Lecture 24
Petri Nets 1

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Models of Software Systems
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This Lecture

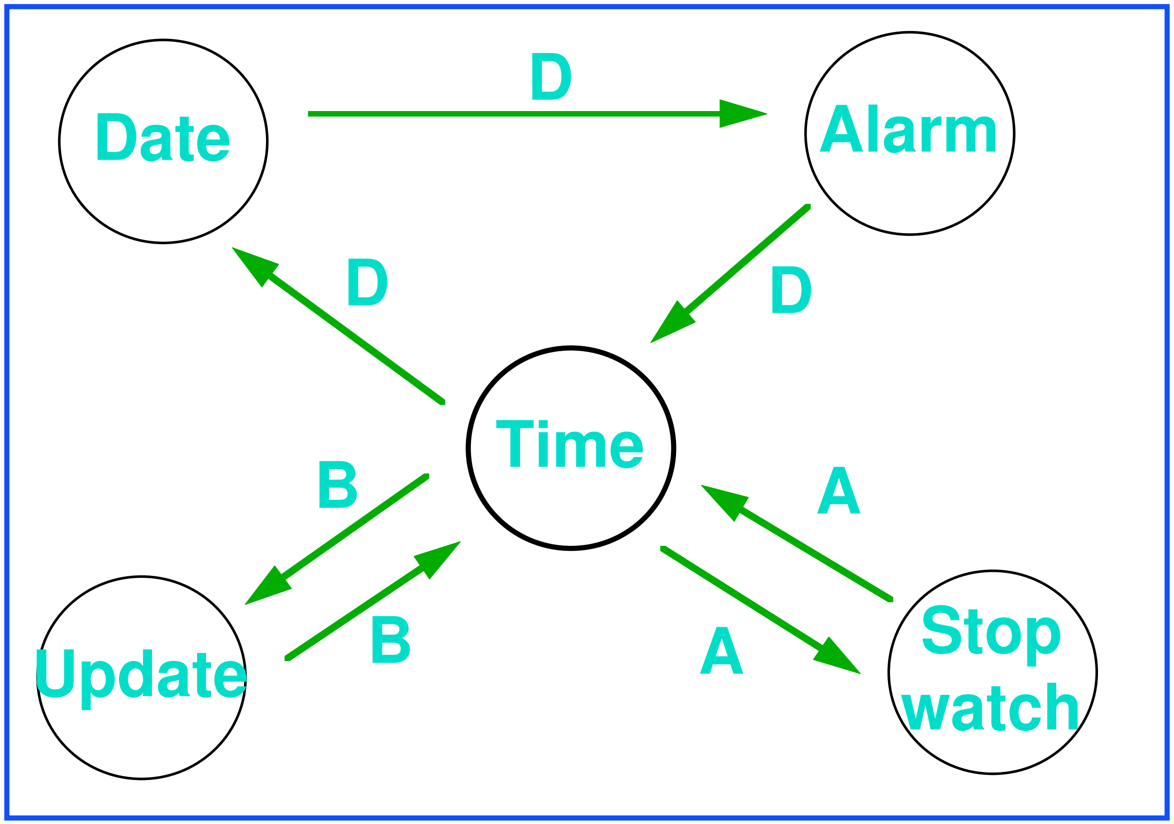
- Modelling concurrency
- Petri Nets - some examples
- Formal definitions
- Special classes of Petri Nets
- More examples

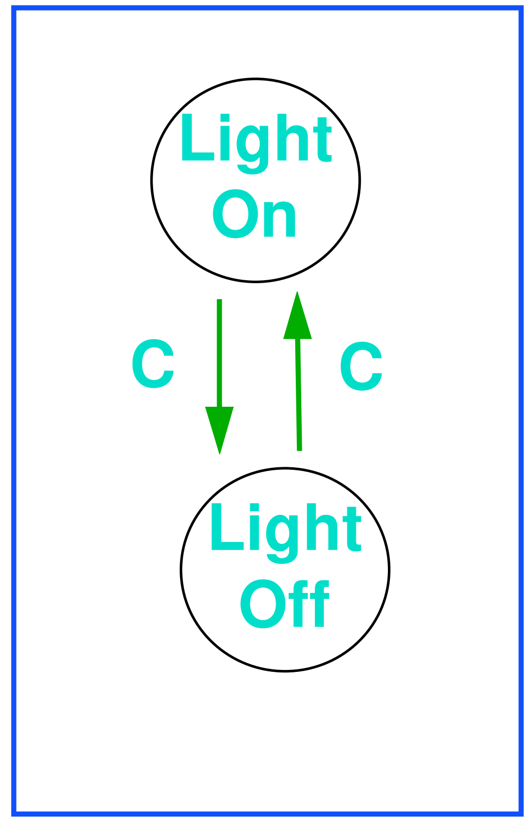
Basic Issues of Concurrency

- **Model of concurrency**
  - e.g interleaving semantics
  - how is combinatorial explosion handled
- **Non-determinism**
  - e.g., to model concurrency
- **Deadlock**
  - e.g, does it arise?
- **Synchronization and communication**
  - how are concurrent processes sychnronized
- **Resource sharing**
  - are there shared resources?
- **Fairness**
  - is it part of the model?

Concurrency and FSAs

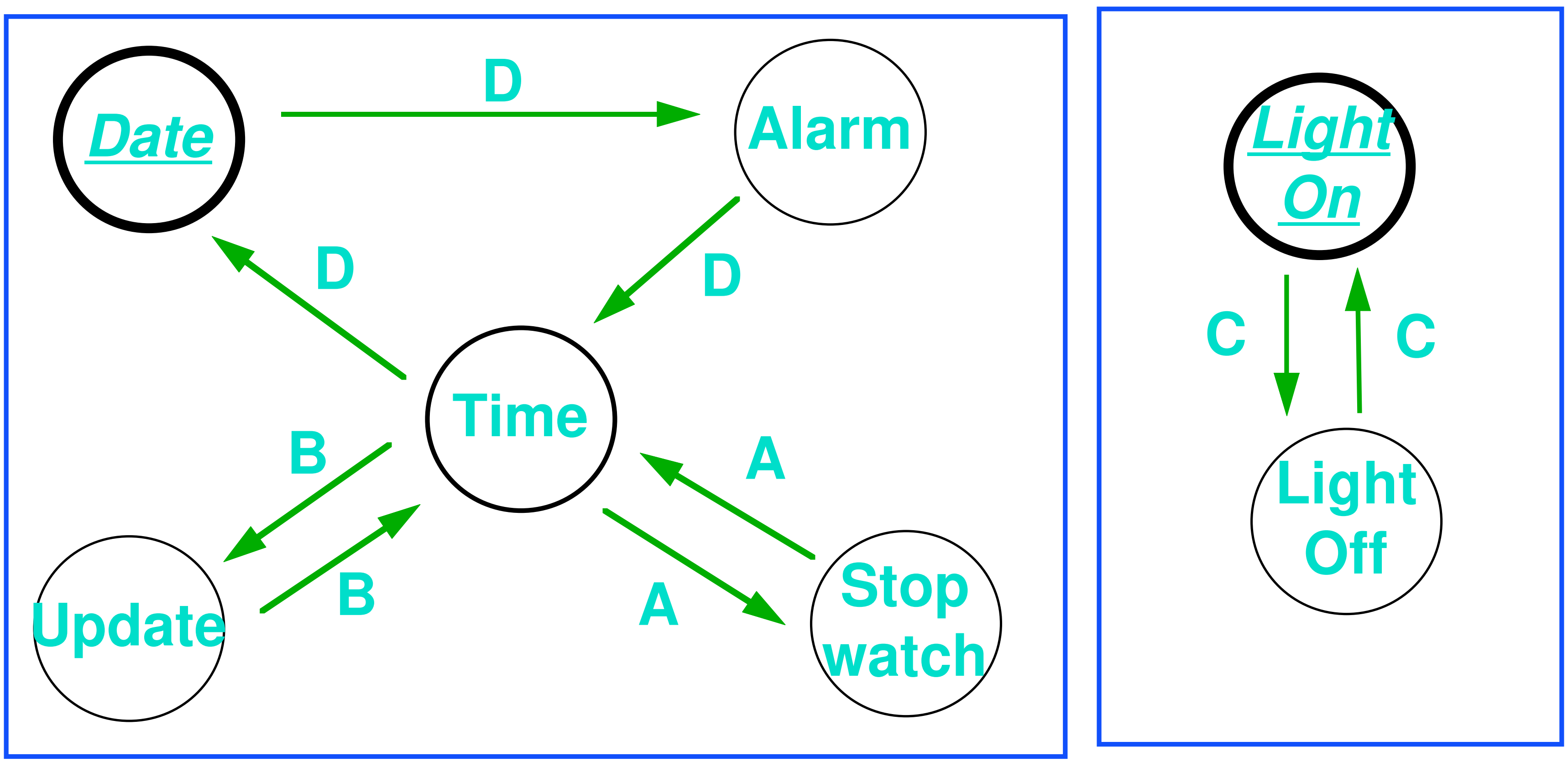
- **Concurrency with Finite State Machines leads to state explosion.**



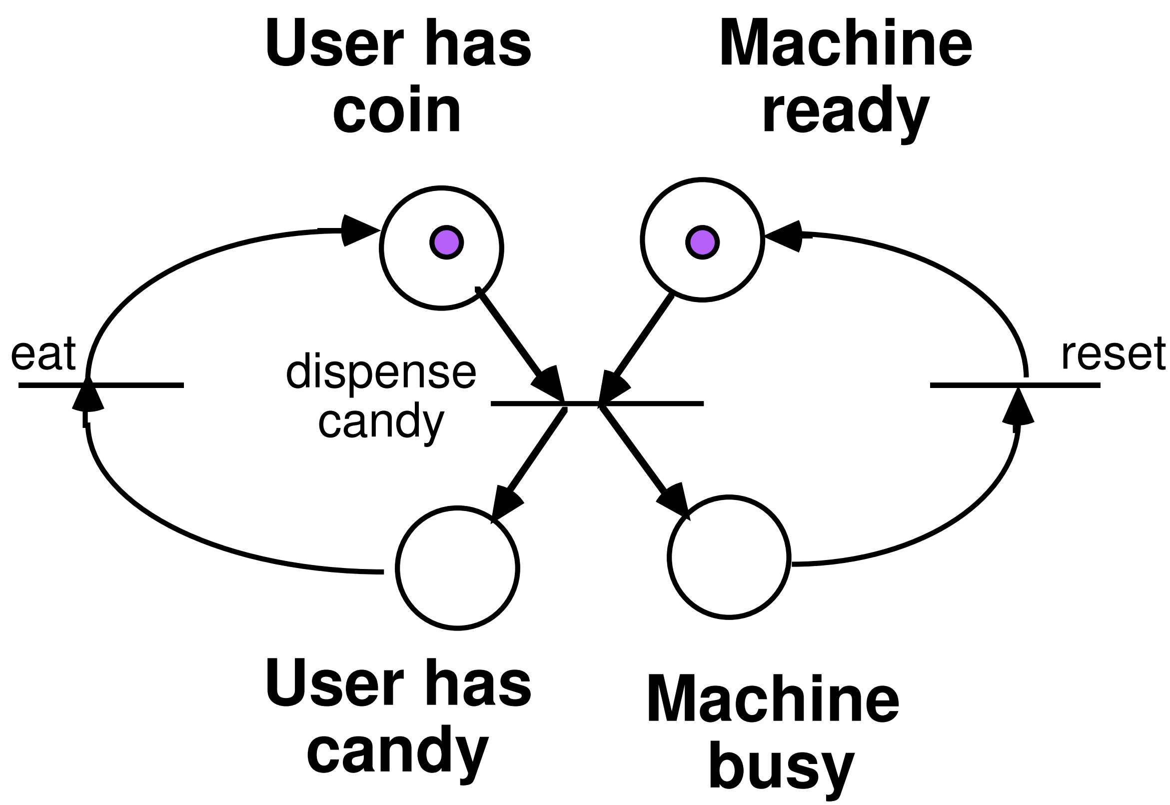


Possible Solution

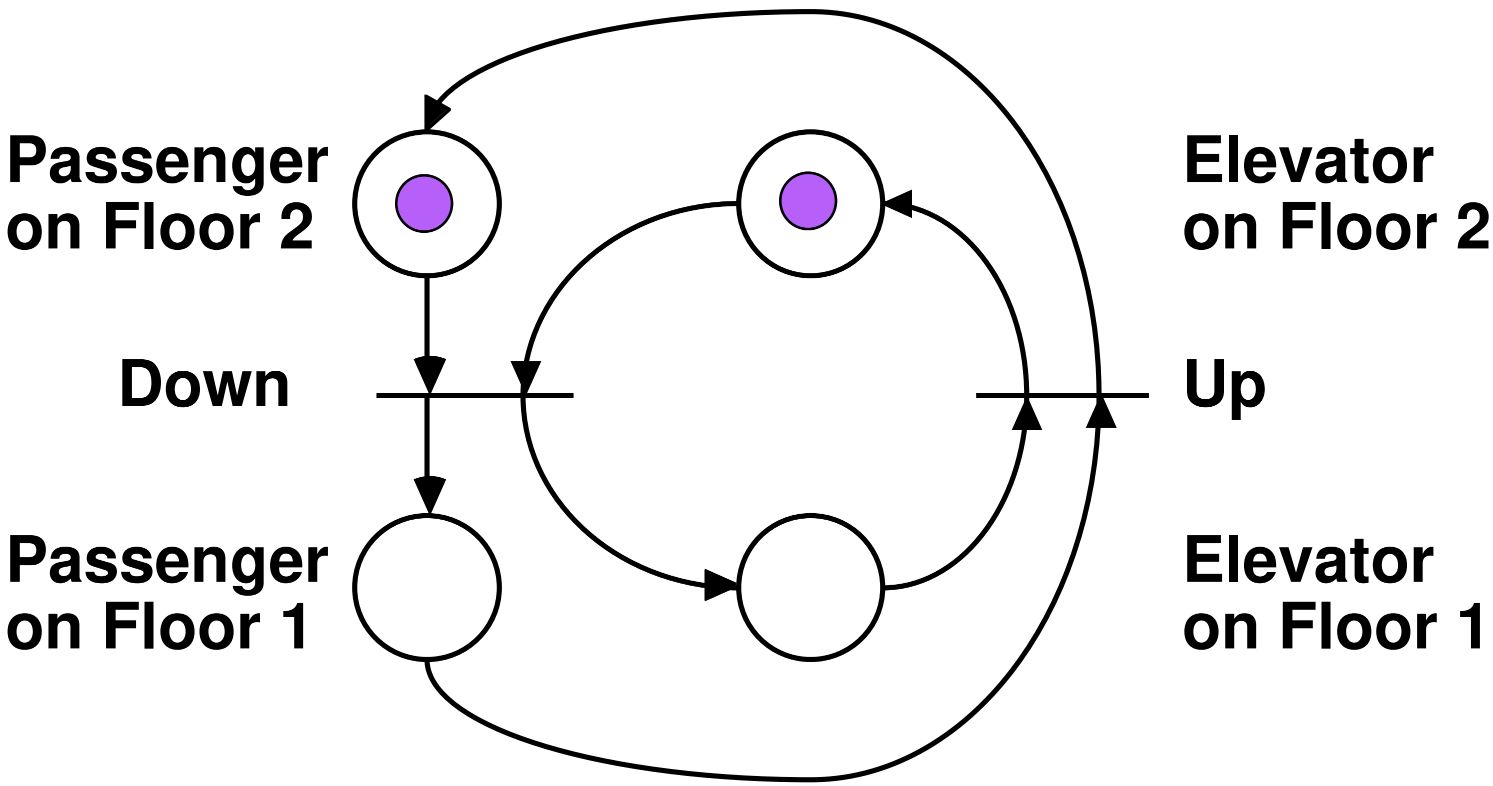
- Allow there to be multiple "state tokens" in a diagram.



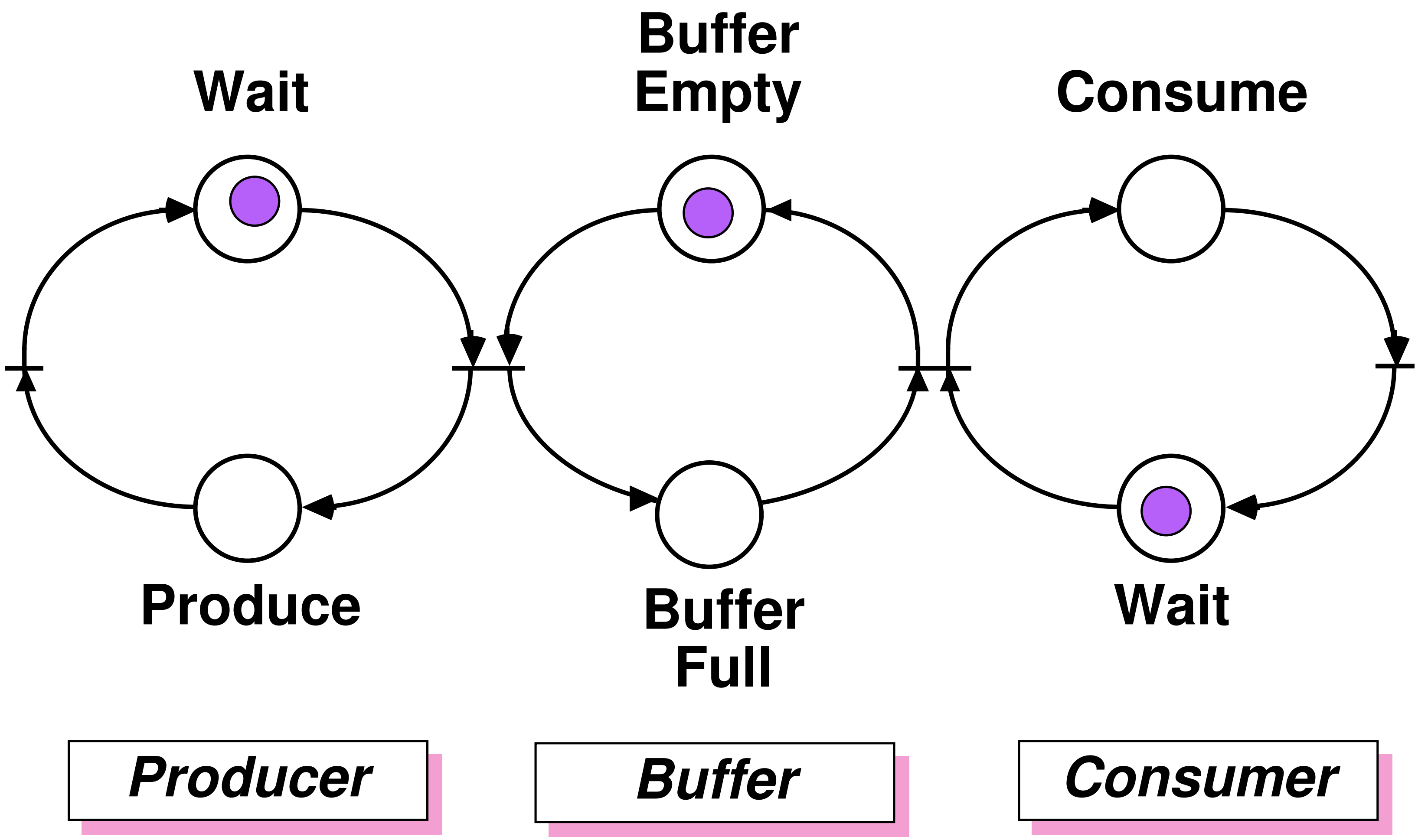
Petri Nets: An Example



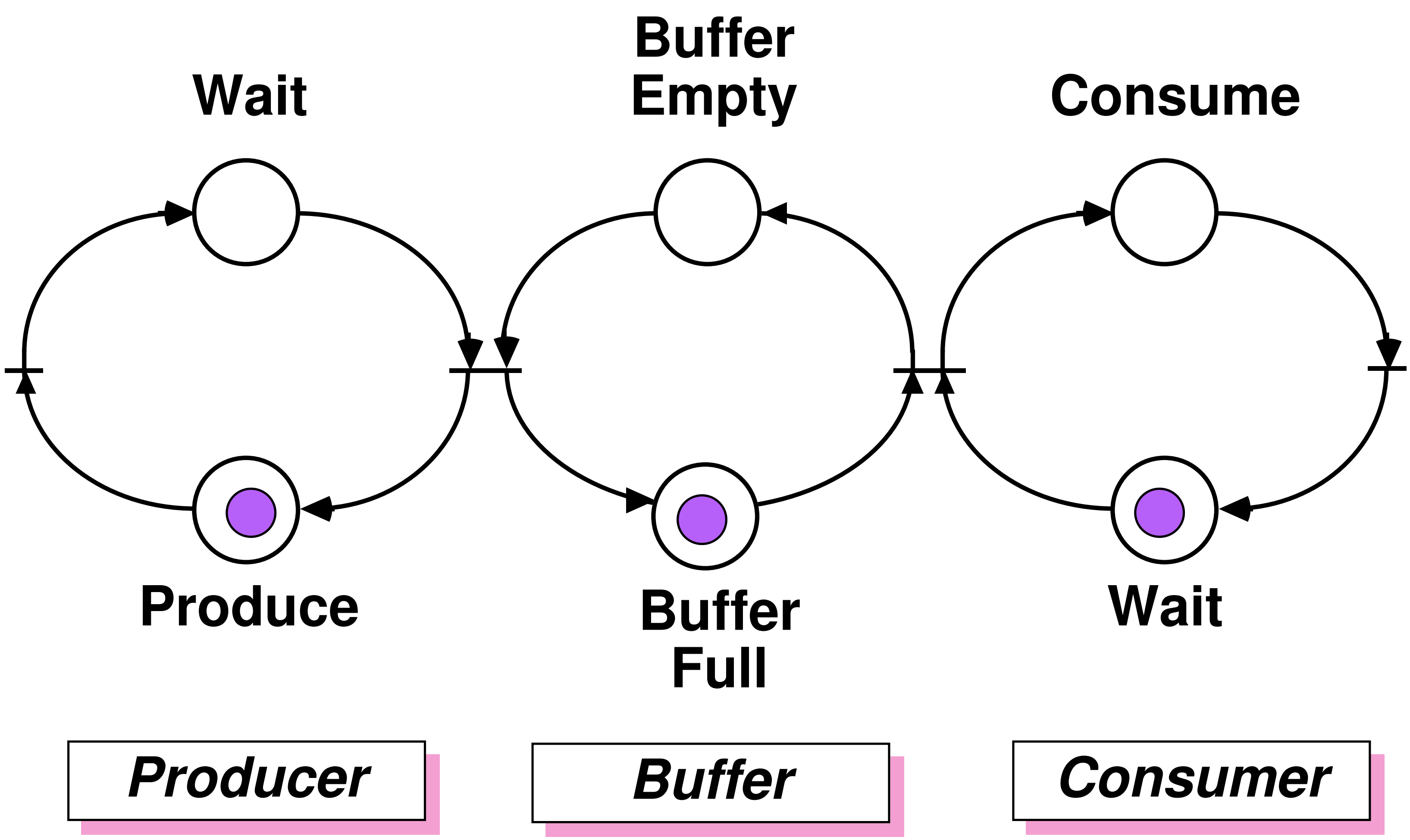
Simple Elevator



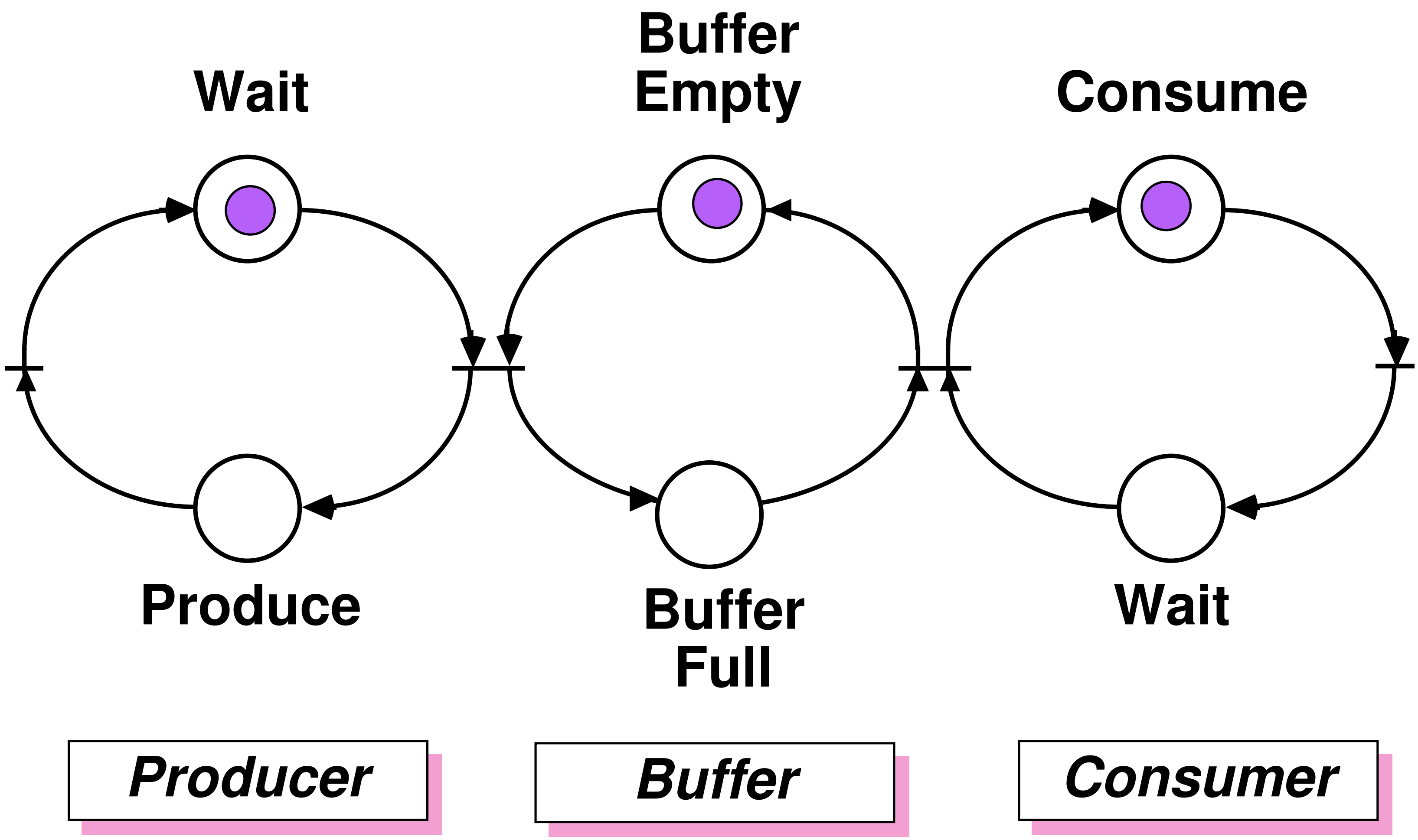
Producer/Consumer



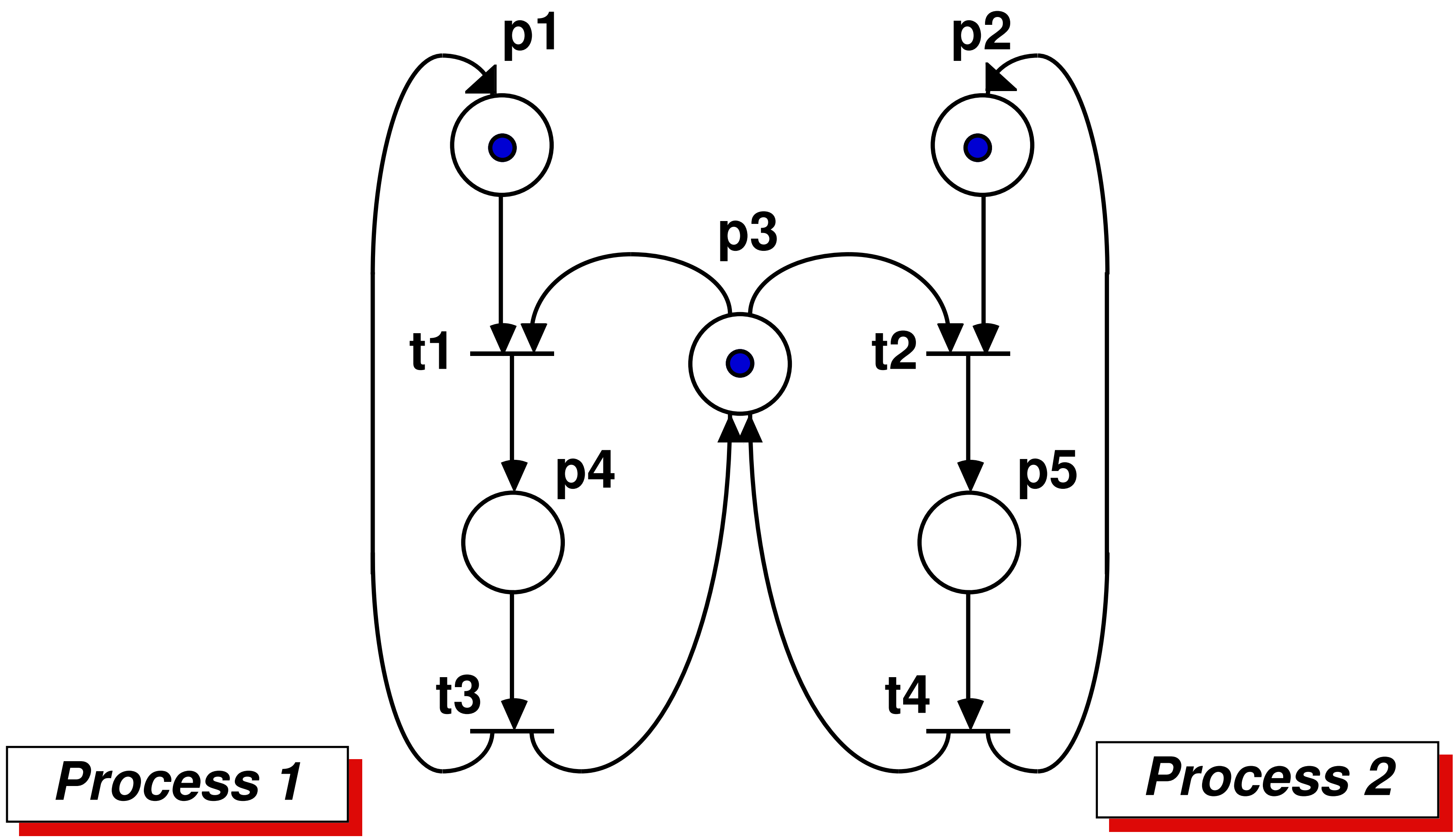
Producer/Consumer (2)



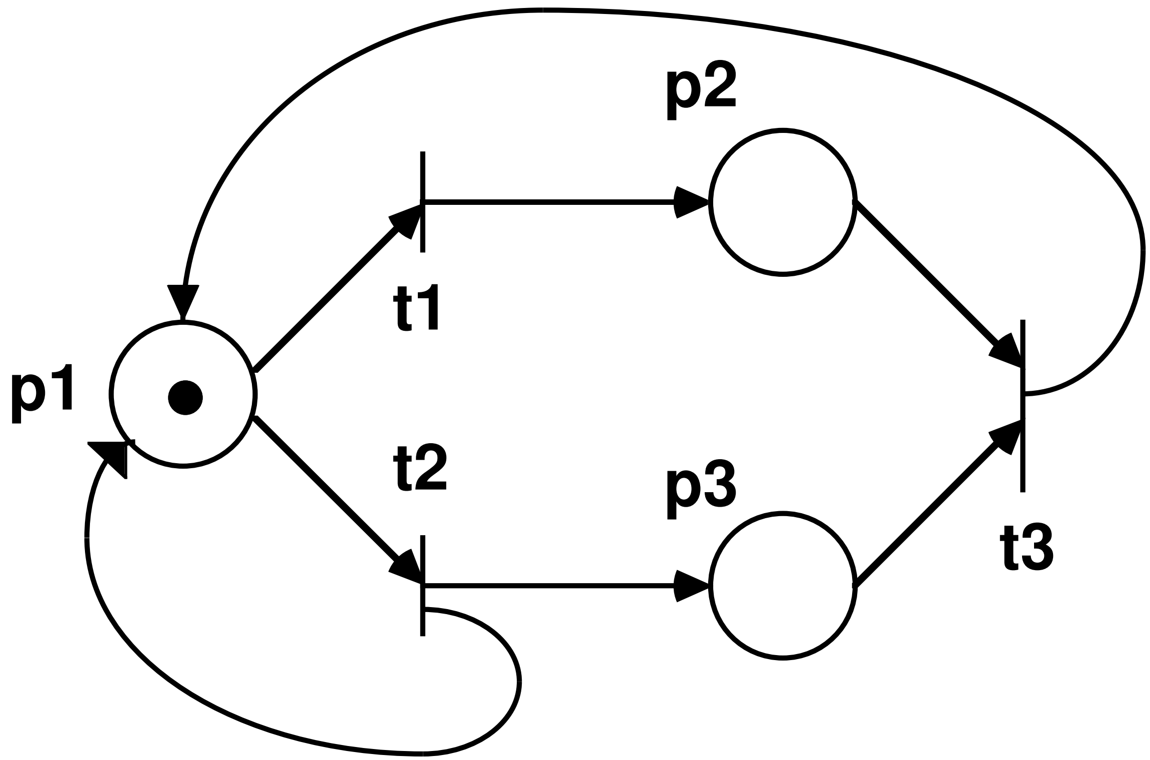
Producer/Consumer (3)



Mutual Exclusion



Another Example



Petri Nets: Formal Definition

- A ***Petri Net*** is a four-tuple (P, T, I, O), where
  - > P is a set of ***places***
  - > T is a set of ***transitions***
  - > I is the **input function**, $I : T \to \mathcal{P}\,P$
    - » identifies the places leading *into* a transition
  - > O is the **output function**, $O : T \to \mathcal{P}\,P$
    - » identifies the places leading *out* of a transition
- **Notes:**
  - > Determines a "bipartite directed graph," where nodes are $P \cup T$ and edges are defined by I & O
  - > Definition implies at most one arc between a transition and any input or output place

Markings & Execution Model

- A ***marking*** M of a Petri net is a mapping that is assignment of ***tokens*** to places.

  $\mathbf{M} : \mathbf{P} \rightarrow \mathrm{N}$
- Petri nets are usually associated with an ***initial marking***.
- A transition is ***enabled*** if each of its input places has at least one token.
- A transition may ***fire*** if it is ***enabled***.
- The firing of a transition
  (a) Removes a token from each input place
  (b) Produces a token on each output place

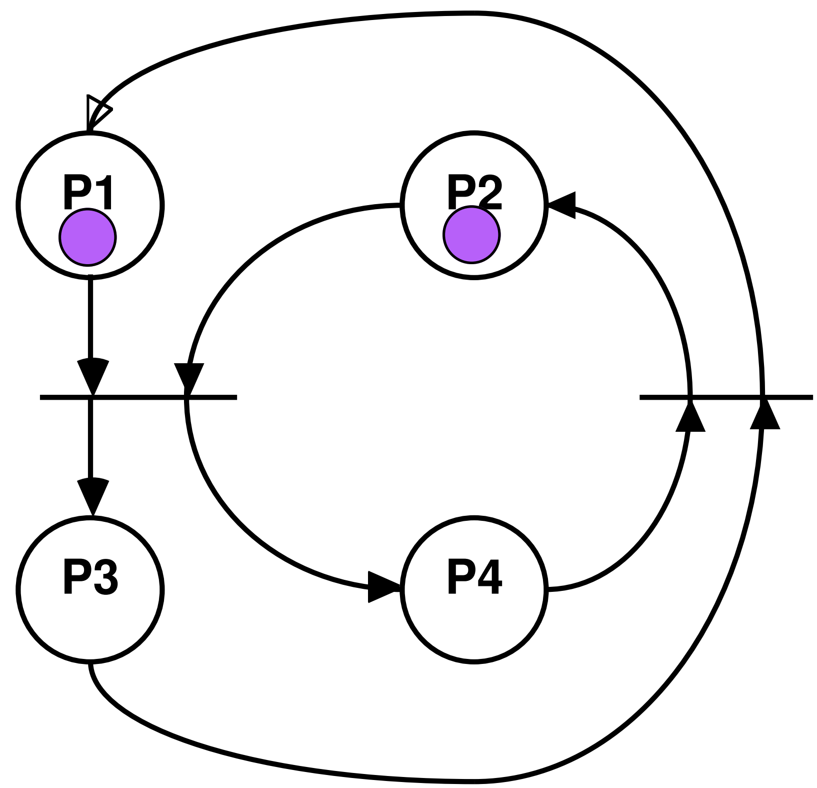
Reachable States

- **The *state space* of a Petri net is the set of all markings (marking = state).**
- **A marking M' is *immediately reachable* from marking M if the firing of some enabled transition in M yields M'.**
- **The *reachable* states are all markings that can be obtained from some initial marking M.**
- **The *reachability set* for a marking M is the set of reachable states. Sometimes indicated by R(M).**

Markings (continued)

- It is often convenient to represent a marking as a vector
- Example:

  Initial marking: (1,1,0,0)
  After 1 step: (0,0,1,1)
  After 2 steps: (1,1,0,0)
  ...
  So reachability set is
  {(1,1,0,0),(0,0,1,1)}



Comparing Markings

- $M \geq M'$ iff each place of M has at least as many tokens as each place of M'
- Example:
  > M = (1,0,0,2)
  > M' = (0,0,0,1)
- Note not all markings are comparable
- Example:
  > M = (1,0,0,2)
  > M' = (0,1,0,1)

As we have seen ...

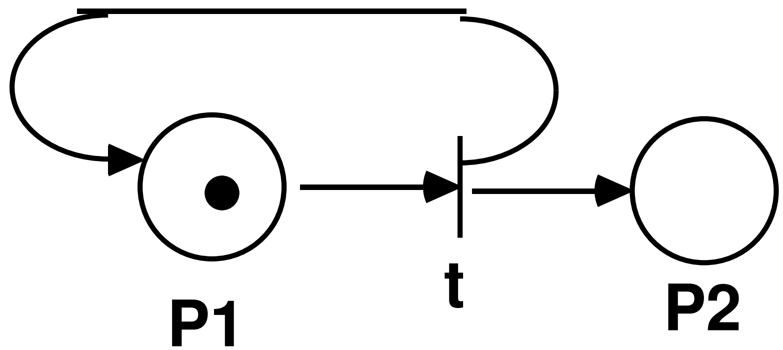
- Petri nets can have nondeterministic transitions.
- That is, two transitions are enabled and one of them will fire nondeterministically.
- This makes it possible for Petri nets to model concurrent execution
  - interleaved firing of transitions
- Additionally Petri nets can model
  - synchronization
  - resource sharing

Special Kinds of Petri Nets

Given an initial marking:

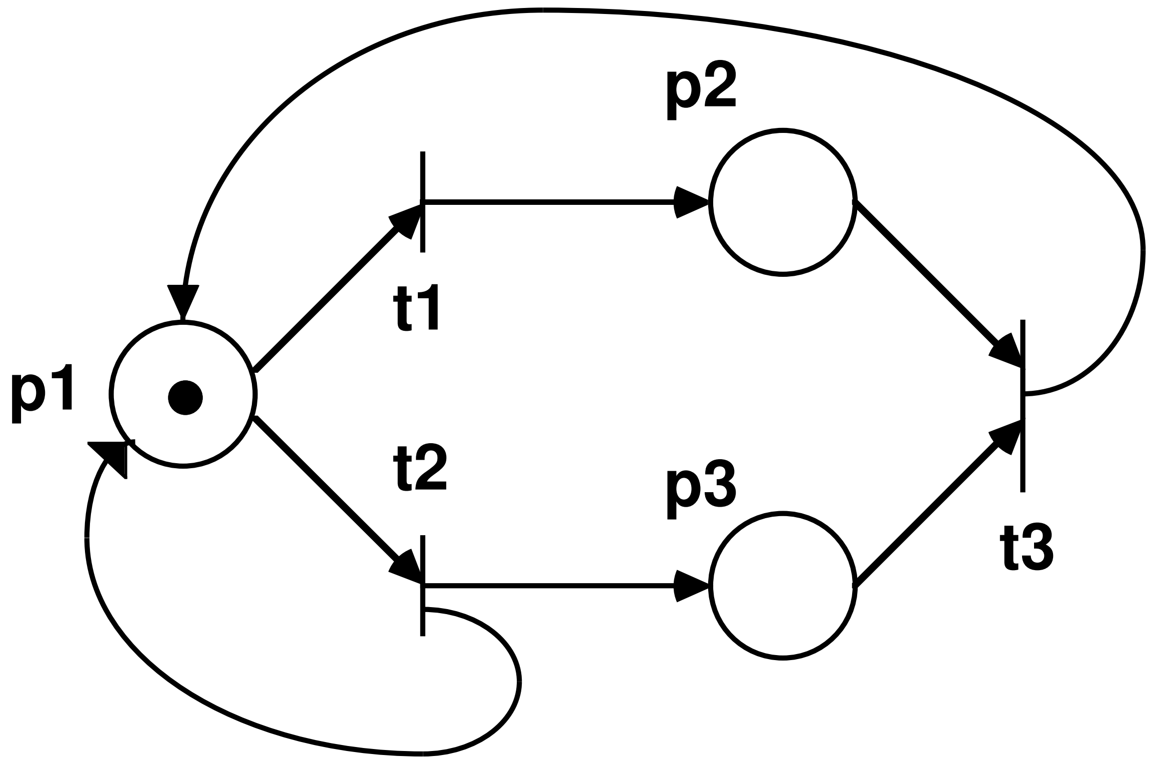
- **Safe net:** No more than one token at any place.
- **K-bounded:** No more than K tokens at any place.
- **Bounded:** K-bounded for some value of K.
- **Conservative:** Number of tokens in net remains constant.

Unbounded Nets

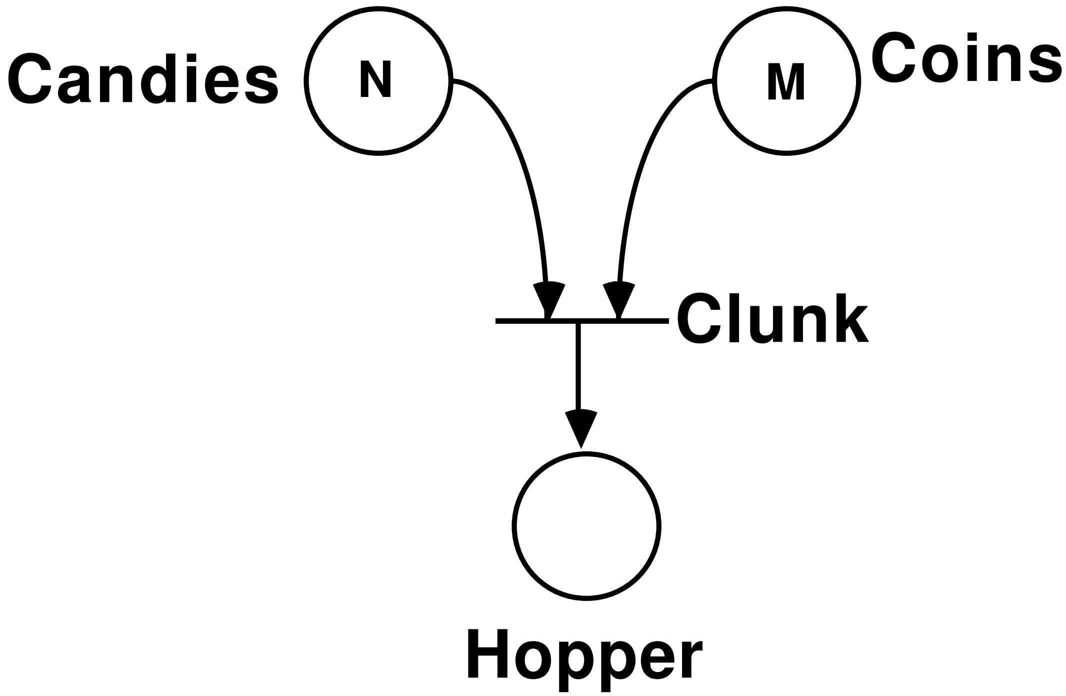


Qn: what is the reachability set?

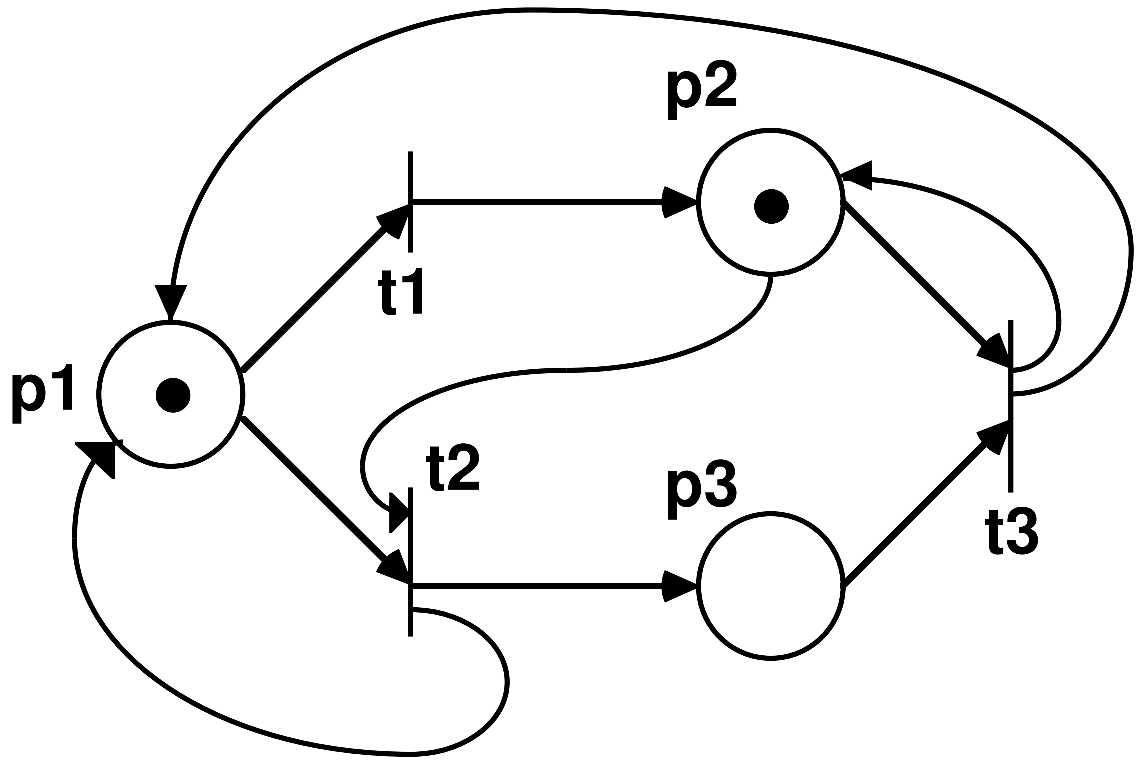
Is This Net Bounded?



Non-Conservative Nets

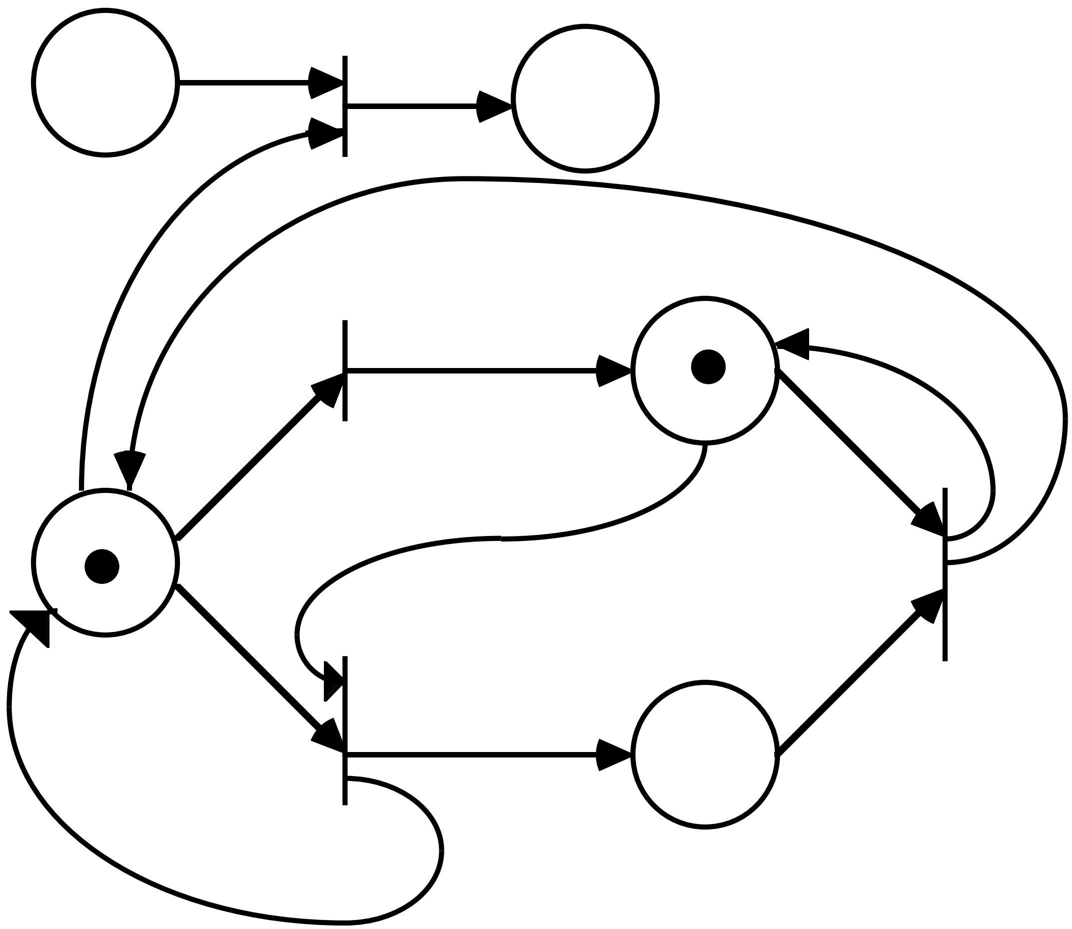


Is This Net Conservative?



Necessary and sufficient conditions?

Is This Net Conservative?



Other Questions

- **Liveness:**
  > Are all transitions potentially fireable?
- **Reachability:**
  > Is marking M' reachable from marking M?
- **Coverability:**
  > Given markings M and M', does there exist a marking M'' reachable from M, such that M'' $\geq$ M' ?

Liveness