Lecture 10
Reasoning about Z Specifications

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The Story Thus Far

- We have seen a brief intro to Z
- Main features
  > use of schemas to encapsulate state

  > use of schemas to describe operations

  > the delta and xi conventions

  > use of schema calculus to build up bigger operations out of smaller ones

Relating Birthday Book to State Machines

S = {known: NAME; birthday: NAME £ DATE |
    known = dom(birthday)}

I = {known: NAME; birthday: NAME £ DATE |
    birthday = $\emptyset$}

A = {AddBirthday(date?:DATE, name?:NAME),
    FindBirthday(date?:DATE)/name!(NAME),
    Remind(date?:DATE)/names!(P Name)}

δ = ...

AddBirthday

AddBirthday(date?:DATE, name?:NAME)

<u>pre</u> name? ∉ known

<u>post</u> birthday' = birthday ∪ { name? ¢ date? }

FindBirthday(date?:DATE)/name!(NAME)

<u>pre</u> name? known

<u>post</u> birthday' = birthday

known’ = known

name! = birthday(date?)

RAddBirthday

RAddBirthday(date?:DATE, name?:NAME)
report!(REPORT)

pre true

post

(name? ∉ known

birthday' = birthday ∪ { name? ¢ date? }

report = ok)

(name? known

birthday' = birthday

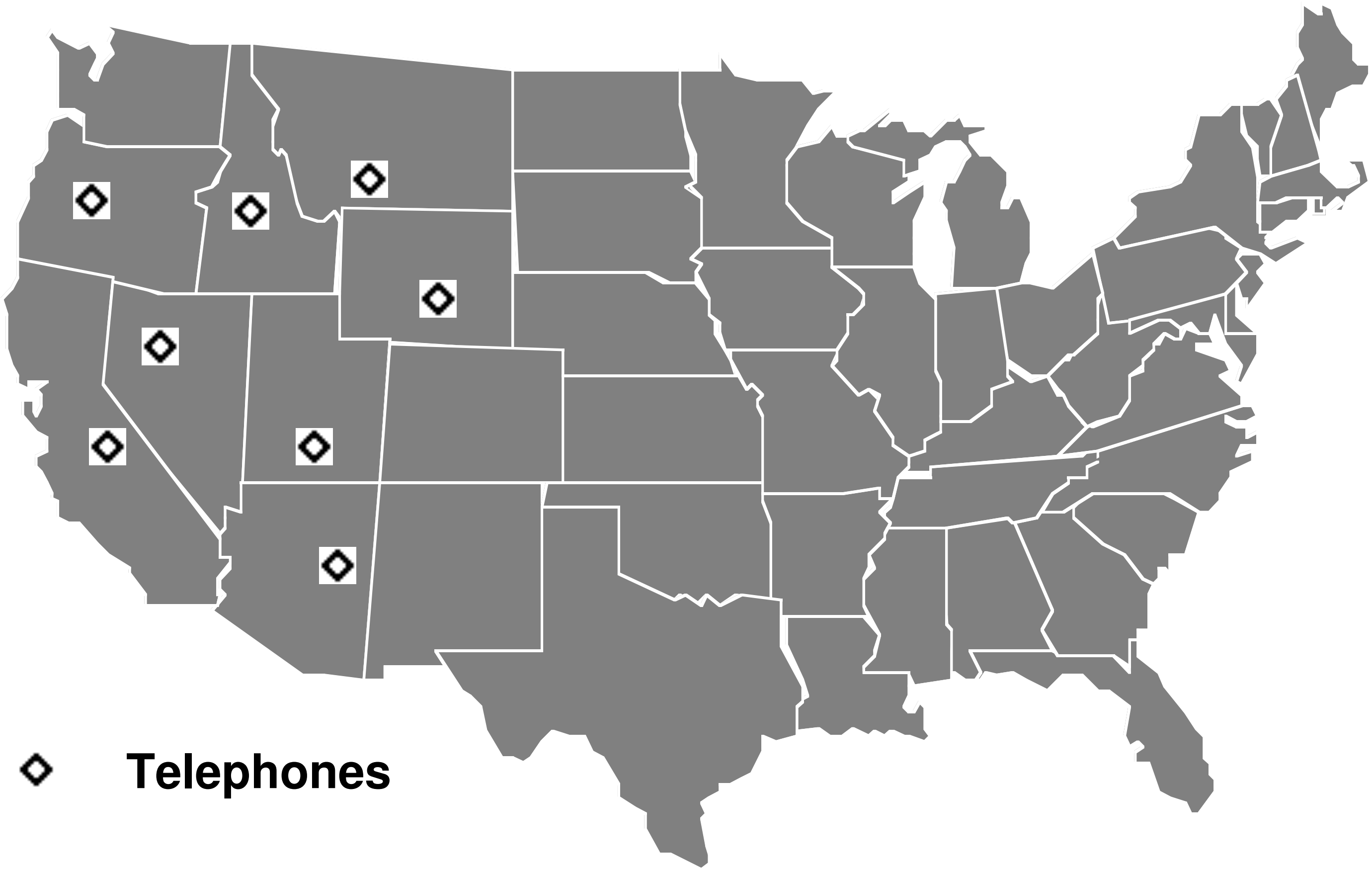
known'= known

report = alreadyknown)

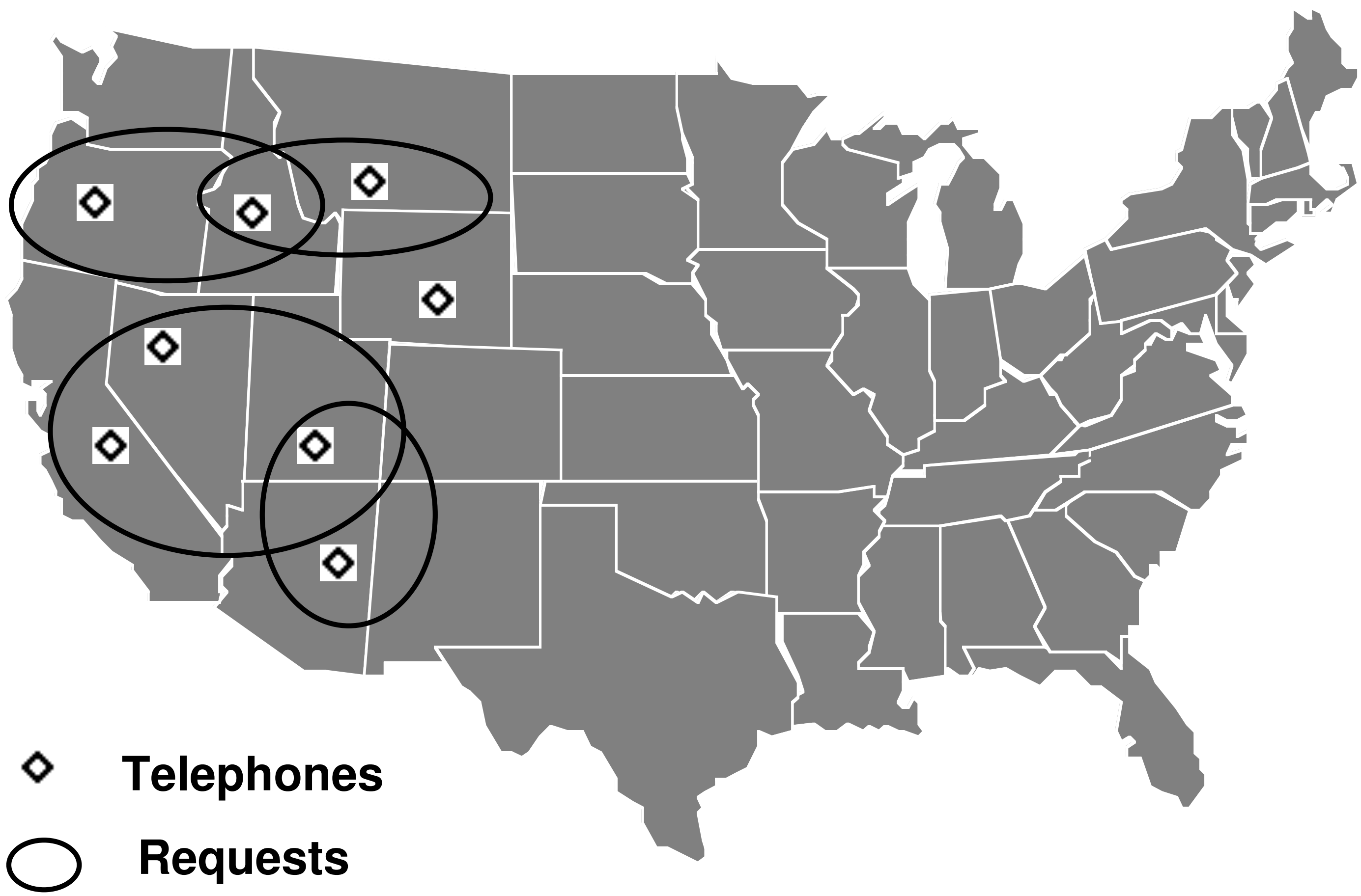
Lecture Outline

- **A Second Example: Telephone Net**
  - > **Adapted from "Telephone Network", by Carroll Morgan**
  - > **In Specification Case Studies, Ian Hayes (ed)**
- **Concepts**
  - > **Use of strong state invariants to make concise models**
  - > **Use of schema calculus to simplify descriptions and associated reasoning**
  - > **Disj, (Generic Definitions)**
  - > **Framing**
  - > **Reasoning with state invariants**

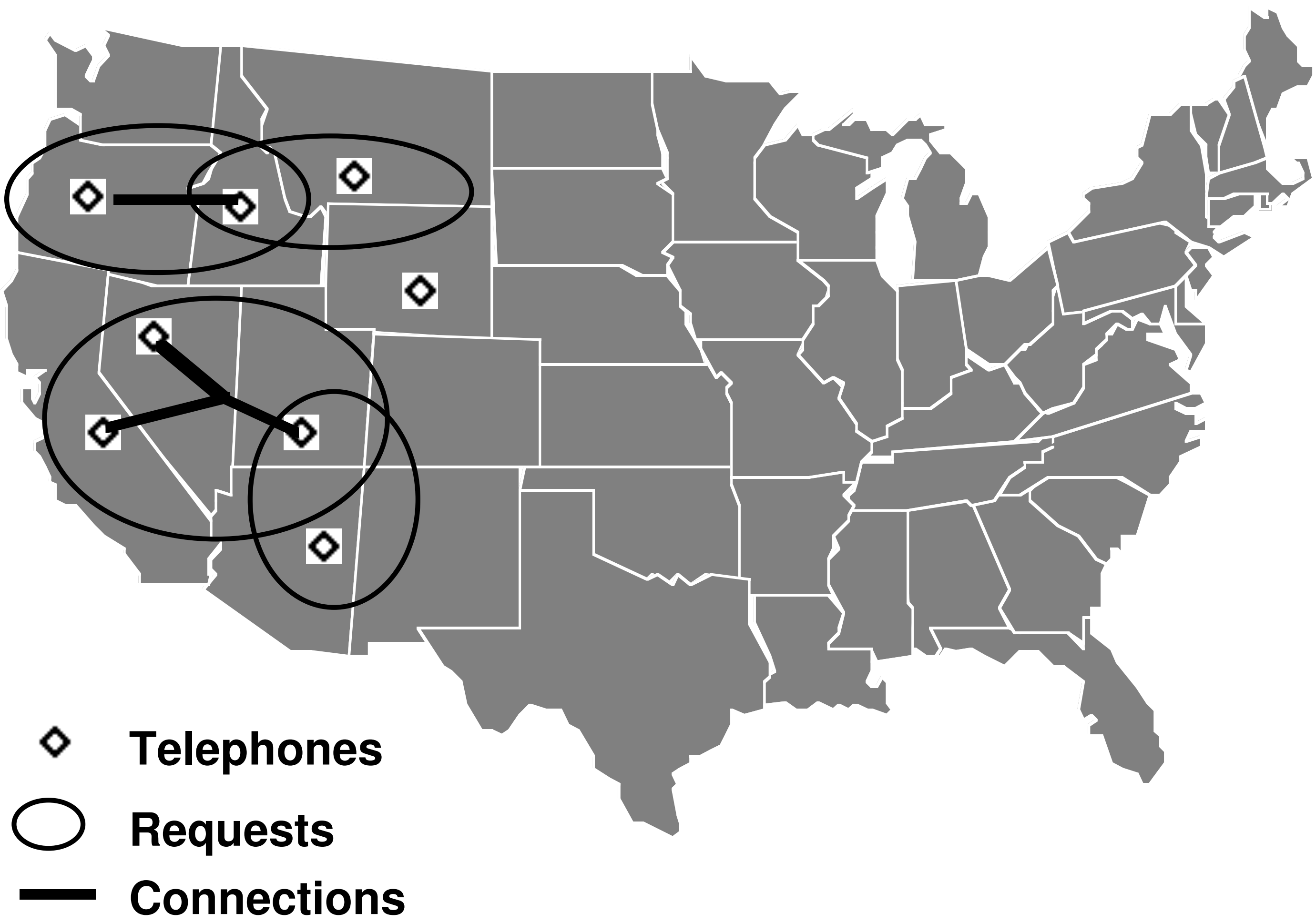
Telephones



Requests for Connection



Connections



Operations

- *Call*: Request a connection between two phones
- *Hangup*: Terminate a connection
- *Busy*: Report whether a phone is busy

State Space

[PHONE]

CONNECTION == $\mathbb{P}$ PHONE

TelephoneNet

reqs, cons: $\mathbb{P}$ CONNECTION

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cons † reqs

***disj* cons**

Example

PHONE = {1, 2, 3 }

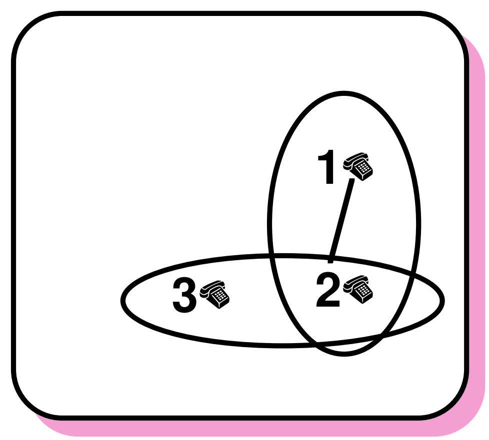
CONNECTIONS =

{ {}, {1}, {2}, {3}, {1,2} {1,3}, {2,3}, {1,2,3} }

TelephoneNet

reqs: { {1,2},{2,3} }

cons: { {1,2} }



Pairwise disjoint sets

Definition: (informal) A set of sets is disjoint iff no pair of those sets has elements in common.

Definition: (formal)

$$disj \text{ cons} \Leftrightarrow$$
$$\forall c1, c2 : cons \bullet c1 \neq c2 \Rightarrow c1 \cap c2 = \emptyset$$

Examples: Which sets are *disj*?

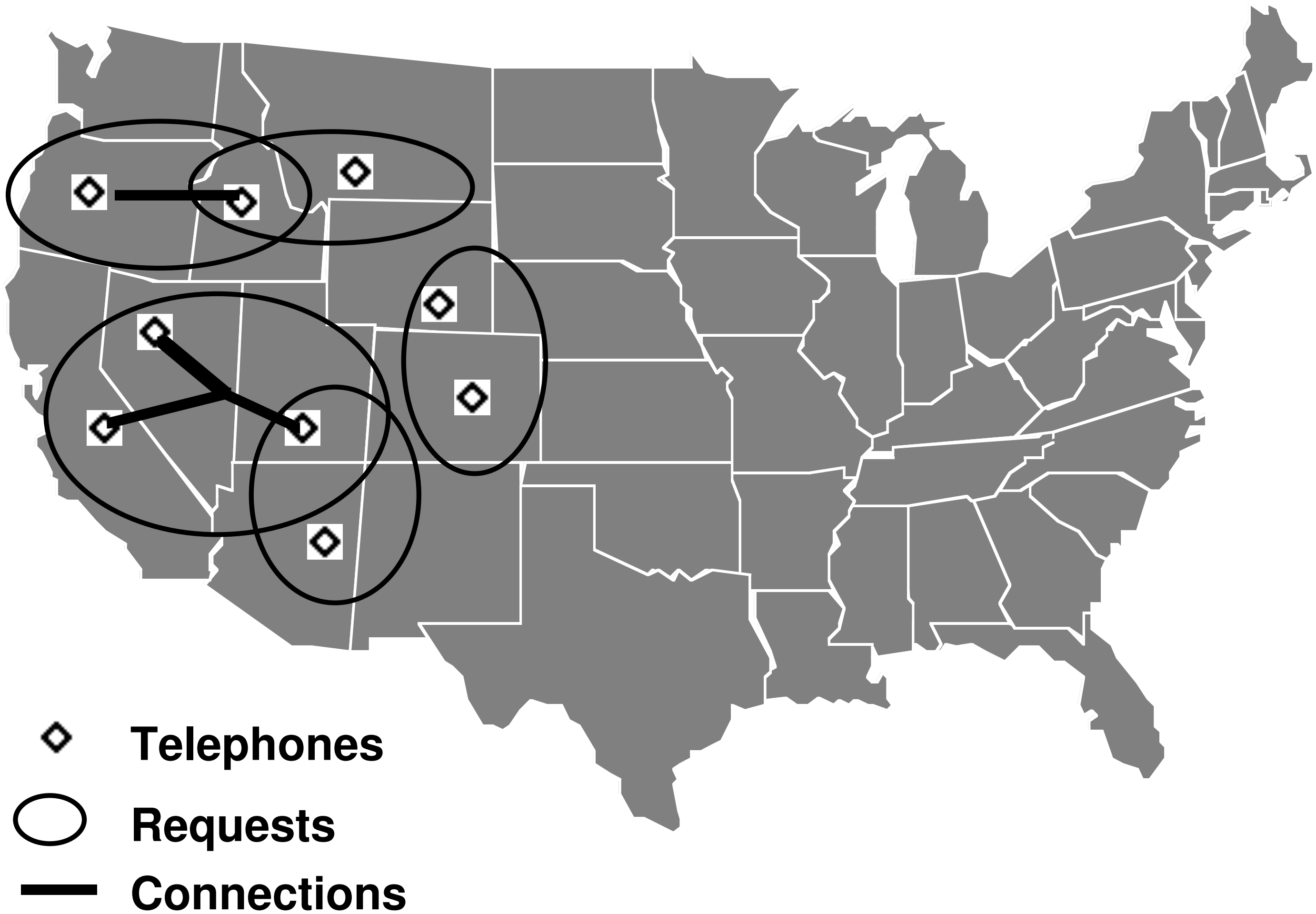
$\{ \{a\}, \{b,c\}, \{d\} \}$

$\{ \{a,b\}, \{c,d\}, \{a,c\} \}$

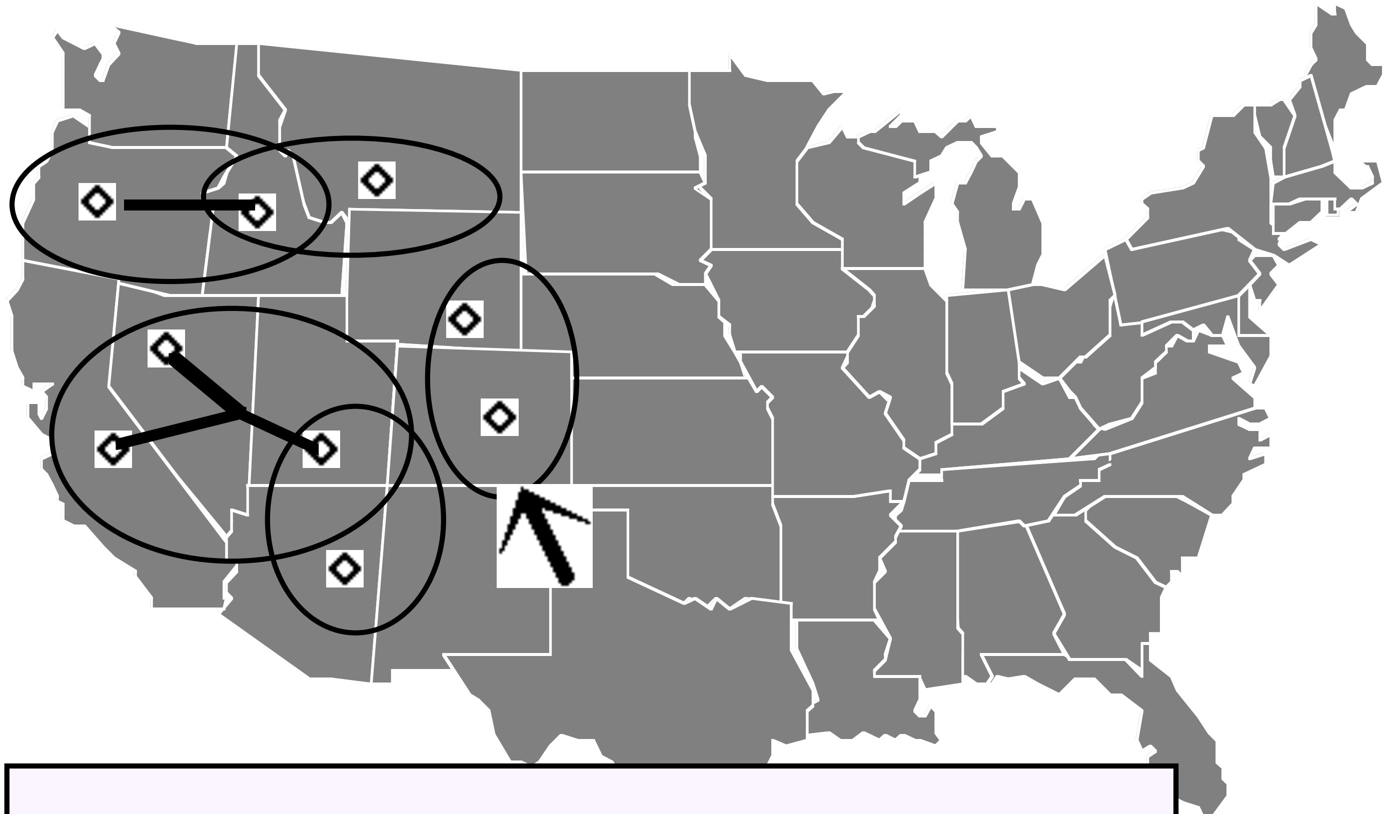
$\{ \emptyset, \{a\} \}$

$\{ \{a,b\}, \{b, a\}\}$

What's Wrong With This Net?



An Inefficient Net



InefficientNet :

$\exists$ **r: reqs \ cons • disj (cons $\cup$ {r})**

Efficient Networks

EfficientTN

TelephoneNet

~ (∃ r: reqs \ cons • disj (cons ∪ {r}))

EfficientTN

reqs, cons: $\mathbb{P}$ CONNECTION

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cons $\dagger$ reqs

disj cons

$\sim (\exists\ r: reqs \setminus cons \bullet disj\ (cons \cup \{r\}))$

Initial Telephone Net

InitTN

EfficientTN'

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reqs' = cons' = ∅

Framing: Event

Event

Δ **EfficientTN**

$\forall$ **c: CONNECTION** $\bullet$

$c \in (cons \cap reqs') \Rightarrow c \in cons'$

This Really Means ...

Event

reqs, cons: $\mathbb{P}$ **PHONE**

reqs', cons': $\mathbb{P}$ **PHONE**

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cons reqs

***disj* cons**

cons' reqs'

***disj* cons'**

$\sim(\exists\, r: reqs \setminus cons \bullet disj\ (cons \cup \{r\}))$

$\sim(\exists\, r: reqs' \setminus cons' \bullet disj\ (cons' \cup \{r\}))$

$\forall\, c: CONNECTION \bullet c \in (cons \cap reqs') \Rightarrow c \in cons'$

But we would not be expected to write it out this way.

Call

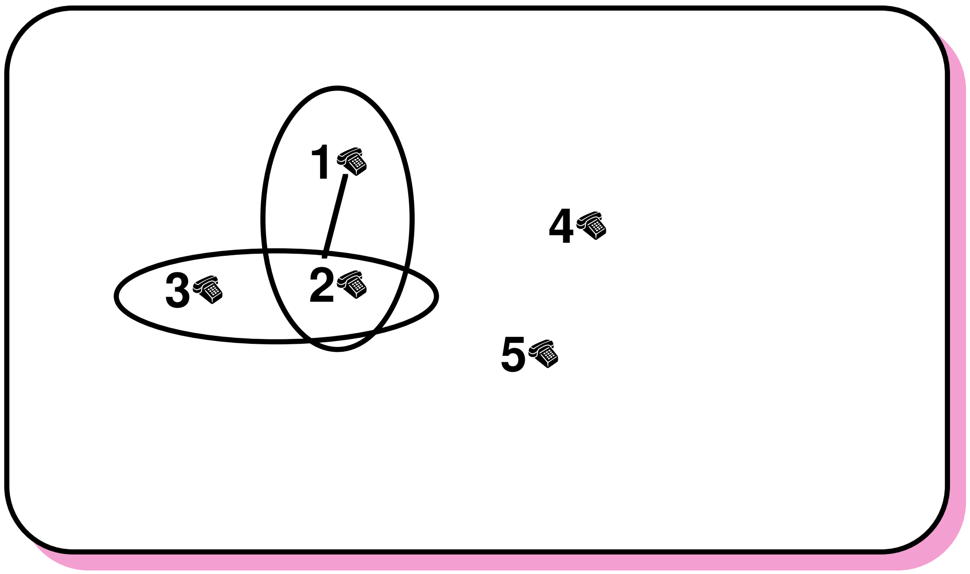
Call

Event

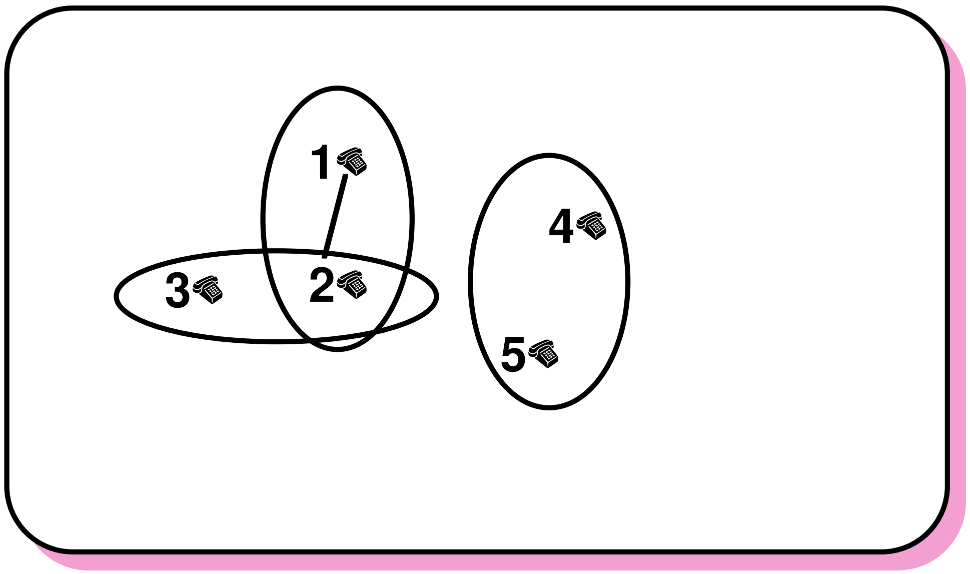
ph?, dialled? : PHONE

$$reqs' = reqs \cup \{ \{ph?, dialled?\} \}$$

Example



Example



Must the new request become a connection?

Hangup

Hangup

Event

ph? : PHONE

reqs' = reqs \ { c: cons | ph? $\in$ c }

Is this realistic?

Busy

YesOrNo ::== Yes | No

Busy

Event

ph?: PHONE

busy!: YesOrNo

$reqs' = reqs$

$busy! = Yes \Leftrightarrow ph? \in \bigcup cons$

What happens to cons?

Reasoning about the Specification

- **Theorem: (Informal) If an operation doesn't change any of the requests then it doesn't change any of the connections.**
- **Proof: (Informal)**
  1. **The constraint on Event tells us that any original connection won't be broken if it is still requested.**
  2. **If an operation doesn't change the requests, all original connections will still be requested.**
  3. **Hence, no connections are broken.**
  4. **Also, no new connections are added because if a connection could be added afterwards, it could have been added before.**
  5. **So connections aren't changed.**

The Fuzz Type Checker

- One of the benefits of being precise about types is that we can type-check our descriptions
- We will use the Fuzz Type Checker to do this.
- It takes Latex descriptions of Z specifications as input and produces error reports and other useful type information.
- To type check a file spec.tex, invoke

  fuzz spec.tex
- To get type information about spec.tex, invoke

  fuzz -t spec.tex