Models of Software Systems

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15-671

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Lecture 1
Course Introduction

- Course Mechanics
- Course Rationale & Objectives
- Overview of Topics
- What are Formal Models?

General Information

- **Instructors:**

David Garlan	Jeannette Wing
WeH 8020	WeH 8219
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- **TA:**
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- **Secretary:**
  Cary Lund, WeH 8106, 268-3853, cary@cs

Computing and Communication

- You will need an account on a SCS machine.
  - > Forms available after class.
- Class bboard: cmu.cs.class.cs771
  - > Read it regularly, post questions and ideas
- E-mail to instructors is fine anytime
- Central location for class files: TBD

Readings

- **Textbooks:**
  - *Software Engineering Mathematics*, by J.C.P. Woodcock and M. Loomis
  - *Communicating Sequential Processes*, by C.A.R. Hoare.
- **Supplementary papers:**
  - Distributed as needed.
- **Reading should be done before class.**

Homework

- **Weekly homework assignments**
  - > Discussion with others encouraged
  - > Final write-up must be your own work
  - > Due at beginning of class
  - > Pass/no-pass (✔, –)
  - > Can redo by next class
  - > Class presentations as needed
- **Lab exercises as needed**
  - > To give practice with tools
  - > Ungraded

Grading

- Homework exercises (65%)
  - About one per week (5% each)
- Exams (30%)
  - Midterm take-home (10%)
  - Final in-class (20%)
- Instructor discretion (5%)

Course Rationale

- Engineering depends on use of precise models for characterizing systems
- Benefits:
  - > Abstraction
    - » highlight important details
  - > Analysis and reasoning
    - » understand the things we produce
  - > Communication
    - » unambiguous descriptions
  - > Design underpinnings
    - » expose choices within design space

Relationship to Other Core Courses

- **Methods**
  - > You will be learning to use formal methods, and, in particular, Z.
- **Management**
  - > Some models used to represent/analyze processes
  - > Metric models
- **Analysis**
  - > Many of the analyses are formal
  - > Others build on the foundations of this course
- **Architecture**
  - > You will develop formal models of architectures

Relationship to Studio

- Formal models are particularly important for safety critical systems
- You will have the opportunity to find areas in which models are needed
- In the past, most studios have found direct application of formal modelling skills

Some of the Models and Logics Used in Software Engineering

- Propositional and Predicate Logic
- State Machines
- Abstract State Machines (Z)
- Algebraic Models
- Models of Concurrency
- Petri Nets
- Trace models and Process Algebras (CSP)
- Temporal Logics

This Course

- Part 1: Models of Sequential Systems
  - Propositional and Predicate Logic
  - State Machines
  - Z as a concrete notation for state machines
  - Module interface notations
  - Algebraic Models
- Part 2: Models of Concurrency
  - Parallel state machines
  - Trace models and Process Algebras (CSP)
  - Petri Nets
  - Temporal Logics

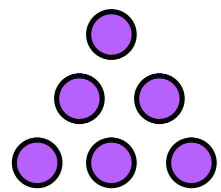
Objectives

- At the end of the course you should be able to
  - Understand the strengths and weaknesses of the models and logics covered in class.
  - Select and describe formal models for certain classes of systems.
  - Describe relations between different levels of abstraction.
  - Prove elementary properties about systems described by the models covered in the course.
  - Have prerequisite mathematical foundations for the other core courses.

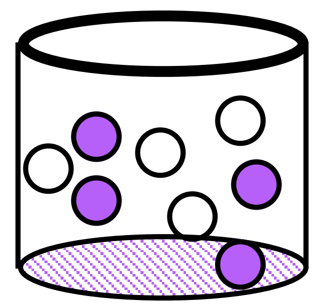
Viewpoint

- Mathematical abstractions (models) can be used to represent real systems
- Formal reasoning can improve our ability to design and build systems
  - > Uncover design flaws
  - > Precisely define requirements
  - > Produce rational implementations
- Different models and logics have different strengths and weaknesses

Example

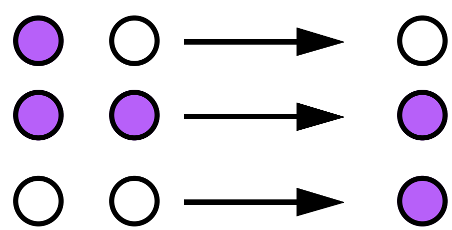


stock of black balls



jar of balls

Rules



The Problem

<u>Question 1:</u> **Does the game stop?**

<u>Question 2:</u> **If so, can you say anything about the color of the last ball by knowing the original configuration?**

Testing

A Formal Model

b = black w = white f = transition function

$$f(b, w) = \begin{array}{ll} (b - 2 + 1, & w) \\ (b - 1, & w - 1 + 1) \\ (b + 1, & w - 2) \end{array}$$

$$f(b, w) = \begin{array}{ll} (b - 1, & w) \\ (b - 1, & w) \\ (b + 1, & w - 2) \end{array}$$

$f(0, 1) = (0, 1)$ $f(1, 0) = (1, 0)$

Example : Reasoning

Theorem 1: The game stops.

Theorem 2:

f* (b, w) = if odd (w)
then (0, 1)
else (1, 0).

(Qn: How might you prove this?)

Moral

- Abstraction allows you to focus on the important aspects of a problem.
- Mathematics allows you to reason about solutions to the problem.
- (You can earn lots of money if you know the right mathematics.)

What is a Formal System?

Informally the basic elements are:

1. A *language* for describing things of interest

   example: $(b, w) = (b - 1, w - 1 + 1)$

2. Some *rules* for reasoning about "statements" in the language

   example: $(b - 1, w - 1 + 1) = (b - 1, w)$

3. Ways of *assigning meaning* to these statements

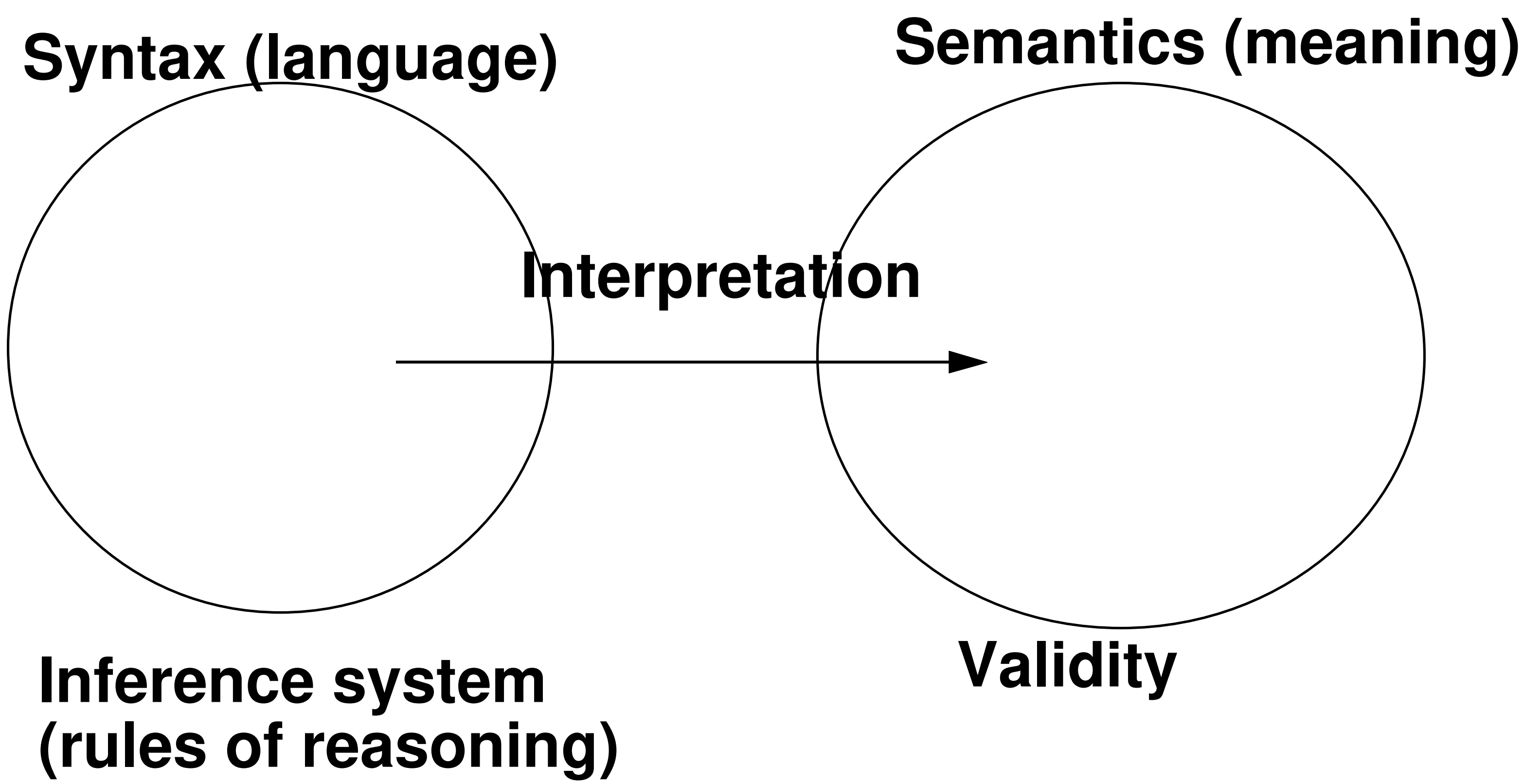
   example: b = number of black balls

4. A *body of results* that have already been demonstrated

   example: facts about numbers

   if b is even then $b-2$ is even

Basic Elements



Syntax: Formal Languages

- All formal systems start by giving

  1. an *alphabet* -- the symbols to be used
  2. syntactic *rules* -- a set of rules for constructing sentences in that system.

- Legal sentences are sometimes called the *well-formed formulae* (wff) of the language

- Examples of wffs:

  Sets: $(A \cup B) \cap (A \cup C)$

  Propositional logic: $(p \Rightarrow q) \wedge (\sim p \vee q)$

  Predicate Logic: $\forall x{:}S \bullet Stooge(x)$

Example

Two formal languages with the same alphabet but different syntactic rules:

1. language of real numbers

- alphabet: {0,1,2,3,4,5,6,7,8,9,.}
- some wffs: "345.678" "2.123" "0.421"
- not wffs: "3.4.5"

2. language of section numbers

- alphabet: {0,1,2,3,4,5,6,7,8,9,.}
- some wffs: "3.4.5" "2" "0.421"
- not wffs: ".5" "3."

(Qn: is ".5" an real number wff?)

Grammars

- Syntactic rules need their own language for description -- called a “meta-language”
- We will use a kind of “BNF”

```
real      = integer | decimal | integer, decimal
decimal   = “.” , integer
integer   = digit | digit, integer
digit     = “0” | “1” | “2” | “3” | ... | “8” | “9”
```

Semantics

- To give meaning to the symbols we
  - pick a semantic domain
  - provide an interpretation (or meaning function) that explains how sentences in the syntactic domain are given meanings in the semantic domain
- Examples

  Number theory: "1" –> one

  Sets: {1,2} the collection containing the numbers 1 and 2.

  Propositional logic: R –> It is Raining.

  Predicate logic: duck(Martha) –> Martha is a duck.

Inference Systems

- A deductive apparatus for manipulating wffs.
  - > Syntactic manipulation: no reference to any interpretation
- Two components:
  - > Axioms: wffs that can be written down without reference to any other wffs in the language.
  - > Inference rules: rules that allow us to produce wffs as *immediate consequences* of other wffs.
- Formal System =

  language + inference system

Proofs and Theorems

- A *proof* in a formal system, F, is a sequence of wffs in the associated formal language in which each wff is
  - > an axiom, or
  - > an immediate consequence of one or more preceding wffs (as determined by the inference rules of the system).
- A wff that can be proved (i.e., appears in a proof) in F is called a *theorem*
- If W can be proven, we write: |- W

Example

Let F be the formal system with

alphabet = {*,@,#}

grammar =

- **sentence = stars, “@”, stars, “#”, stars**
- **stars = “*” | stars, “*”**

deductive apparatus =

- **Axiom: *@*#****
- **Rule: If a @ b # c is a wff (where a, b, c are stars lists), then a @ b * # c * is an immediate consequence**

Example (continued)

Theorem 1.1: * @ ** # *****

Proof:

1. * @ * # **	Axiom
2. * @ ** # ***	Inf rule applied to 1
3. * @ *** # ****	Inf rule applied to 2
4. * @ **** # *****	Inf rule applied to 3

(Qn: can you prove ** @ *** # ***** ?)

Example (continued)

- We can give an interpretation to these symbols:

  $*$ $\rightarrow$ 1

  $**$ $\rightarrow$ 2

  ...

  @ $\rightarrow$ +

  # $\rightarrow$ =

- Theorem 1.1 means 1 + 4 = 5
- For this interpretation every theorem is true
- But not every true addition fact can be proved

Consistency and Completeness

- An interpretation of a formal system in which wffs denote statements which can be true or false is
  - > *consistent* (or *sound*) if every theorem of the system interprets to a true statement
  - > *complete* if every true statement can be proved as a theorem
- Most formal systems we encounter will be sound for the standard interpretations
- Many of the systems will not be complete

Derivations

- (Informal) A derivation of the wff W in formal system F from a set P of wffs (called premises), is a proof in which a wff in the proof can be
  - an axiom
  - a premise
  - an immediate consequence of previous wffs
- A derivation is like a proof in a formal system whose axioms have been augmented with the premise.
- We write P |- W

Example

- **Derive the following:**

  *** @ ** @ ***** |- *** @ *** @ ******

  Derivation:

  1. *** @ ** @ *****   Premise
  2. *** @ *** @ ******   2 Rule 1

- **Derive the following:**

  * @ ** @ ***** |- * @ *** @ ******

  Derivation:

  1. * @ ** @ *****   Premise
  2. * @ *** @ ******   2 Rule 1

- **We can prove nonsense from false premises**