# PRE and Loop Invariant Code Motion

15-745 Spring 2009

## Common Subexpression Elimination

Find computations that are always performed at least twice on an execution path and eliminate all but the first

Usually limited to algebraic expressions

- put in some cannonical form

Almost always improves performance

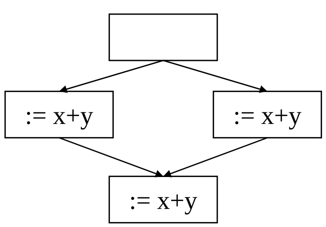
- except when?



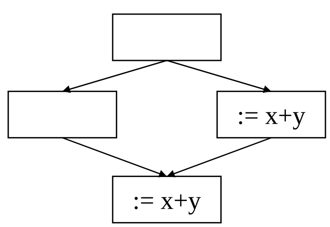
## CSE Limitation

Searches for “totally” redundant expressions

- An expression is totally redundant if it is recomputed along all paths leading to the redundant expression
- An expression is partially redundant if it is recomputed along some but not all paths




***fully redundant***




***partially redundant***



## Loop-Invariant Code Motion

Moves computations that produce the same value on every iteration of a loop outside of the loop

When is a statement loop invariant?

- when all its operands are loop invariant...

# Loop Invariance

An operand is loop-invariant if

1. it is a constant,
2. all defs (use ud-chain) are located outside the loop, or
3. has a single def (ud-chain again) which is inside the loop and that def is itself loop invariant

Can use iterative algorithm to compute loop invariant statements



# Loop Invariant Code **Motion**

Naïve approach: move all loop-invariant statements to the preheader

Not always valid for statements which define variables

If statement s defines *v*, can only move s if

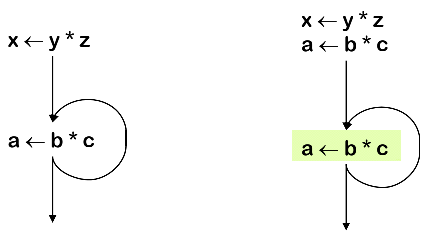
- s dominates **all** uses of *v* in the loop
- s dominates all loop exits

***Why?***



# Loop Invariant Code Motion

Loop invariant expressions are a form of partially redundant expressions. Why?

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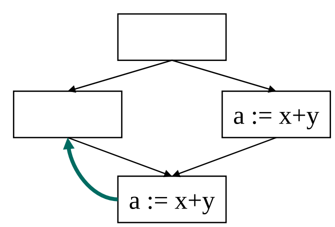
# Partial Redundancy Elimination

Moves computations that are at least partially redundant to their optimal computation points and eliminates totally redundant ones

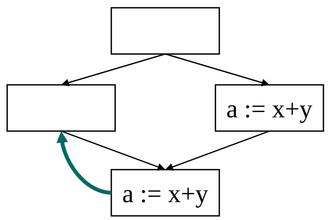
Encompasses CSE and loop-invariant code motion



# Optimal Computation Point

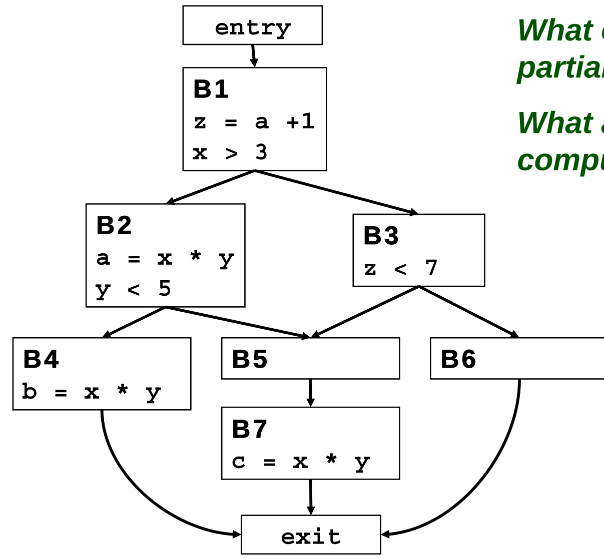
Optimal?

- Result used and never recalculated
- Expression placed late as possible
  *Why?*






# PRE Example

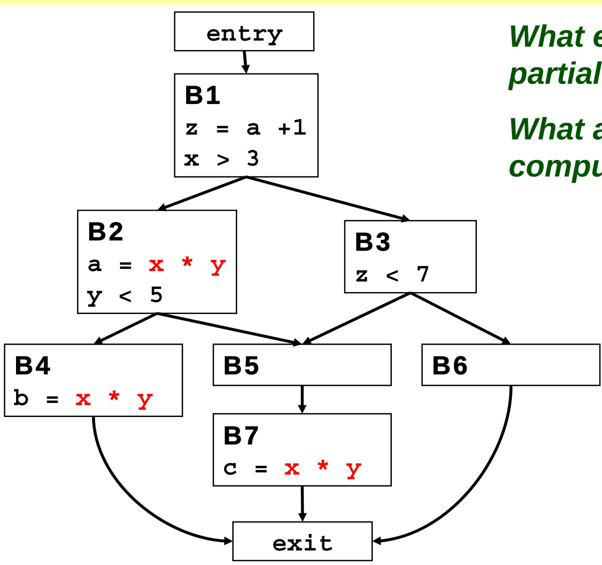



***What expression is partially redundant?***

***What are the optimal computation points?***



# PRE Example

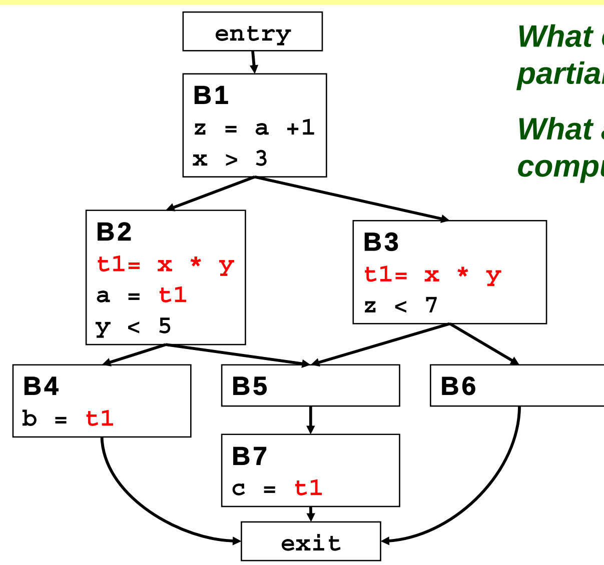



***What expression is partially redundant?***

***What are the optimal computation points?***



# PRE Example

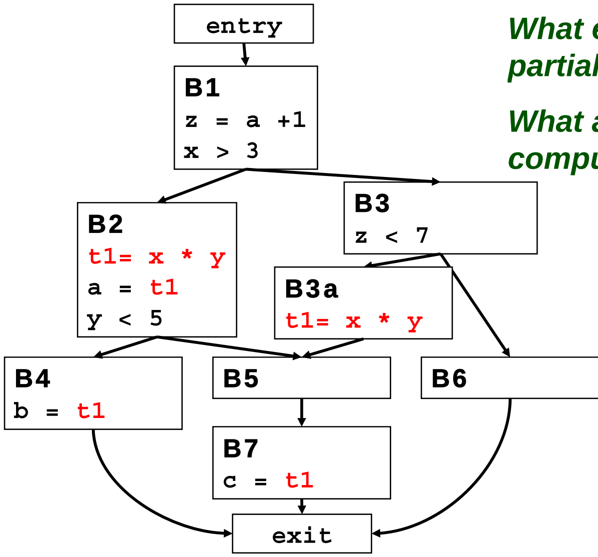



***What expression is partially redundant?***

***What are the optimal computation points?***

Not quite perfect

# PRE Example




***What expression is partially redundant?***

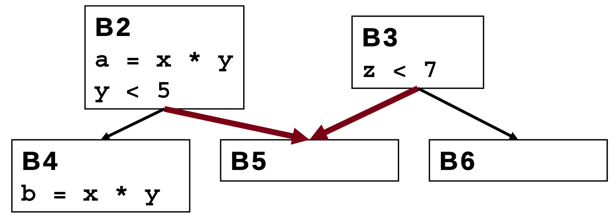
***What are the optimal computation points?***



# Critical Edge Splitting

In order for PRE to work well, we must split critical edges

A *critical edge* is an edge that connects a block with multiple successors to a block with multiple predecessors



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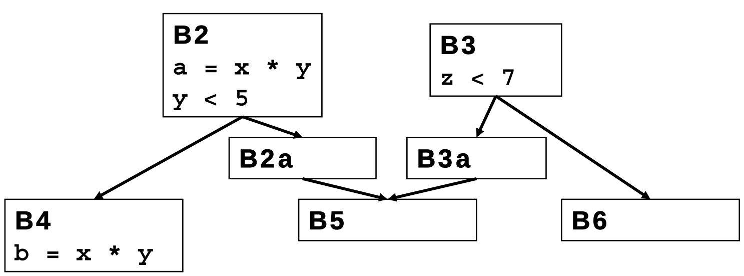




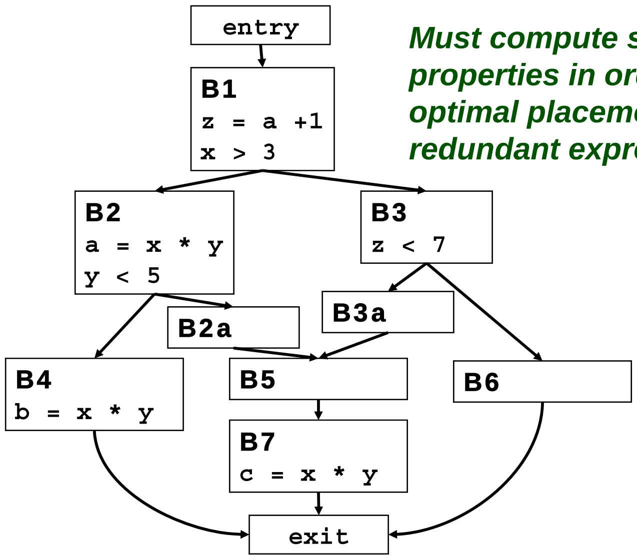

# PRE History

PRE was first formulated as a *bidirectional* data flow analysis by Morel and Renvoise in 1979

Knoop, Rüthing, and Steffen came up with a way to do it using several unidirectional analysis in 1992 (called their approach *lazy code motion*)

- this is a much simpler method
- but it is still very complicated

# PRE Example




***Must compute several data flow properties in order to find optimal placement for partially redundant expressions***



# General Approach to analysis

- Computationally Optimal Placement
  - Anticipatable
    computing exp is useful along any path to exit
  - Earliest
    p is the earliest point to compute exp
  - Compute exp Ant ∩ Earliest
  - Increases register Pressure
- Lazy Code Motion



# General Approach to analysis

- Computationally Optimal Placement
- Lazy Code Motion
  - Latest
    Cannot move exp past p on any path
  - Isolated
    all uses of exp follow immediately after p
  - Compute exp at Latest ∩ ~Isolated



# Local Transparency (TRANSloc)

An expression's value is *locally transparent* in a block if there are no assignments in the block to variables within the expression

- ie, expression not killed

| Block | TRANSloc |
|---|---|
| entry | {a+1,x*y} |
| B1 | {a+1,x*y} |
| B2 | {x*y} |
| B2a | {a+1,x*y} |
| B3 | {a+1,x*y} |
| B3a | {a+1,x*y} |
| B4 | {a+1,x*y} |
| B5 | {a+1,x*y} |
| B6 | {a+1,x*y} |
| B7 | {a+1,x*y} |
| exit | {a+1,x*y} |

# Local Anticipatable (ANTloc)

An expression's value is *locally anticipatable* in a block if

- there is a computation of the expression in the block
- the computation can be safely moved to the beginning of the block

| Block | ANTloc |
|---|---|
| entry | { } |
| B1 | {a+1} |
| B2 | {x*y} |
| B2a | { } |
| B3 | { } |
| B3a | { } |
| B4 | {x*y} |
| B5 | { } |
| B6 | { } |
| B7 | {x*y} |
| exit | { } |



# Globally Anticipatable (ANT)

An expression's value is *globally anticipatable* on entry to a block if

- every path from this point includes a computation of the expression
- it would be valid to place a computation of an expression anywhere along these paths

***This is like liveness, only for expressions***



# Globally Anticipatable (ANT)

$$ANTin(i) = ANTloc(i) \cup \left(TRANSloc(i) \cap ANTout(i)\right)$$

$$ANTout(i) = \bigcap_{j \in succ(i)} ANTin(j)$$

$$ANTout(exit) = \{\}$$

| Block | ANTin | ANTout |
|---|---|---|
| entry | {a+1} | {a+1} |
| B1 | {a+1} | { } |
| B2 | {x*y} | {x*y} |
| B2a | {x*y} | {x*y} |
| B3 | { } | { } |
| B3a | {x*y} | {x*y} |
| B4 | {x*y} | { } |
| B5 | {x*y} | {x*y} |
| B6 | { } | { } |
| B7 | {x*y} | { } |
| exit | { } | { } |



# Earliest (EARL)

An expression's value is *earliest* on entry to a block if

- **no** path from entry to the block evaluates the expression to produce the same value as evaluating it at the block's entry would

***Intuition:***

***at this point if we compute the expression we are computing something completely new***

***says nothing about usefulness of computing expression***

# Earliest (EARL)

$$EARLin(i) = \bigcup_{j \in pred(i)} EARLout(j)$$

$$EARLout(i) = \overline{TRANSloc(i)} \cup \left(\overline{ANTin(i)} \cap EARLin(i)\right)$$

$$EARLin(entry) = U$$

| Block | EARLin | EARLout |
|---|---|---|
| entry | {a+1,x*y} | {x*y} |
| B1 | {x*y} | {x*y} |
| B2 | {x*y} | {a+1} |
| B2a | {a+1} | {a+1} |
| B3 | {x*y} | {x*y} |
| B3a | {x*y} | { } |
| B4 | {a+1} | {a+1} |
| B5 | {a+1} | {a+1} |
| B6 | {x*y} | {x*y} |
| B7 | {a+1} | {a+1} |
| exit | {a+1,x*y} | {a+1,x*y} |



# Computationally Optimal

It is computationally optimal to compute exp at entry to block if

$$\exp \in ANTin(block) \cap EARLin(block)$$

But, it may increase register pressure.



# Delayedness (DELAY)

An expression is *delayed* on entry to a block if

- All paths from entry to block contain a anticipatable and early computation of exp (could be this block) AND all uses of exp follow this block.
- I.e., exp can be delayed to at least this block.



# Delayedness (DELAY)

$$DELAYin(i) = \left(ANTin(i) \cap EARLin(i)\right) \cup \bigcap_{j \in pred(i)} DELAYout(j)$$

$$DELAYout(i) = \overline{ANTloc(i)} \cap DELAYin(i)$$

$$DELAYin(entry) = ANTin(entry) \cap EARLin(entry)$$

| Block | ANTin(i) ∩ EARLin(i) |
|---|---|
| entry | {a+1} |
| B2 | {x*y} |
| B3a | {x*y} |

| Block | DELAYin | DELAYout |
|---|---|---|
| entry | {a+1} | {a+1} |
| B1 | {a+1} | { } |
| B2 | {x*y} | { } |
| B2a | { } | { } |
| B3 | { } | { } |
| B3a | {x*y} | {x*y} |
| B4 | { } | { } |
| B5 | { } | { } |
| B6 | { } | { } |

# Lateness (LATE)

An expression is *latest* on entry to a block if

- it is the optimal point for computing the expression and
- on every path from the block entry to exit, any other optimal computation point occurs after an expression computation in the original flowgraph

***i.e., there is no "later" placement for this expression***



# Latestness (LATE)

$$LATEin(i) = DELAYin(i) \cap \left( ANTloc(i) \cup \overline{\bigcap_{j \in succ(i)} DELAYin(j)} \right)$$

| Block | LATEin |
|---|---|
| entry | { } |
| B1 | {a+1} |
| B2 | {x*y} |
| B2a | { } |
| B3 | { } |
| B3a | {x*y} |
| B4 | { } |
| B5 | { } |
| B6 | { } |
| B7 | { } |
| exit | { } |



# Isolatedness (ISOL)

An optimal placement in a block for the computation of an expression is *isolated* iff

- on every path from a successor of the block to the exit block, every original computation is preceded by the optimal placement point



# Isolatedness (ISOL)

$$ISOLin(i) = LATEin(i) \cup \left( \overline{ANTloc(i)} \cap ISOLout(i) \right)$$

$$ISOLout(i) = \bigcap_{j \in succ(i)} ISOLin(j)$$

$$ISOLout(exit) = \{\}$$

| Block | ISOLin | ISOLout |
|---|---|---|
| entry | { } | { } |
| B1 | {a+1} | { } |
| B2 | {x*y} | { } |
| B2a | { } | { } |
| B3 | { } | { } |
| B3a | {x*y} | { } |
| B4 | { } | { } |
| B5 | { } | { } |
| B6 | { } | { } |
| B7 | { } | { } |
| exit | { } | { } |

## Optimal Placement

The set of expression for which a given block is the optimal computation point is the set of expressions that are latest and not isolated

$$OPT(i) = LATEin(i) \cap \overline{ISOLout(i)}$$



## Redundant Computations

The set of redundant expressions in a block consist of those used in the block that are neither isolated nor latest

$$REDN(i) = ANTloc(i) \cap \overline{LATEin(i) \cup ISOLout(i)}$$



## OPT and REDN

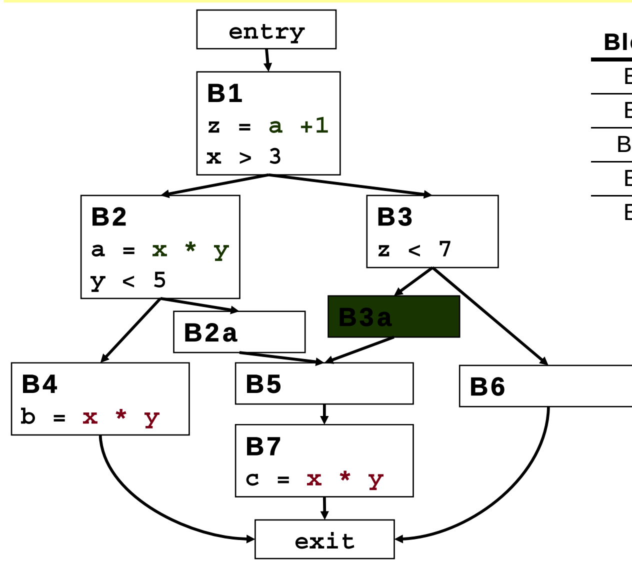
| Block | OPT | REDN |
|---|---|---|
| entry | {} | {} |
| B1 | {a+1} | {} |
| B2 | {x*y} | {} |
| B2a | {} | {} |
| B3 | {} | {} |
| B3a | {x*y} | {} |
| B4 | {} | {x*y} |
| B5 | {} | {} |
| B6 | {} | {} |
| B7 | {} | {x*y} |
| exit | {} | {} |

***insert these (if necessary)***

***remove these***



## PRE Example




| Block | OPT | REDN |
|---|---|---|
| B1 | {a+1} | {} |
| B2 | {x*y} | {} |
| B3a | {x*y} | {} |
| B4 | {} | {x*y} |
| B7 | {} | {x*y} |

# PRE Example

*4 data flow analyses later…*