Lecture 13

Introduction to

Static Single Assignment (SSA)

(Slides courtesy of Seth Goldstein.)

Values ≠ Locations

```
…
for (i=0; i++; i<10) {
   … = … i …;
   …
}
for (i=j; i++; i<20) {
   …  = i …
}
```

Def-use chains help solve the problem.

Def-Use Chains are Expensive

```
foo(int i, int j) {
   …
   switch (i) {
   case 0: x=3;break;
   case 1: x=1; break;
   case 2: x=6; break;
   case 3: x=7; break;
   default: x = 11;
   }
   switch (j) {
   case 0: y=x+7; break;
   case 1: y=x+4; break;
   case 2: y=x-2; break;
   case 3: y=x+1; break;
   default: y=x+9;
   }
   …
```

In general,

N defs

M uses

⇒ O(NM) space and time

One solution: limit each variable to ONE definition site

Def-Use Chains are Expensive

```
foo(int i, int j) {
   …
   switch (i) {
   case 0: x=3; break;
   case 1: x=1; break;
   case 2: x=6;
   case 3: x=7;
   default: x = 11;
   }
   x1 is one of the above x's
   switch (j) {
   case 0: y=x1+7;
   case 1: y=x1+4;
   case 2: y=x1-2;
   case 3: y=x1+1;
   default: y=x1+9;
   }
   …
```

One solution: limit each variable to ONE definition site

Advantages of SSA

- Makes du-chains explicit
- Makes dataflow analysis easier
- Improves register allocation
  - Automatically builds "webs"
  - Makes building interference graphs easier
- For most programs reduces space/time requirements

SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - each use uses the most recently defined var.
  - (Similar to Value Numbering)

Straight-line SSA

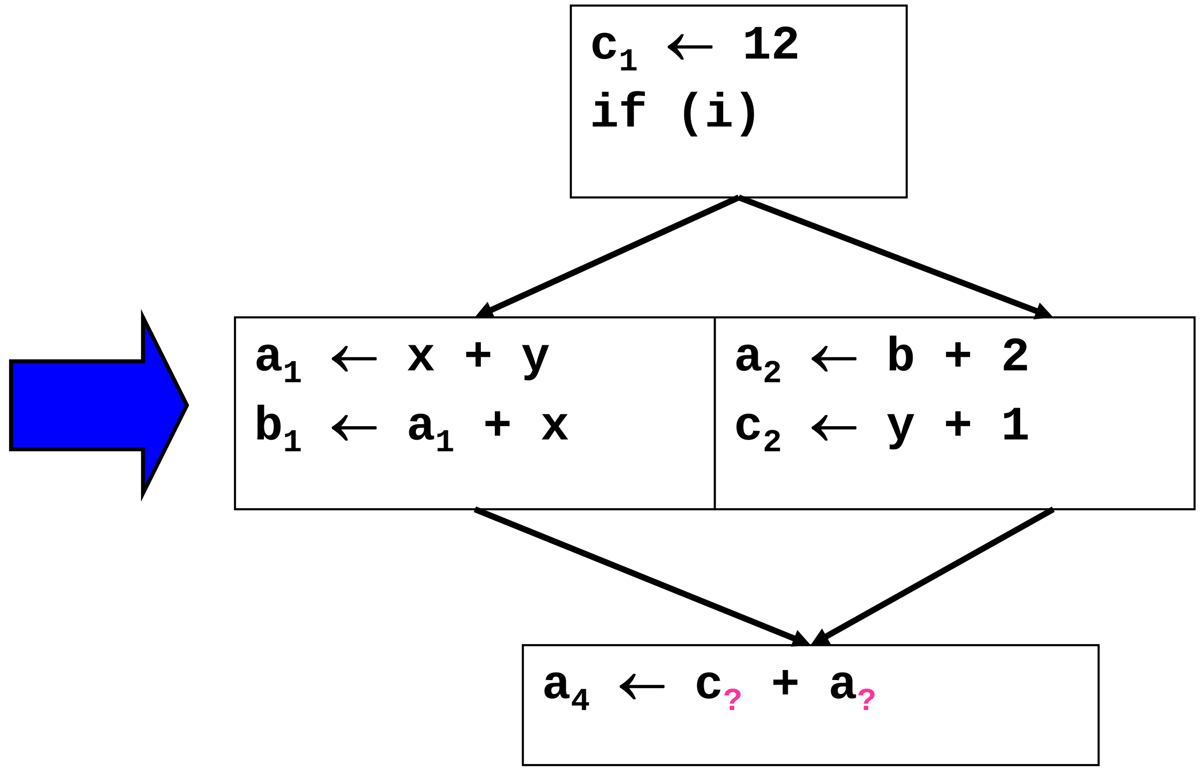
$$
\begin{aligned}
a &\leftarrow x + y \\
b &\leftarrow a + x \\
a &\leftarrow b + 2 \\
c &\leftarrow y + 1 \\
a &\leftarrow c + a
\end{aligned}
\qquad \Rightarrow \qquad
\begin{aligned}
a_1 &\leftarrow x + y \\
b_1 &\leftarrow a_1 + x \\
a_2 &\leftarrow b_1 + 2 \\
c_1 &\leftarrow y + 1 \\
a_3 &\leftarrow c_1 + a_2
\end{aligned}
$$

SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - each use uses the most recently defined var.
  - (Similar to Value Numbering)
- What about at joins in the CFG?

Merging at Joins

```
c ← 12
if (i) {
   a ← x + y
   b ← a + x
} else {
   a ← b + 2
   c ← y + 1
}
a ← c + a
```



SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
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  - assign to a fresh variable at each stmt.
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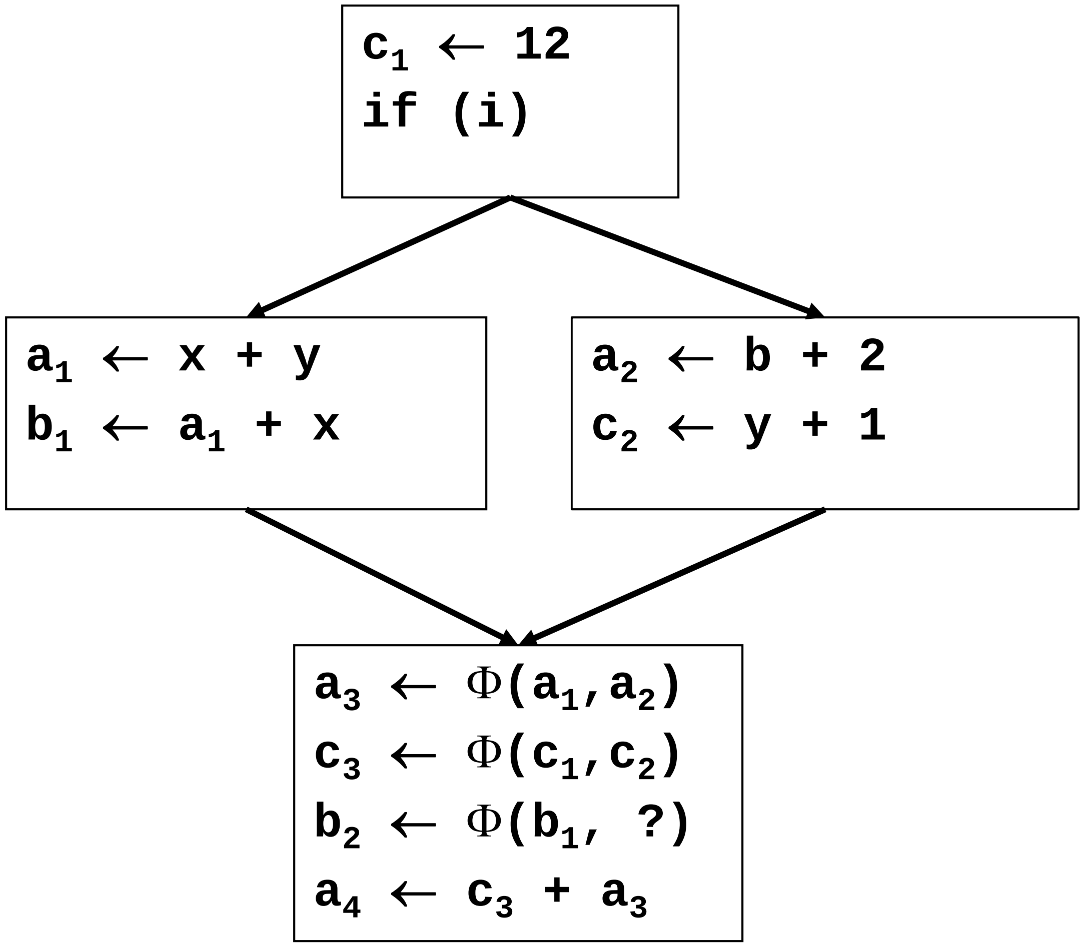
  - (Similar to Value Numbering)
- What about at joins in the CFG?
  - Use a notational fiction: a Φ function

Merging at Joins



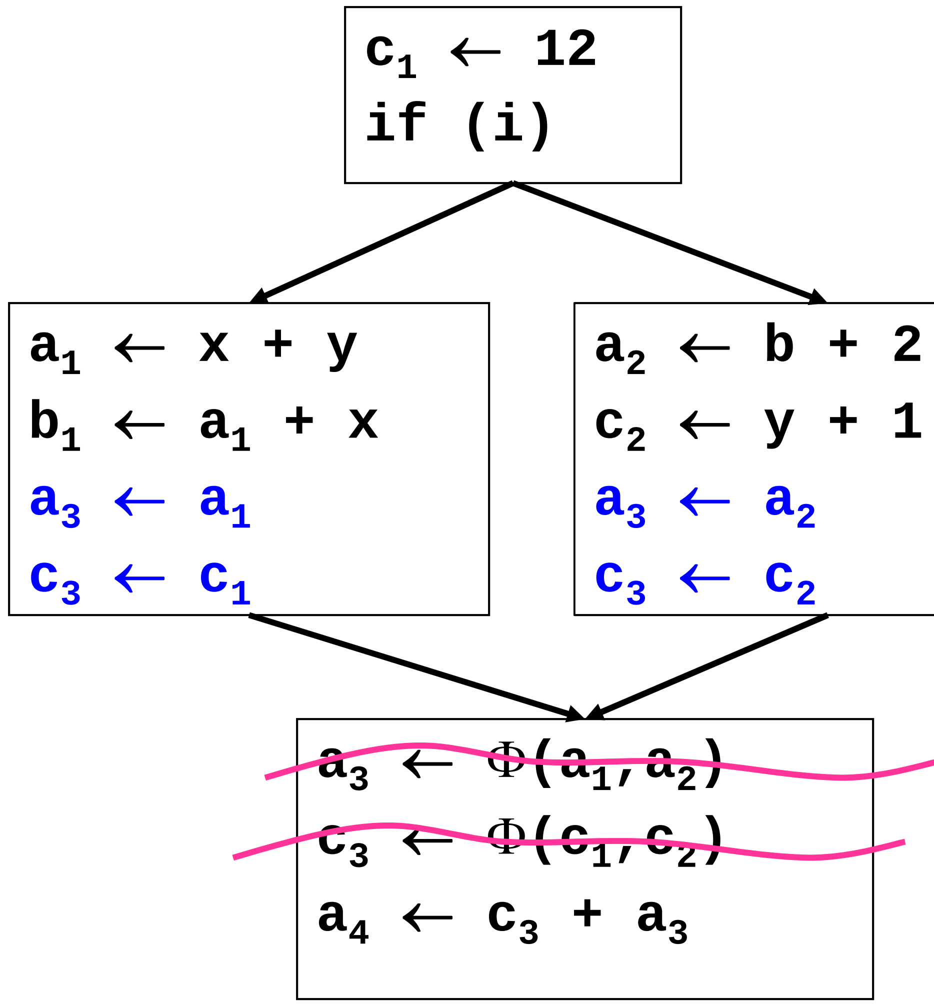
The Φ function

- Φ merges multiple definitions along multiple control paths into a single definition.
- At a basic block with *p* predecessors, there are *p* arguments to the Φ function.

$$x_{new} \leftarrow \Phi(x_1,\ x_1,\ x_1,\ \dots,\ x_p)$$

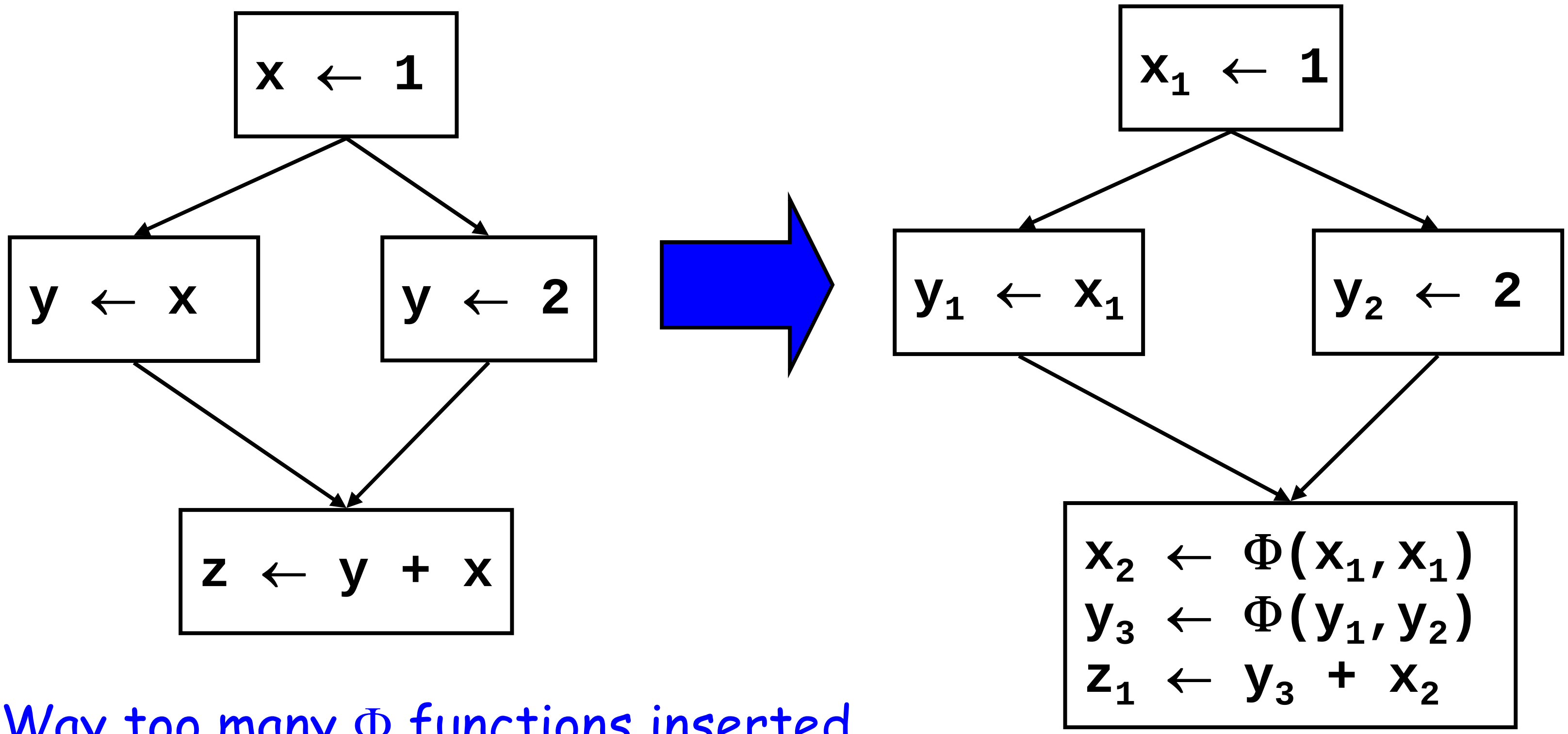
- How do we choose which x_i to use?
  - We don't really care!
  - If we care, use moves on each incoming edge

"Implementing" Φ



Trivial SSA

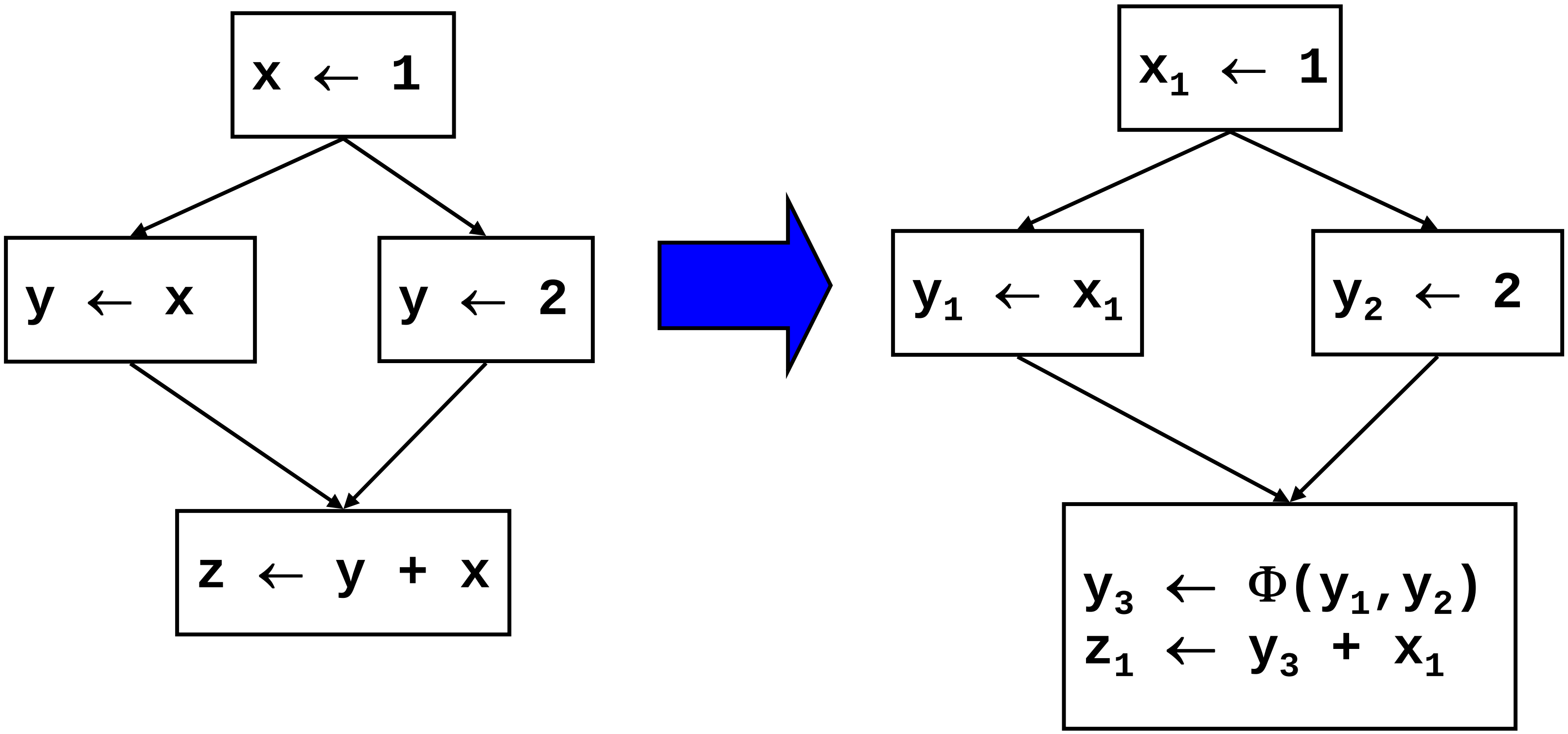
- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.



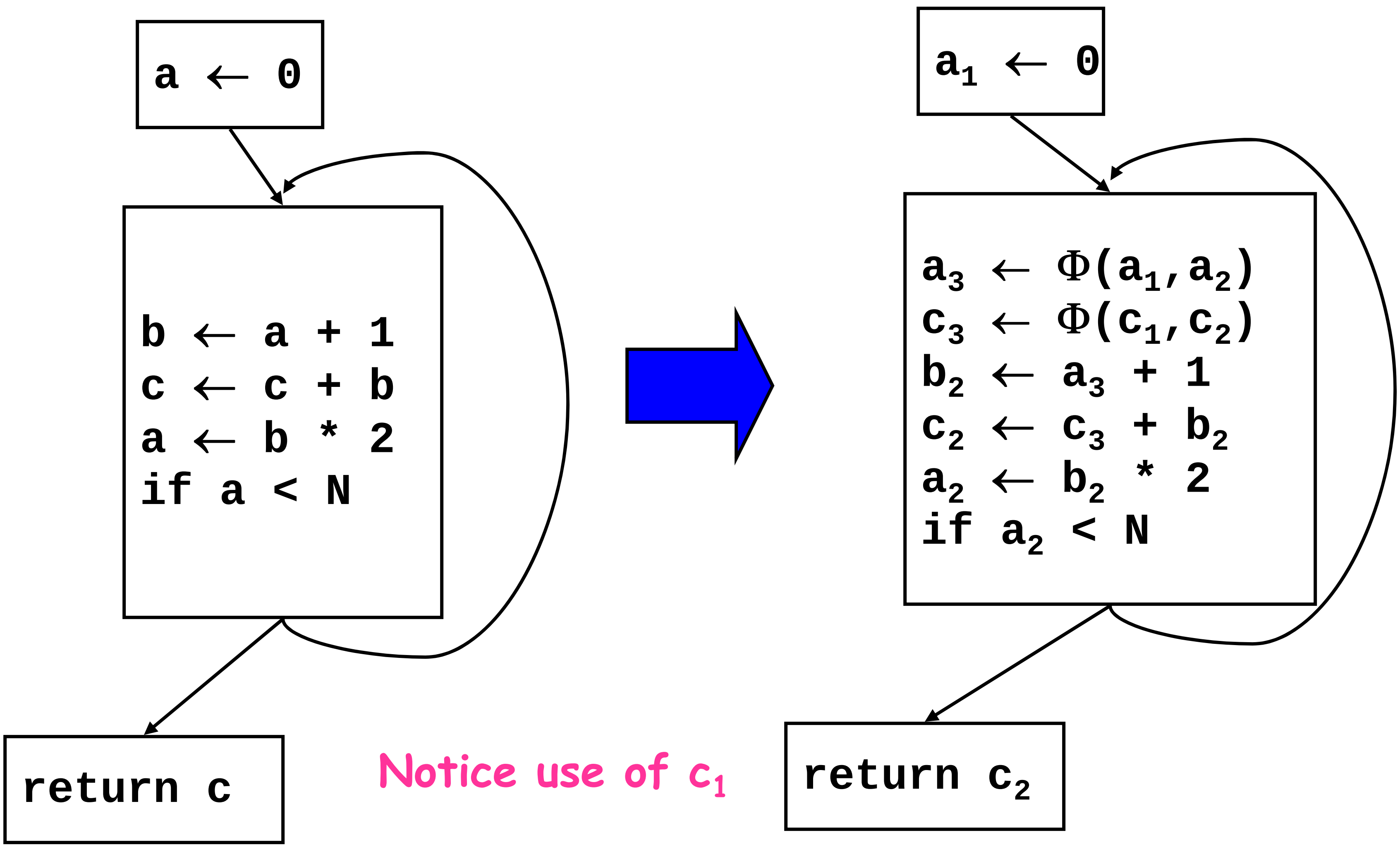
Way too many Φ functions inserted.

Minimal SSA

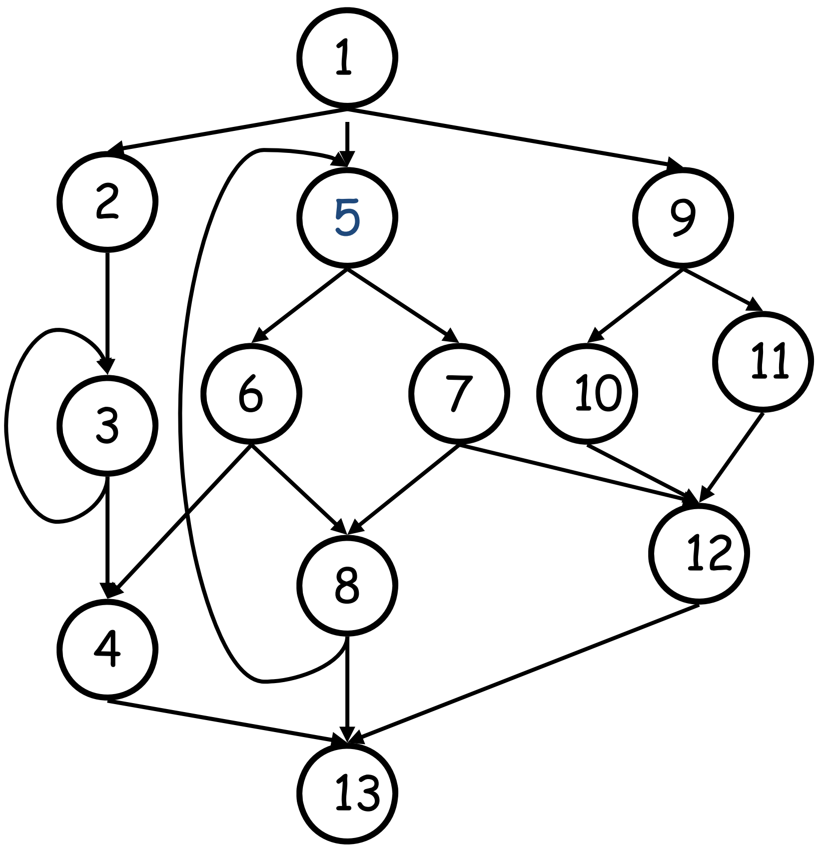
- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables with multiple outstanding defs.



Another Example



When Do We Insert Φ?



CFG

If there is a def of **a** in block 5, which nodes need a Φ()?

When do we insert Φ?

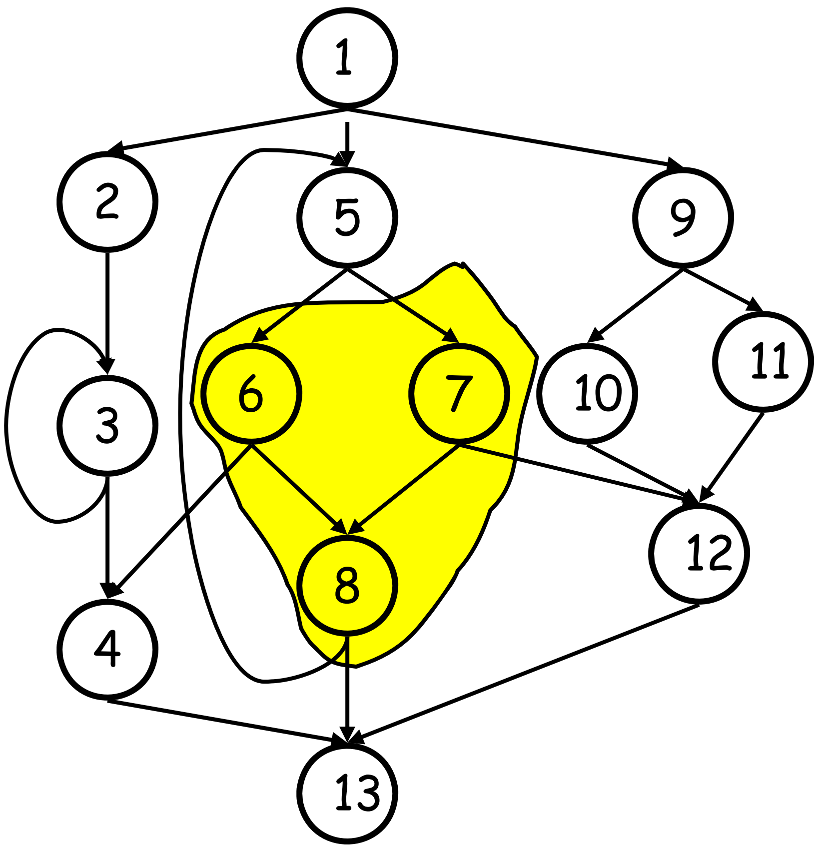
- We insert a Φ function for variable **A** in block Z iff:
  - **A** was defined more than once before
    - (i.e., **A** defined in **X** and Y AND X $\neq$ Y)
  - There exists a non-empty path from x to z, P_{xz}, and a non-empty path from y to z, P_{yz}, s.t.
    - $P_{xz} \cap P_{yz} = \{ z \}$
    - $z \notin P_{xq}$ or $z \notin P_{yr}$ where $P_{xz} = P_{xq} \rightarrow z$ and $P_{yz} = P_{yr} \rightarrow z$
- Entry block contains an implicit def of all vars
- Note: $A = \Phi(\ldots)$ is a def of A

Dominance Property of SSA

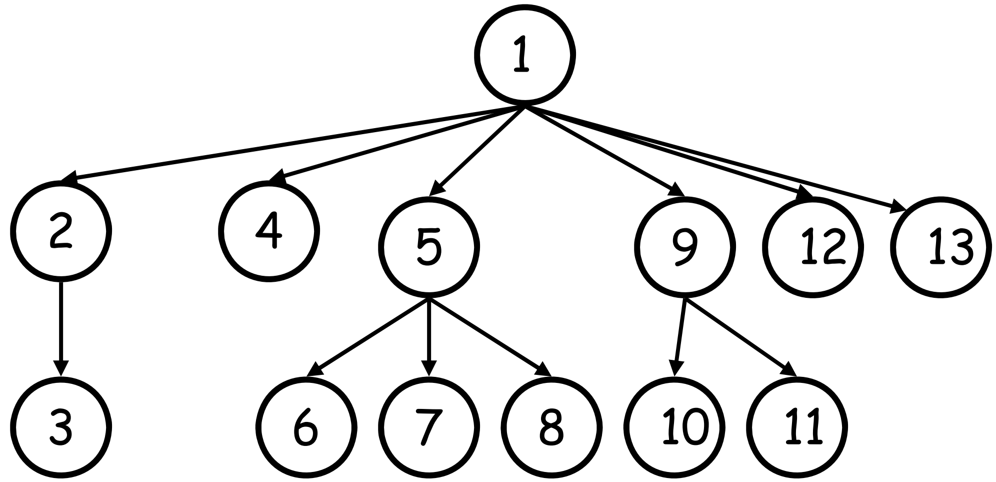
- In SSA, definitions dominate uses.
  - If x_i is used in $x \leftarrow \Phi(\ldots, x_i, \ldots)$, then BB($x_i$) dominates i^{th} predecessor of BB(PHI)
  - If x is used in $y \leftarrow \ldots x \ldots$, then BB(x) dominates BB(y)

- We can use this for an efficient algorithm to convert to SSA

Dominance



CFG

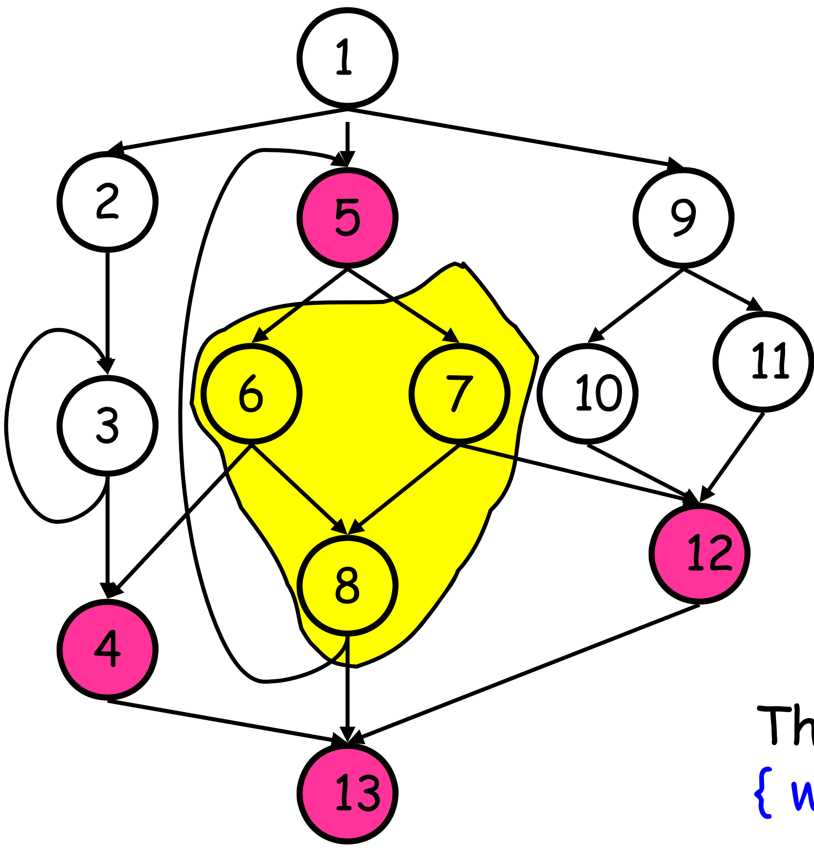


D-Tree

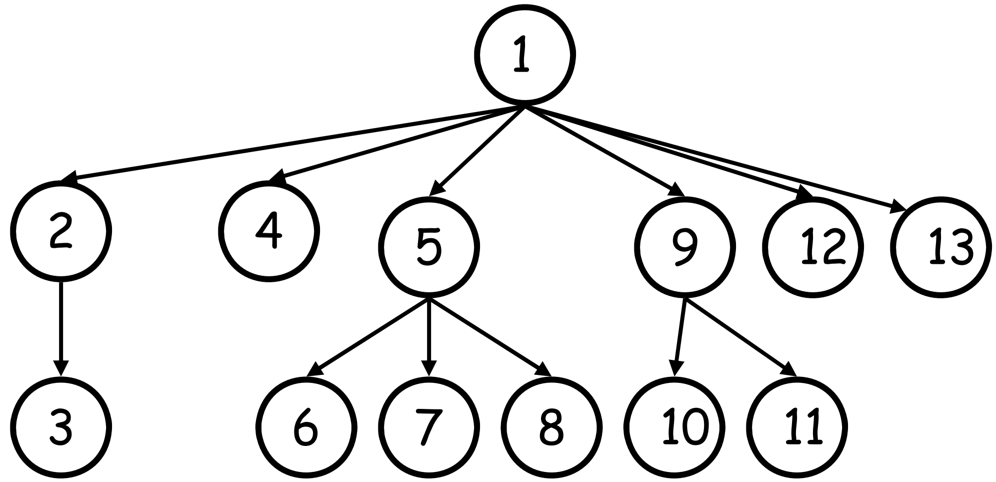
If there is a def of a in block 5, which nodes need a $\Phi()$?

x strictly dominates w (x sdom w) iff x dom w AND $x \neq w$

Dominance Frontier



CFG

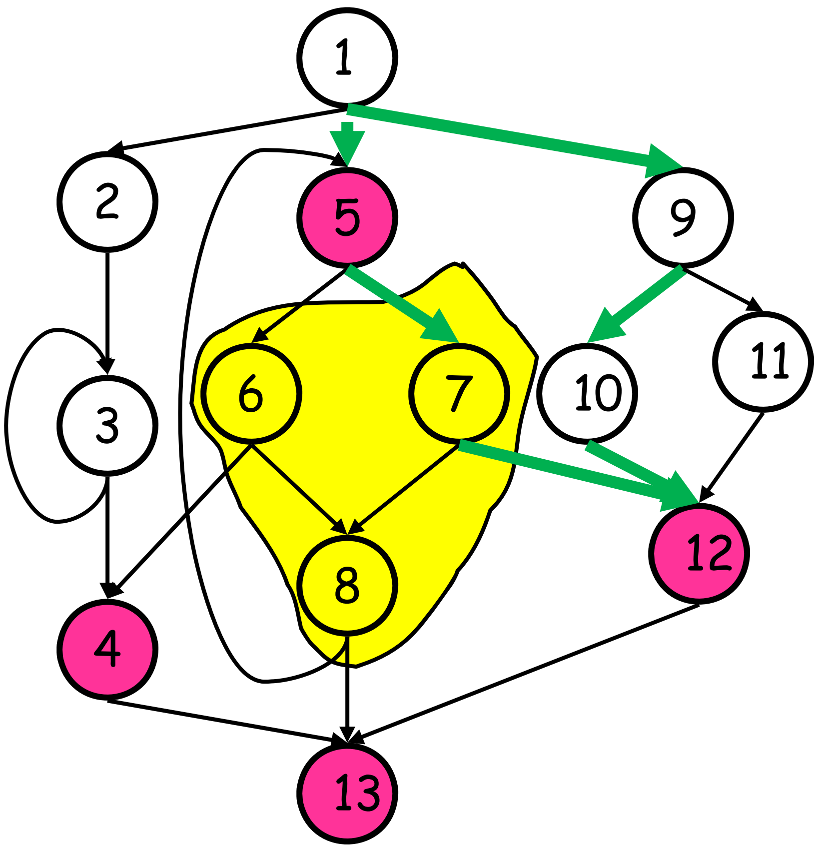


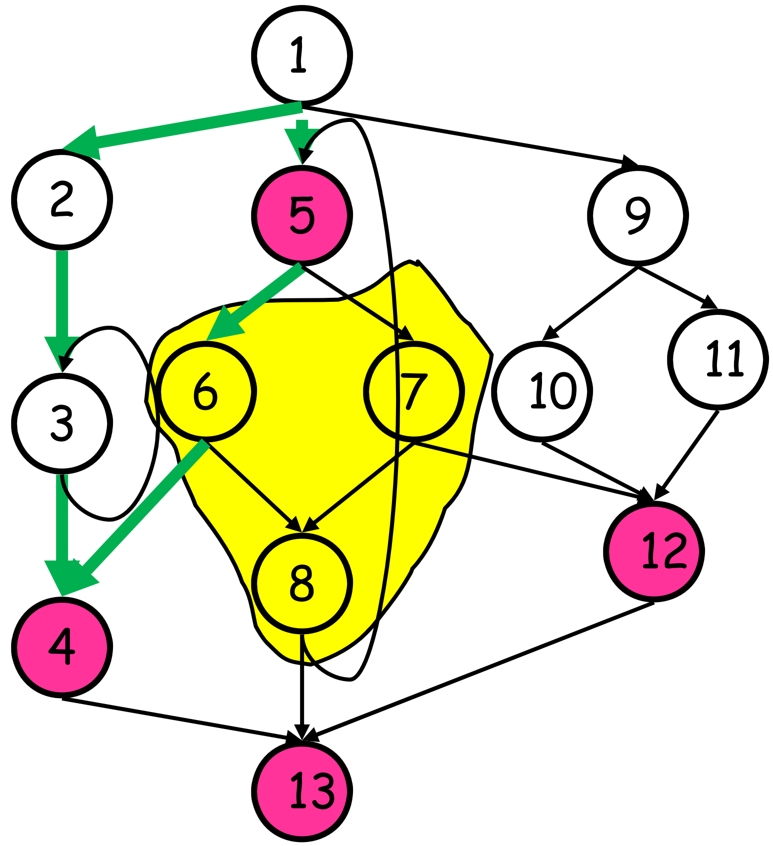
D-Tree

The Dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

x strictly dominates w (x sdom w) iff x dom w AND x ≠ w

Dominance Frontier and Path Convergence





Using Dominance Frontier to Compute SSA

- place all $\Phi()$
- Rename all variables

Using Dominance Frontier to Place $\Phi()$

- Gather all the defsites of every variable
- Then, for every variable
  - foreach defsite
    - foreach node in DominanceFrontier(defsite)
      - if we haven't put $\Phi()$ in node, then put one in
      - if this node didn't define the variable before, then add this node to the defsites

- This essentially computes the Iterated Dominance Frontier on the fly, inserting the minimal number of $\Phi()$ neccesary

Using Dominance Frontier to Place Φ()

```
foreach node n {
   foreach variable v defined in n {
     orig[n] ∪= {v}
     defsites[v] ∪= {n}
   }
}
foreach variable v {
   W = defsites[v]
   while W not empty {
     n = remove node from W
     foreach y in DF[n]
     if y ∉ PHI[v] {
       insert "v ← Φ(v,v,…)" at top of y
       PHI[v] = PHI[v] ∪ {y}
       if v ∉ orig[y]: W = W ∪ {y}
     }
   }
}
```

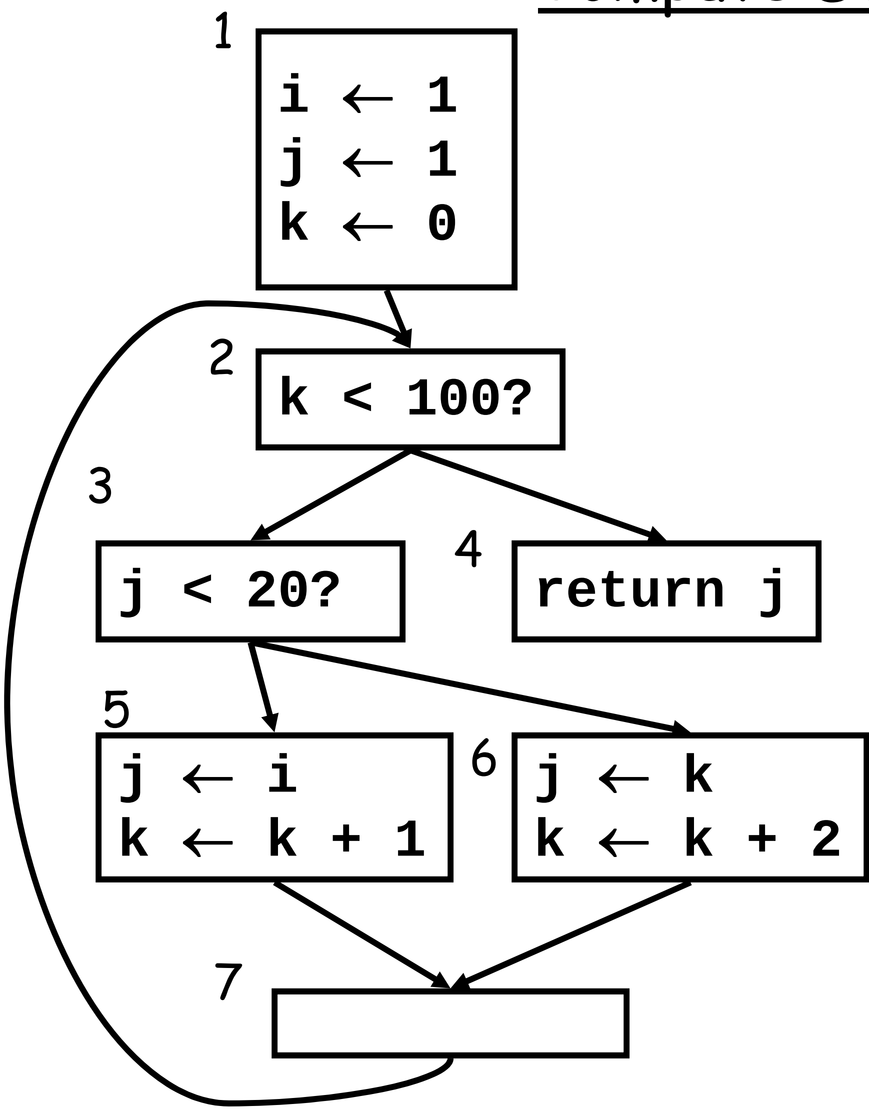
Renaming Variables

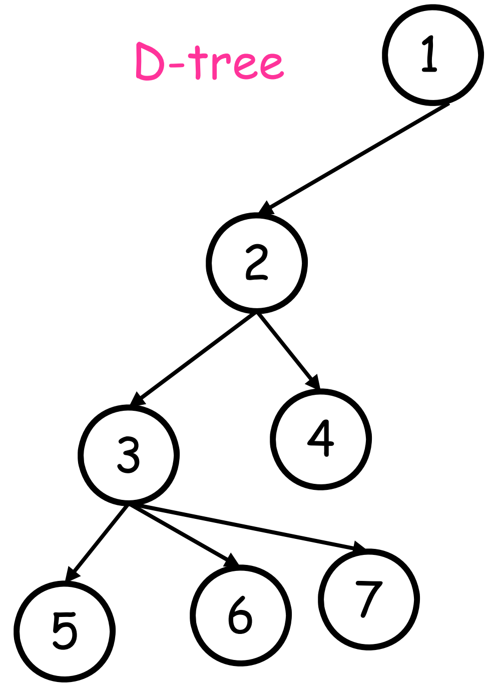
- Algorithm:
  - Walk the D-tree, renaming variables as you go
  - Replace uses with more recent renamed def

- For straight-line code this is easy
- What if there are branches and joins?
  - use the closest def such that the def is above the use in the D-tree

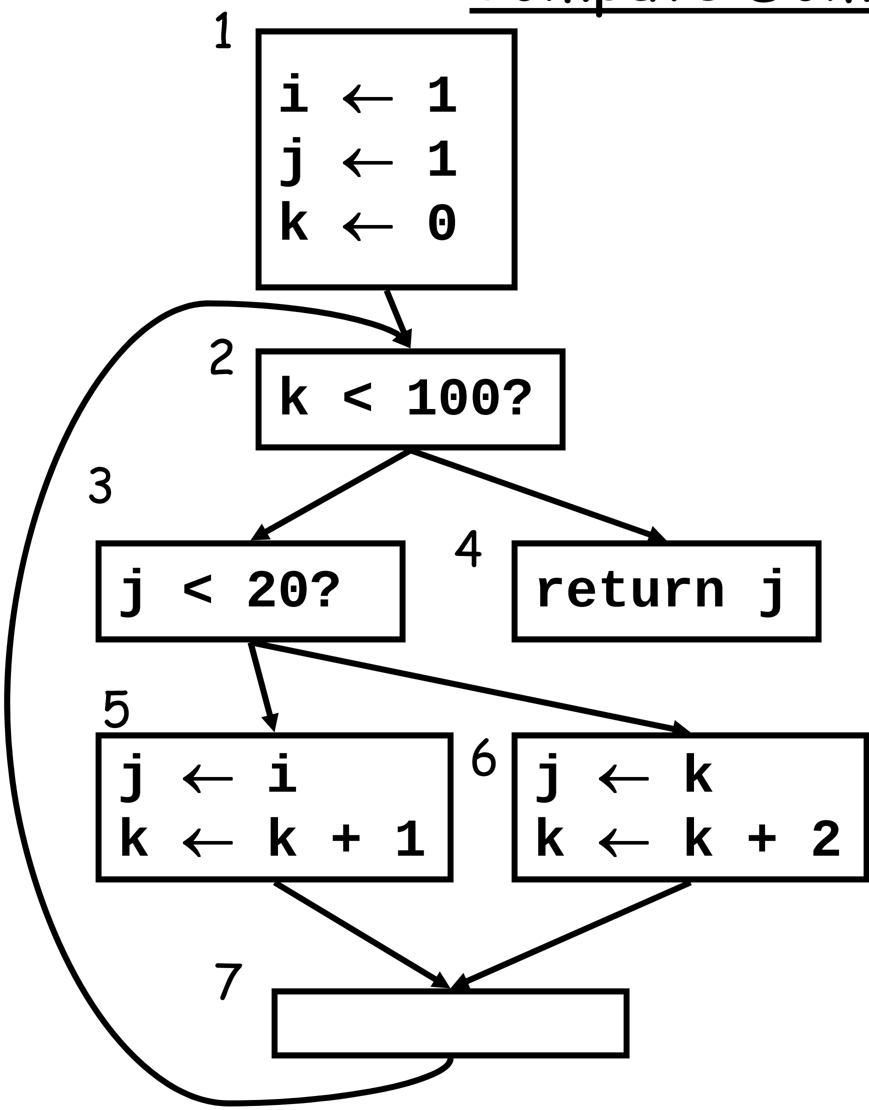
- Easy implementation:
  - for each var: rename (v)
  - rename(v): replace uses with top of stack
    at def: push onto stack
    call rename(v) on all children in D-tree
    for each def in this block pop from stack

Compute Dominance Tree



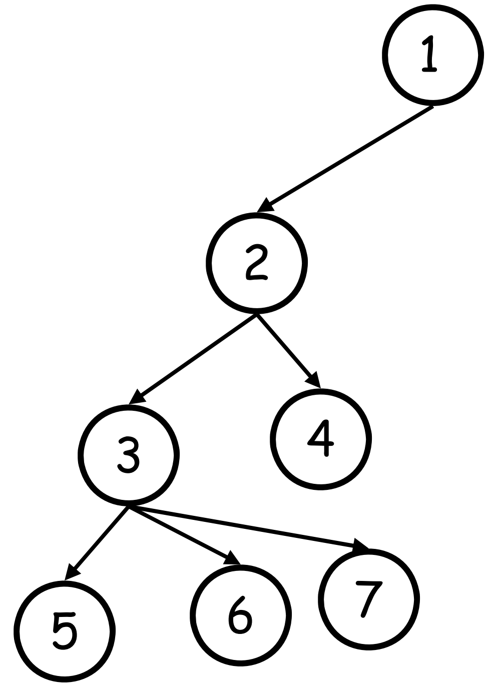


Compute Dominance Frontiers

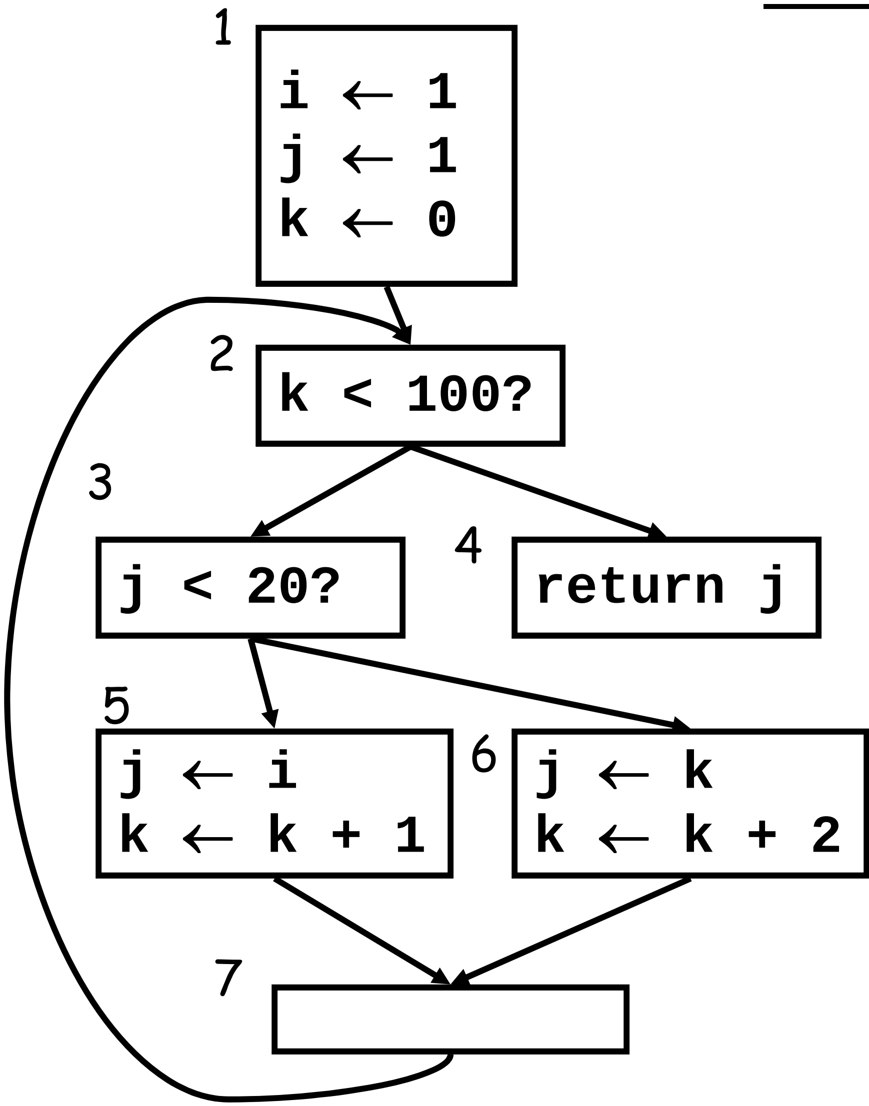


DFs

1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}



Insert Φ()



	DFs		orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

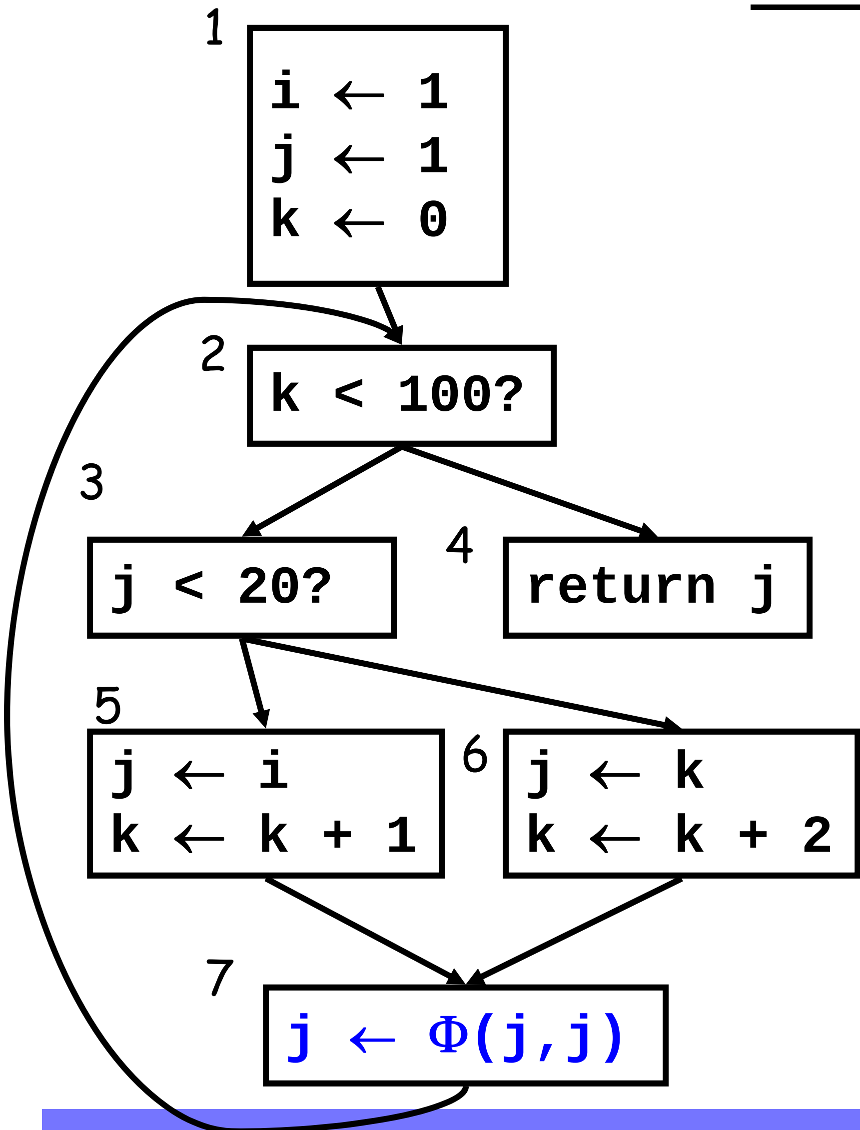
DFs

var i: W={1}

var j: W={1,5,6}

DF{1} DF{5}

Insert Φ()



DFs		orig[n]	
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

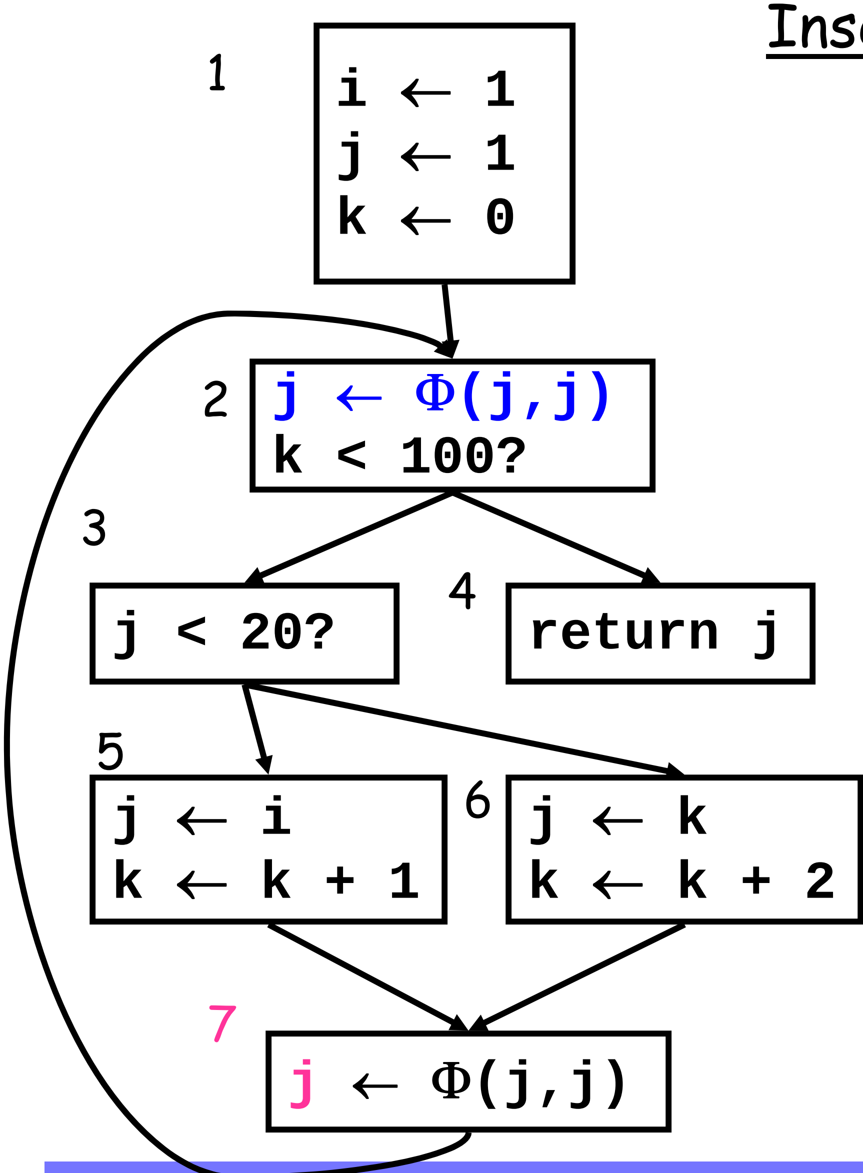
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

var j: W={1,5,6}

DF{1} DF{5}



Insert Φ()

n	DFs	n	orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

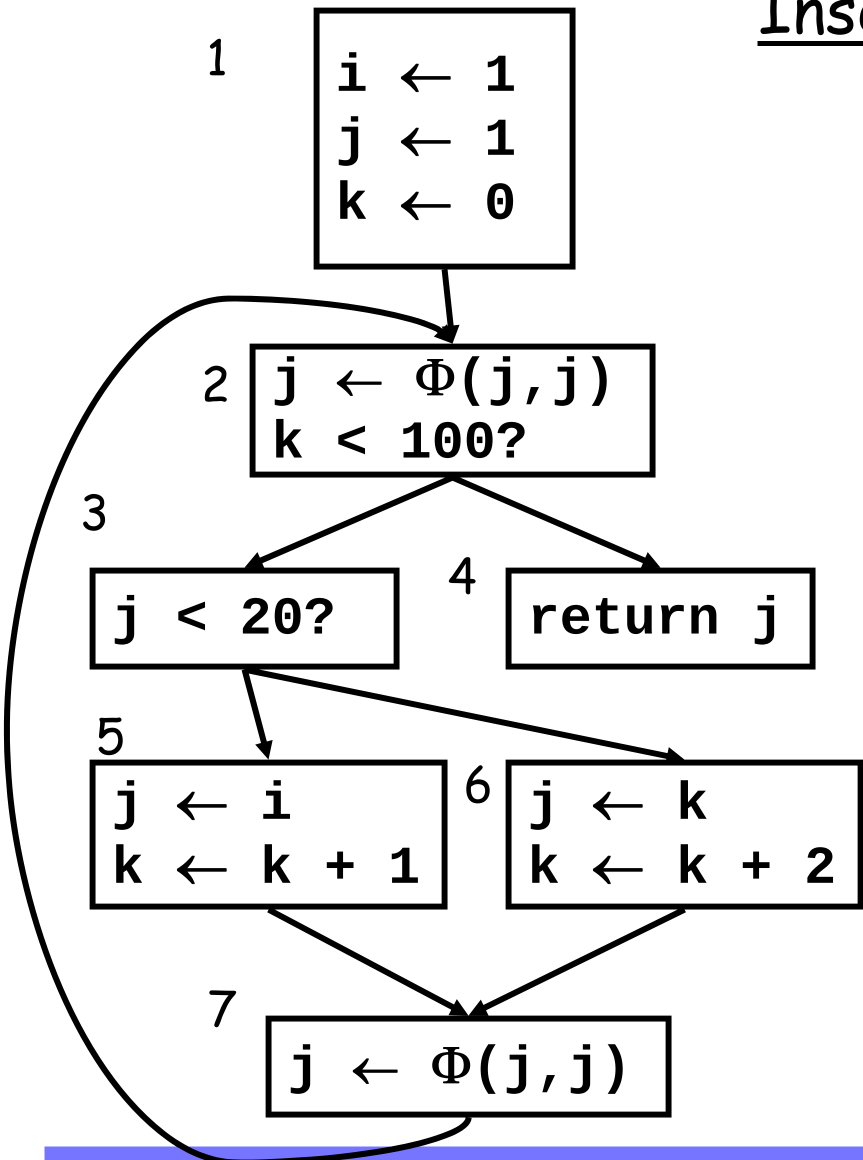
defsites[v]

v	defsites
i	{1}
j	{1,5,6,7}
k	{1,5,6}

DFs

var j: W={1,5,6,7}

DF{1} DF{5} DF{7}



Insert Φ()

DFs		orig[n]	
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

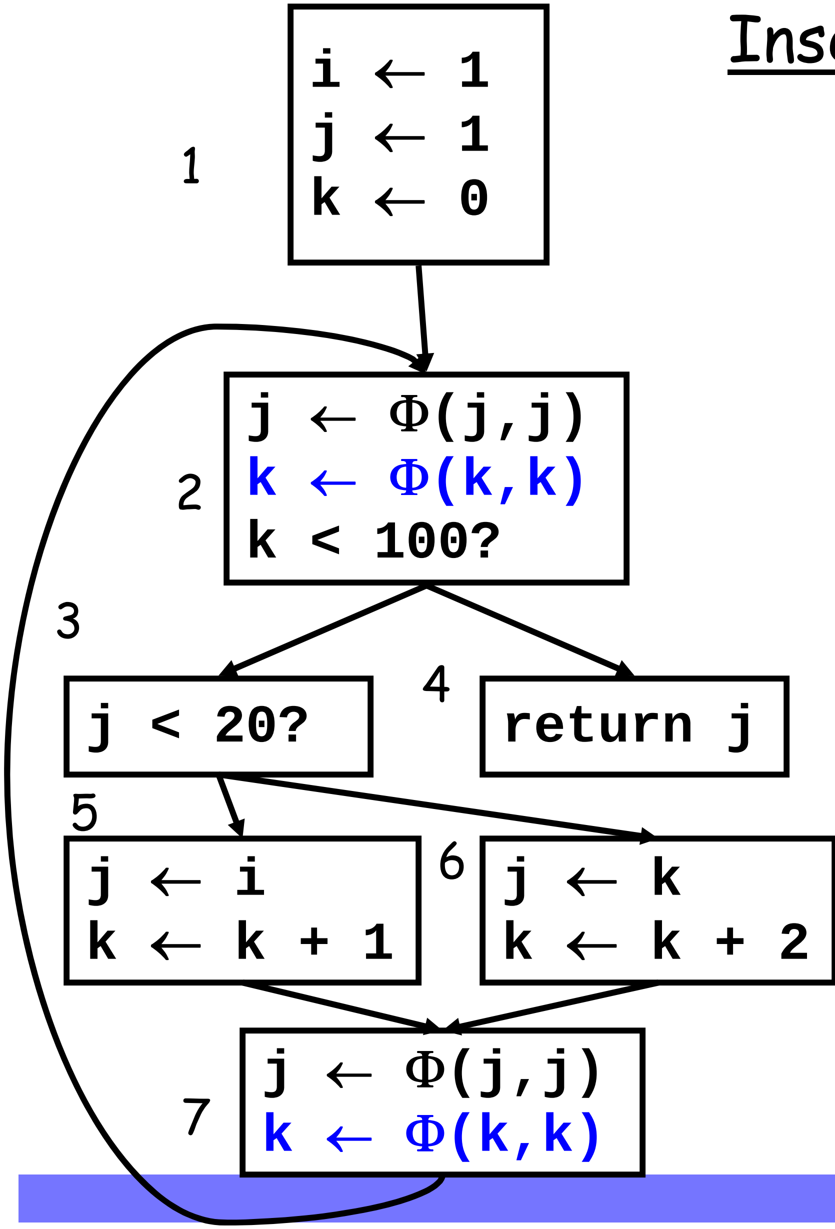
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

var j: W={1,5,6,7}

DF{1} DF{5} DF{7} DF{6}



Insert Φ()

n	DFs	n	orig[n]
1	{}	1	{ i,j,k}
2	{2}	2	{}
3	{2}	3	{}
4	{}	4	{}
5	{7}	5	{j,k}
6	{7}	6	{j,k}
7	{2}	7	{}

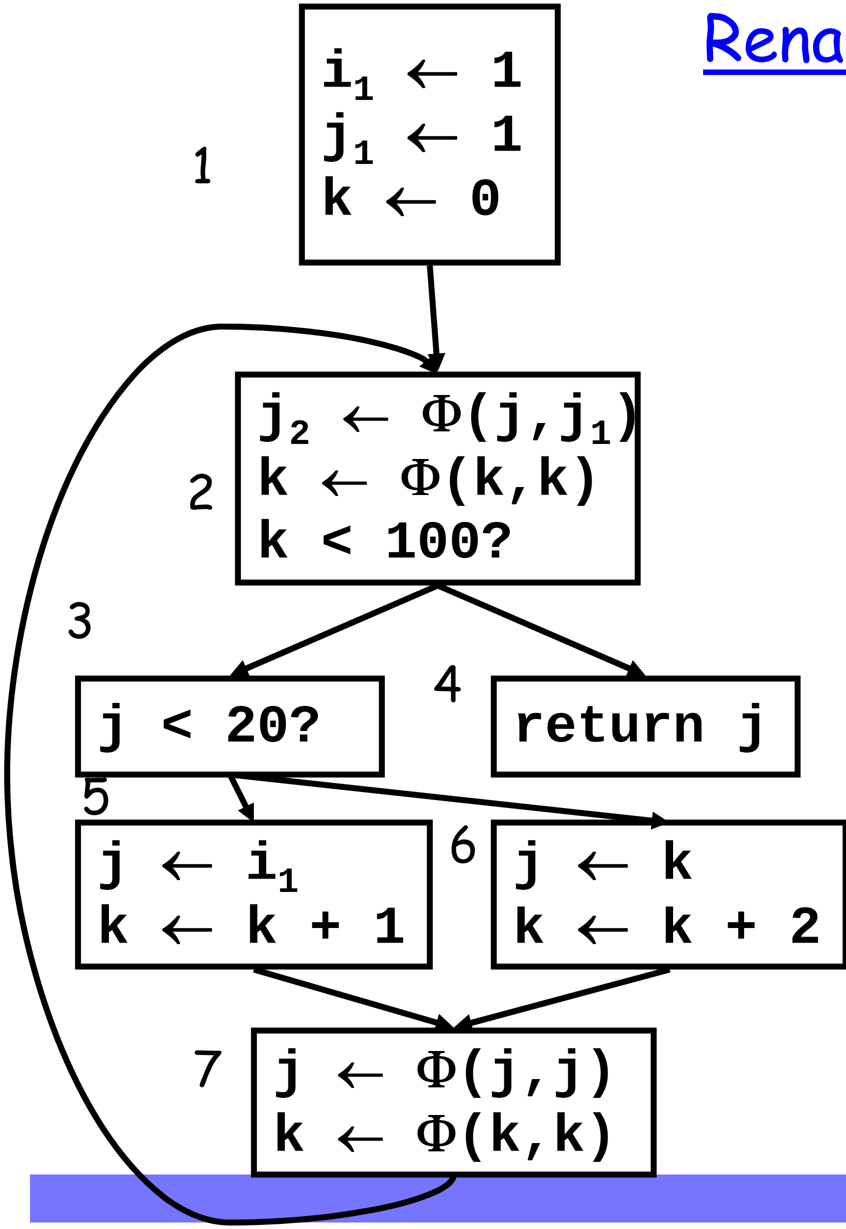
defsites[v]

v	defsites[v]
i	{1}
j	{1,5,6}
k	{1,5,6}

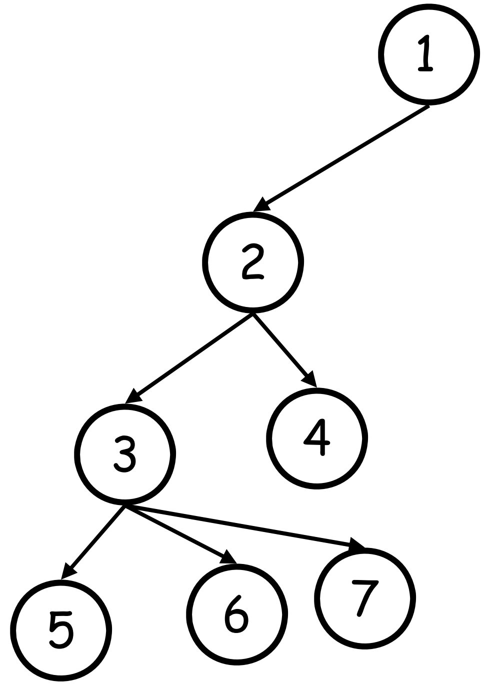
DFs

var k: W={1,5,6}

Rename Vars

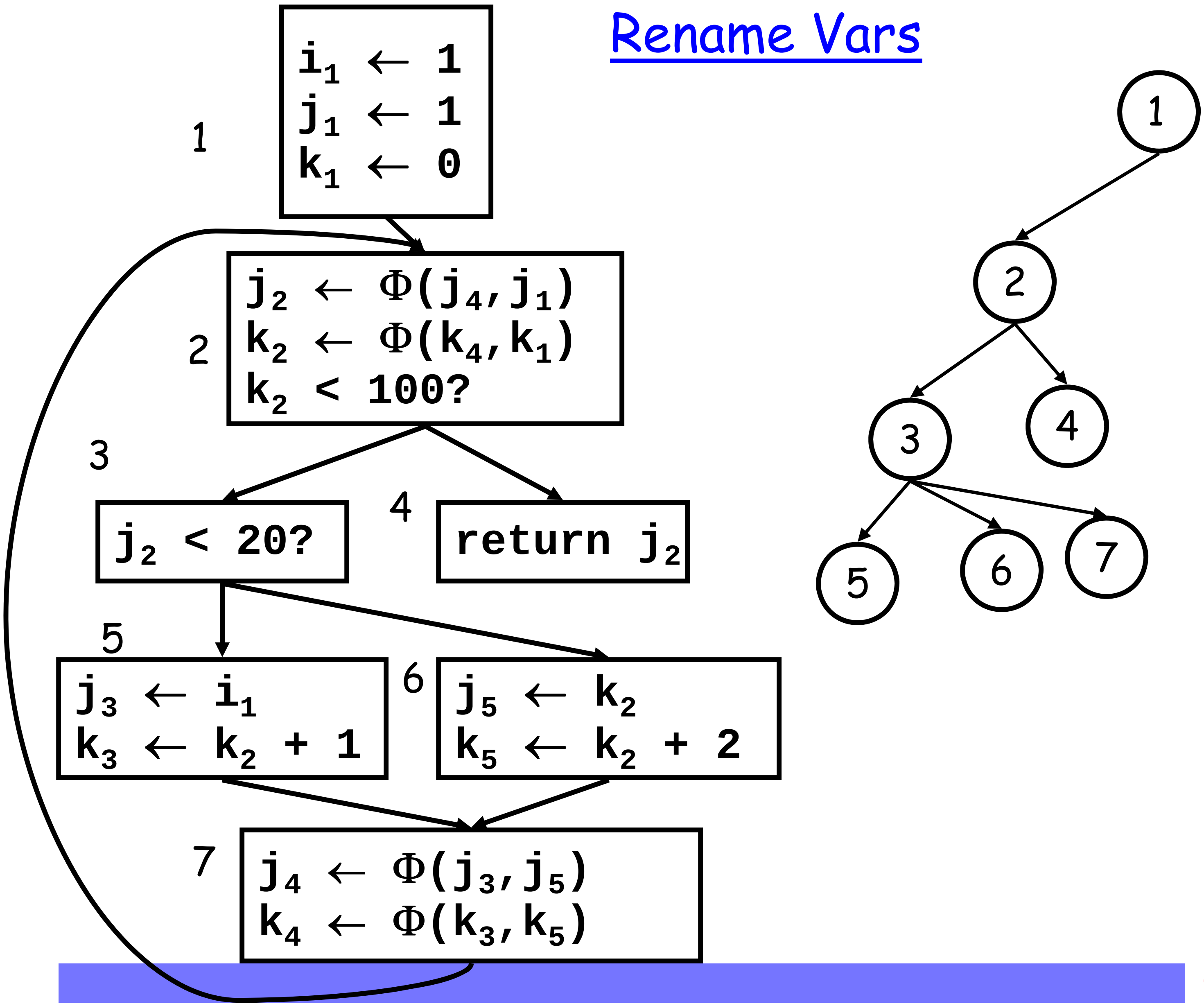


1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k \leftarrow 0$
2
$j_2 \leftarrow \Phi(j, j_1)$
$k \leftarrow \Phi(k, k)$
k < 100?
3
j < 20?
4
return j
5
$j \leftarrow i_1$
$k \leftarrow k + 1$
6
$j \leftarrow k$
$k \leftarrow k + 2$
7
$j \leftarrow \Phi(j, j)$
$k \leftarrow \Phi(k, k)$



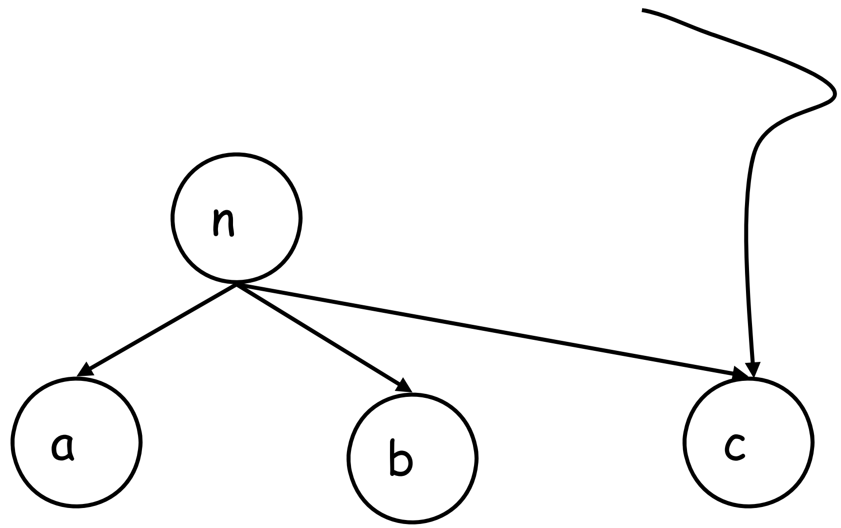
1
2
3
4
5
6
7

Rename Vars



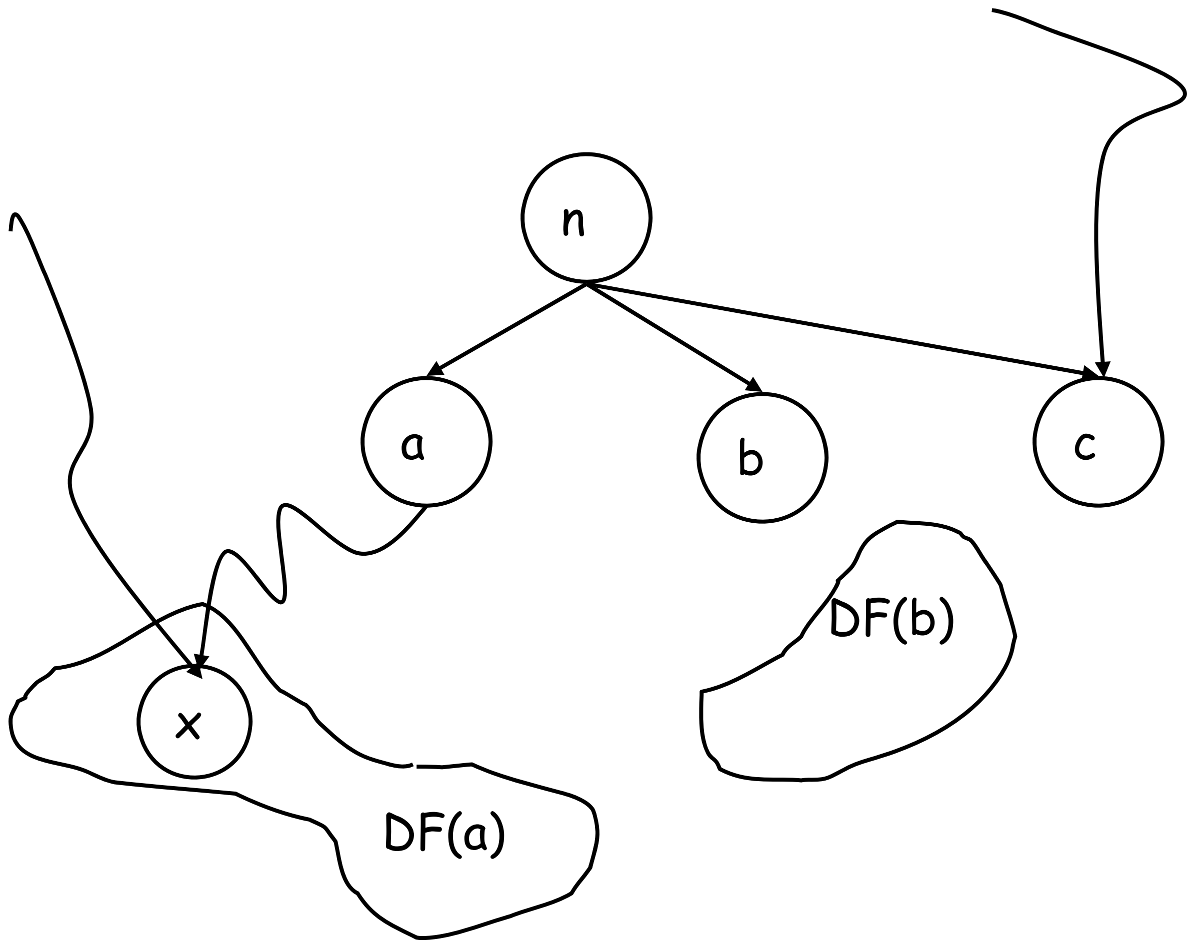
1
i1 ← 1
j1 ← 1
k1 ← 0
2
j2 ← Φ(j4,j1)
k2 ← Φ(k4,k1)
k2 < 100?
3
j2 < 20?
4
return j2
5
j3 ← i1
k3 ← k2 + 1
6
j5 ← k2
k5 ← k2 + 2
7
j4 ← Φ(j3,j5)
k4 ← Φ(k3,k5)
1
2
3
4
5
6
7

Computing DF(n)



n dom a
n dom b
!n dom c

Computing DF(n)



n dom a
n dom b
!n dom c

Computing the Dominance Frontier

The Dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

```
compute-DF(n)
        S = {}
        foreach node y in succ[n]
                if idom(y) ≠ n
                        S = S ∪ { y }
        foreach child of n, c, in D-tree
                compute-DF(c)
                foreach w in DF[c]
                        if !n dom w
                                S = S ∪ { w }
        DF[n] = S
```

SSA Properties

- Only 1 assignment per variable
- Definitions dominate uses