

15-750 — Graduate Algorithms — Spring 2008
Miller and Sinop and Wu

Assignment 5 Due date: Wednesday April 16, 2008

Some Reminders:

- Read the Policies section on the course web site before you start working on this assignment. Collaboration **is** permitted for this assignment.
- You should refrain from using outside sources when solving these problems. For each problem, state whether you have seen it before. If you have questions, contact the course staff.
- We prefer that you type up your solutions (preferably using LaTeX). You may neatly hand-write your solutions, but if we have trouble reading them you will be required to type up future solutions.

1 Commuting Times on a Grid

[20 points]

Let M_n be the n by n grid graph. Suppose the vertex A is in the middle of the left-side and B is on the right-side as shown in Figure 1, where all edges have unit resistance.

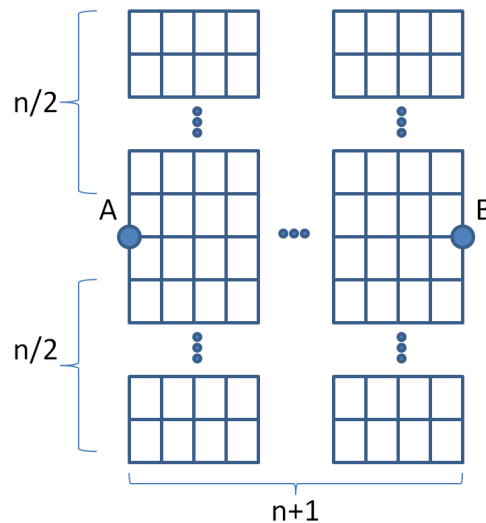


Figure 1: The n by n grid graph with vertex A on the left-side and vertex B on the right.

- Find the commute time between A and B . You need only find asymptotic upper and lower bounds on the commute time.
- Determine the asymptotic commute time between two vertices that are at a distance d apart in M_n .

HINT: Use Rayleigh's Monotonicity Law to upper and lower bound the commute time.

2 Resistive Metric

[20 points]

Let G be a graph of resistors with vertices $\{V_1, \dots, V_n\}$. Let $R(V_i, V_j) = R_{ij}$ be the effective resistance from V_i to V_j .

Show that R is a metric over the vertices of G .

IMPORTANT: Give a real proof. You may prove this fact in terms of electrical networks or in terms of random walks. In either case, be rigorous.

HINT: Think Harmonic Functions.

3 Perfect Elimination Order

[20 points]

A graph $G = (V, E)$ is called perfect graph if there exists a pivot strategy with zero fill. In this problem, we will investigate pivoting strategies for certain perfect graphs.

- (a) Let $u_1 = [s_1, t_1], u_2 = [s_2, t_2], \dots, u_n = [s_n, t_n]$ be n intervals on the real line. Let G be a corresponding interval graph where $\{i, j\} \in E$ iff $[s_i, t_i] \cap [s_j, t_j] \neq \emptyset$. As discussed in lecture such a graph are called an **interval graph**. Find a pivoting strategy with zero fill or perfect elimination order for these graphs.
- (b) We next consider a more general class of graphs. Let T be a tree, and $T_1, T_2, \dots, T_n \subseteq T$ a collection of subtrees of T . We can generalize the notion of interval graphs for these trees as follows. Consider a graph G where the vertices are the subtree and $\{i, j\} \in E$ iff $T_i \cap T_j \neq \emptyset$. Generalize the previous algorithm for these graphs.

(Hint: Use induction on $|T|$. When can you remove a leaf of T without changing the graph?)