Lecture 9
Partial Redundancy Elimination

- Global code motion optimization
- Remove partially redundant expressions
- Loop invariant code motion
- Can be extended to do Strength Reduction
- No loop analysis needed
- Bidirectional flow problem

Redundancy

- A Common Subexpression is a Redundant Computation

\[
\begin{align*}
t_1 & = a + b \\
t_2 & = a + b \\
t_3 & = a + b
\end{align*}
\]

- Occurrence of expression E at P is redundant if E is available there:
  - E is evaluated along every path to P, with no operands redefined since.
- Redundant expression can be eliminated

References

Loop Invariants are Partial Redundancies

- Loop invariant expression is partially redundant
  
  \[ a = \ldots \]

  \[ t_1 = a + b \]

- As before, partially redundant computation can be eliminated if we insert computations to make it fully redundant.
- Remaining copies can be eliminated through copy propagation or more complex analysis of partially redundant assignments.

Partial Redundancy Elimination

- The Method:
  1. Insert Computations to make partially redundant expression(s) fully redundant.
  2. Eliminate redundant expression(s).

- Issues [Outline of Lecture]:
  1. What expression occurrences are candidates for elimination?
  2. Where can we safely insert computations?
  3. Where do we want to insert them?

  For this lecture, we assume one expression of interest, \( a+b \).
  - In practice, with some restrictions, can do many expressions in parallel.

Which Occurrences Might Be Eliminated?

- In CSE,
  - \( E \) is available at \( P \) if it is previously evaluated along every path to \( P \), with no subsequent redefinitions of operands.
  - If so, we can eliminate computation at \( P \).

- In PRE,
  - \( E \) is partially available at \( P \) if it is previously evaluated along at least one path to \( P \), with no subsequent redefinitions of operands.
  - If so, we might be able to eliminate computation at \( P \), if we can insert computations to make it fully redundant.
  - Occurrences of \( E \) where \( E \) is partially available are candidates for elimination.

Finding Partially Available Expressions

- Forward flow problem
  - Lattice = \( \{ 0, 1 \} \), meet is union (\( \cup \)), Top = 0 (not PAVAIL), entry = 0
  - \( PAVOUT[i] = (PAVIN[i] - KILL[i]) \cup AVLOC[i] \)
  - \( PAVIN[i] = \begin{cases} 
  0 & \text{if } i = \text{entry} \\
  PAVOUT[p] & \text{if } i \in \text{preds}(p) \\
  \text{otherwise} & 
  \end{cases} \)

- For a block,
  - Expression is locally available (AVLOC) if downwards exposed.
  - Expression is killed (KILL) if any assignments to operands.

\[ a = \ldots \]
\[ \ldots = a + b \]
\[ a = \ldots \]
\[ \ldots = a + b \]
Partial Availability Example

For expression $a + b$.

- $a = \ldots$
- $t_1 = a + b$
- $t_2 = a + b$
- $a = \ldots$

$KILL = 1$
$AVLOC = 0$
$PAVIN = PAVOUT = \ldots$

$KILL = 1$
$AVLOC = 0$
$PAVIN = PAVOUT = \ldots$

$KILL = 0$
$AVLOC = 1$
$PAVIN = PAVOUT = \ldots$

$KILL = 1$
$AVLOC = 1$
$PAVIN = PAVOUT = \ldots$

- Occurrence in loop is partially redundant.

Finding Anticipated Expressions

Backward flow problem
- $\text{Lattice} = \{0, 1\}$, meet is intersect ($\cap$), top = 1 (PANT), exit = 0

- $\text{ANTIN}[i] = \text{ANTLOC}[i] \cup (\text{ANTOUT}[i] \cdot \text{KILL}[i])$
- $\text{ANTOUT}[i] = \begin{cases} 0 & i = \text{exit} \\ \cap \text{ANTIN}[s] & \text{otherwise} \end{cases}$

For a block,
- Expression locally anticipated (ANTLOC) if upwards exposed.

- $a = \ldots$
- $\ldots = a + b$
- $a = \ldots$

- $a = \ldots$
- $\ldots = a + b$
- $a = \ldots$

Where Can We Insert Computations?

- Safety: never introduce a new expression along any path.
- Performance: never increase the # of computations on any path.

$t_1 = a + b$
$t_2 = a + b$
$t_3 = a + b$

- Insertion could introduce exception, change program behavior.
- If we can add a new basic block, can insert safely in most cases.
- Solution: insert expression only where it is anticipated.

Anticipation Example

For expression $a + b$.

- $a = \ldots$
- $t_1 = a + b$
- $t_2 = a + b$
- $a = \ldots$

$KILL = 1$
$ANTIN = \ldots$
$ANTLOC = 0$
$ANTOUT = \ldots$

$KILL = 0$
$ANTIN = \ldots$
$ANTLOC = 1$
$ANTOUT = \ldots$

$KILL = 1$
$ANTIN = \ldots$
$ANTLOC = 0$
$ANTOUT = \ldots$

- Expression is anticipated at end of first block.
- Computation may be safely inserted there.
Where Do We Want to Insert Computations?

- Morel-Renvoise and variants: “Placement Possible”
  - Dataflow analysis shows where to insert:
    - **PPIN**: “Placement possible at entry of block or before.”
    - **PPOUT**: “Placement possible at exit of block or before.”
  - Insert at earliest place where **PP** = 1.
  - Only place at end of blocks,
    - **PPIN** really means “Placement possible or not necessary in each predecessor block.”
    - Don’t need to insert where expression is already available.
  - **INSERT[i]** = **PPOUT[i]** \(\cap \neg** PPIN[i]** \(\cup \neg AVOUT[i] \) \(\cap \neg AVOUT[i] \)

- Remove (upwards-exposed) computations where **PPIN**=1.
  - **DELETE[i]** = **PPIN[i]** \(\cap\) **ANTLOC[i]**

Formulating the Problem

- **PPOUT**: we want to place at output of this block only if
  - we want to place at entry of all successors
- **PPIN**: we want to place at input of this block only if (all of):
  - we have a local computation to place, or a placement at the end of this block which we can move up
  - we want to move computation to output of all predecessors where expression is not already available (don’t insert at input)
  - we can gain something by placing it here (**PAVIN** heuristic)
- **PAVIN**
- **PPIN**: we want to place at start of this block only if (all of):
  - we have a local computation to place, or a placement at the end of this block which we can move up
  - we want to move computation to output of all predecessors where expression is not already available (don’t insert at input)
  - we gain something by moving it up (**PAVIN** heuristic)
- **Forward or Backward?** BOTH!
- Problem is bidirectional, but lattice \([0, 1]\) is finite, so
  - as long as transfer functions are monotone, it converges.

Computing “Placement Possible”

- **PPOUT**: we want to place at output of this block only if
  - we want to place at entry of all successors
  - **PPOUT[i]** \(= \begin{cases} 0 & i \in \text{exit} \\ \bigcap_{s \in \text{succ}(i)} \text{PPIN}[s] & \text{otherwise} \end{cases} \)
- **PPIN**: we want to place at start of this block only if (all of):
  - we have a local computation to place, or a placement at the end of this block which we can move up
  - we want to move computation to output of all predecessors where expression is not already available (don’t insert at input)
  - we gain something by moving it up (**PAVIN** heuristic)
- **PPIN[i]** \(= \begin{cases} 0 & i \in \text{exit} \\ \bigcap_{p \in \text{preds}(i)} \text{ANTLOC}[i] \cup \text{PPOUT}[i] \cup \text{KILL}[i] \cup \text{AVOUT}[i] & \text{otherwise} \end{cases} \)
"Placement Possible" Example 1

- \(a = \ldots\)
- \(t_1 = a + b\)
- \(a = \ldots\)
- \(t_2 = a + b\)

- \(KILL = 1\)
- \(PAVIN = 0\)
- \(PPIN = \)
- \(AVLOC = 0\)
- \(AVOUT = 0\)
- \(PPOUT = \)
- \(ANTLOC = 0\)
- \(KILL = 0\)
- \(AVLOC = 1\)
- \(AVOUT = 0\)
- \(PPIN = \)
- \(PPOUT = \)
- \(PPOUT = \)

"Placement Possible" Example 2

- \(a = \ldots\)
- \(t_1 = a + b\)
- \(a = \ldots\)
- \(t_2 = a + b\)

- \(KILL = 1\)
- \(PAVIN = 0\)
- \(PPIN = \)
- \(AVLOC = 1\)
- \(AVOUT = 1\)
- \(PPOUT = \)
- \(ANTLOC = 0\)
- \(KILL = 0\)
- \(AVLOC = 0\)
- \(AVOUT = 1\)
- \(PPIN = \)
- \(PPOUT = \)
- \(PPOUT = \)

"Placement Possible" Correctness

- \textbf{Convergence} of analysis: transfer functions are monotone.
- \textbf{Safety}: Insert only if anticipated.

\[
\begin{align*}
PPIN[i] & \subseteq (PPOUT[i] \cup \text{KILL}[i]) \cup \text{ANTLOC}[i] \\
PPOUT[i] & = \begin{cases}
0 & i = \text{exit} \\
\bigcap_{s \in \text{succ}(i)} PPIN[s] & \text{otherwise}
\end{cases}
\end{align*}
\]

- INSERT \(\subseteq\) PPOUT \(\subseteq\) ANTOUT, so insertion is safe.
- \textbf{Performance}: never increase the \# of computations on any path
  - \(\text{DELETE} = \text{PPIN} \cap \text{ANTLOC}\)
  - On every path from an INSERT, there is a DELETE.
  - The number of computations on a path does not increase.

Morel-Renvoise Limitations

- Movement usefulness tied to PAVIN heuristic
  - Makes some useless moves, might increase register lifetimes:

- Doesn't find some eliminations:

- Bidirectional data flow difficult to compute.
Related Work

- Don't need heuristic
  - Dhamdhere, Drechsler-Stadel, Knoop, et al.
  - use restricted flow graph or allow edge placements.
- Data flow can be separated into unidirectional passes
  - Dhamdhere, Knoop, et al.
- Improvement still tied to accuracy of computational model
  - Assumes performance depends only on the number of computations along any path.
  - Ignores resource constraint issues: register allocation, etc.
  - Knoop, et al. give "earliest" and "latest" placement algorithms which begin to address this.
- Further issues:
  - more than one expression at once, strength reduction, redundant assignments, redundant stores.