Indexing with B-trees

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Problem

Given a large collection of records,

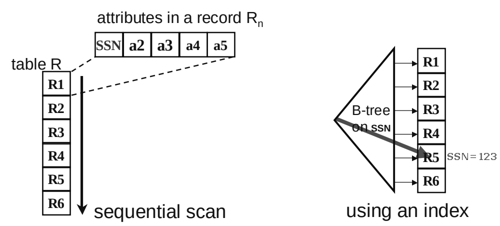
find **similar/interesting** things,
i.e.,
allow fast, approximate queries

Indexing

- primary key indexing
  - B-trees and variants
  - (static) hashing
  - extendible hashing
- secondary key indexing
- spatial access methods
- text
- ...

Primary Key Indexing

- find employee with ssn=123

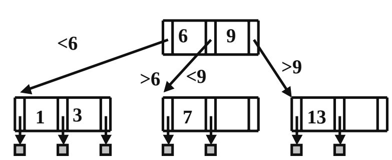


B-trees

- **Most successful** family of index schemes
  - B-trees
  - B^+-trees
  - B^*-trees
- Can be used for
  - primary/secondary, or
  - clustering/non-clustering index.
- Balanced "n-way" search trees

B-trees: Example

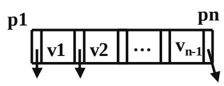
Here is a B-tree of order 3:



B-tree Properties

In a B-tree of order n:

- key order preserved
- at most n pointers
- at least n/2 pointers (except root)
- all leaves at the same level
- if number of pointers is k, node has exactly k-1 keys
- (leaves are empty)

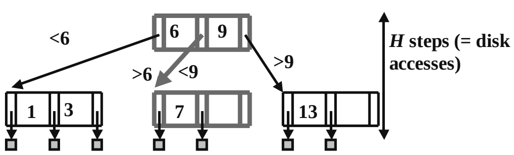


B-tree Properties (cont.)

- “block aware” nodes: each node -> disk page
- O(log (N)) for everything! (ins/del/search)
- typically, if m = 50 - 100, then 2 - 3 levels
- utilization >= 50%, guaranteed; on average 69%

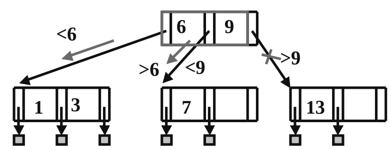
Exact-Match Queries

E.g., ssn=8



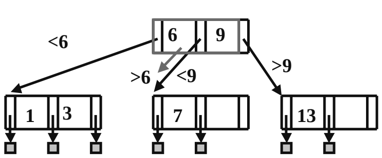
Range Queries

E.g., ***5<salary<8***



Proximity Queries

E.g., nearest neighbor searches: ***salary ~ 8***

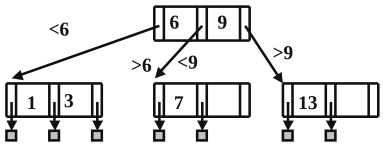


B-trees: Insertion

- Insert in leaf; on overflow, push middle up (recursively)
- split: preserves B - tree properties

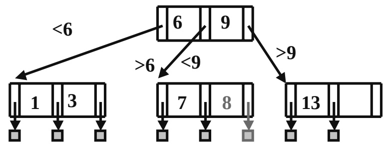
B-trees: Insertion (cont.)

Easy case: Tree T0; insert ‘**8**’



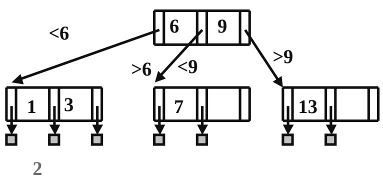
B-trees: Insertion (cont.)

Tree T0; insert ‘**8**’



B-trees: Insertion (cont.)

Hardest case: Tree T0; insert ‘**2**’



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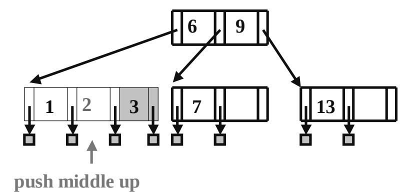

B-trees: Insertion (cont.)

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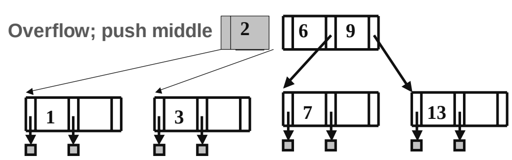

B-trees: Insertion (cont.)

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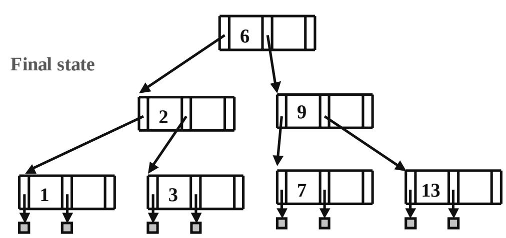


B-trees - insertion

- Q: What if there are two middles? (eg, order 4)
- A: either one is fine

B-trees: Insertion Sketch

Algorithm:

1. insert in leaf
2. on overflow:
   push middle up (recursively – 'propagate split')

- Split preserves all B - tree properties (!!)
- Notice how it grows:
  height increases when root overflows & splits
- Automatic, incremental re-organization

Algorithm: Insertion of Key 'K'

```
find the correct leaf node 'L';
  if ( 'L' overflows ) {
    split 'L' by pushing middle key up to parent 'P';
    if ('P' overflows) {
      repeat the split recursively;
    } else {
     add key 'K' in node 'L';  // maintain key order in 'L'
  }
```


B-trees: Deletion

Rough outline of algorithm:
- Delete key;
- on underflow, may need to merge

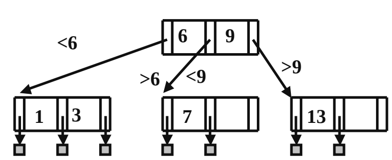
In practice, some implementors just allow underflows to happen…

B-trees: Deletion Cases

- <u>Case 1</u>
  delete a key at a leaf – no underflow
- <u>Case 2</u>
  delete non-leaf key – no underflow
- <u>Case 3</u>
  delete leaf-key; underflow, and 'rich sibling'
- <u>Case 4</u>
  delete leaf-key; underflow, and 'poor sibling'

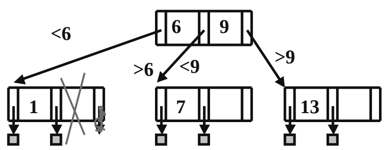
B-trees: Deletion Case 1

- Case 1: delete a key at a leaf
- Easiest case: no underflow (delete 3 from T0)



B-trees: Deletion Case 1 (cont.)

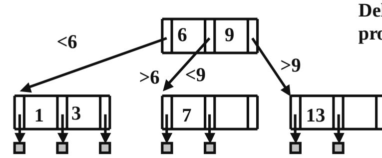
Easiest case: Tree T0; delete ‘3’



B-trees: Deletion Case 2

- Case2: delete a key at a non-leaf
- no underflow (eg., delete 6 from T0)

Delete & promote, ie:



B-trees: Deletion Case 2 (cont.)

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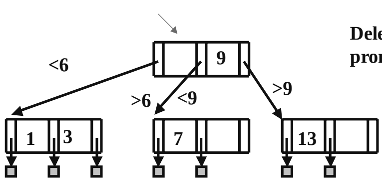

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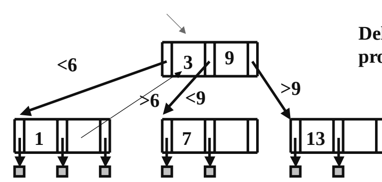

B-trees: Deletion Case 2 (cont.)

- Case2: delete a key at a non-leaf
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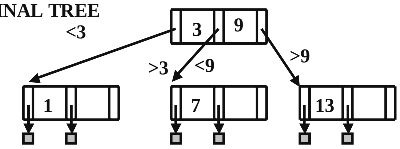
FINAL TREE



B-trees: Deletion Case 2 (cont.)

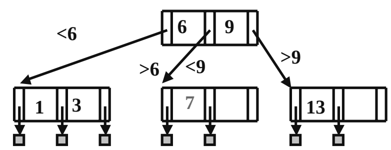
- Case2: delete a key at a non-leaf – no underflow (eg., delete 6 from T0)
- Q: How to promote?
- A: pick the largest key from the left sub-tree (or the smallest from the right sub-tree)
- Observation: every deletion eventually becomes a deletion of a leaf key

B-trees: Deletion Cases (cont.)

- Case1: delete a key at a leaf – no underflow
- Case2: delete non-leaf key – no underflow
- ⇨ Case3: delete leaf-key; underflow, and 'rich sibling'
- Case4: delete leaf-key; underflow, and 'poor sibling'

B-trees: Deletion Case 3

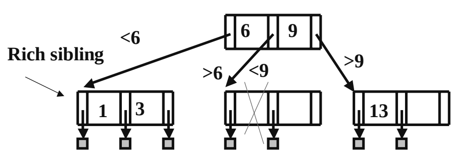
- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



B-trees: Deletion Case 3 (cont.)

Case3: underflow & 'rich sibling'
- e.g., delete 7 from T0

Delete & borrow



B-trees: Deletion Case 3 (cont.)

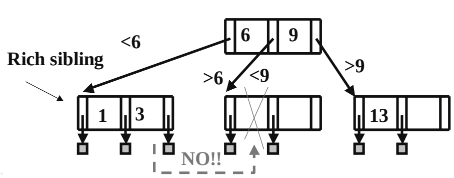
- Case3: underflow & 'rich sibling'

- 'rich' = can give a key, without underflowing
- 'borrowing' a key: THROUGH the PARENT!

B-trees: Deletion Case 3 (cont.)

Case3: underflow & 'rich sibling'
- e.g., delete 7 from T0

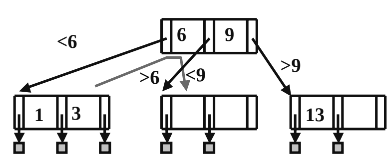
Delete & borrow



B-trees: Deletion Case 3 (cont.)

Case3: underflow & 'rich sibling'
- e.g., delete 7 from T0

Delete & borrow

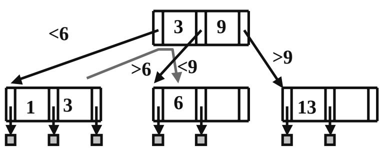


B-trees: Deletion Case 3 (cont.)

Case3: underflow & ‘rich sibling’

- e.g., delete 7 from T0

Delete & borrow

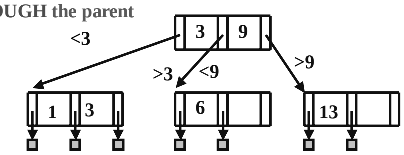


B-trees: Deletion Case 3 (cont.)

Case3: underflow & ‘rich sibling’

- e.g., delete 7 from T0

Delete & borrow THROUGH the parent



B-trees: Deletion Cases (cont.)

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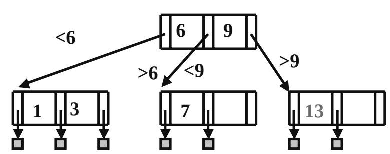
- ⇨ Case4: delete leaf-key; underflow, and ‘poor sibling’

B-trees: Deletion Case 4

Case4: underflow & ‘poor sibling’

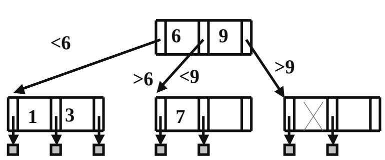
- eg., delete 13 from T0)



B-trees: Deletion Case 4 (cont.)

Case4: underflow & ‘poor sibling’

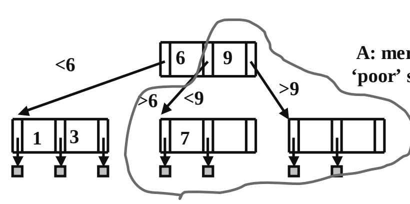
- eg., delete 13 from T0)



B-trees: Deletion Case 4 (cont.)

Case4: underflow & ‘poor sibling’

- eg., delete 13 from T0)



A: merge w/ ‘poor’ sibling

B-trees: Deletion Case 4 (cont.)

Case4: underflow & ‘poor sibling’

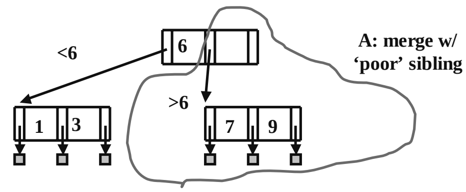
- eg., delete 13 from T0)

- Merge, by pulling a key from the parent
- exact reversal from insertion: ‘split and push up’, vs. ‘merge and pull down’
- Ie.:

B-trees: Deletion Case 4 (cont.)

Case4: underflow & ‘poor sibling’

- eg., delete 13 from T0)



B-trees: Deletion Case 4 (cont.)

Case4: underflow & ‘poor sibling’

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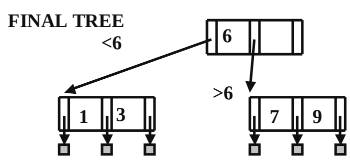


B-trees: Deletion Case 4 (cont.)

- Case 4: underflow & ‘poor sibling’
- -> ‘pull key from parent, and merge’
- Q: What if the parent underflows?
- A: repeat recursively

Algorithm: Deletion of Key ‘K’

```
locate key ‘K’, in node ‘N’
  if( ‘N’ is a non-leaf node) {
    delete ‘K’ from ‘N’;
    find the immediately largest key ‘K1’;
      /* which is guaranteed to be on a leaf node ‘L’ */
    copy ‘K1’ in the old position of ‘K’;
    invoke DELETION on ‘K1’ from the leaf node ‘L’;
  else {
/* ’N’ is a leaf node */
```

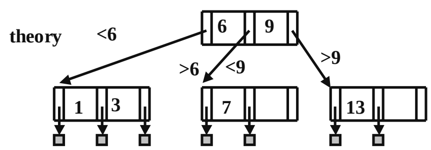
…next slide…

Deletion of Key ‘K’ (cont.)

```
if( ’N’ underflows ){
      let ’N1’ be the sibling of ’N’;
      if( ’N1’ is "rich"){   /* ie., N1 can lend us a key */
        borrow a key from ’N1’  THROUGH parent node;
      } else {    /* N1 is 1 key away from underflowing */
            MERGE: pull key from parent ’P’, merge it
            with keys of ’N’ and ’N1’  into new node;
        if( ’P’ underflows) { repeat recursively }
      }
  }
```

B-trees in Practice

In practice:
- no empty leaves;
- pointers to records



B-trees in Practice (cont.)

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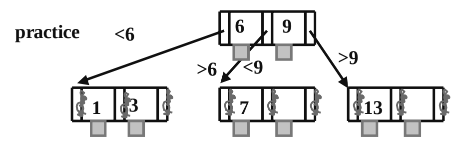

B-trees in Practice (cont.)

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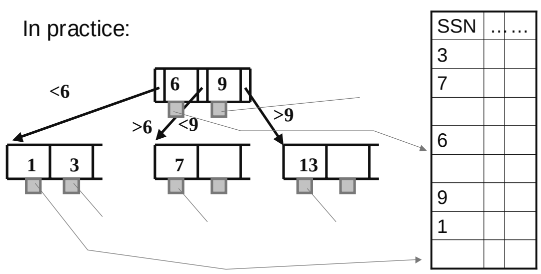
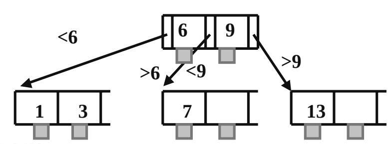


B-trees in Practice (cont.)

In practice, the formats are:
- leaf nodes: (v1, rp1, v2, rp2, … vn, rpn)
- Non-leaf nodes: (p1, v1, rp1, p2, v2, rp2, …)

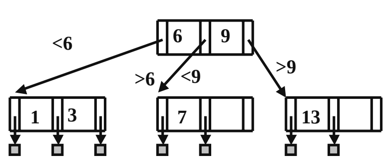


Overview

- B – trees
- **B+ - trees, B*-trees**

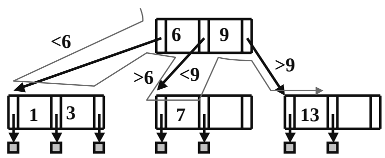
B+ trees: Motivation

B-tree – print keys in sorted order:



B+ trees: Motivation (cont.)

B-tree needs back-tracking – how to avoid it?



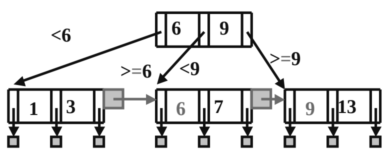
Solution: B+ - trees

- Facilitate sequential ops
- They string all leaf nodes together

AND

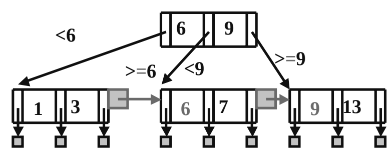
- replicate keys from non-leaf nodes, to make sure every key appears at the leaf level

B+ trees



B+ trees: Insertion

E.g., insert '**2**'

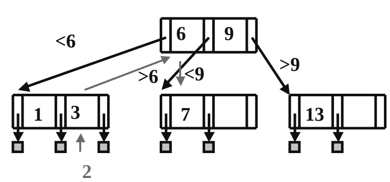


B*-trees: Motivation

- Splits drop utilization to 50%
- May increase height
- How to avoid them?

B*-trees: Deferred Split!

Instead of splitting, LEND keys to sibling!
(through PARENT, of course!)



B*-trees: deferred split!

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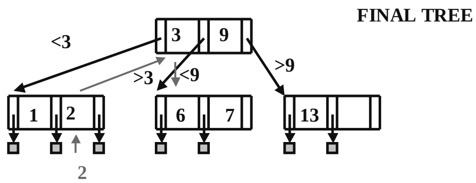


B*-trees: Advantages

- Tree becomes
  - Shorter,
  - More packed,
  - Faster
- Rare case: improve together
  - space utilization
  - speed

- BUT: What if sibling has no room for 'lending'?

B*-trees: deferred split!

- BUT: What if sibling has no room for 'lending'?

- <u>2-to-3 split</u>
  1. get the keys from the sibling
  2. pool them with ours (and a key from the parent)
  3. split in **3**

- Details: too messy (and even worse for deletion)

Conclusions

- Main ideas: recursive; block-aware; on overflow -> split; defer splits
- All B-tree variants have excellent, **O(logN) worst-case performance for ins/del/search**
- It's the prevailing indexing method

Performance Aspects of B-trees

Two parameters matter:

- Height H (maximum search path)

$$H = 1 + \lceil \log_{F^*}(\lceil N/C^* \rceil) \rceil$$

  - N is the number of tuples
  - C* is the average number of entries in a leaf node, and
  - F* is the average number of entries in an index node.

- Size S (number of pages tree occupies)

$$S = \sum_i (F^*)^{i-1},\ 1 \le i < H$$

Reducing the Number of Leafs

- Increase page size (hard)
- Shorten data length (values, tuples, pointers)

- Is it worthwhile to change the tuples to TIDs?

- No – extra page accesses!

From Gray&Reuter: $1.1 \le \log_{F^*} X$ must hold

i.e., average fan-out really small or tuples > 1K

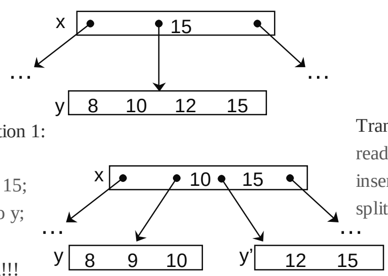
Increasing the Fanout

- Compression
  - Prefix – store differences (suffixes)
  - Suffix – store prefixes
- Prefix compression: sequential scan
  - “anchor” keys

Lehman and Yao – CC on B-trees

- “safe” node: node with <2k entries
- “unsafe” node: node with =2k entries
- Simple CC won’t do. Why?

Example



Transaction 1:
read x;
look for 15;
get ptr to y;

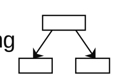
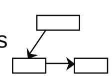
ERROR!!!

Transaction 2:
read x; read y;
insert 9
split y into y+y’

Previous B-tree CC algorithms

- Samadi 1976
  - lock the whole subtree of affected node
- Bayer & Schkolnick 1977
  - parameters on degree/type of consistency required
  - writer-exclusion locks (readers may proceed) upper
  - exclusive locks on modified nodes
- Miller & Snyder 1978
  - pioneer and follower locks
  - locked region moves up the tree
  - no modifications

B^{link}-tree

- Node + P2k+1 – pointer to next node at the same level of tree
- Rightmost node’s B-link is NULL
- IDEA:
- Splitting  is implemented as 
- legal to have “left twin” and no parent

Advantages

- Allows for “temporary fix” until all pointers are added correctly
- Link pointers should be used infrequently
  - because splitting a node is a “special case”
- “Level traversal” comes for free as a side effect

Algorithms

- Search
  - No locks needed for reads
  - Just move right as well as down
- Insertions
  - Well-ordered locks
  - Use stack to remember ancestors
  - Split while preserving links
- Deletions
  - No underflows, no merging