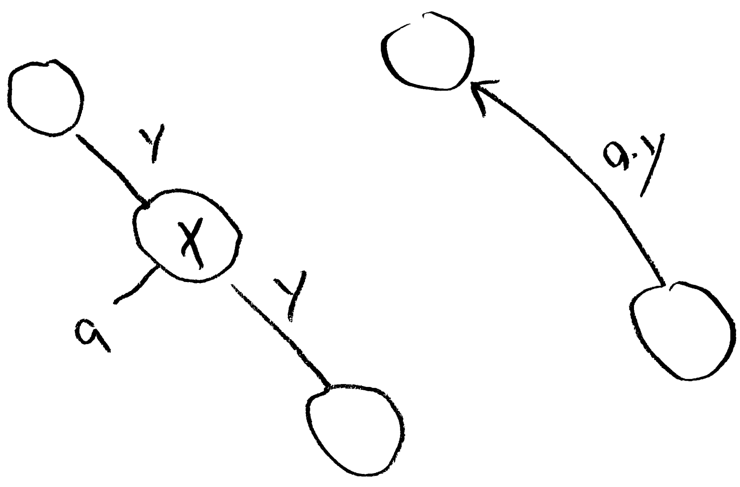
$O(n^2)$ Work

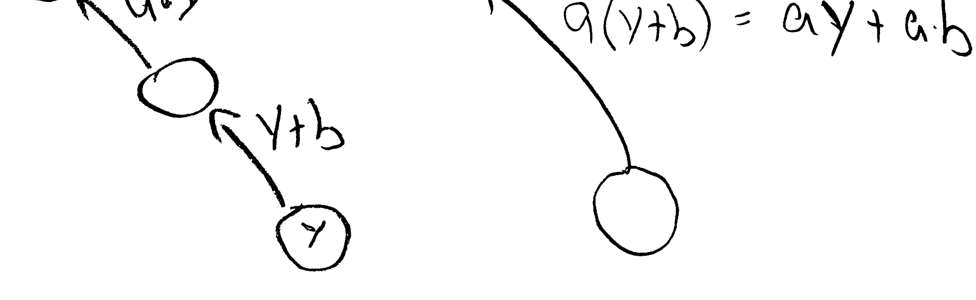
Keeping nodes with only one value busy!

Here we view the tree edges as transformers.

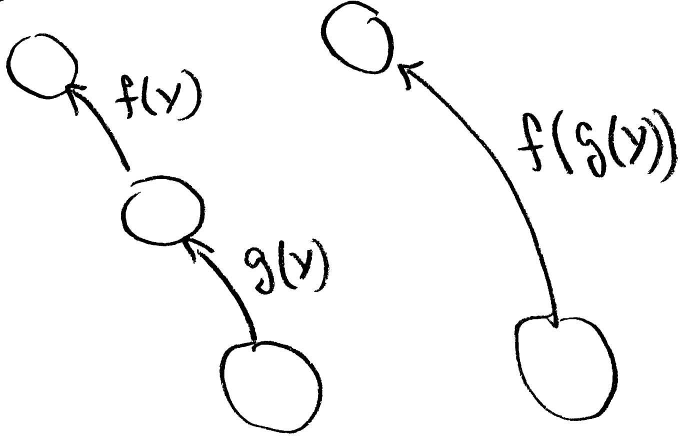
Init: The edge are the identity.







The general case.



$$f(y) = ay + b \qquad g(y) = cy + d$$

$$f(g(y)) = a(cy+d) + b = \overline{a \cdot c}\, y + (ad + b)$$

Note: fcns $ay+b$ are closed under compositions

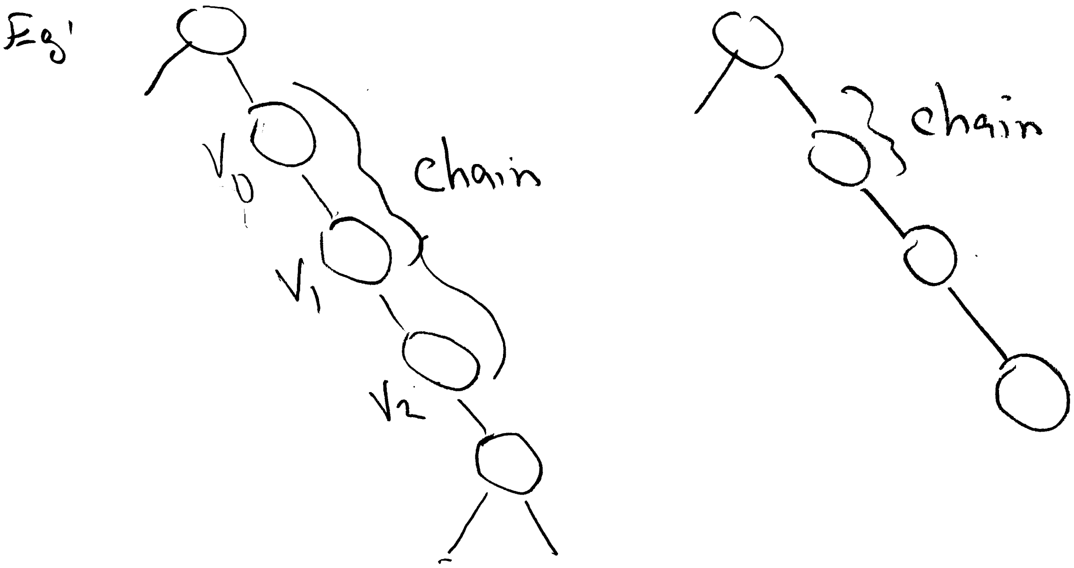
We can also remove an independent set of 1-child nodes (degree 2 nodes)

Very similiar to pivoting in Gaussian Elim!

Def $v_0 \cdots v_k$ is a chain if:

1) v_{i+1} is only child of v_i $0 \le i < k$.
2) v_k has only one child & it is not a leaf

Eg:



The Independent Set

1) All leaves
2) Max independent set from each maximal chain

Parallel Tree Contraction

RAKE $\equiv$ remove all leaves

COMPRESS $\equiv$ replace each maximal chain of length k with one of length $\lfloor k/2 \rfloor$.

CONTRACT = {RAKE, COMPRESS}

Thm $|\text{CONTRACT}(T)| \leq 2/3 |T|$

pf Def V_0 = leaves of T

$V_1 \subseteq V$ with 1 child

$V_2 \subseteq V$ with $2 \leq$ # children

$C \subseteq V_1$ with child in V_0

<u>Claim</u> $|V_0| > |V_2|$

<u>pf</u> induct on size of T.

Claim: $|V_0| \geq |C|$

<u>Def</u> $R_a = V_0 \cup V_2 \cup C$

$Com = V_1 - R_a$

$RAKE(R_a) \subseteq V_2 \cup C \Rightarrow |RAKE(R_a)| \leq 2/3 |R_a|$

<u>Note</u> Com $\equiv$ union of maximal chains

$|COMPRESS(Com)| \leq \frac{1}{2} |Com)$

<u>Cor</u> After $\log_{3/2} n$ CONTRACTS T is empty

A Simple Randomized List-Ranking

Assume Linked-list is doubly linked.

Alg Splicing-Out

1) Make $n/\log n$ queues of size $\log n$ (1 queue/proc)
2) Set sex of all nodes to M.
3) Reset sex of each queue-top to random sex.
4) In || <u>If</u> top is F and points to M <u>then</u> "splice-out" top.
5) Repeat 3)&4) while some queue not empty.

<u>Thm</u> After $O(\log n)$ rounds all queues are empty with high probability.

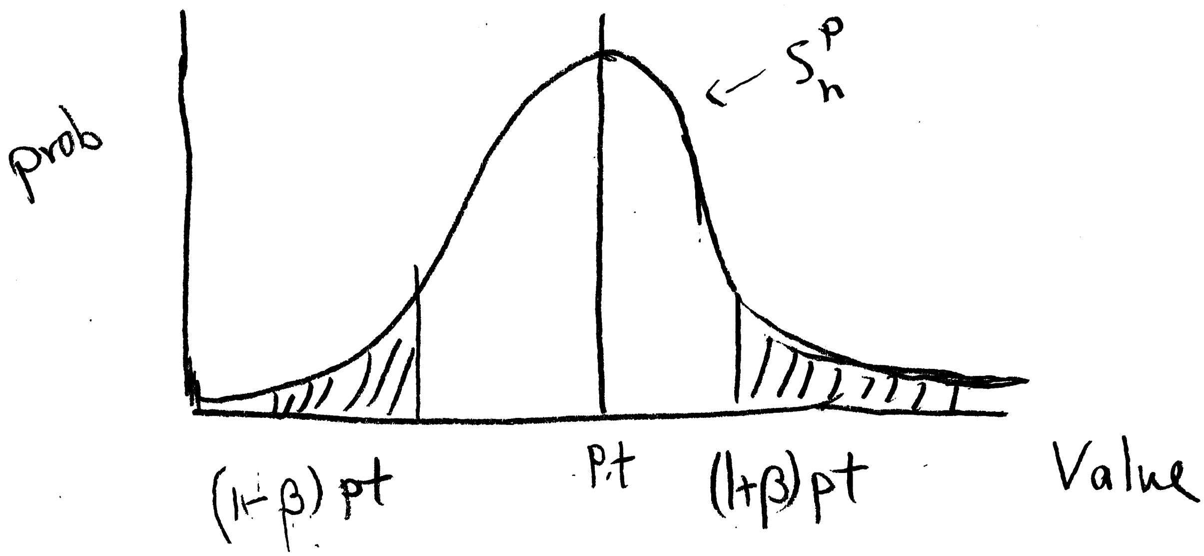
Chernoff Bounds

Let $X_1 \cdots X_t$ be independent 0/1 random variables

Assume $\text{Prob}(X_i = 1) = p$

The binomial random variable is

$$S_n^p = X_1 + \cdots + X_t$$



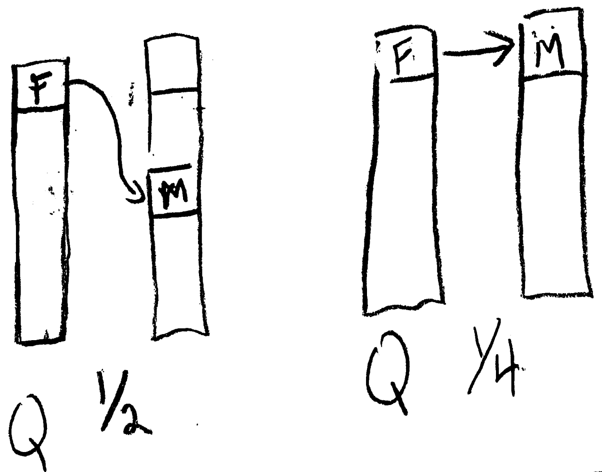
$$\text{Expect}(S_t^p) = \sum E(X_1) = p.t$$

Thm $\text{Prob}\left(S_t^p < (1-\beta)pt\right) < e^{-\beta^2 pt/2} \quad \forall\, 0 \le \beta < 1$

Thm $\text{Prob}\left(S_t^p > (1+\beta)pt\right) < e^{-\beta^2 pt/3} \quad \forall\, 0 \le \beta < 1$

Lets fix one of the queues, say, Q.

At a given round the prob Top is spliced-out is $\geq 1/4$



View prob as:

We have a coin Prob(Head) = $1/4$ Prob(Tail) = $3/4$

Question: After t flips what is

$$\text{Prob}[\#\text{heads} < \log n] ?$$

Suppose we pick t s.t. Expect #heads = $4 \log n$

ie $t = 16 \log n$

We apply Chernoff with

$p = 1/4$, $t = 16 \log n$ $\beta = 3/4$

$$\text{Prob}\left(S_t^p < (1-\beta)pt\right) < e^{-\beta^2 pt/2}$$

$$\text{Prob}\left(S_t^p < \log n\right) < e^{-(3/4)^2 (\frac{1}{4}) 16 (\log n)/2}$$

$$= e^{-9/8 \log n} = n^{-9/8}$$

Thus Prob that some queue is not empty after $t = 16 \log n$ rounds $< (n/\log n)\, n^{-9/8}$

$$< n^{-1/8}$$