Lecture 24
Petri Nets 1

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Models of Software Systems
15-671
Fall 1995

This Lecture

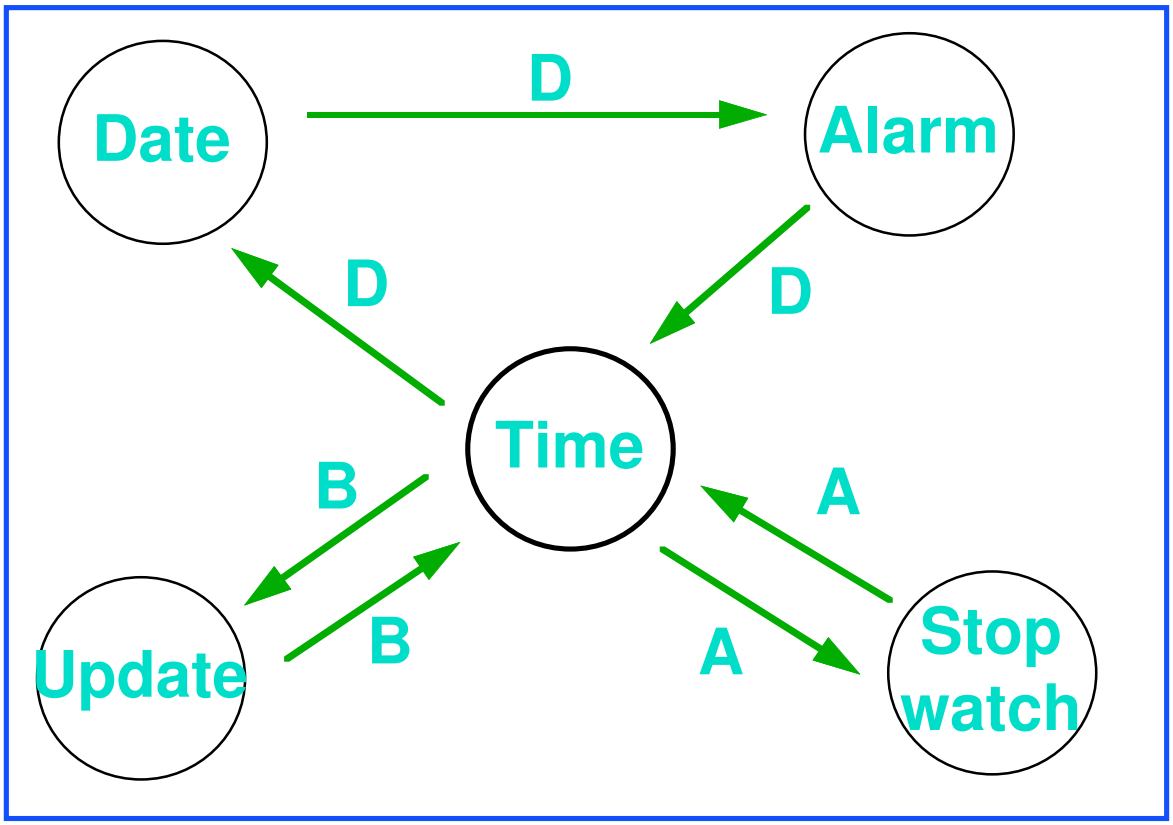
- Modelling concurrency
- Petri Nets - some examples
- Formal definitions
- Special classes of Petri Nets
- More examples

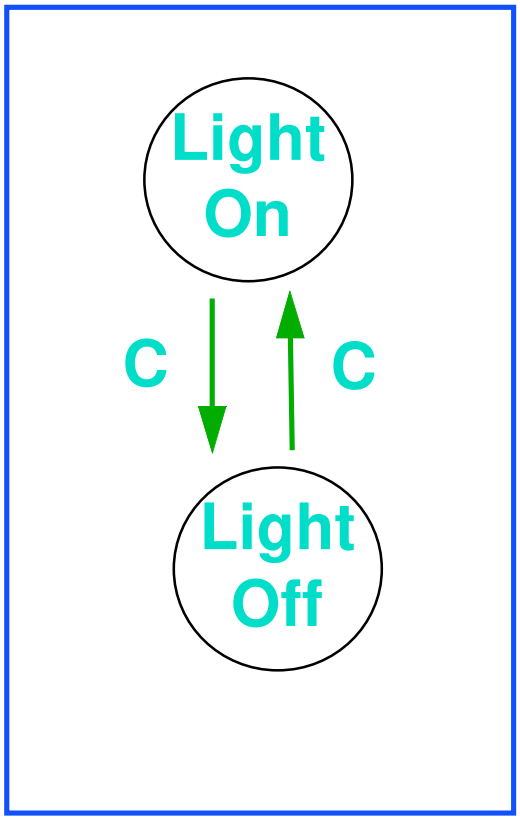
Basic Issues of Concurrency

- **Model of concurrency**
  - > e.g interleaving semantics
  - > how is combinatorial explosion handled
- **Non-determinism**
  - > e.g., to model concurrency
- **Deadlock**
  - > e.g, does it arise?
- **Synchronization and communication**
  - > how are concurrent processes sychnronized
- **Resource sharing**
  - > are there shared resources?
- **Fairness**
  - > is it part of the model?

Concurrency and FSAs

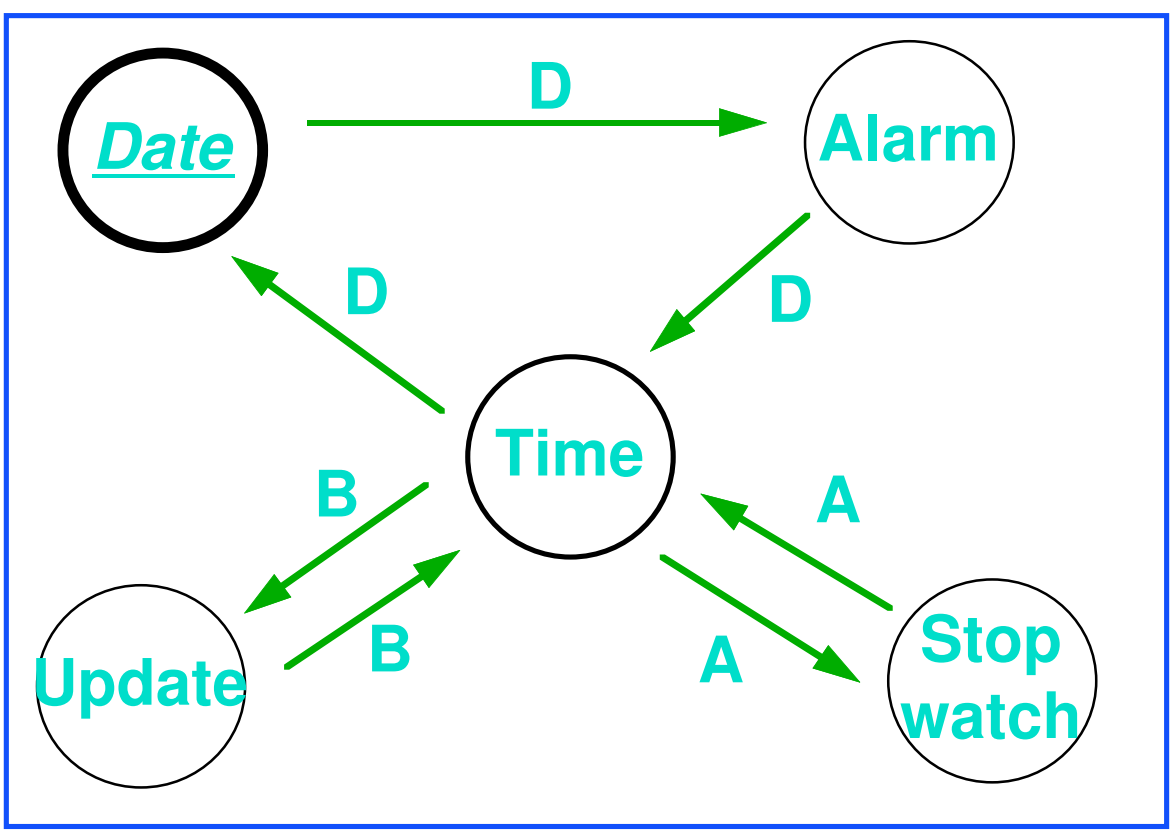
- **Concurrency with Finite State Machines leads to state explosion.**

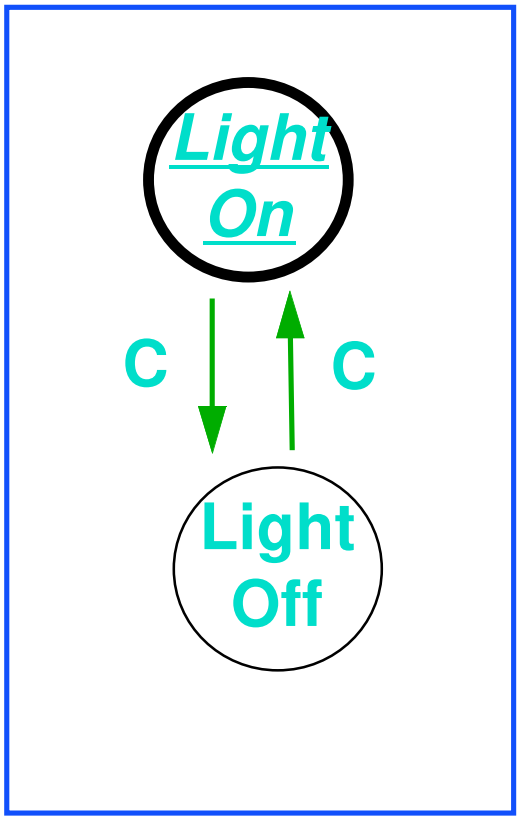




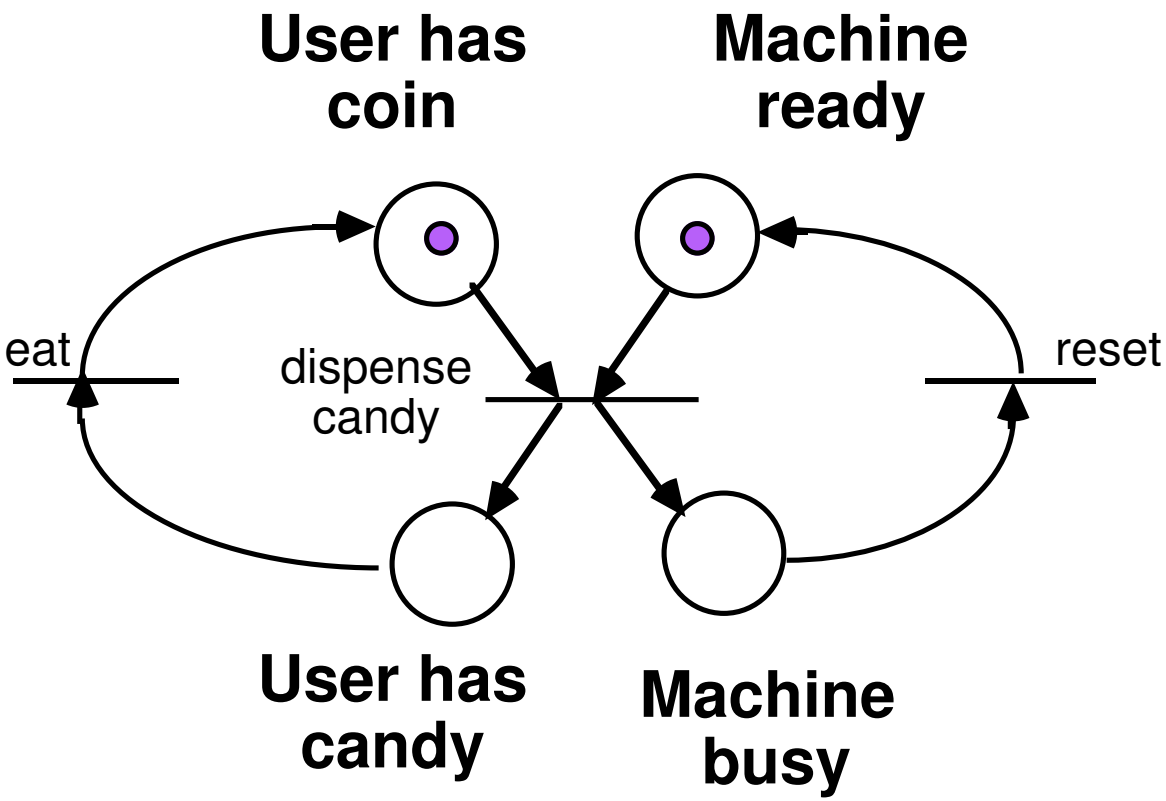
Possible Solution

- **Allow there to be multiple "state tokens" in a diagram.**

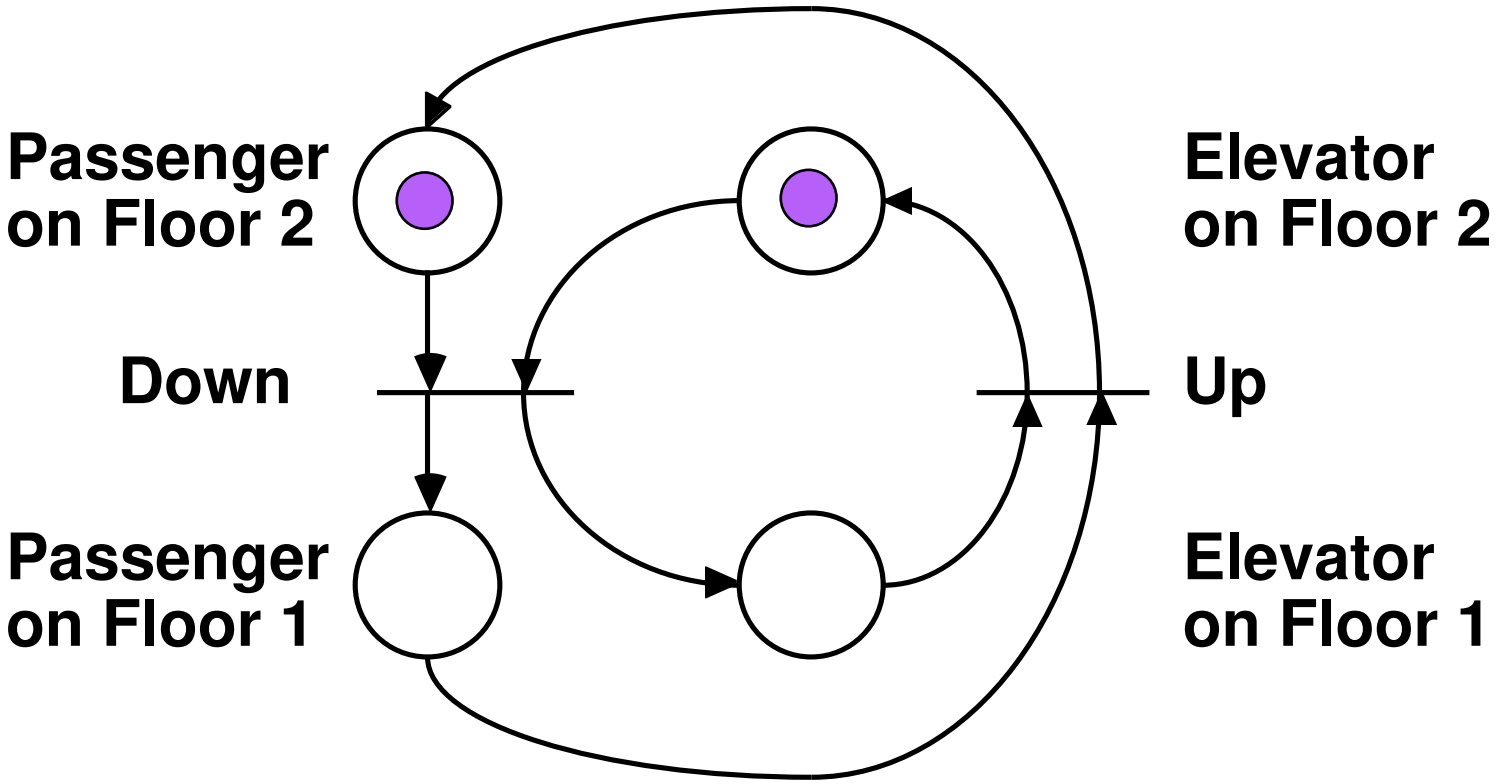




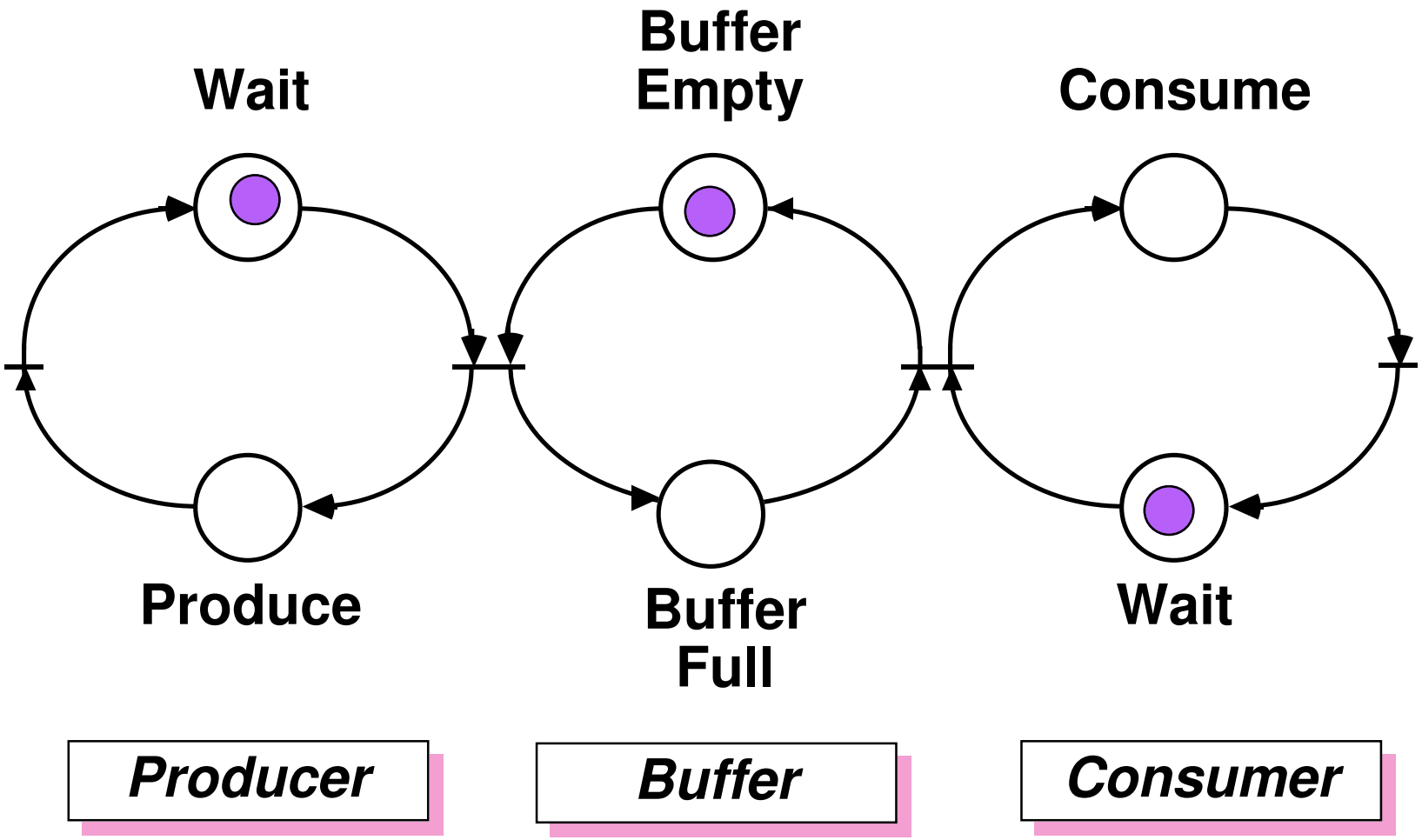
Petri Nets: An Example



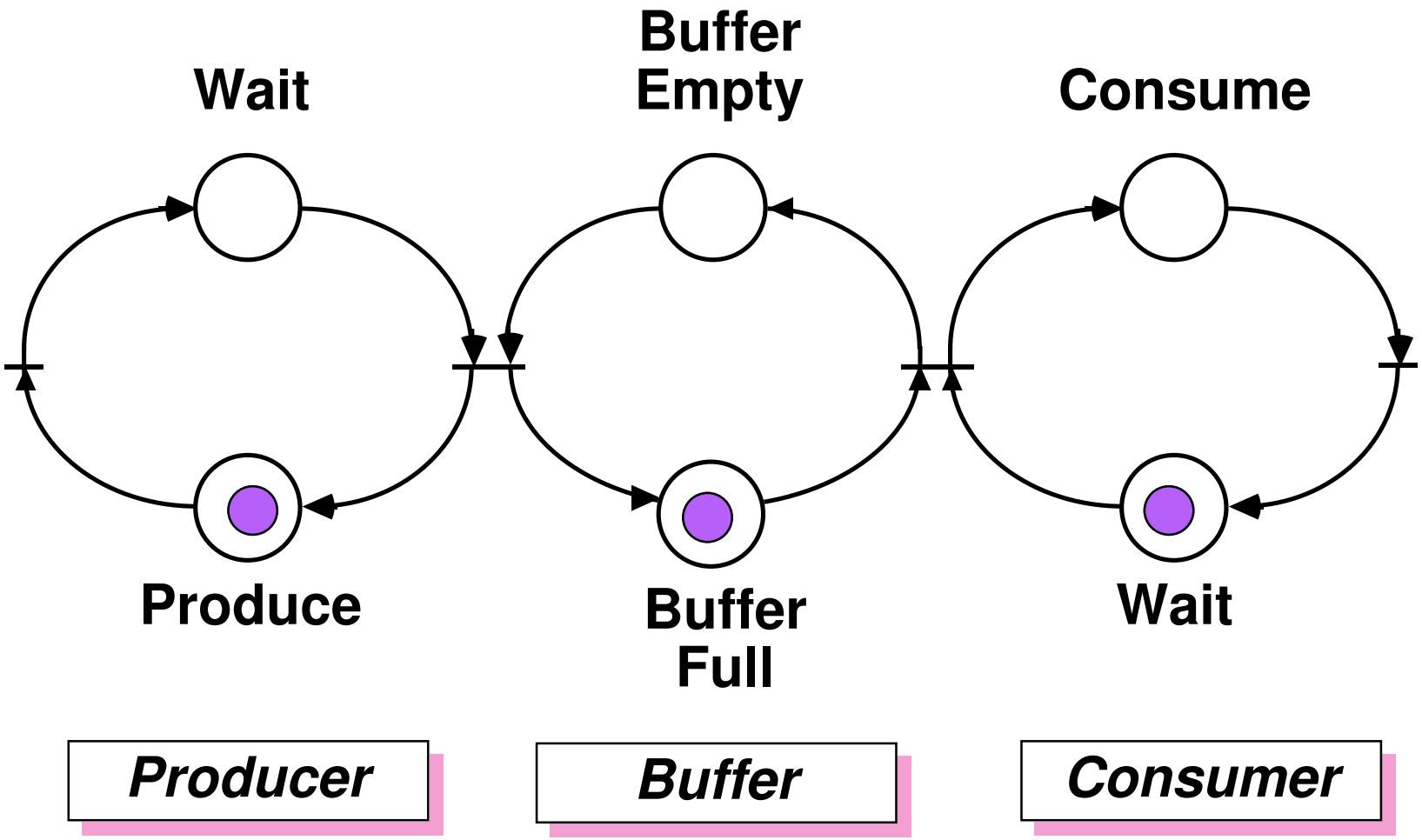
Simple Elevator



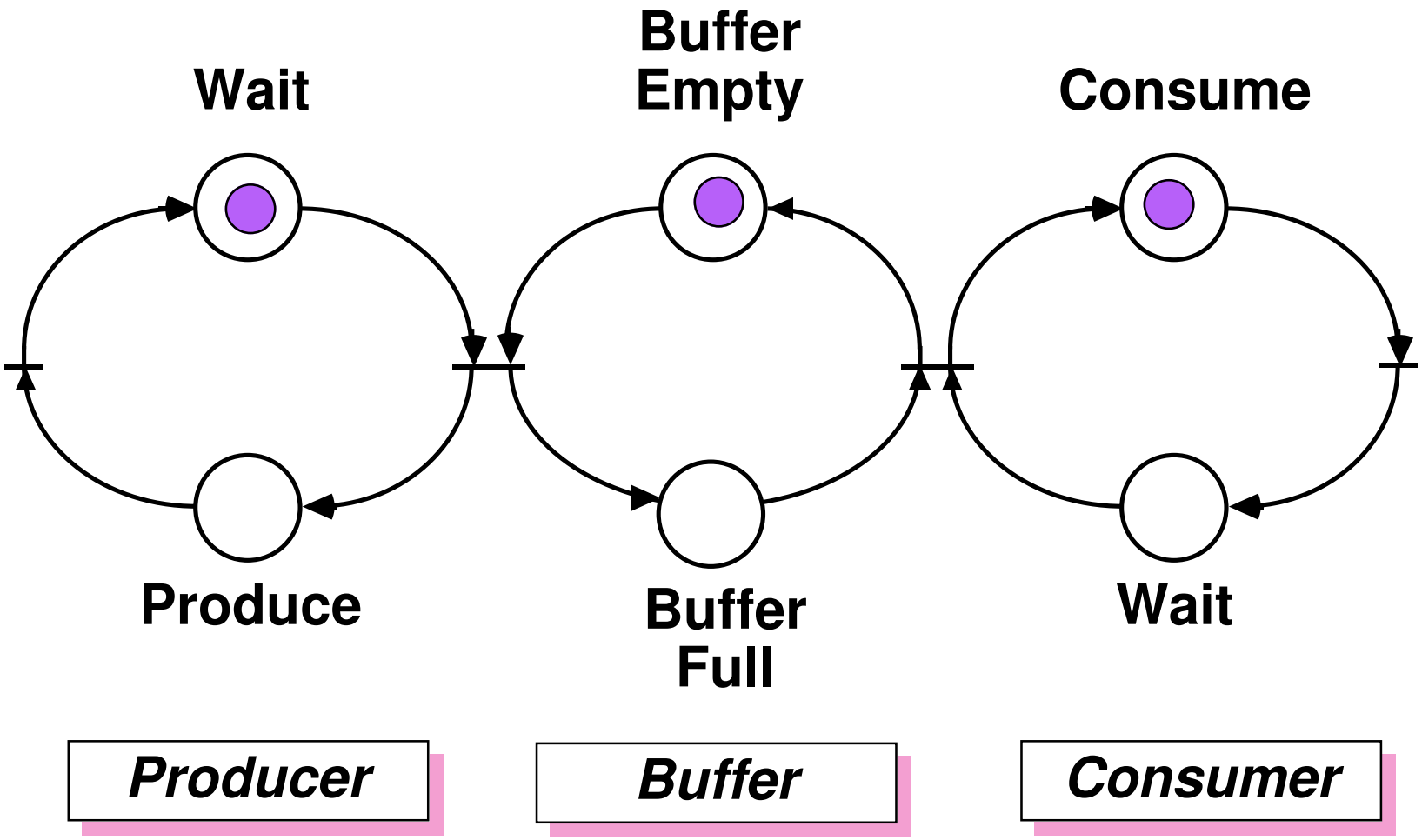
Producer/Consumer



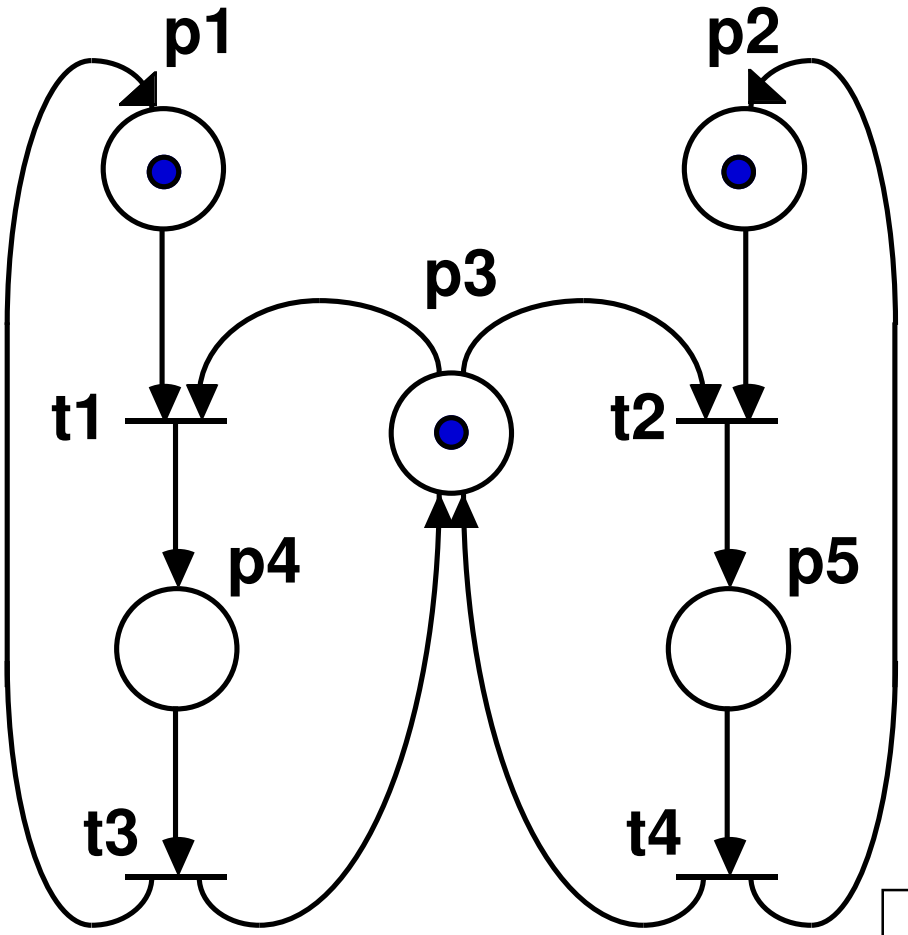
Producer/Consumer (2)



Producer/Consumer (3)



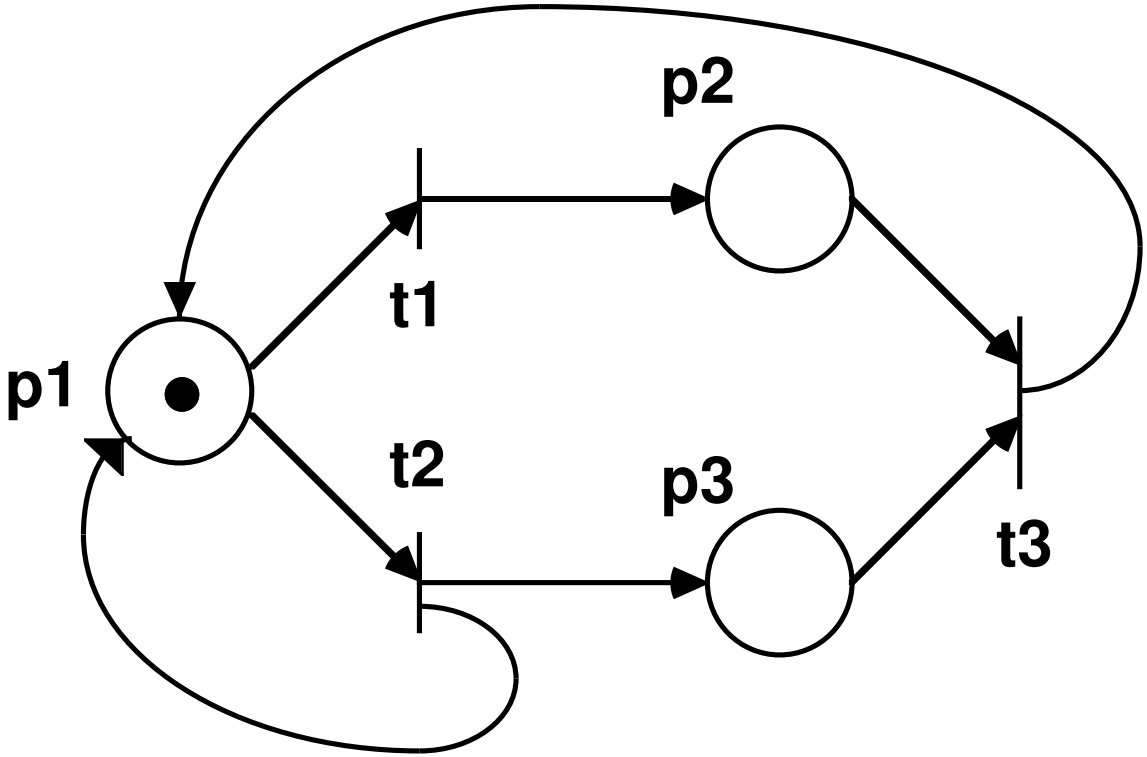
Mutual Exclusion



Process 1

Process 2

Another Example



Petri Nets: Formal Definition

- A ***Petri Net*** **is a four-tuple (P, T, I, O), where**
  - **P is a set of *places***
  - **T is a set of *transitions***
  - **I is the input function,** $I : T \rightarrow P^P$
    - **identifies the places leading *into* a transition**
  - **O is the output function,** $O: T \rightarrow P^P$
    - **identifies the places leading *out of* a transition**
- **Notes:**
  - **Determines a "bipartite directed graph," where nodes are** $P \cup T$ **and edges are defined by I & O**
  - **Definition implies at most one arc between a transition and any input or output place**

Markings & Execution Model

- A ***marking*** M of a Petri net is a mapping that is assignment of ***tokens*** to places.

  $M : P \rightarrow N$

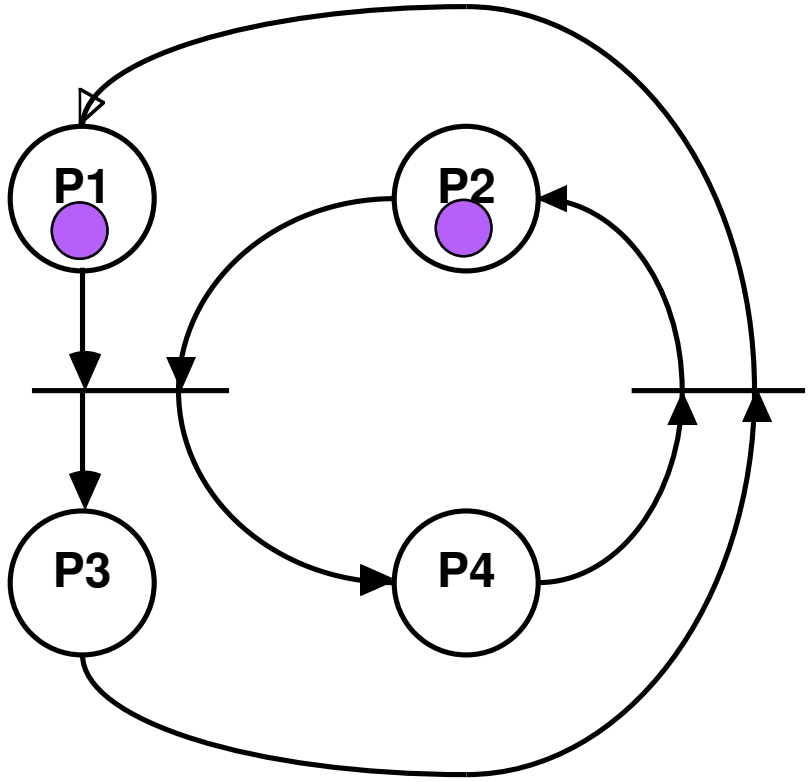
- Petri nets are usually associated with an ***initial marking***.
- A transition is ***enabled*** if each of its input places has at least one token.
- A transition may ***fire*** if it is ***enabled***.
- The firing of a transition
  (a) Removes a token from each input place
  (b) Produces a token on each output place

Reachable States

- The ***state space*** of a Petri net is the set of all markings (marking = state).
- A marking M' is ***immediately reachable*** from marking M if the firing of some enabled transition in M yields M'.
- The ***reachable*** states are all markings that can be obtained from some initial marking M.
- The ***reachability set*** for a marking M is the set of reachable states. Sometimes indicated by R(M).

Markings (continued)

- It is often convenient to represent a marking as a vector
- Example:
  - Initial marking: (1,1,0,0)
  - After 1 step: (0,0,1,1)
  - After 2 steps: (1,1,0,0)
  - ...
  - So reachability set is {(1,1,0,0),(0,0,1,1)}



Comparing Markings

- M $\geq$ M' iff each place of M has at least as many tokens as each place of M'
- Example:
  - M = (1,0,0,2)
  - M' = (0,0,0,1)
- Note not all markings are comparable
- Example:
  - M = (1,0,0,2)
  - M' = (0,1,0,1)

As we have seen ...

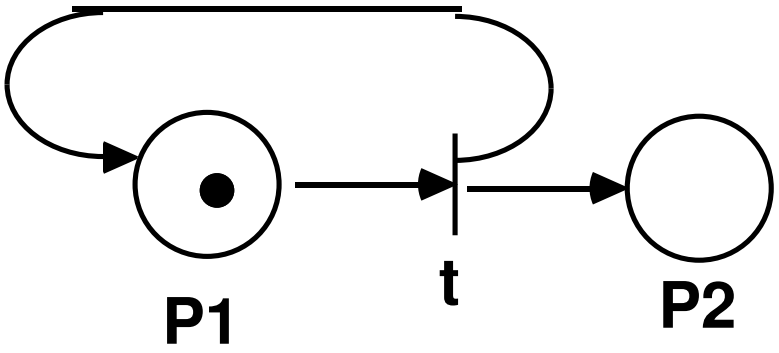
- Petri nets can have nondeterministic transitions.
- That is, two transitions are enabled and one of them will fire nondeterministically.
- This makes it possible for Petri nets to model concurrent execution
  - interleaved firing of transitions
- Additionally Petri nets can model
  - synchronization
  - resource sharing

Special Kinds of Petri Nets

Given an initial marking:

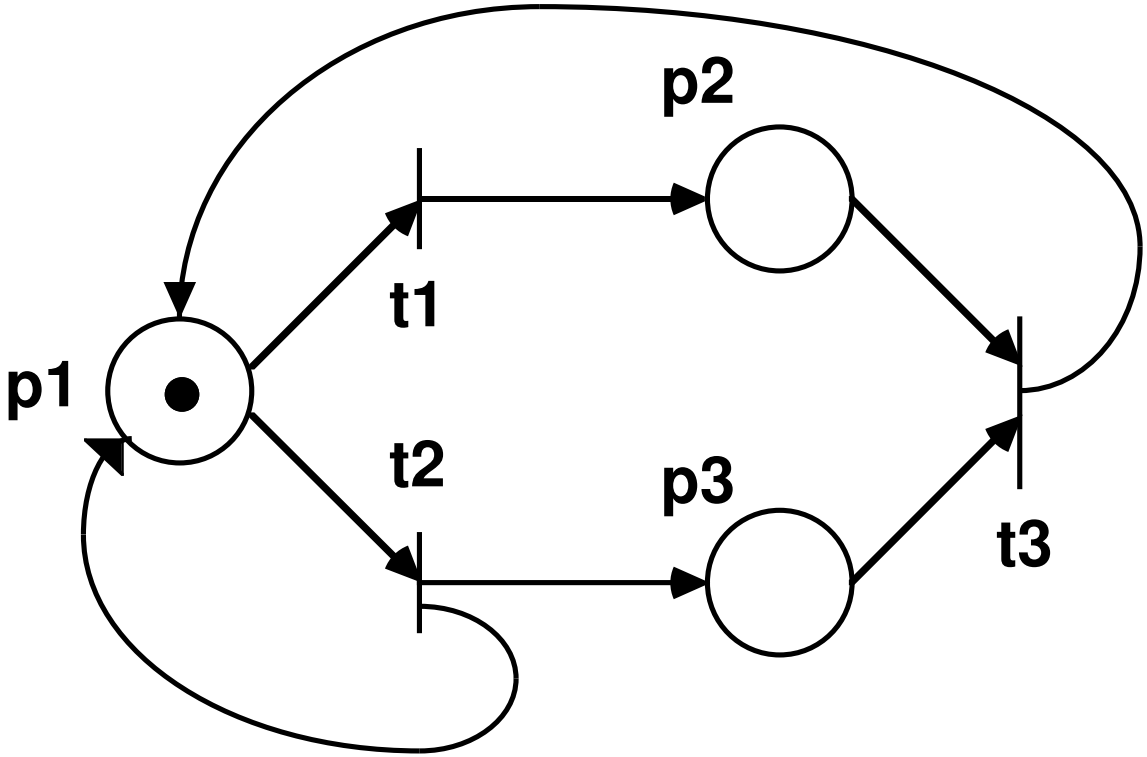
- **Safe net:** No more than one token at any place.
- **K-bounded:** No more than K tokens at any place.
- **Bounded:** K-bounded for some value of K.
- **Conservative:** Number of tokens in net remains constant.

Unbounded Nets

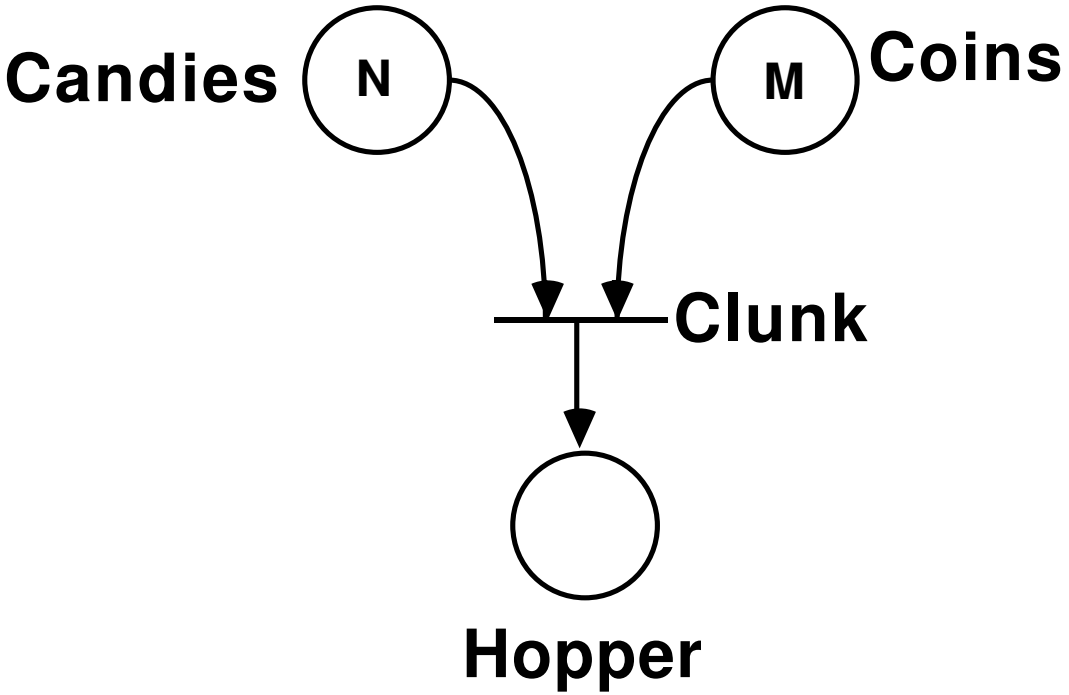


Qn: what is the reachability set?

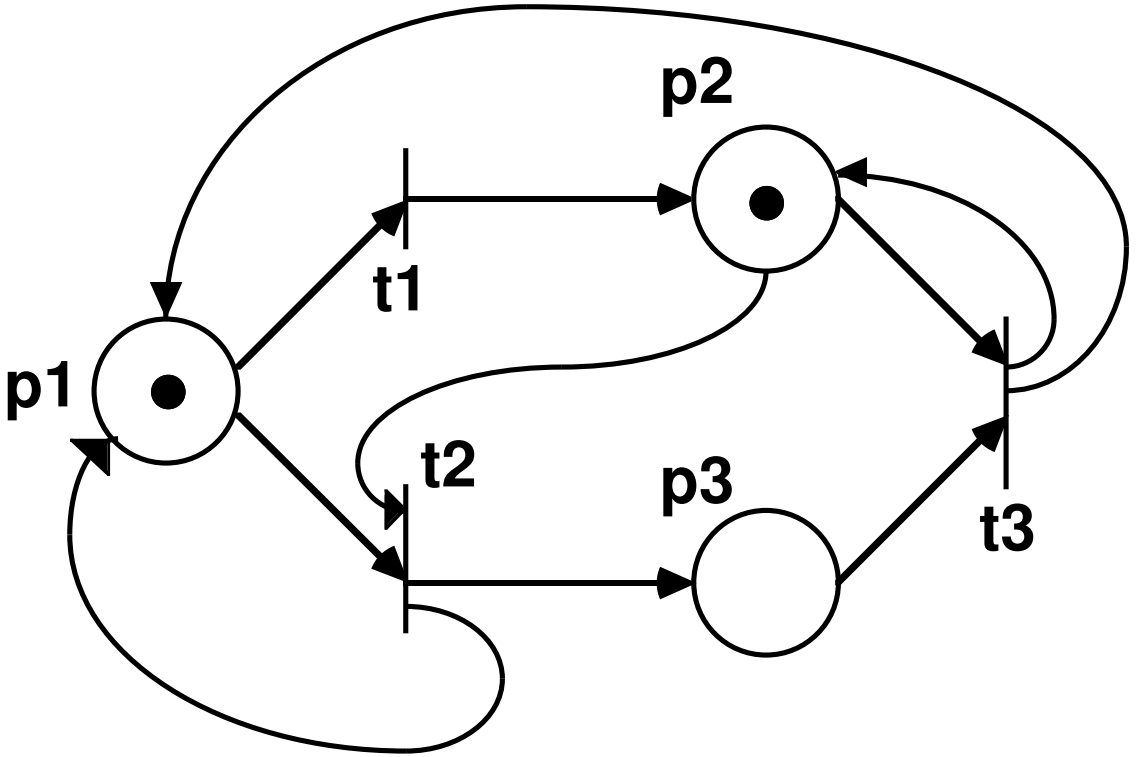
Is This Net Bounded?



Non-Conservative Nets

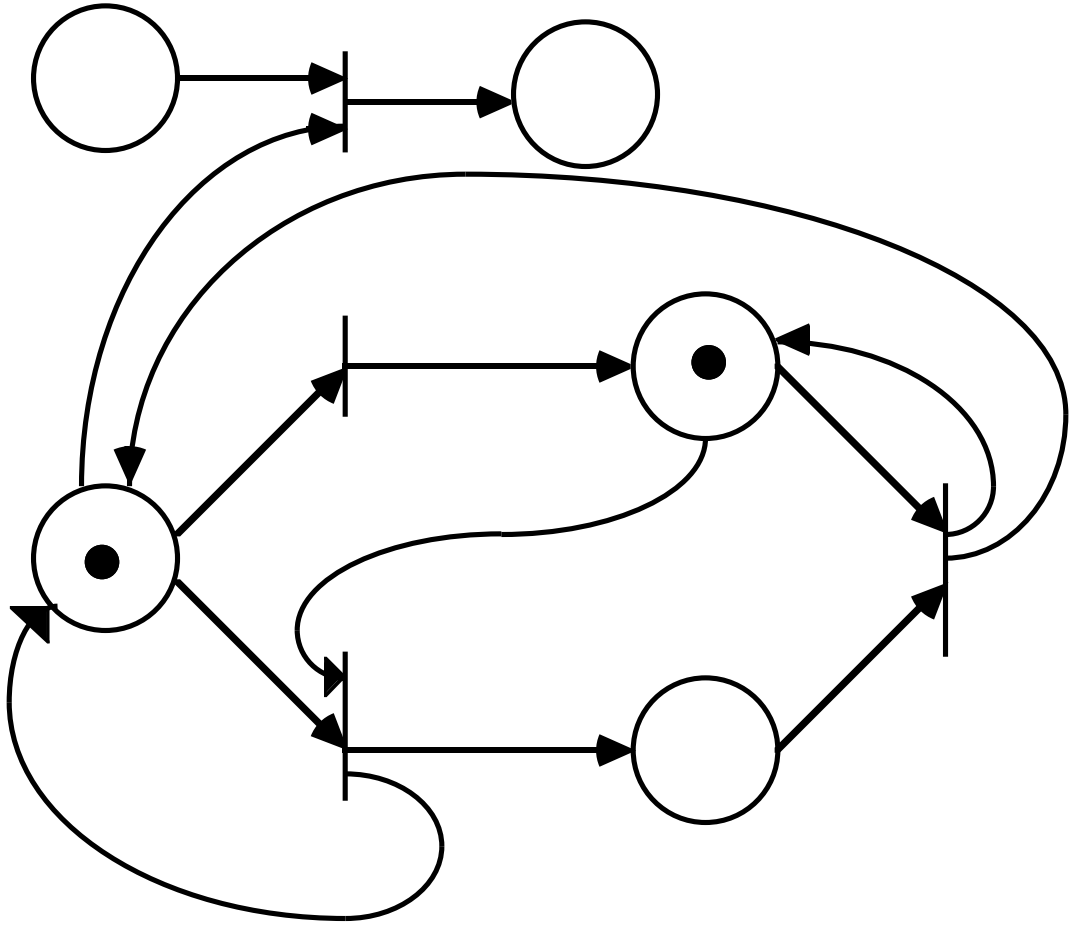


Is This Net Conservative?



Necessary and sufficient conditions?

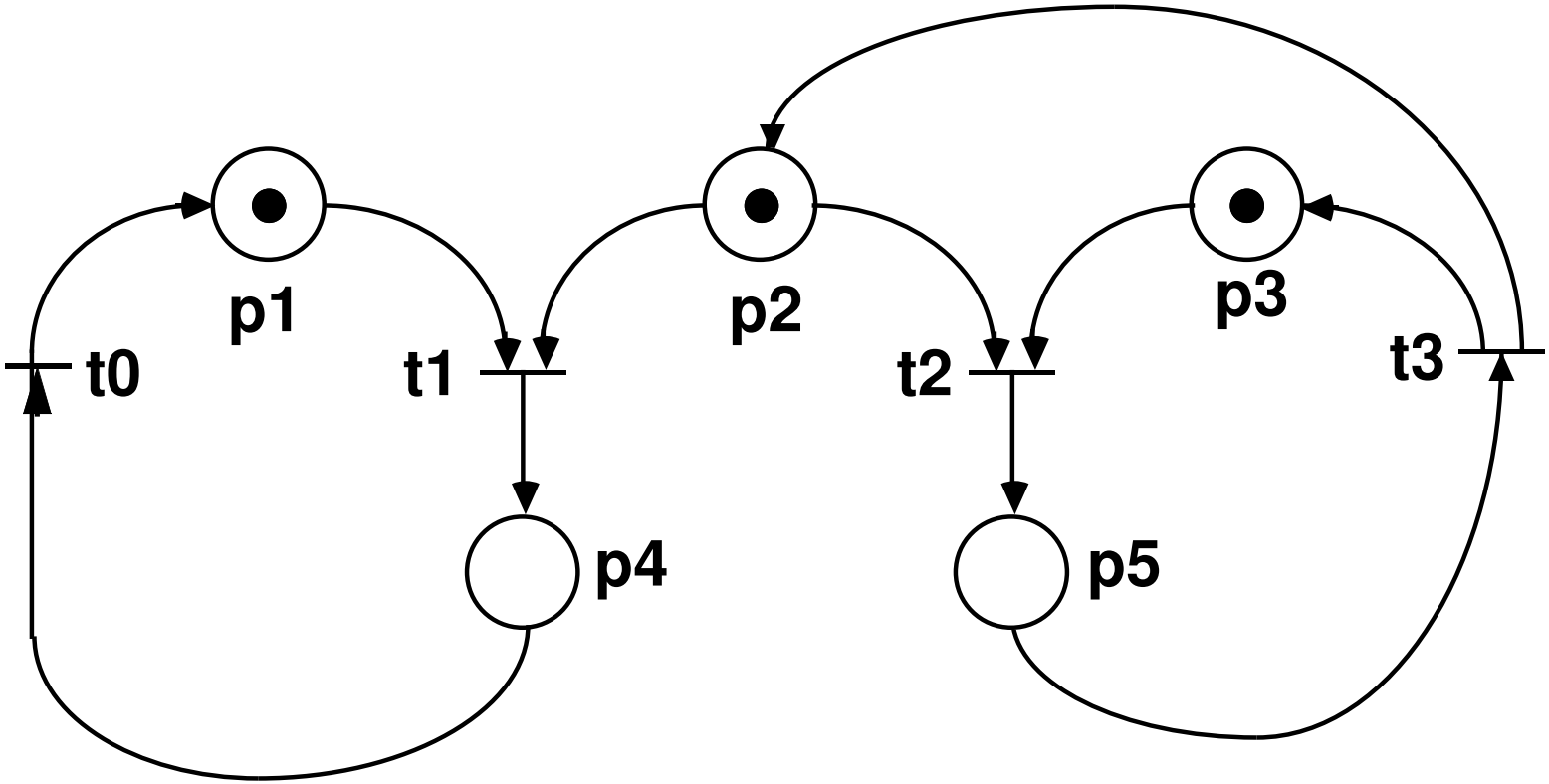
Is This Net Conservative?



Other Questions

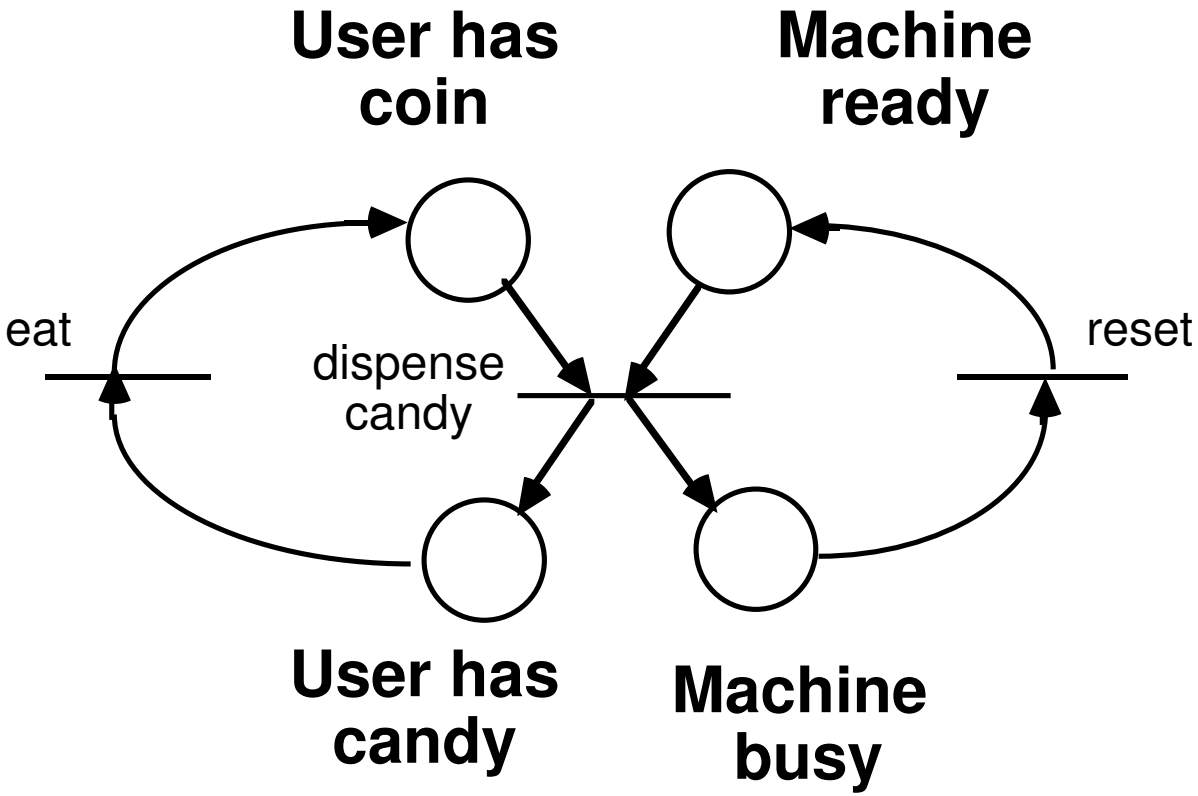
- **Liveness:**
  - > **Are all transitions potentially fireable?**
- **Reachability:**
  - > **Is marking M' reachable from marking M?**
- **Coverability:**
  - > **Given markings M and M', does there exist a marking M'' reachable from M, such that M'' ≥ M' ?**

Liveness



Last Slide

Simple Example



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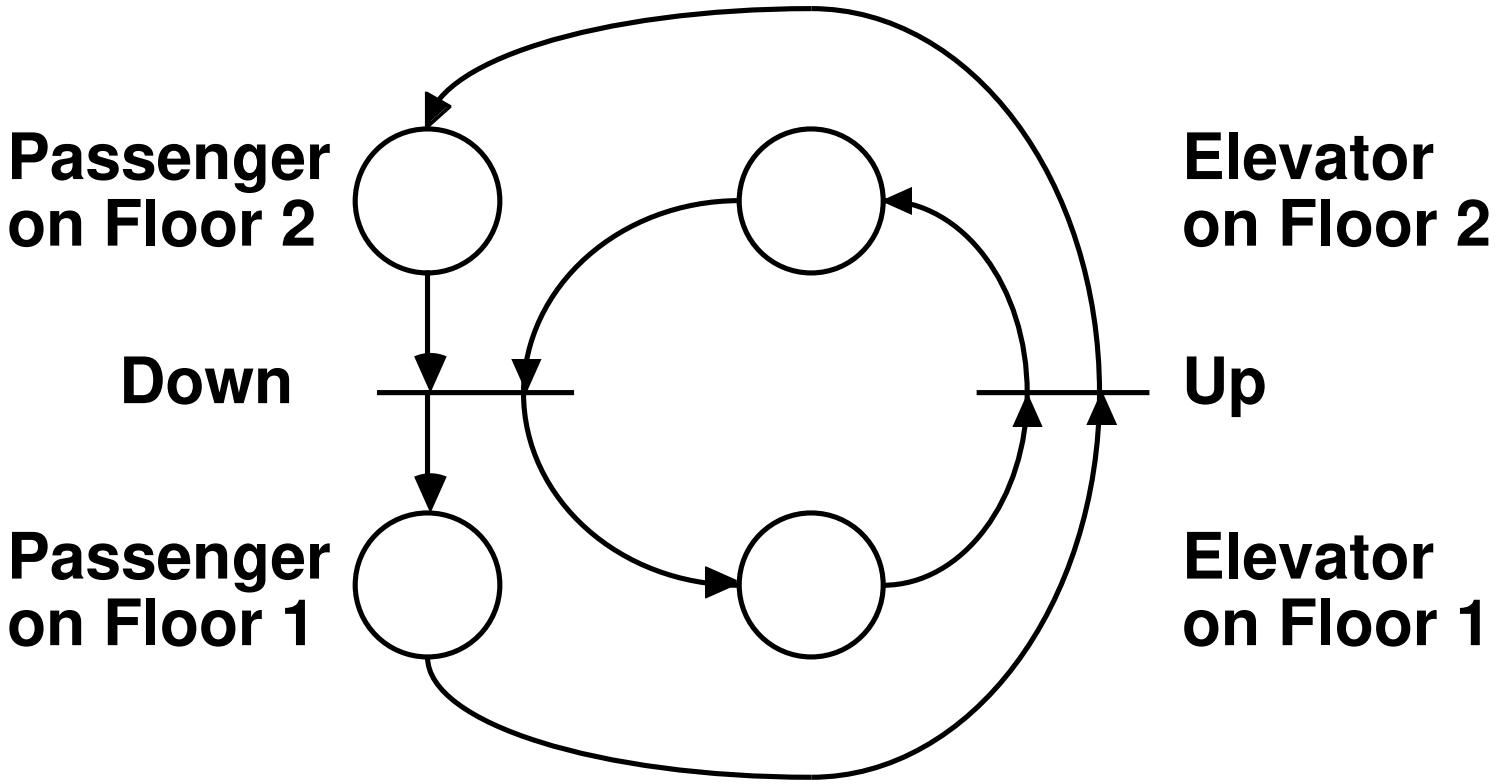

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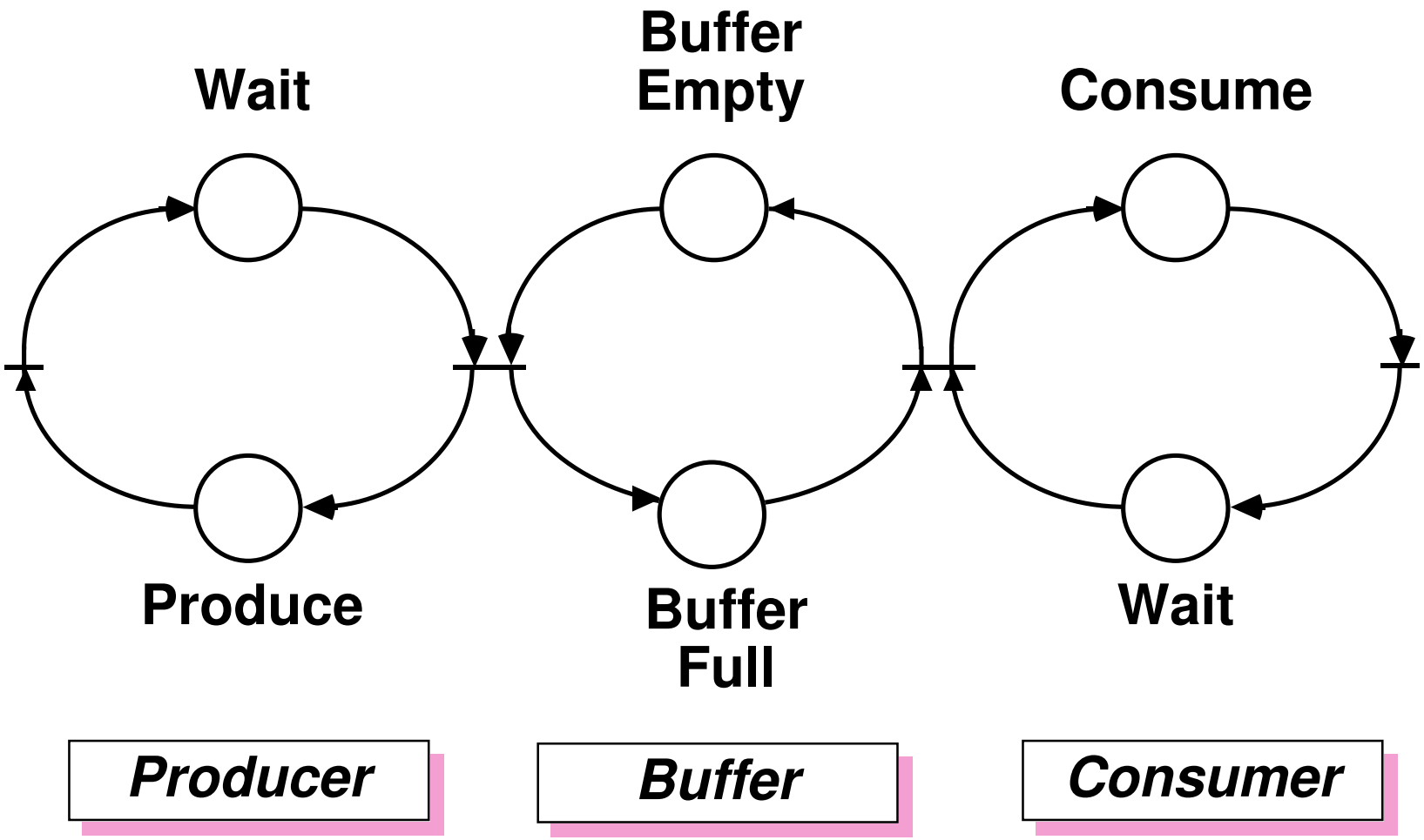
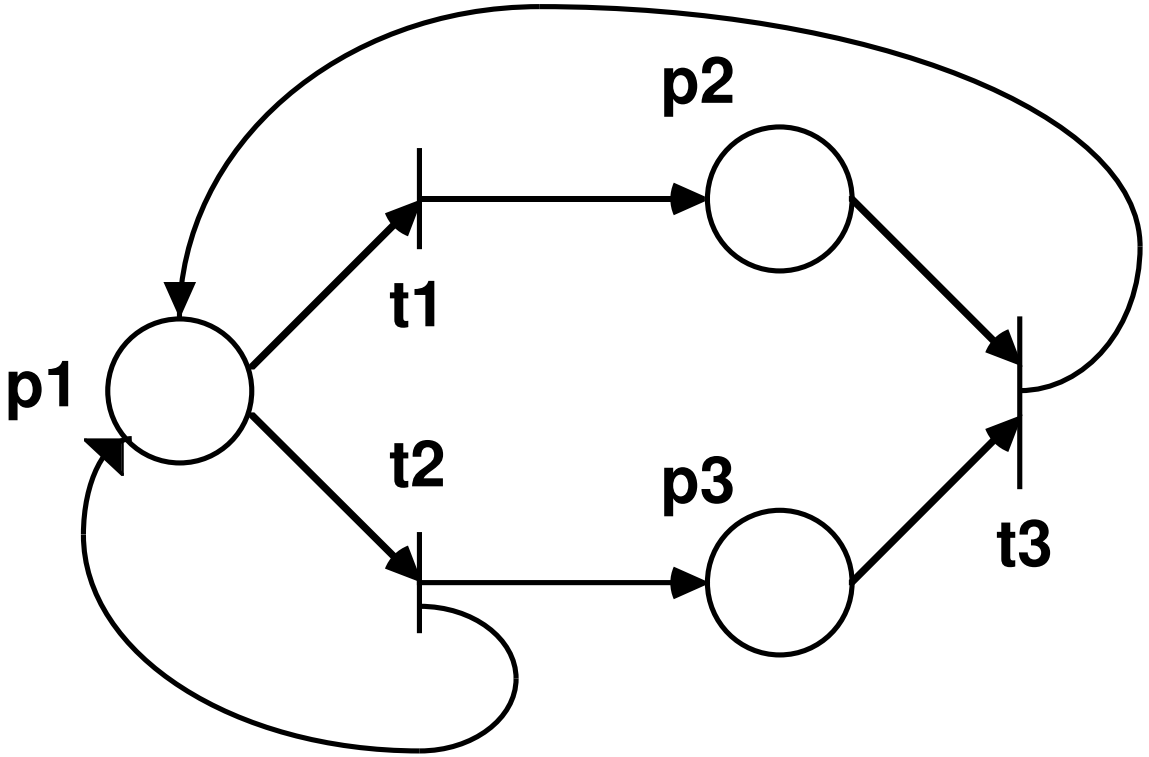


Is This Net Bounded?



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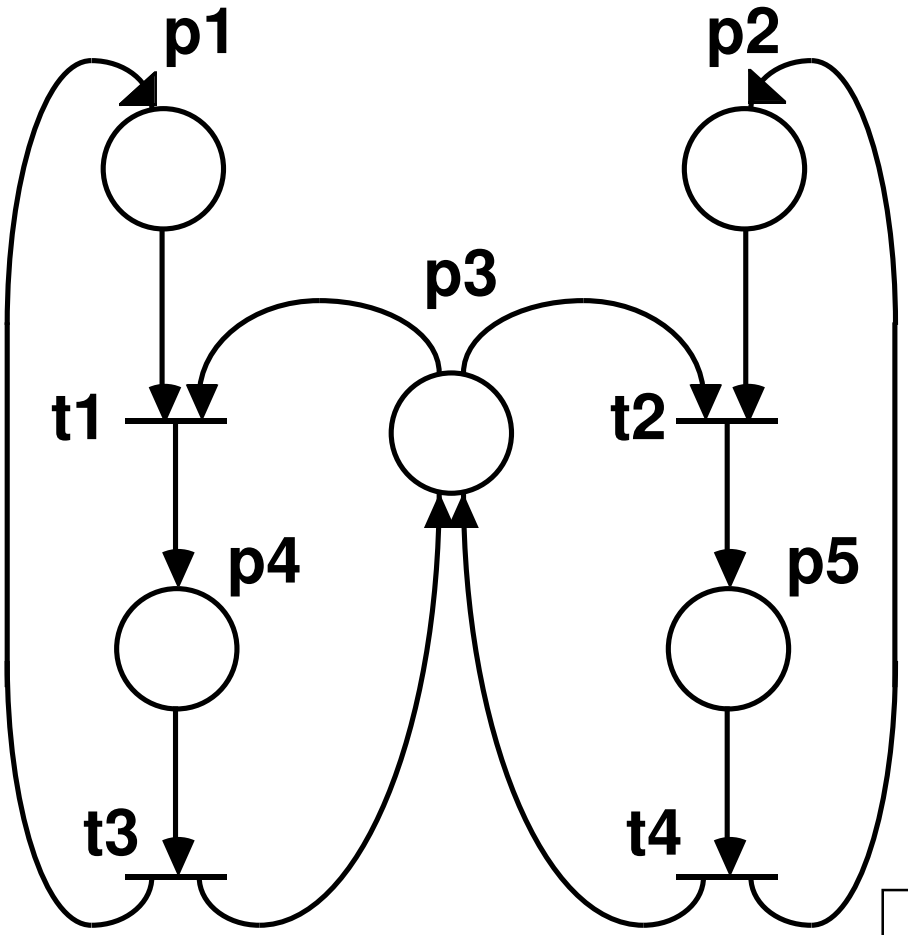
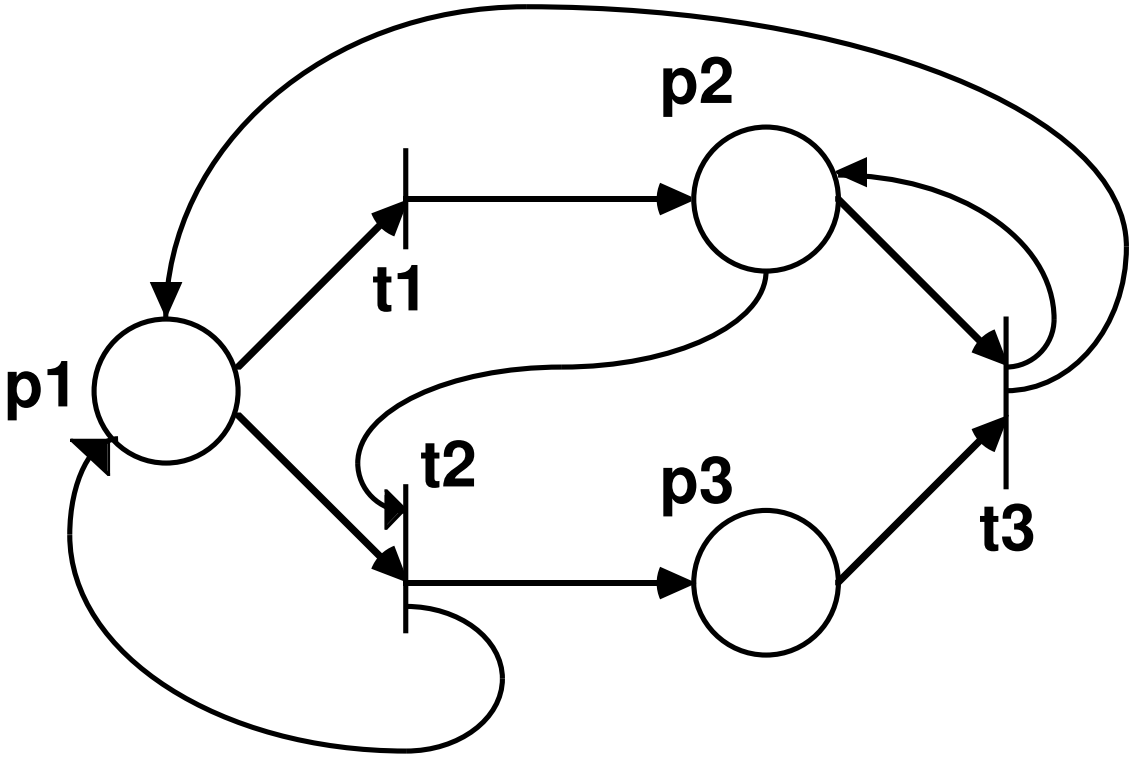


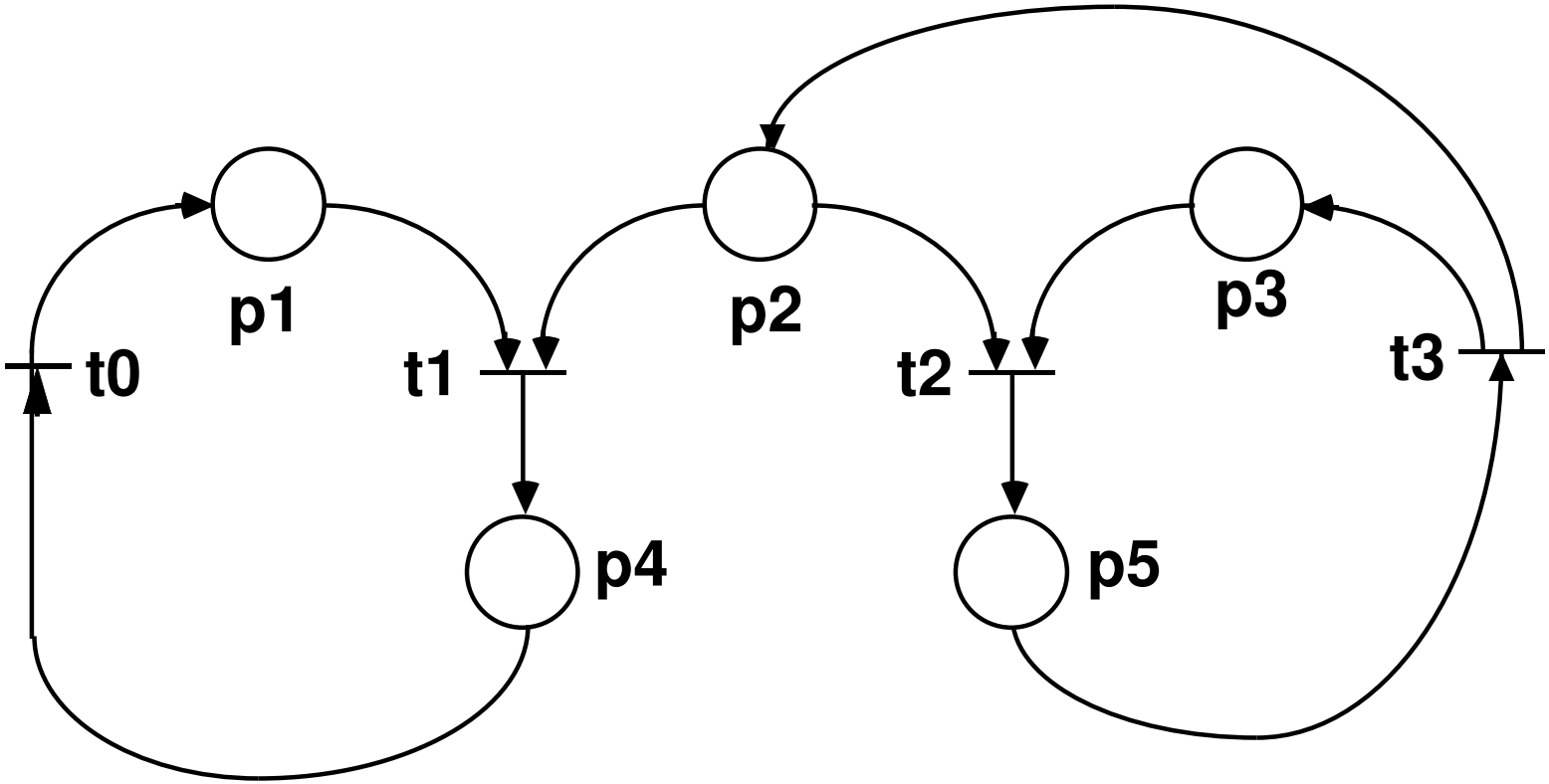
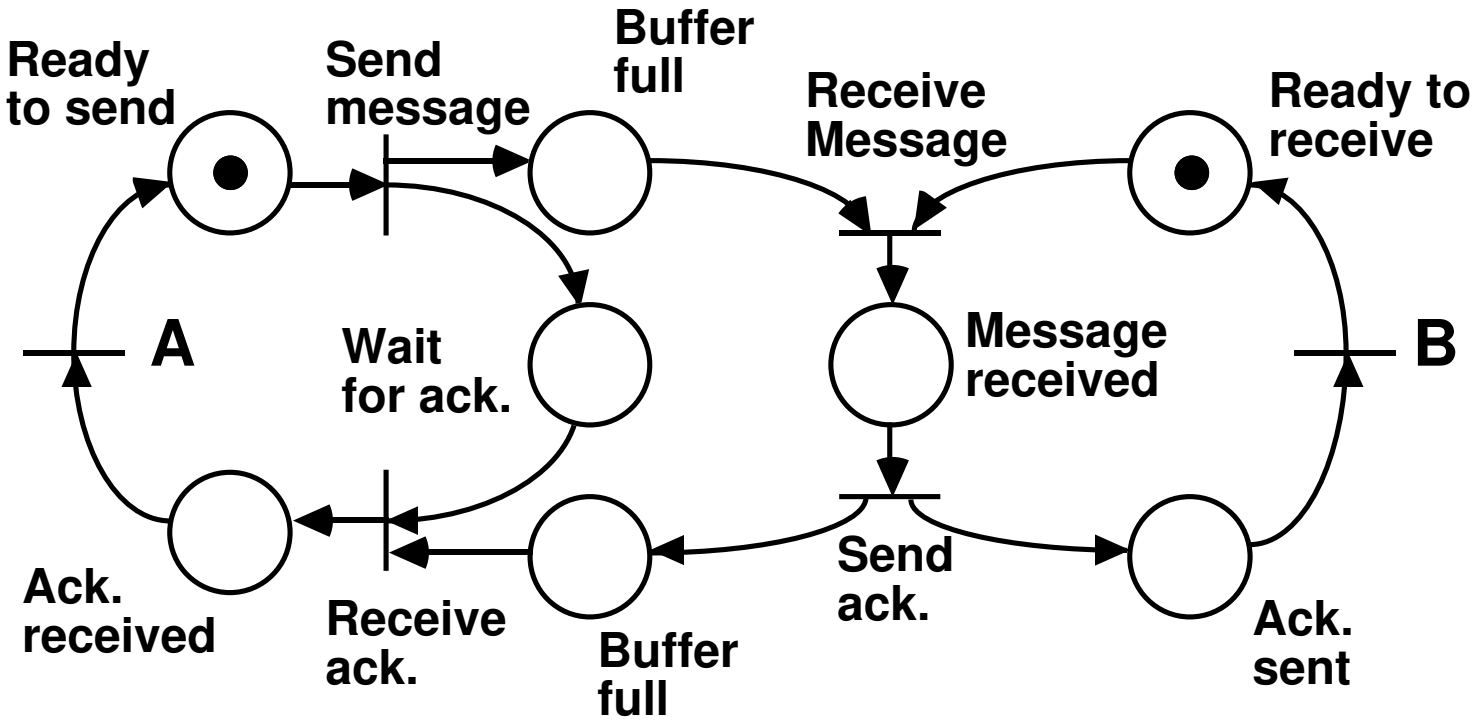
Process 1

Process 2

Is This Net Conservative?



Liveness



Another Example