

15-745 Lecture 2

Dataflow Analysis
Basic Blocks
Related Optimizations
SSA

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A sample program

```
int fib10(void) {  
    int n = 10;  
    int older = 0;  
    int old = 1;  
    if (What are those numbers?) {  
        int i;  
  
        if (n <= 1) return n;  
        for (i = 2; i < n; i++) {  
            result = old + older;  
            older = old;  
            old = result;  
        }  
        return result;  
    }  
    1:   n <- 10  
    2:   older <- 0  
    3:   old <- 1  
    4:   result <- 0  
    5:   if n <= 1 goto 14  
    6:   i <- 2  
    7:   if i > n goto 13  
    8:   result <- old + older  
    9:   older <- old  
   10:  old <- result  
   11:  i <- i + 1  
   12:  JUMP 7  
   13:  return result  
   14:  return n
```

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Dataflow Analysis

- Last time we looked at code transformations
 - Constant propagation
 - Copy propagation
 - Common sub-expression elimination
 - ...
- Today, dataflow analysis:
 - How to determine if it is **legal** to perform such an optimization
 - (Not doing analysis to determine if it is **beneficial**)

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Simple Constant Propagation

- Can we do SCP?
- How do we recognize it?
- What aren't we doing?
- Metanote:
 - keep opts simple!
 - Use combined power

```
1:   n <- 10  
2:   older <- 0  
3:   old <- 1  
4:   result <- 0  
5:   if n <= 1 goto 14  
6:   i <- 2  
7:   if i > n goto 13  
8:   result <- old + older  
9:   older <- old  
10:  old <- result  
11:  i <- i + 1  
12:  JUMP 7  
13:  return result  
14:  return n
```

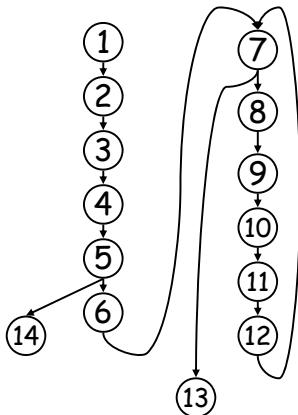
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Reaching Definitions

- A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.



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```

1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
  
```

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Reaching Definitions (ex)

- 1 reaches 5, 7, and 14
- 2 reaches 8
- Older in 8 is reached by
 - 2
 - 9

```

1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
  
```

- Tells us which definitions reach a particular use (ud-info)

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Reaching Definitions (ex)

- 1 reaches 5, 7, and 14

14, Really?

Meta-notes:

- (almost) always conservative
- only know what we know
- Keep it simple:
 - What opt(s), if run before this would help
 - What about:
 - 1: x <- 0
 - 2: if (false) x<-1
 - 3: ... x ...
 - Does 1 reach 3?
 - What opt changes this?

```

1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
  
```

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Calculating Reaching Definitions

- A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.

- Build up RD stmt by stmt
- Stmt s, "d: v <- x op y", generates d
- Stmt s, "d: v <- x op y", kills all other defs(v)

Or,

- $\text{Gen}[s] = \{ d \}$
- $\text{Kill}[s] = \text{defs}(v) - \{ d \}$

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Gen and kill for each stmt

	Gen	kill
1: n <- 10	1	
2: older <- 0	2	9
3: old <- 1	3	10
4: result <- 0	4	8
5: if n <= 1 goto 14		
6: i <- 2	6	11
7: if i > n goto 13		
8: result <- old + older	8	4
9: older <- old	9	2
10: old <- result	10	3
11: i <- i + 1	11	6
12: JUMP 7		
13: return result		
14: return n		

How can we determine the defs that reach a node?

We can use:

- control flow information
- gen and kill info

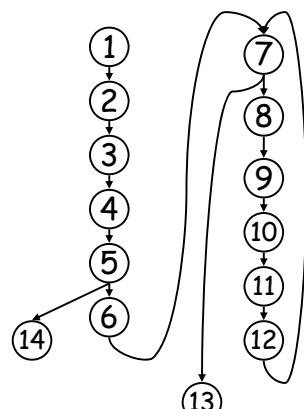
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What is pred[n]?

- Pred[n] are all nodes that can reach n in the control flow graph.
- E.g., pred[7] = { 6, 12 }



```

1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
  
```

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Computing in[n] and out[n]

- In[n]: the set of defs that reach the beginning of node n
- Out[n]: the set of defs that reach the end of node n

$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

- Initialize $in[n]=out[n]=\{\}$ for all n
- Solve iteratively

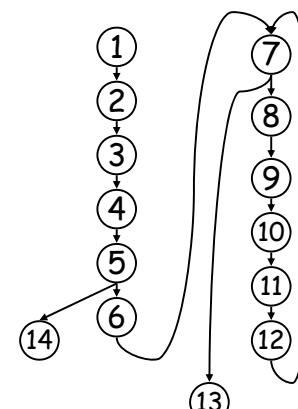
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What order to eval nodes?

- Does it matter?
- Lets do: 1,2,3,4,5,14,6,7,13,8,9,10,11,12



```

1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
  
```

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Example:

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	Kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8		
5: if n <= 1 goto 14				
6: i <- 2	6	11		
7: if i > n goto 13				
8: result <- old + older	8	4		
9: older <- old	9	2		
10: old <- result	10	3		
11: i <- i + 1	11	6		
12: JUMP 7				
13: return result				
14: return n				

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Example (pass 1)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	Kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14				
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13				
8: result <- old + older	8	4	1-4,6	1-3,6,8
9: older <- old	9	2	1-3,6,8	1,3,6,8,9
10: old <- result	10	3	1,3,6,8,9	1,6,8-10
11: i <- i + 1	11	6	1,6,8-10	1,8-11
12: JUMP 7				
13: return result				
14: return n				1-4

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Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \quad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	Kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14			1-4	1-4
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13			1-4,6,8-11	1-4,6,8-11
8: result <- old + older	8	4	1-4,6,8-11	1-3,6,8-11
9: older <- old	9	2	1-3,6,8-11	1,3,6,8-11
10: old <- result	10	3	1,3,6,8-11	1,6,8-11
11: i <- i + 1	11	6	1,6,8-11	1,8-11
12: JUMP 7			1,8-11	1,8-11
13: return result			1-4,6	1-4,6
14: return n			1-4	1-4

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An Improvement: Basic Blocks

- No need to compute this one stmt at a time

- For straight line code:

- In[s1; s2] = in[s1]
- Out[s1; s2] = out[s2]

- Can we combine the gen and kill sets into one set per BB?

Gen Kill

- | | | |
|---------------|---|-----|
| 1: i <- 1 | 1 | 8,4 |
| 2: j <- 2 | 2 | |
| 3: k <- 3 + i | 3 | 11 |
| 4: i <- j | 4 | 1,8 |
| 5: m <- i + k | 5 | |
- Gen[BB]={2,3,4,5}
 - Kill[BB]={1,8,11}
 - Gen[s1;s2]=
 - Kill[s1;s2]=

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BB sets

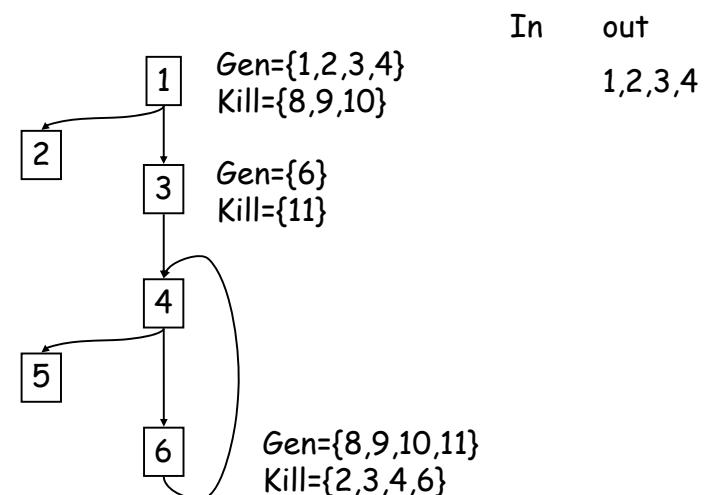
	Gen	kill	
1	1: n <- 10	1	
	2: older <- 0	2	9
	3: old <- 1	3	10
	4: result <- 0	4	8
	5: if n <= 1 goto 14		1,2,3,4 8,9,10
3	6: i <- 2	6	11
4	7: if i > n goto 13		11
	8: result <- old + older	8	4
	9: older <- old	9	2
6	10: old <- result	10	3
	11: i <- i + 1	11	6
	12: JUMP 7		8-11 2-4,6
2	13: return result		
5	14: return n		

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BB sets

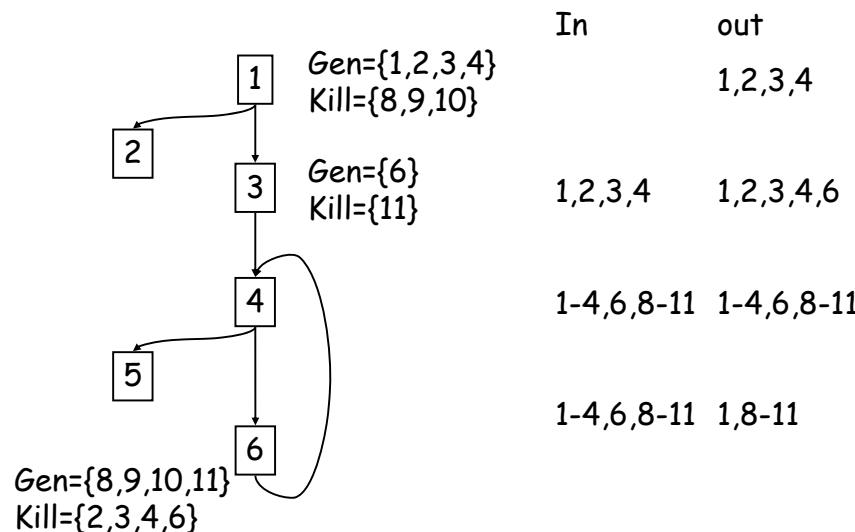


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BB sets



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Forward Dataflow

- Reaching definitions is a forward dataflow problem:
It propagates information from preds of a node to the node
- Defined by:
 - Basic attributes: (gen and kill)
 - Transfer function: $\text{out}[b] = F_{bb}(\text{in}[b])$
 - Meet operator: $\text{in}[b] = M(\text{out}[p])$ for all $p \in \text{pred}(b)$
 - Set of values (a lattice, in this case powerset of program points)
 - Initial values for each node b
- Solve for fixed point solution

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How to implement?

- Values?
- Gen?
- Kill?
- F_{bb} ?
- Order to visit nodes?
- When are we done?
 - In fact, do we know we terminate?

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Implementing RD

- Values: bits in a bit vector
- Gen: 1 in each position generated, otherwise 0
- Kill: 0 in each position killed, otherwise 1
- F_{bb} : $out[b] = (in[b] \mid gen[b]) \& kill[b]$
- Init $in[b]=out[b]=0$
- When are we done?
- What order to visit nodes? Does it matter?

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RD Worklist algorithm

```
Initialize: in[B] = out[b] = ∅  
Initialize: in[entry] = ∅  
Work queue, W = all Blocks in topological order  
while (|W| != 0) {  
    remove b from W  
    old = out[b]  
    in[b] = {over all pred(p) ∈ b} ∪ out[p]  
    out[b] = gen[b] ∪ (in[b] - kill[b])  
    if (old != out[b]) W = W ∪ succ(b)  
}
```

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Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

1: n <- 10	1	
2: older <- 0	1	1,2
3: old <- 1	1,2	1,2,3
4: result <- 0	1-3	1-4
5: if n <= 1 goto 14	1-4	1-4
6: i <- 2	1-4	1-4,6
7: if i > n goto 13	1-4,6,8-11	1-4,6,8-11
8: result <- old + older	1-4,6,8-11	1-3,6,8-11
9: older <- old	1-3,6,8-11	1,3,6,8-11
10: old <- result	1,3,6,8-11	1,6,8-11
11: i <- i + 1	1,6,8-11	1,8-11
12: JUMP 7	1,8-11	1,8-11
13: return result	1-4,6	1-4,6
14: return n	1-4	1-4

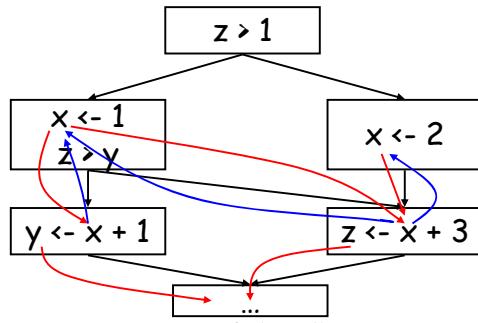
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Def-use chains are valuable too

- Def-use chain: for each definition of var x , a list of all uses of that definition
- Computed from liveness analysis, a backward dataflow problem
- Def-use and use-def are different



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Using RD for Simple Const. Prop.

1: n <- 10	1	
2: older <- 0	1	1,2
3: old <- 1	1,2	1,2,3
4: result <- 0	1-3	1-4
5: if n <= 1 goto 14	1-4	1-4
6: i <- 2	1-4	1-4,6
7: if i > n goto 13	1-4,6,8-11	1-4,6,8-11
8: result <- old + older	1-4,6,8-11	1-3,6,8-11
9: older <- old	1-3,6,8-11	1,3,6,8-11
10: old <- result	1,3,6,8-11	1,6,8-11
11: i <- i + 1	1,6,8-11	1,8-11
12: JUMP 7	1,8-11	1,8-11
13: return result	1-4,6	1-4,6
14: return n	1-4	1-4

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Better Constant Propagation

- What about:

```

x <- 1
if (y > z)
  x <- 1
a <- x
  
```

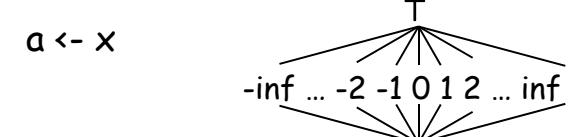
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Better Constant Propagation

- What about: $x <- 1$
if ($y > z$)
 $x <- 1$



Lattice

- Meet: $a \sqcap b = \max(a, b)$
- $\perp \sqcap a = a$
- $c \sqcap c = c$
- $\perp \sqcap d = \perp$ (if $c \neq d$)

- Init all vars to: bot or top?

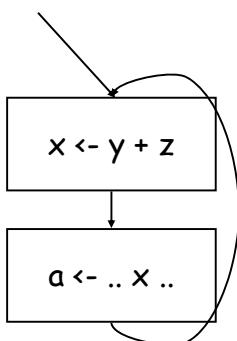
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Loop Invariant Code Motion

- When can expression be moved out of a loop?



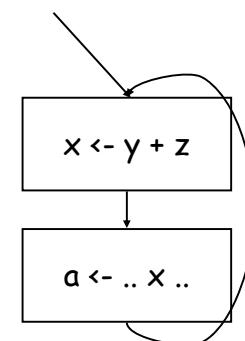
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Loop Invariant Code Motion

- When can expression be moved out of a loop?
- When all reaching definitions of operands are outside of loop, expression is loop invariant
- Use ud-chains to detect
- Can du-chains be helpful?



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Liveness (def-use chains)

- A variable x is live-out of a stmt s if x can be used along some path starting a s , otherwise x is dead.
- Why is this important?
- How can we frame this as a dataflow problem?

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Liveness as a dataflow problem

- This is a backwards analysis
 - A variable is live out if used by a successor
 - Gen: For a use: indicate it is live coming into s
 - Kill: Defining a variable v in s makes it dead before s (unless s uses v to define v)
 - Lattice is just live (top) and dead (bottom)
- Values are variables
- $In[n] = \text{variables live before } n$
= $out[n] - kill[n] \cup gen[n]$
- $Out[n] = \text{variables live after } n$
= $\bigcup_{S \in SUCC(n)} In[S]$

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Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?

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Dead Code Elimination

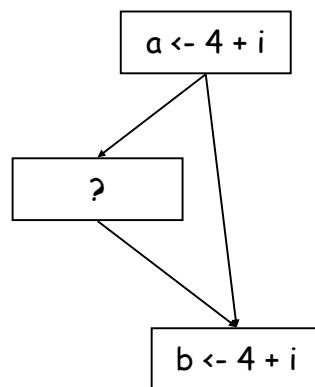
- Code is dead if it has no effect on the outcome of the program.
- When is code dead?
 - When the definition is dead, and
 - When the instruction has no side effects
- So:
 - run liveness
 - Construct def-use chains
 - Any instruction which has no users and has no side effects can be eliminated

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When can we do CSE?



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Available Expressions

- $X+Y$ is "available" at statement S if
 - $x+y$ is computed along every path from the start to S AND
 - neither x nor y is modified after the last evaluation of $x+y$

$a \leftarrow b+c$
 $b \leftarrow a-d$
 $c \leftarrow b+c$
 $d \leftarrow a-d$

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Computing Available Expressions

- Forward or backward?
- Values?
- Lattice?
- $\text{gen}[b] =$
- $\text{kill}[b] =$
- $\text{in}[b] =$
- $\text{out}[b] =$
- initialization?

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Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- $\text{gen}[b] =$ if b evals expr e and doesn't define variables used in e
- $\text{kill}[b] =$ if b assigns to x, then all exprs using x are killed.
- $\text{out}[b] = \text{in}[b] - \text{kill}[b] \cup \text{gen}[b]$
- $\text{in}[b] =$ what to do at a join point?
- initialization?

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Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- $\text{gen}[b] =$ if b evals expr e and doesn't define variables used in e
- $\text{kill}[b] =$ if b assigns to x, exprs(x) are killed
 $\text{out}[b] = \text{in}[b] - \text{kill}[b] \cup \text{gen}[b]$
- $\text{in}[b] =$ An expr is avail only if avail on ALL edges, so: $\text{in}[b] = \cap$ over all $p \in \text{pred}(b)$, $\text{out}[p]$
- Initialization
 - All nodes, but entry are set to ALL avail
 - Entry is set to NONE avail

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Constructing Gen & Kill

Stmt	Gen	Kill
$t \leftarrow x \text{ op } y$	$\{x \text{ op } y\} - \text{kill}[s]$	{exprs containing t}
$t \leftarrow M[a]$	$\{M[a]\} - \text{kill}[s]$	
$M[a] \leftarrow b$		
$f(a, \dots)$		{ $M[x]$ for all x}
$t \leftarrow f(a, \dots)$		

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Constructing Gen & Kill

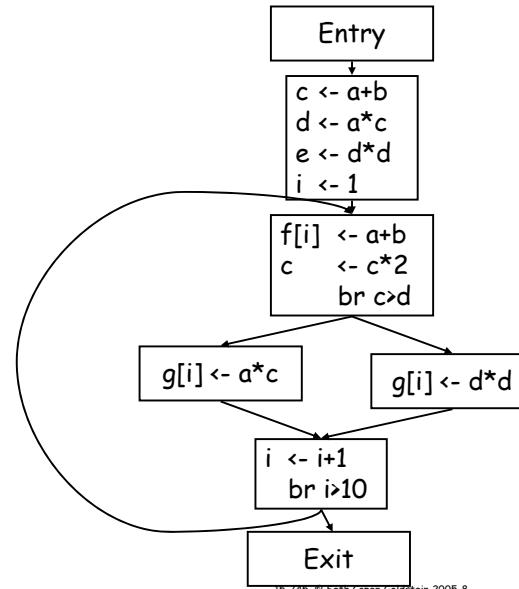
Stmt	Gen	Kill
$t \leftarrow x \text{ op } y$	$\{x \text{ op } y\}$ -kill[s]	{exprs containing t}
$t \leftarrow M[a]$	$\{M[a]\}$ -kill[s]	{exprs containing t}
$M[a] \leftarrow b$	{}	{for all x, $M[x]$ }
$f(a, \dots)$	{}	{for all x, $M[x]$ }
$t \leftarrow f(a, \dots)$	{}	{exprs containing t for all x, $M[x]$ }

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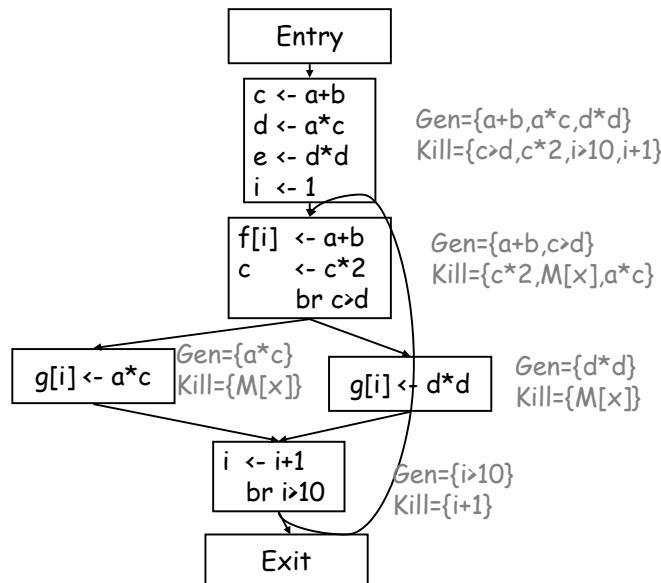
Example



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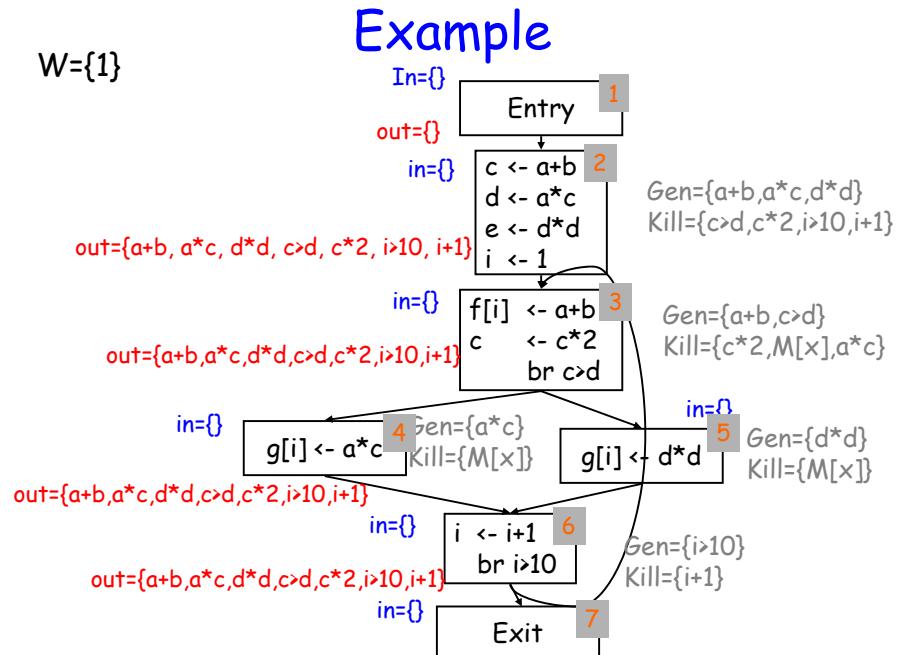
Example



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 $W=\{1\}$ 

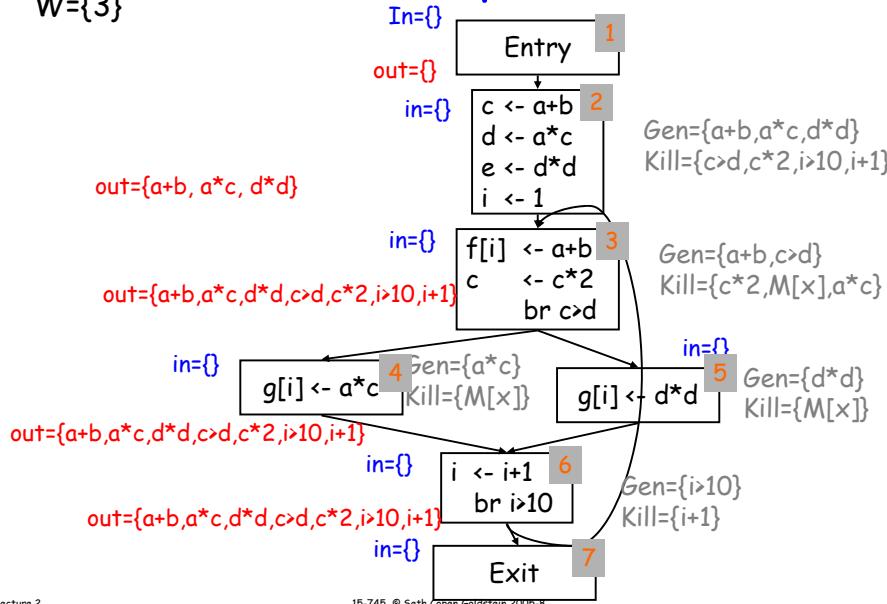
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$W=\{3\}$

Example

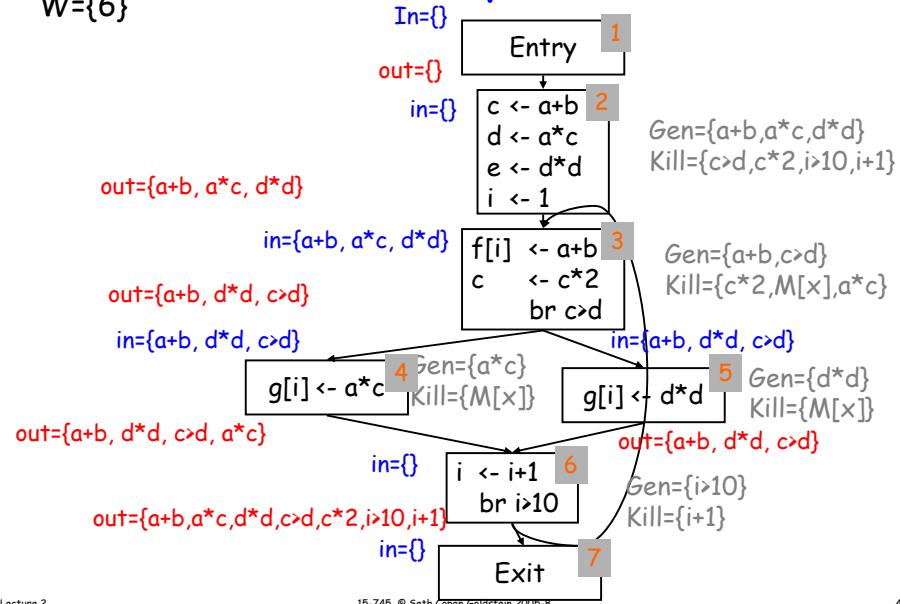


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Example

$W=\{6\}$

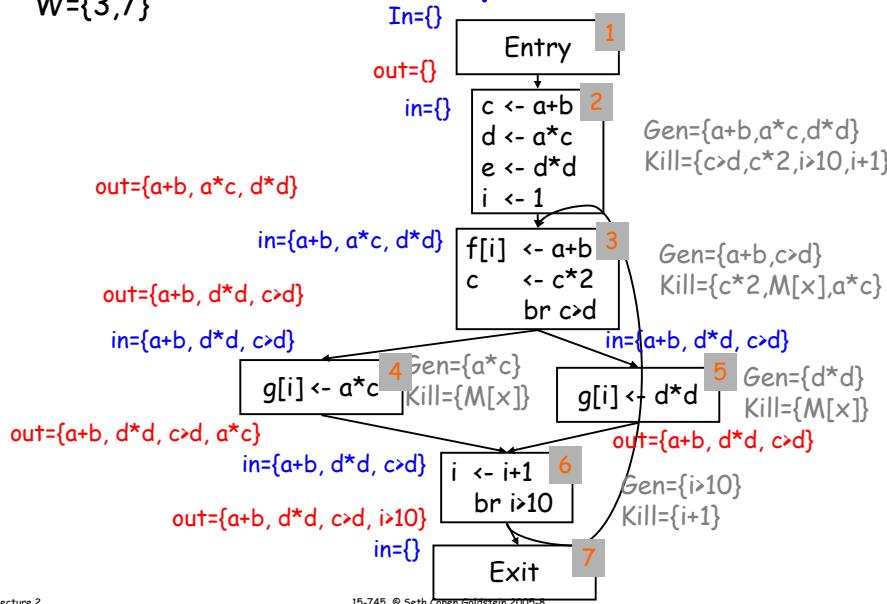


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$W=\{3,7\}$

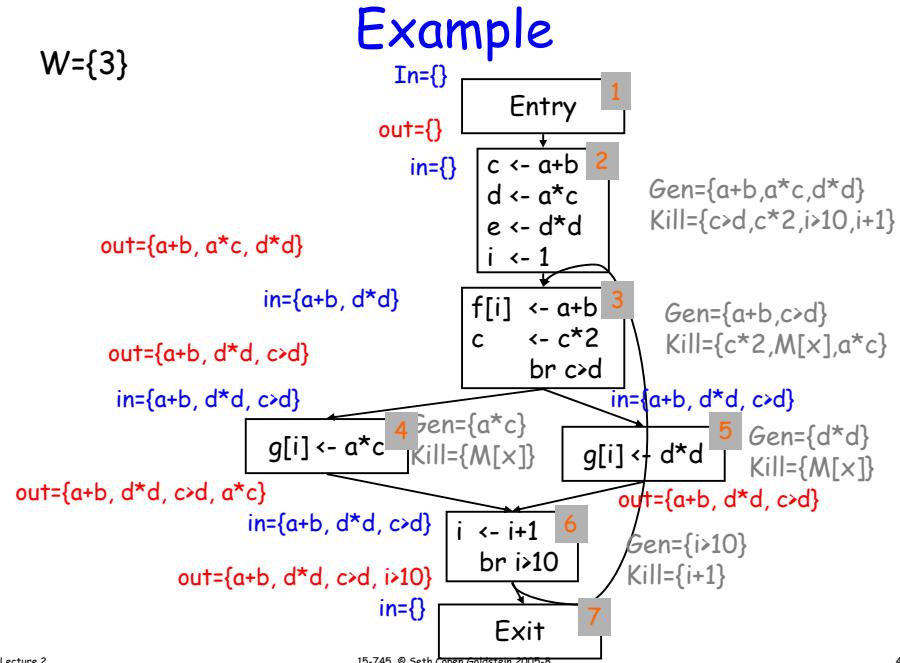
Example



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$W=\{3\}$



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CSE

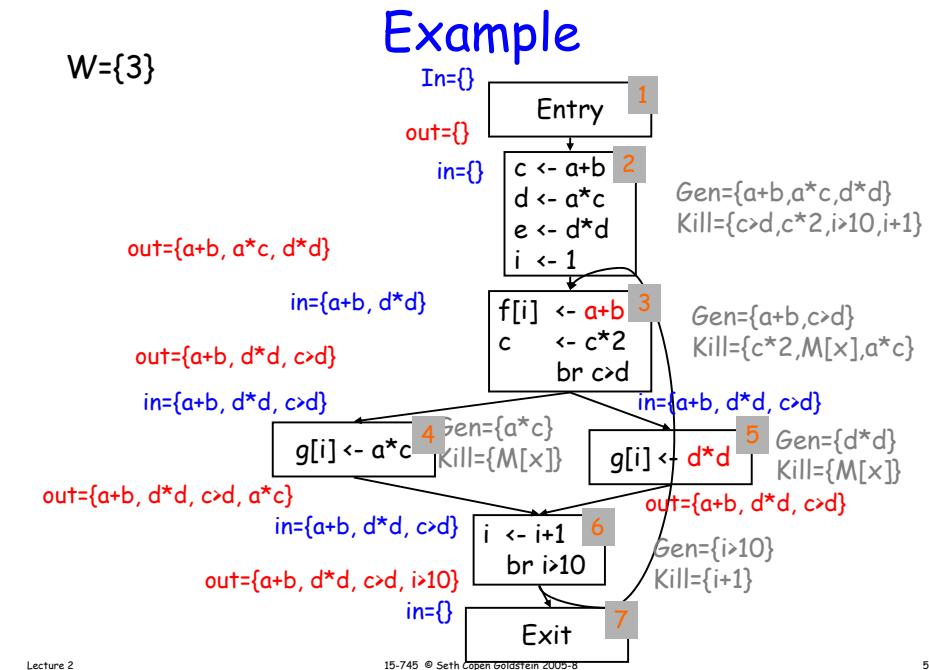
- Calculate Available expressions
- For every stmt in program
 - If expression, $x \text{ op } y$, is available {
 - Compute reaching expressions for $x \text{ op } y$ at this stmt
 - foreach stmt in RE of the form $t \leftarrow x \text{ op } y$
 - rewrite at: $t' \leftarrow x \text{ op } y$
 - $t \leftarrow t'$
- }
- replace $x \text{ op } y$ in stmt with t'

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$W=\{3\}$



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Calculating RE

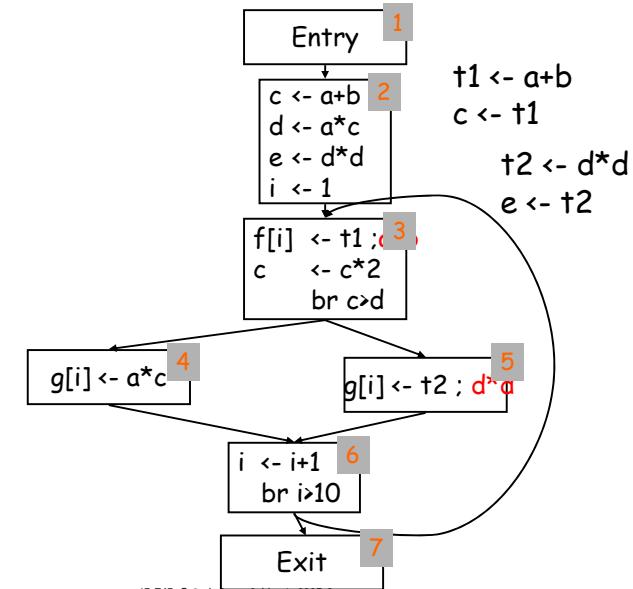
- Could be dataflow problem, but not needed enough, so ...
- To find RE for $x \text{ op } y$ at stmt S
 - traverse cfg backward from S until
 - reach $t \leftarrow x + y$ (& put into RE)
 - reach definition of x or y

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Example



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Dataflow Summary

	Union	intersection
Forward	Reaching defs	Available exprs
Backward	Live variables	

Later in course we look at bidirectional dataflow

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Def-Use chains are expensive

```
foo(int i, int j) {  
    ...  
    switch (i) {  
        case 0: x=3; break;  
        case 1: x=1; break;  
        case 2: x=6; break;  
        case 3: x=7; break;  
        default: x = 11;  
    }  
    switch (j) {  
        case 0: y=x+7; break;  
        case 1: y=x+4; break;  
        case 2: y=x-2; break;  
        case 3: y=x+1; break;  
        default: y=x+9;  
    }  
    ...  
}
```

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Def-Use chains are expensive

```
foo(int i, int j) {  
    ...  
    switch (i) {  
        case 0: x=3;  
        case 1: x=1;  
        case 2: x=6;  
        case 3: x=7;  
        default: x = 11;  
    }  
    switch (j) {  
        case 0: y=x+7;  
        case 1: y=x+4;  
        case 2: y=x-2;  
        case 3: y=x+1;  
        default: y=x+9;  
    }  
    ...  
}
```

In general,
N defs
M uses
 $\Rightarrow O(NM)$ space and time

A solution is to limit each
var to ONE def site

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Def-Use chains are expensive

```
foo(int i, int j) {  
    ...  
    switch (i) {  
        case 0: x=3; break;  
        case 1: x=1; break;  
        case 2: x=6;  
        case 3: x=7;  
        default: x = 11;  
    }  
x1 is one of the above x's  
    switch (j) {  
        case 0: y=x1+7;  
        case 1: y=x1+4;  
        case 2: y=x1-2;  
        case 3: y=x1+1;  
        default: y=x1+9;  
    }  
}
```

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A solution is to limit each
var to ONE def site

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Advantages of SSA

- Makes du-chains explicit
- Makes dataflow analysis easier
- Improves register allocation
 - Automatically builds Webs
 - Makes building interference graphs easier
- For most programs reduces space/time requirements

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SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
 - assign to a fresh variable at each stmt.
 - Each use uses the most recently defined var.
 - (Similar to Value Numbering)

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Straight-line SSA

```
a ← x + y  
b ← a + x  
a ← b + 2  
c ← y + 1  
a ← c + a
```



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Straight-line SSA

$a \leftarrow x + y$	$a_1 \leftarrow x + y$
$b \leftarrow a + x$	$b_1 \leftarrow a_1 + x$
$a \leftarrow b + 2$	$a_2 \leftarrow b_1 + 2$
$c \leftarrow y + 1$	$c_1 \leftarrow y + 1$
$a \leftarrow c + a$	$a_3 \leftarrow c_1 + a_2$



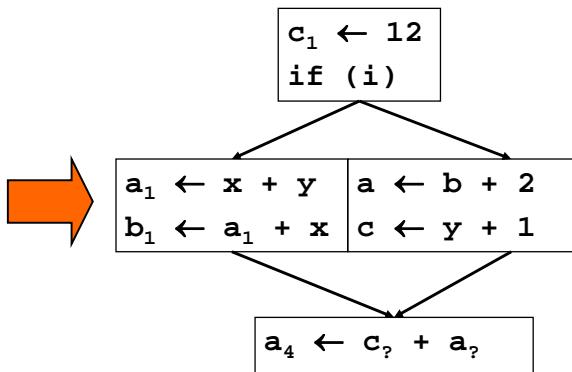
SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
 - assign to a fresh variable at each stmt.
 - Each use uses the most recently defined var.
 - (Similar to Value Numbering)
- **What about at joins in the CFG?**

Merging at Joins

```

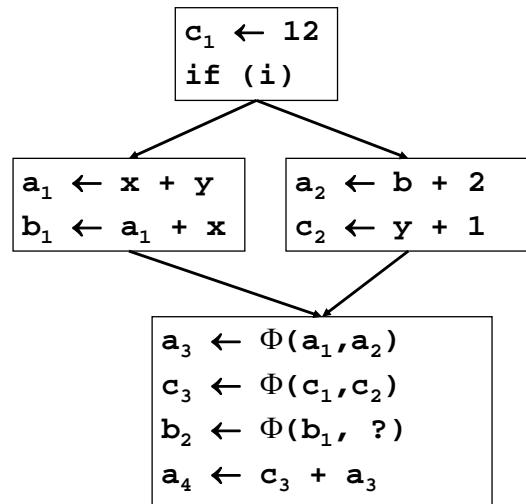
c ← 12
if (i) {
  a ← x + y
  b ← a + x
} else {
  a ← b + 2
  c ← y + 1
}
a ← c + a
    
```



SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
 - assign to a fresh variable at each stmt.
 - Each use uses the most recently defined var.
 - (Similar to Value Numbering)
- **What about at joins in the CFG?**
 - Use a notional fiction: A Φ function

Merging at Joins



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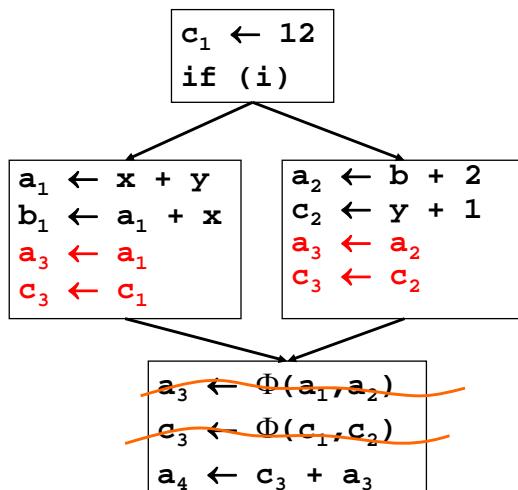
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"Implementing" Φ



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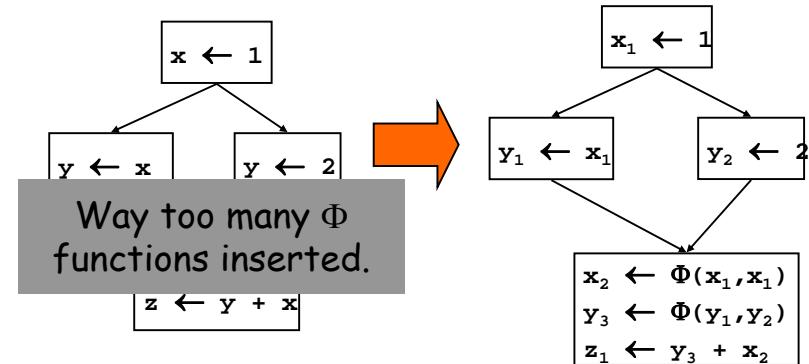
The Φ function

- Φ merges multiple definitions along multiple control paths into a single definition.
- At a BB with p predecessors, there are p arguments to the Φ function.

$$x_{\text{new}} \leftarrow \Phi(x_1, x_2, x_3, \dots, x_p)$$
- How do we choose which x_i to use?
 - We don't really care!
 - If we care, use moves on each incoming edge

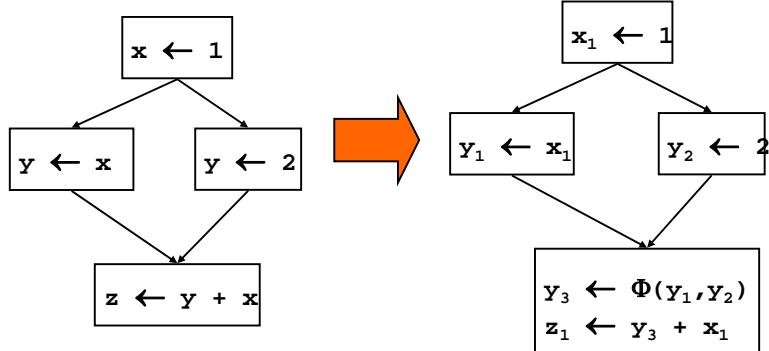
Trivial SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.



Minimal SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all variables with **multiple outstanding defs**.

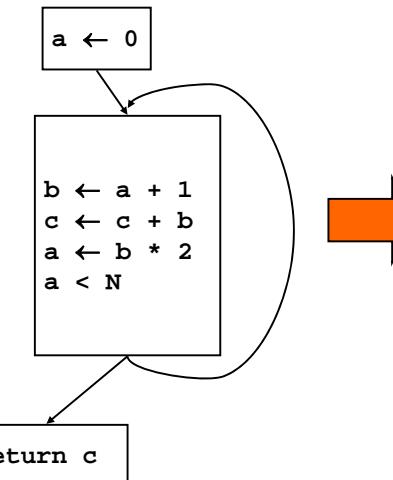


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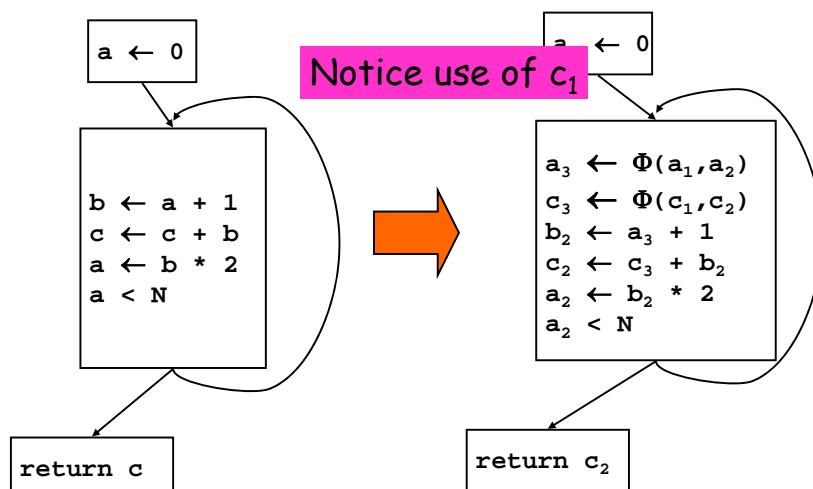
Another Example



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Another Example



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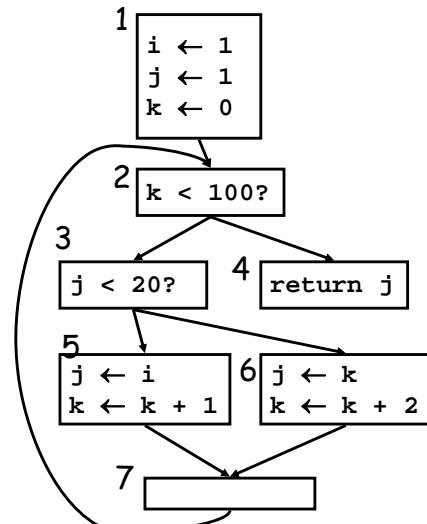
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Lets optimize the following:

```

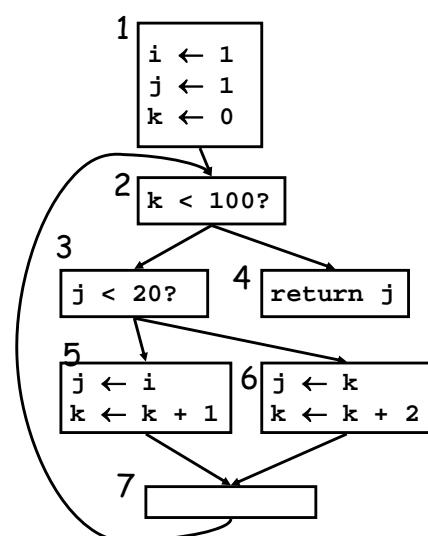
i=1;
j=1;
k=0;
while (k<100) {
    if (j<20) {
        j=i;
        k++;
    } else {
        j=k;
        k+=2;
    }
}
return j;
  
```



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First, turn into SSA



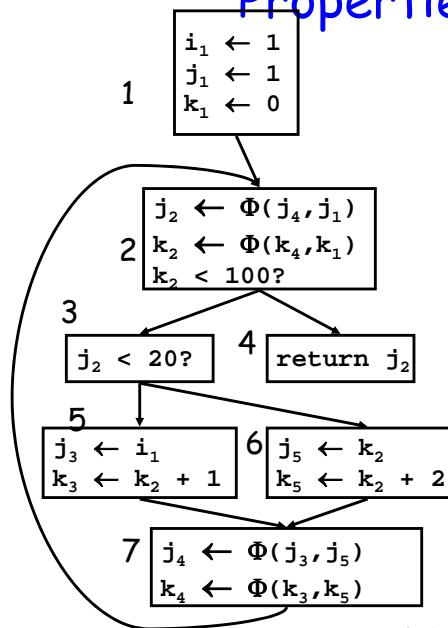
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Properties of SSA

- Only 1 assignment per variable
- definitions dominate uses
- Can we use this to help with constant propagation?



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Constant Propagation

- If " $v \leftarrow c$ ", replace all uses of v with c
- If " $v \leftarrow \Phi(c, c, c)$ " replace all uses of v with c

```
w <- list of all defs
while !W.isEmpty {
    Stmt S <- W.removeOne
    if S has form "v <- \Phi(c,...,c)"
        replace S with V <- c
    if S has form "v <- c" then
        delete S
    foreach stmt U that uses v,
        replace v with c in U
    W.add(U)
}
```

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Other stuff we can do?

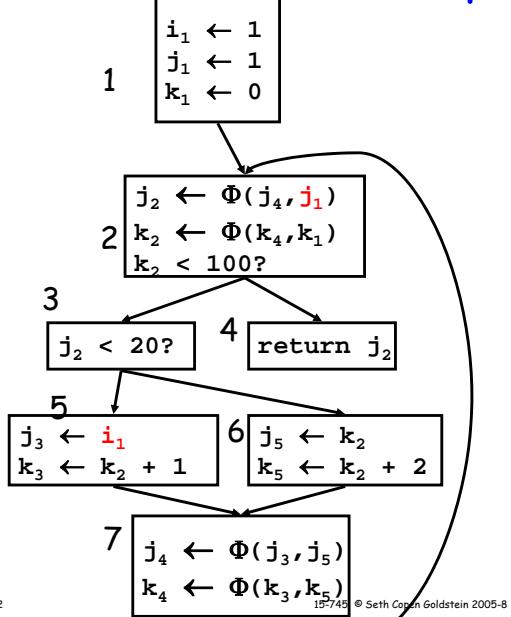
- Copy propagation
delete " $x \leftarrow \Phi(y)$ " and replace all x with y
delete " $x \leftarrow y$ " and replace all x with y
- Constant Folding
(Also, constant conditions too!)
- Unreachable Code
Remember to delete all edges from unreachable block

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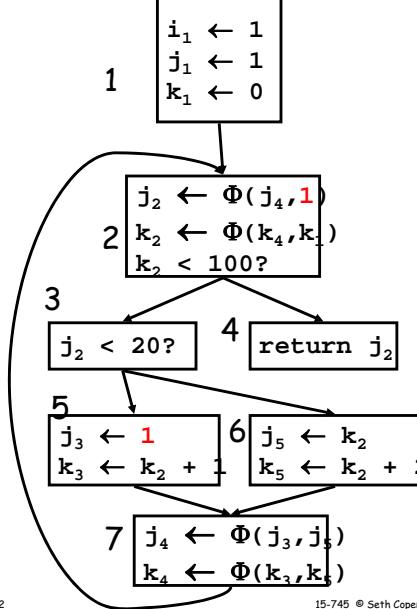
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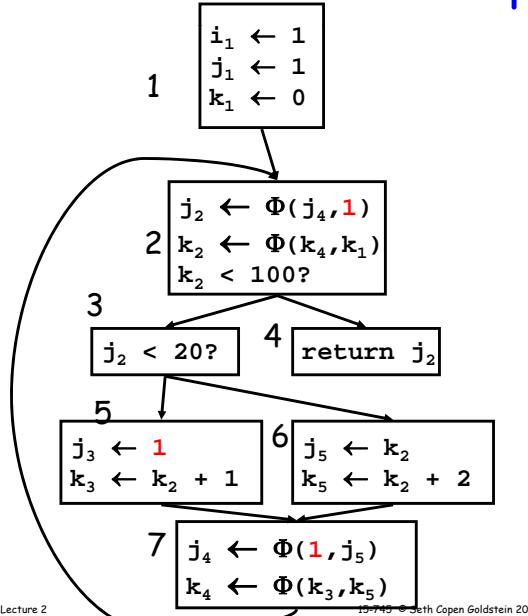
Constant Propagation



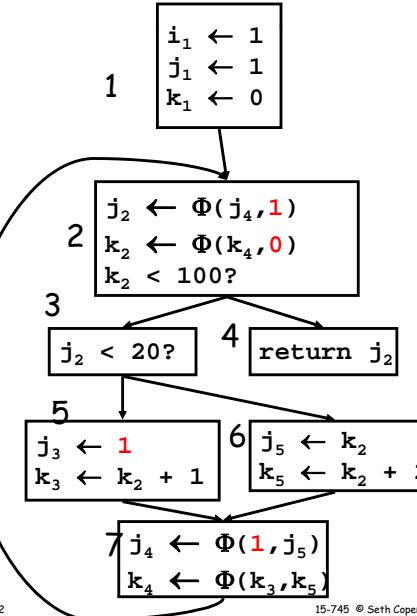
Constant Propagation



Constant Propagation



Constant Propagation



But, so what?

You will have to wait
til next time :)

Summary

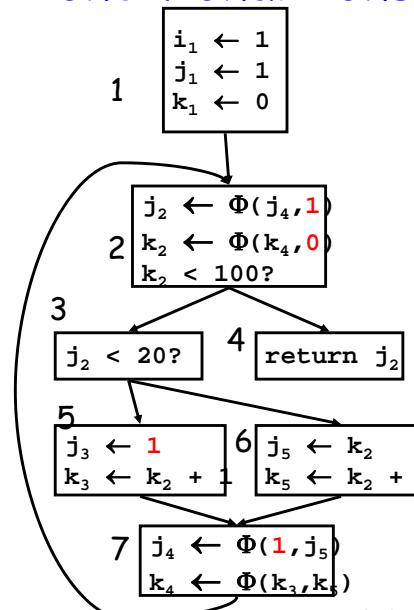
- Dataflow framework
 - Lattice, meet, direction, transfer function, initial values
- Du-chains, ud-chains
- CSE
- SSA
 - One static definition per variable
 - Φ -functions

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Conditional Constant Propagation



- Does block 6 ever execute?
- Simple CP can't tell
- CCP can tell:
 - Assumes blocks don't execute until proven otherwise
 - Assumes Values are constants until proven otherwise

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Tracks:

- Blocks (assume unexecuted until proven otherwise)
- Variables (assume not executed, only with proof of assignments of a non-constant value do we assume not constant)

Use a lattice for variables:

TOP = we have evidence that variable can hold different values at different times

integers = we have seen evidence that the var has been assigned a constant with the value

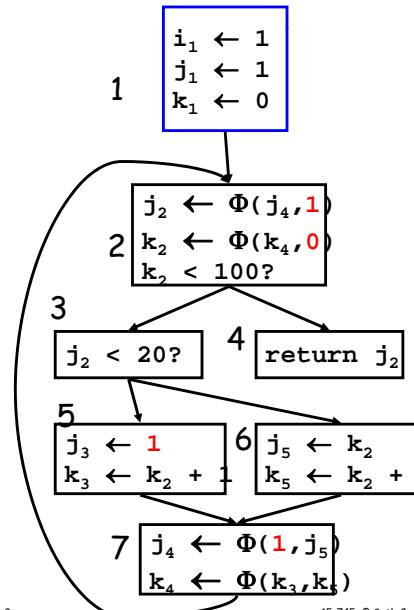
BOT = not executed

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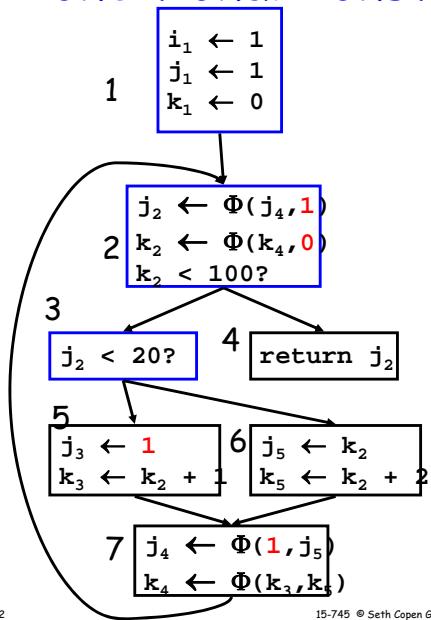
Conditional Constant Propagation



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Conditional Constant Propagation

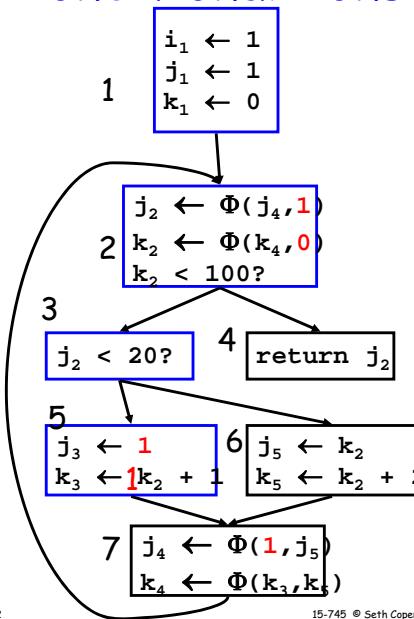


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Conditional Constant Propagation

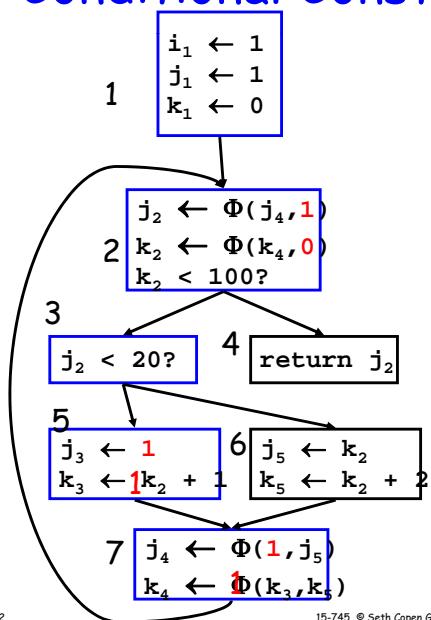


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Conditional Constant Propagation

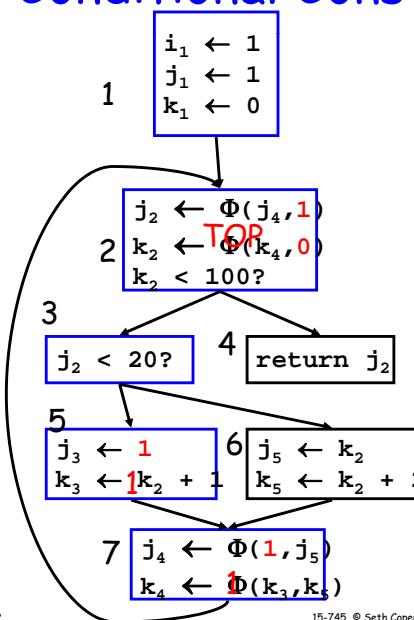


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Conditional Constant Propagation

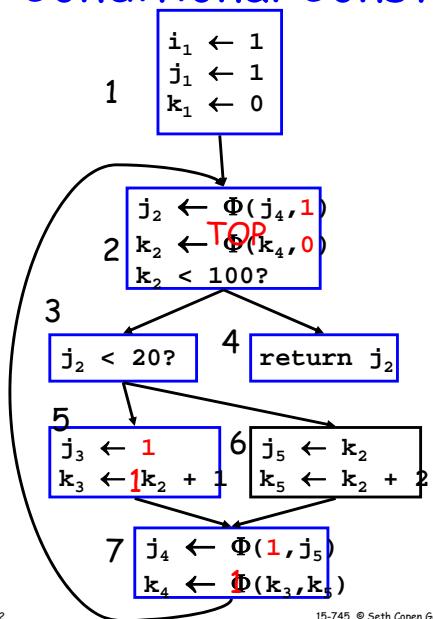


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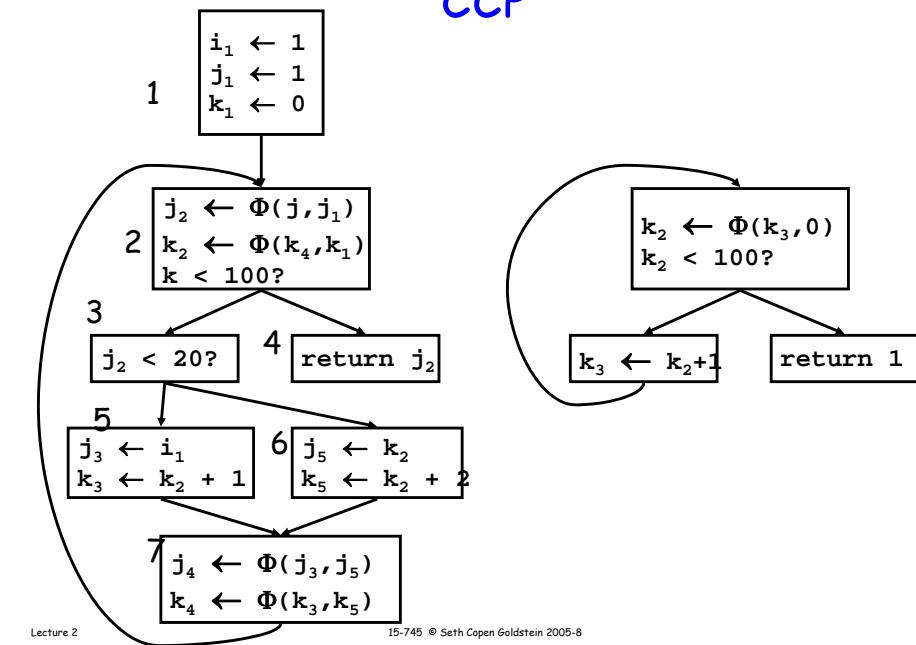
Conditional Constant Propagation



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CCP



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