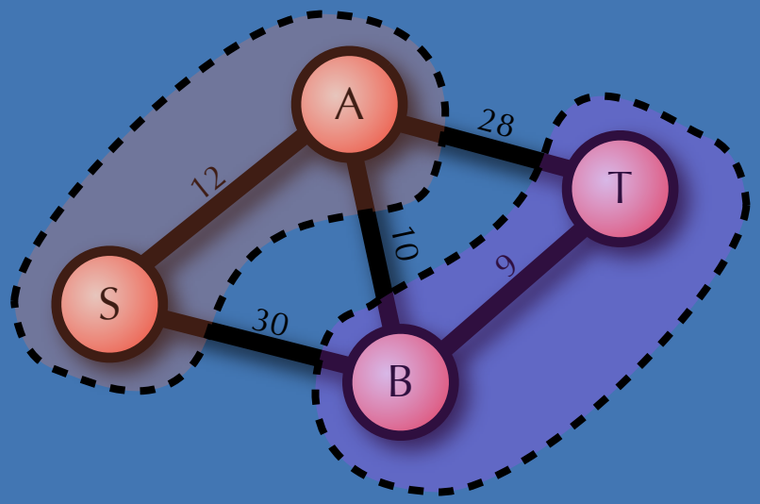
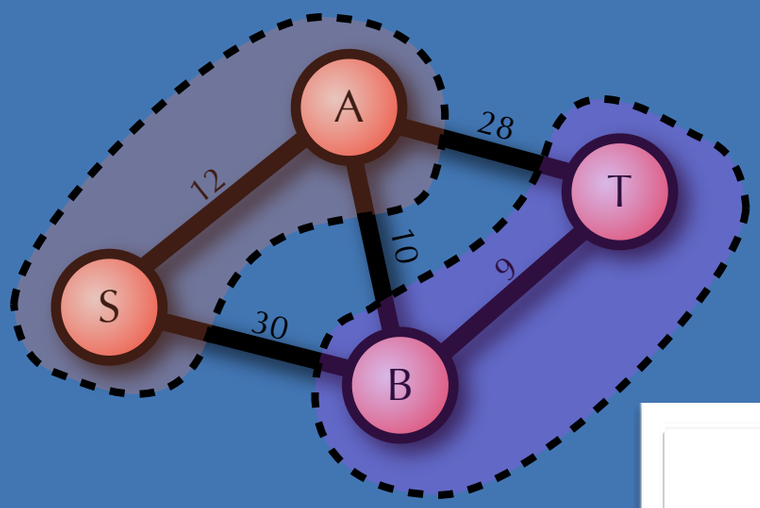


Graph Cuts

...and Images

Adrien Treuille
Carnegie Mellon University

Overview



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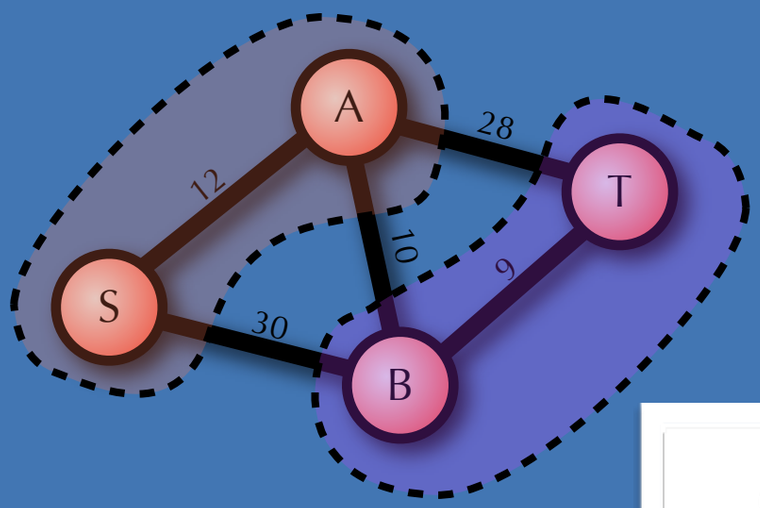

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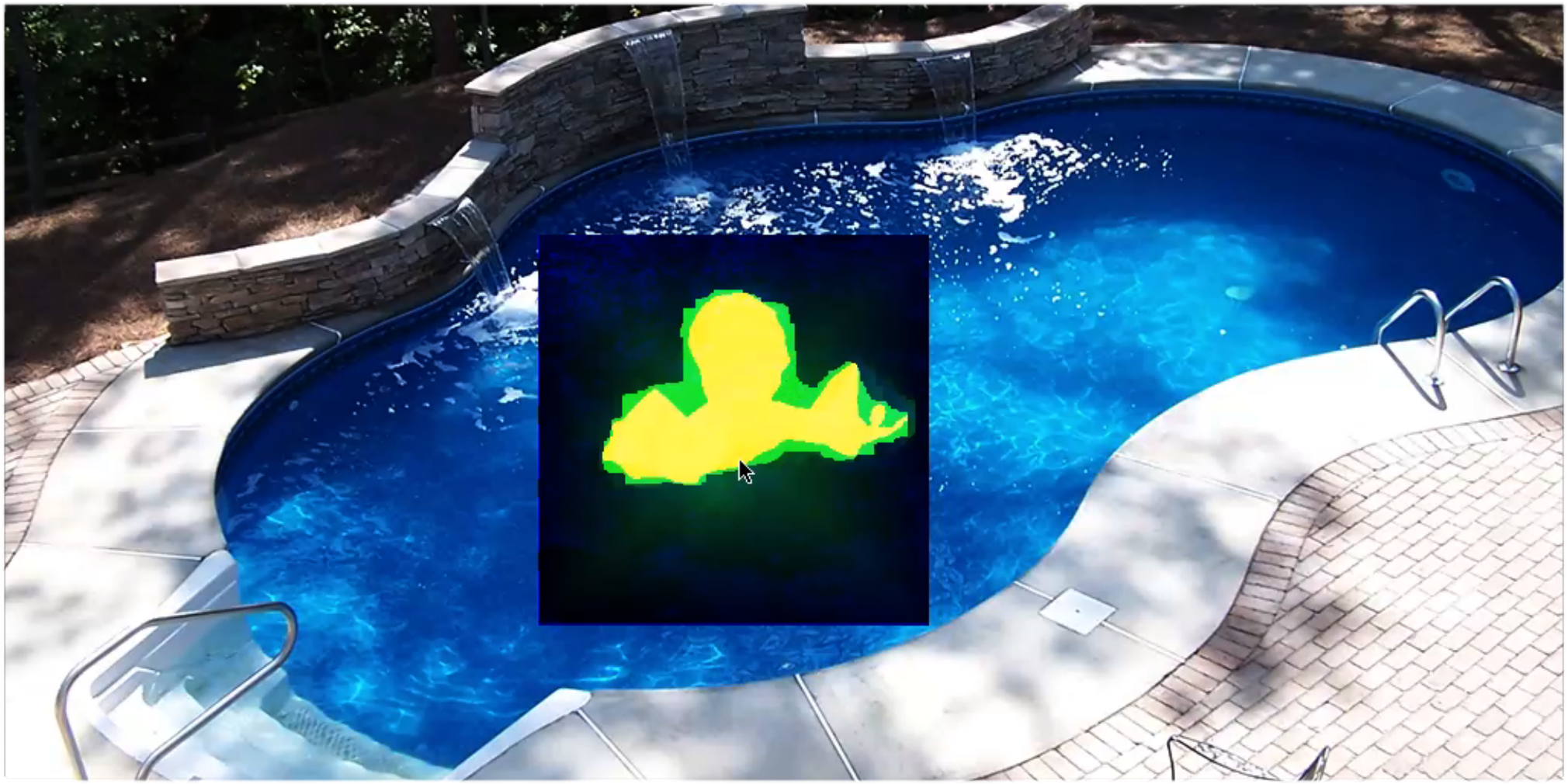
Start Here...

Move the Foreground

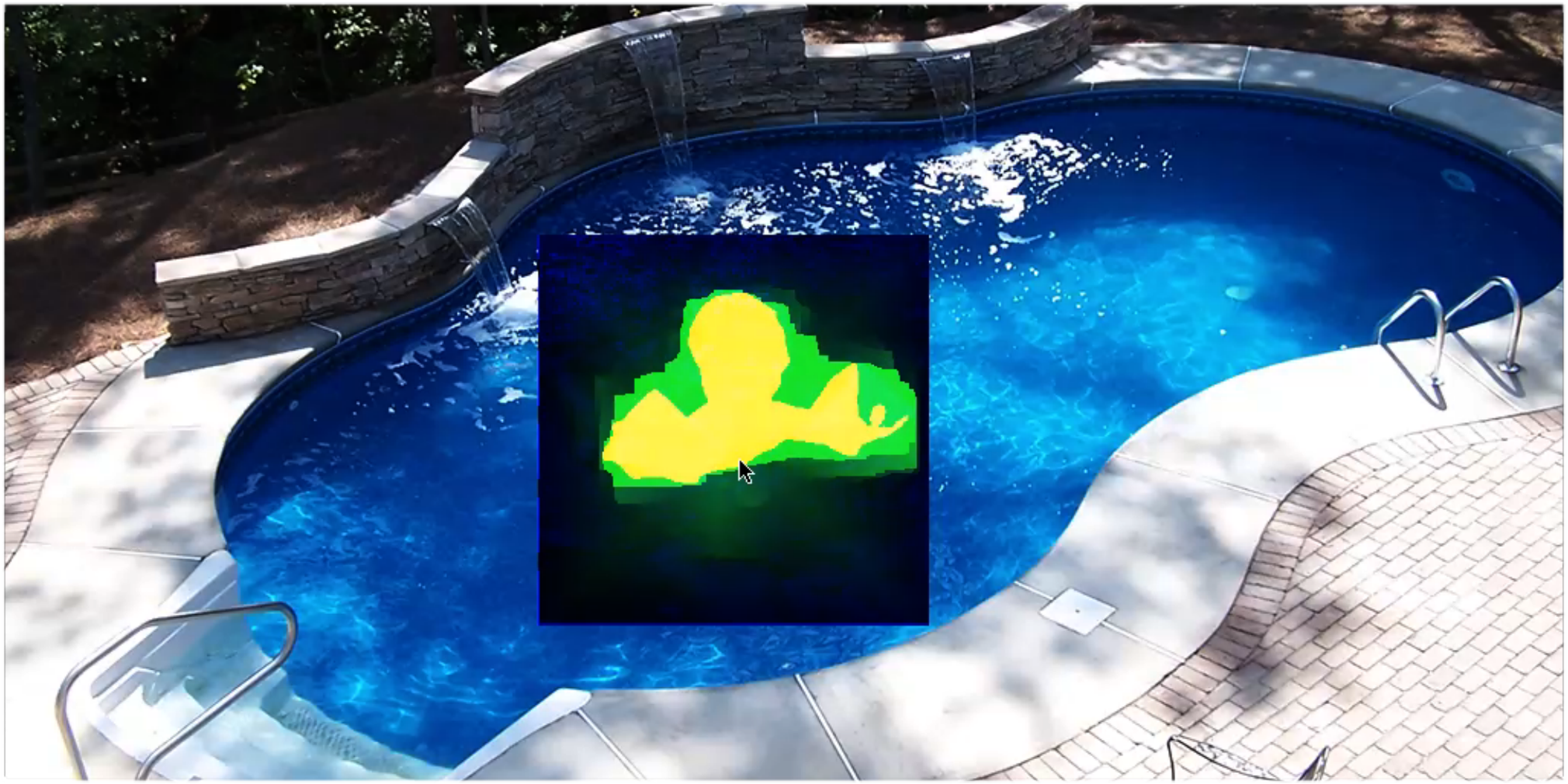
The "Minimum" Mask

Graph Cuts Starting

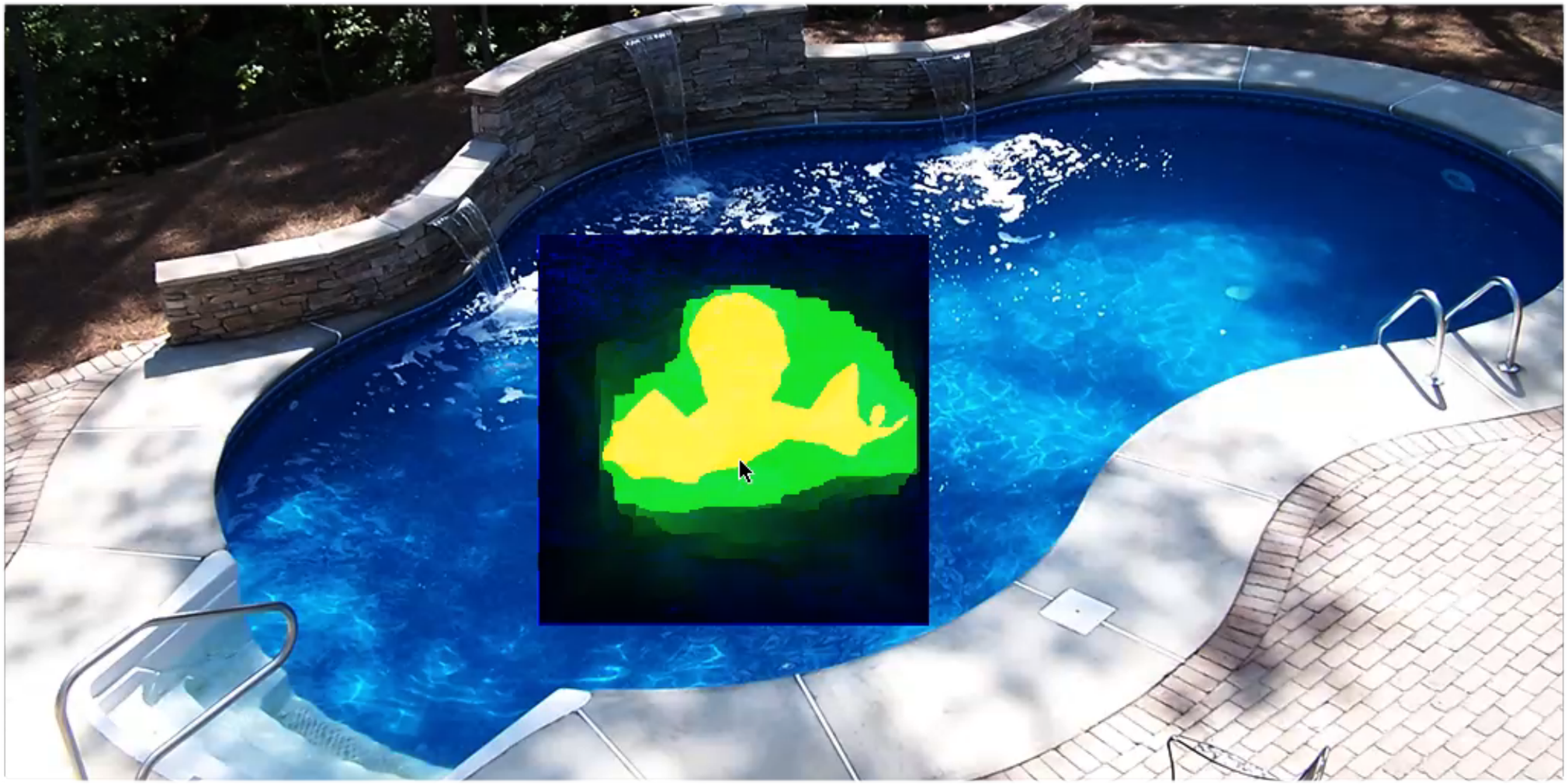
More Graph Cuts...



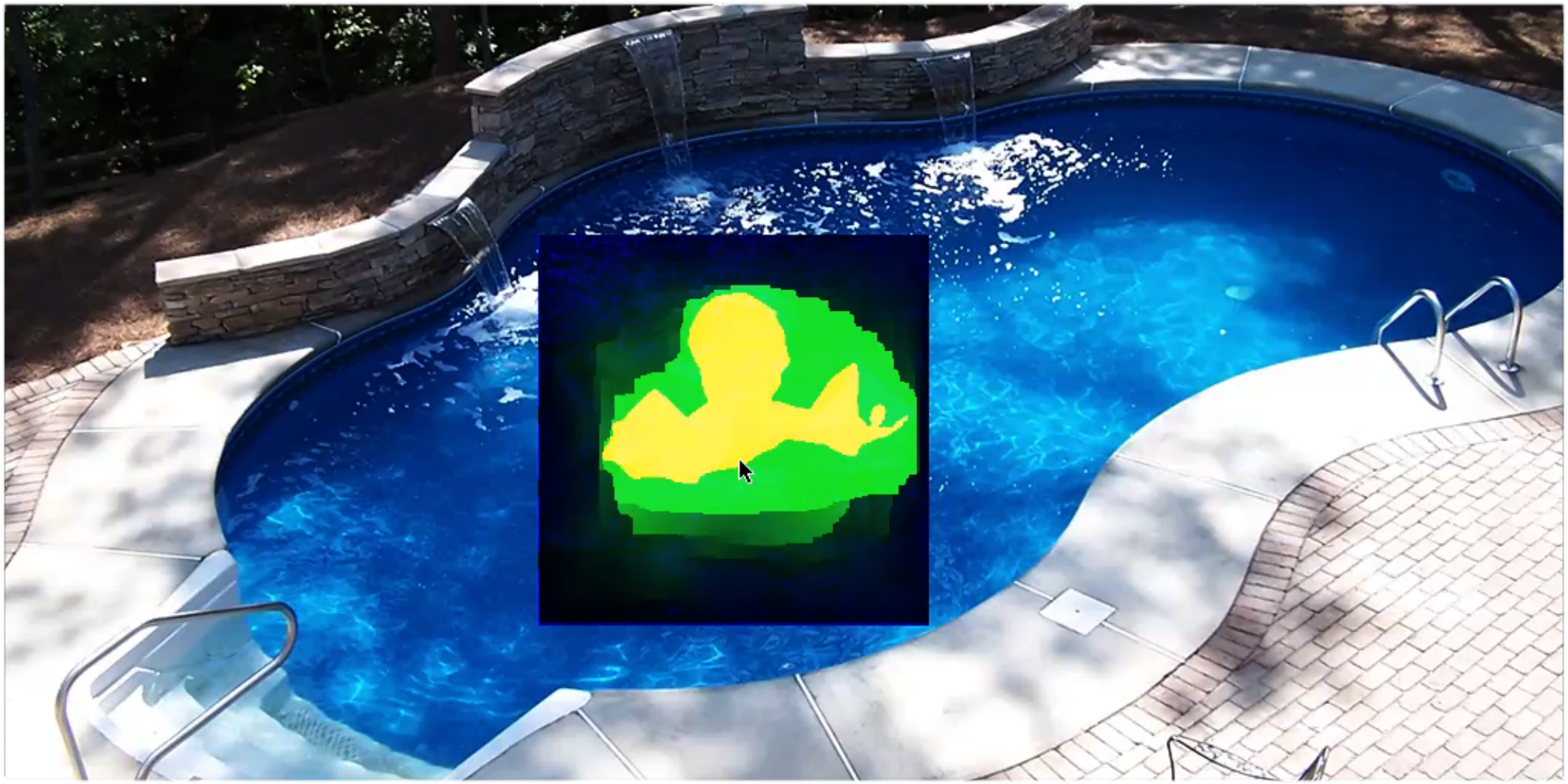
More Graph Cuts...



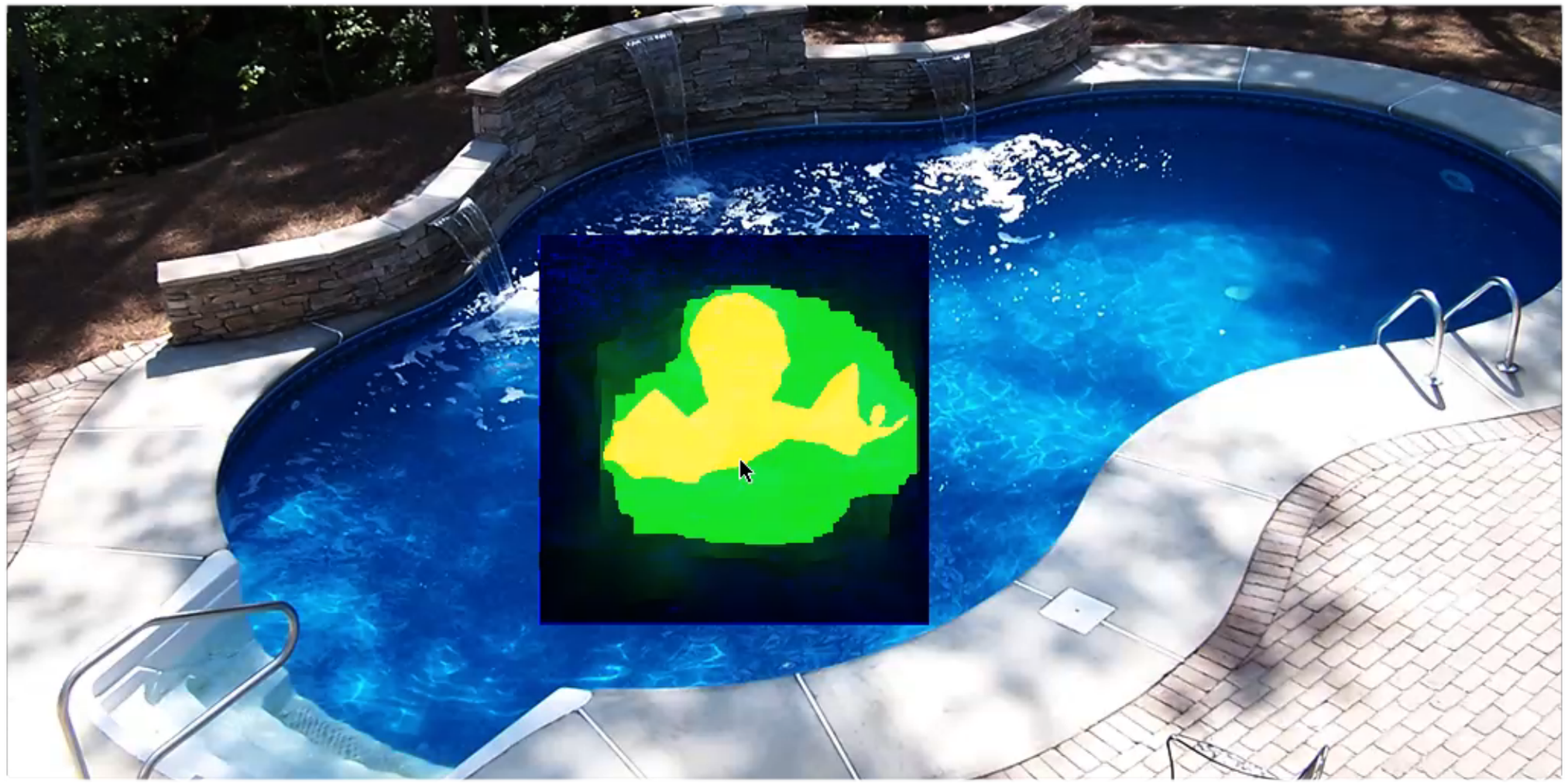
More Graph Cuts...



More Graph Cuts...



Graph Cuts Finished!



Paste in the Foreground

More Poisson Blending

More Poisson Blending

More Poisson Blending

...it's never really done.

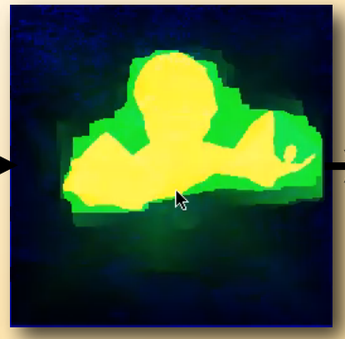
Overview of P3

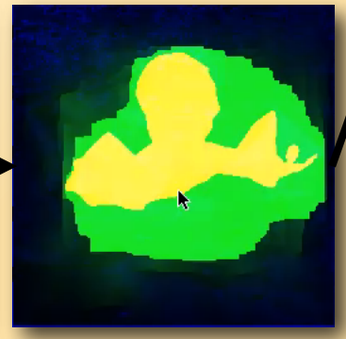
Graph Cuts

Background

Foreground

Minimum Mask





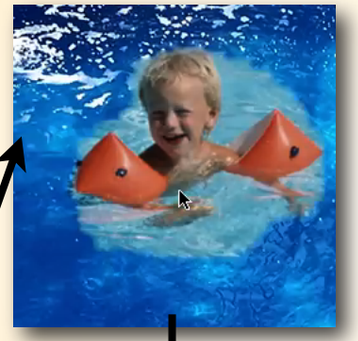
Iteration 0

More Iterations

Optimal Mask

Poisson Blending

Pasted Pixels



Iteration 0

More Iterations

More Iterations

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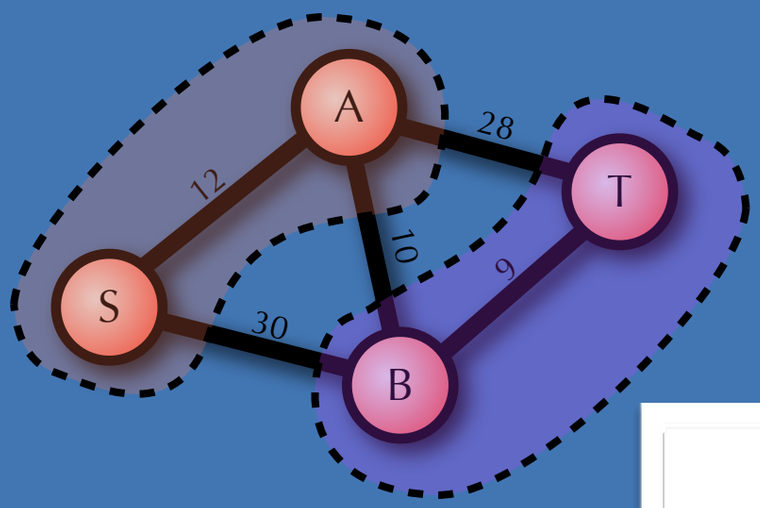

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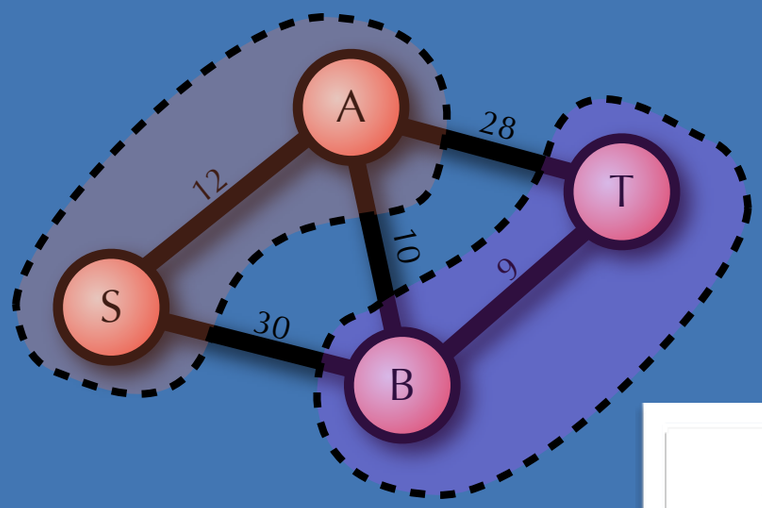

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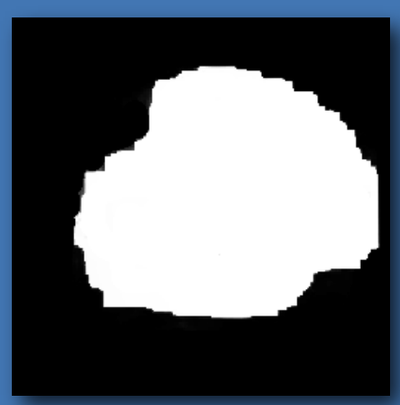
The Question

Background

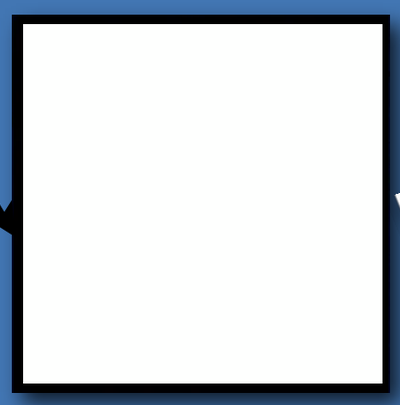
Foreground

Minimal Mask



Optimal Mask

Optimal Combination

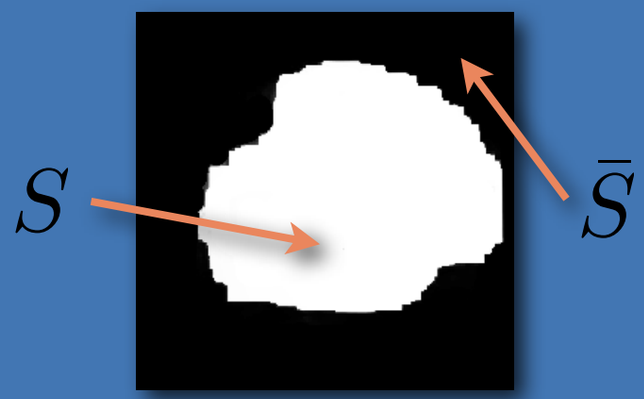


Maximal Mask

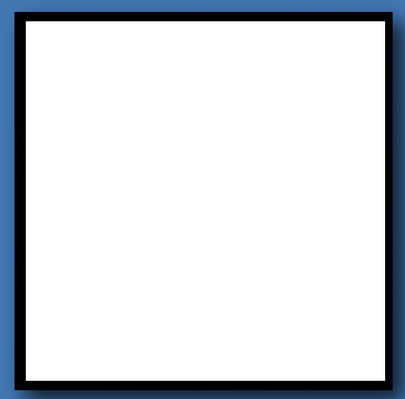
How can we pick the optimal mask?

Mask Objective

Minimal Mask



Optimal Mask



Maximal Mask

$$S_{MIN} \subseteq S_{OPT} \subseteq S_{MAX}$$

$$\phi(S) = \begin{cases} \\ \\ \end{cases}$$

Intermediate Masks

Let's just look at pixels adjacent to the "cut."

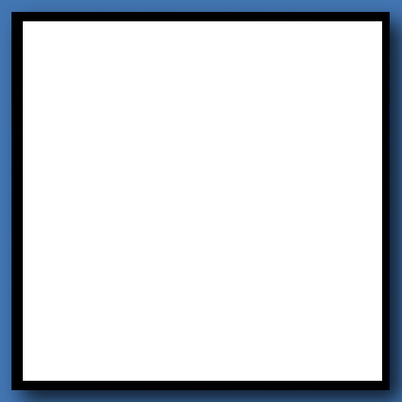
- **Low values** mean that it's a **good** place to cut.
- **High values** mean that it's a **bad** place to cut.

Where do we put good and bad values?

Mask Objective

Minimal Mask

Optimal Mask



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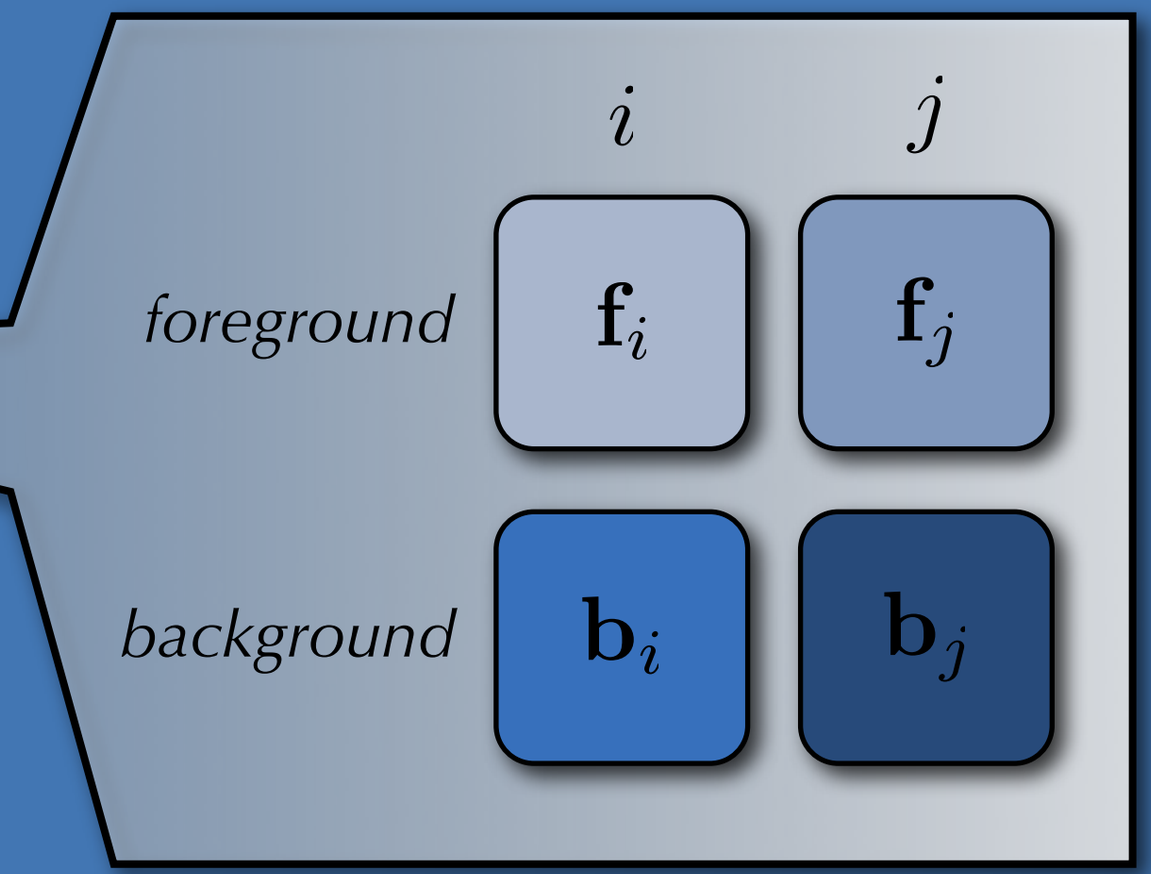
Pairwise Objective



Color Disparity

$$w_{ij} = |\mathbf{f}_i - \mathbf{b}_i| + |\mathbf{f}_j - \mathbf{b}_j|$$

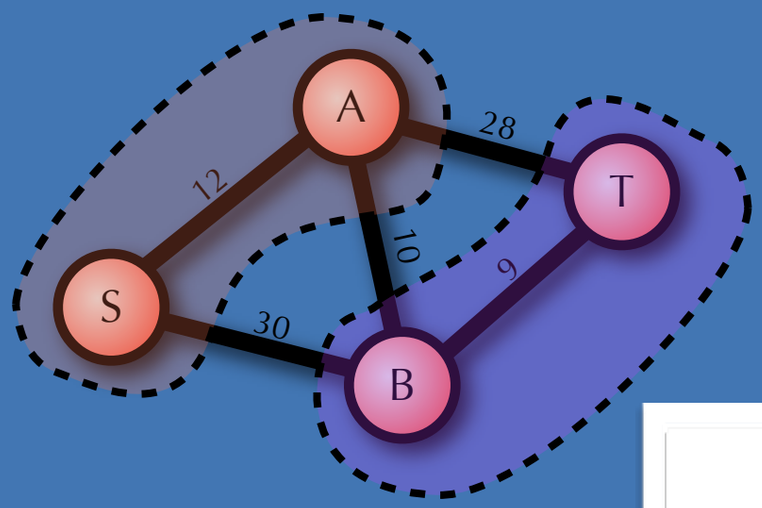
Gradient Disparity

$$w_{ij} = |(\mathbf{f}_i - \mathbf{f}_j) - (\mathbf{b}_i - \mathbf{b}_j)|$$

What others might there be?

Image Saliency

Overview



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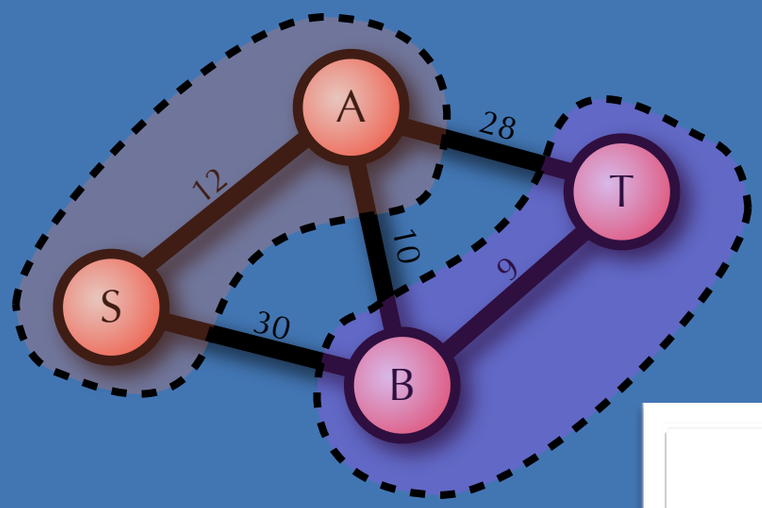

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Image Graph Cuts

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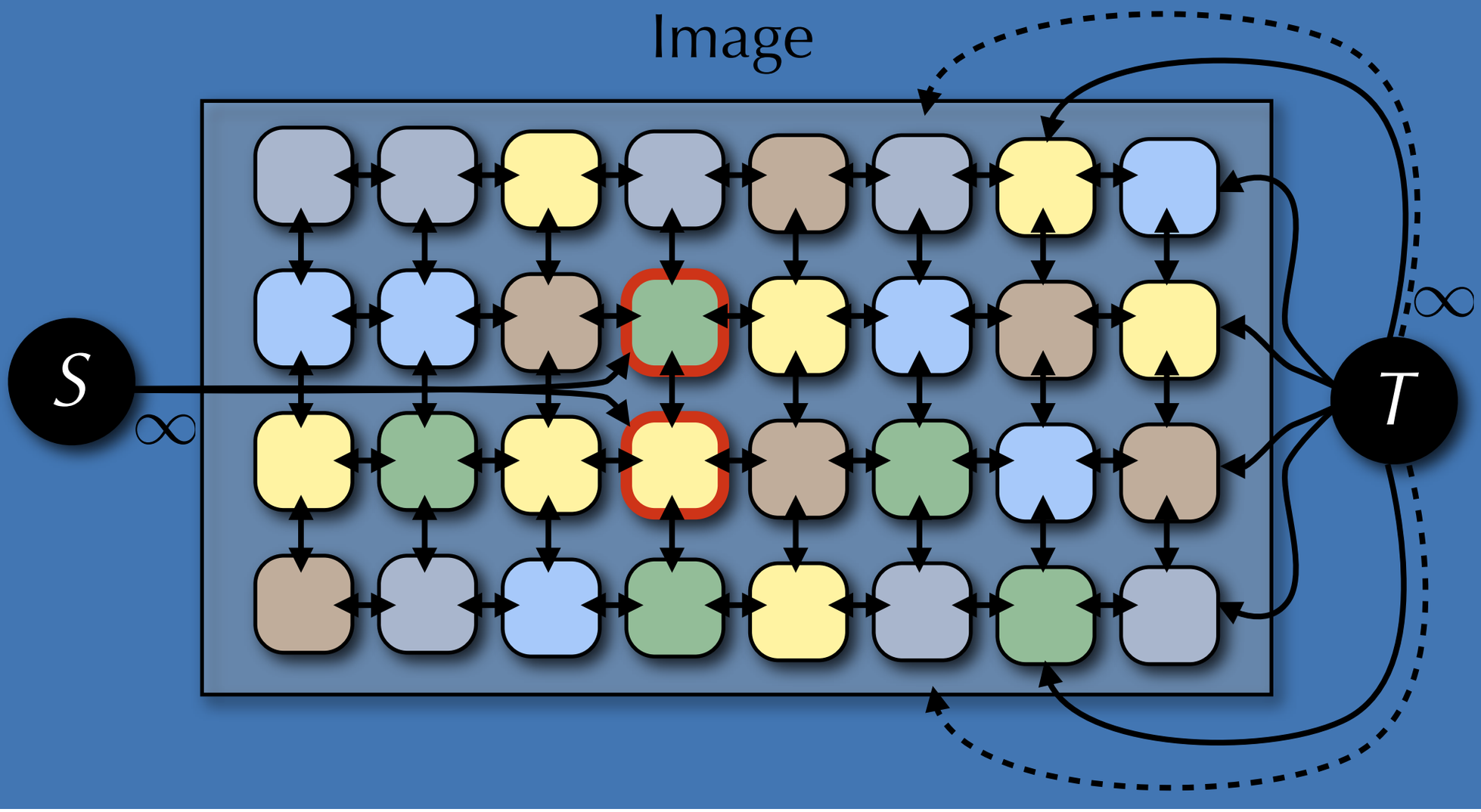

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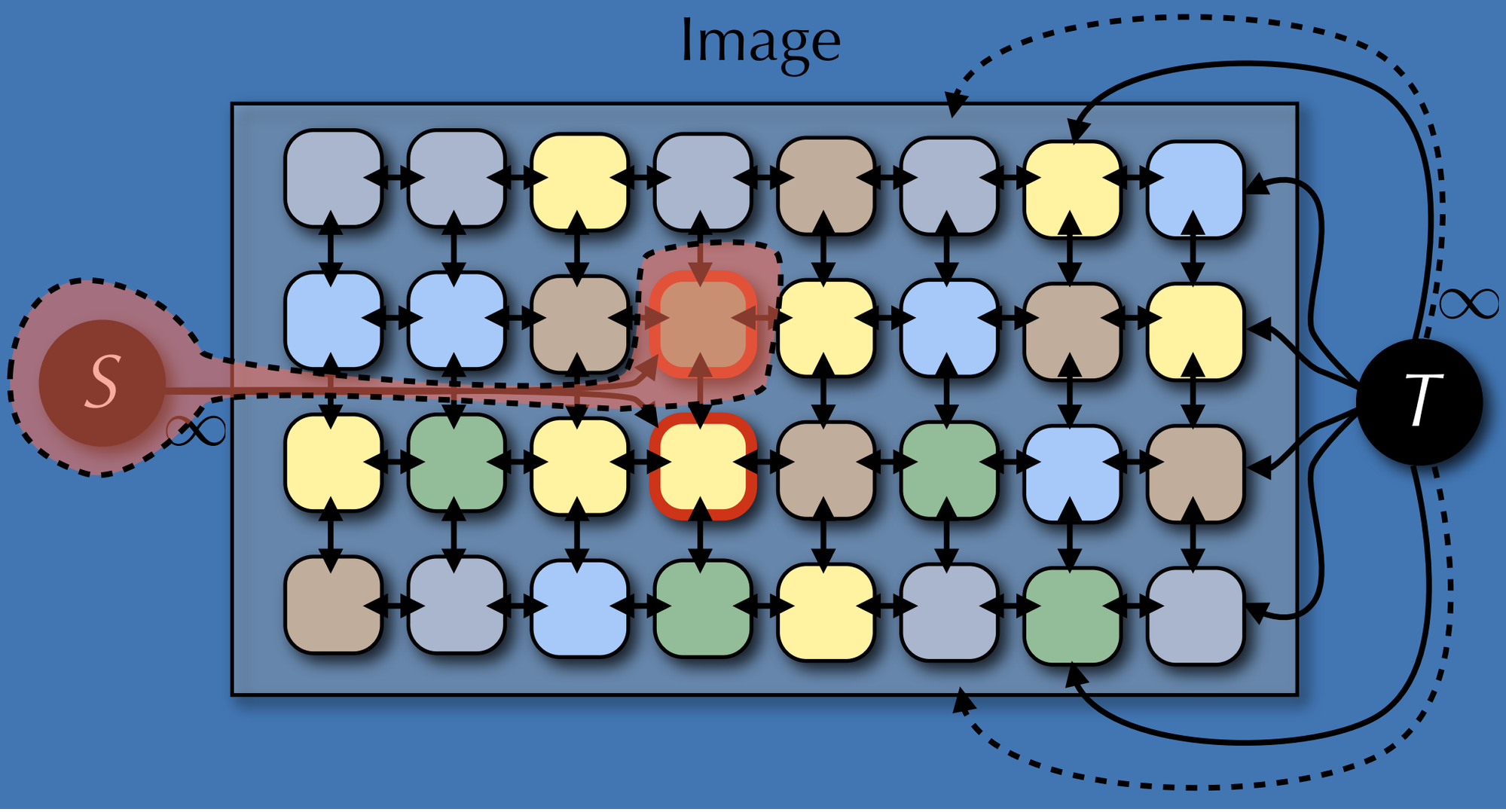

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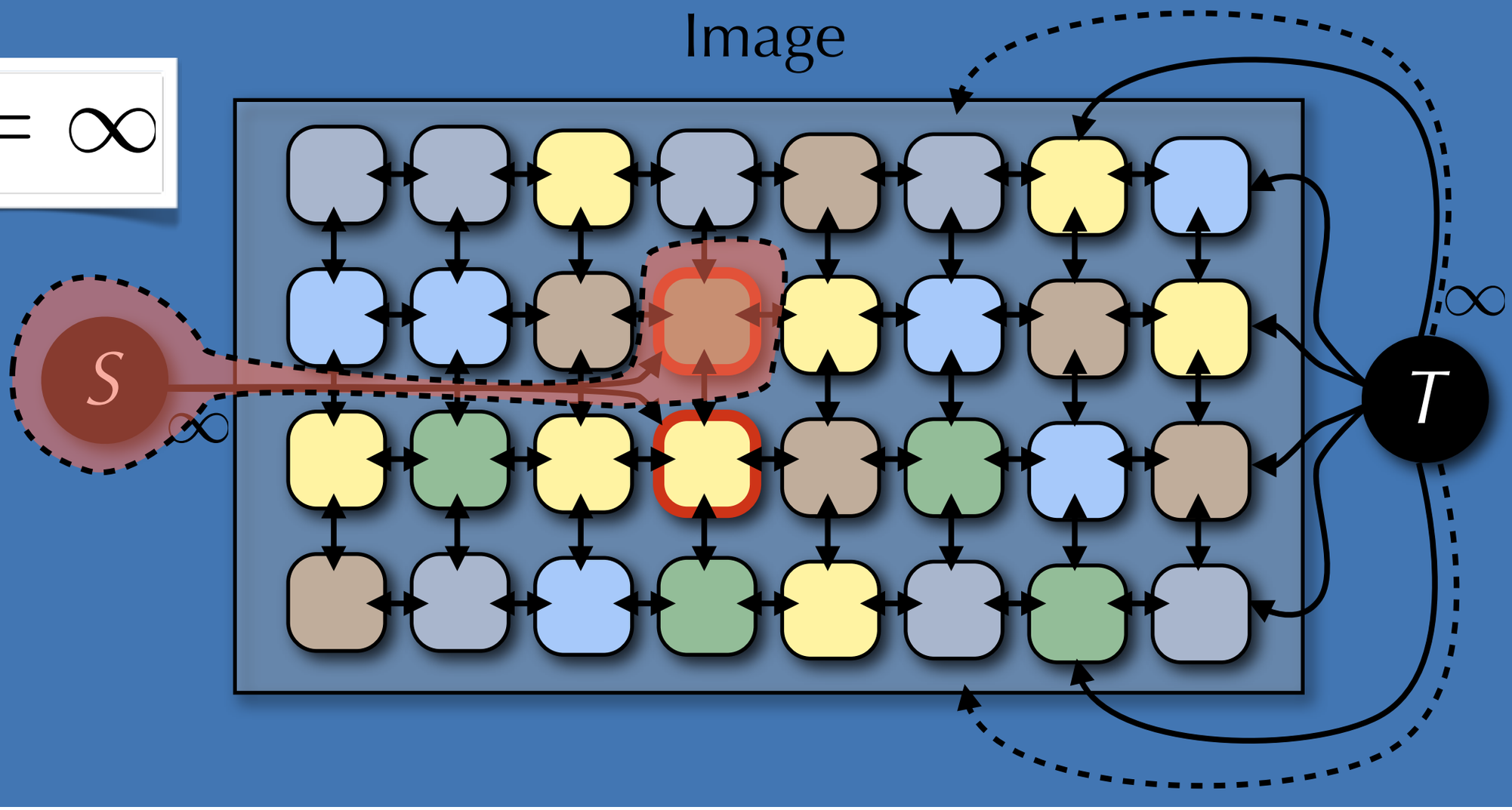
$\phi = \infty$



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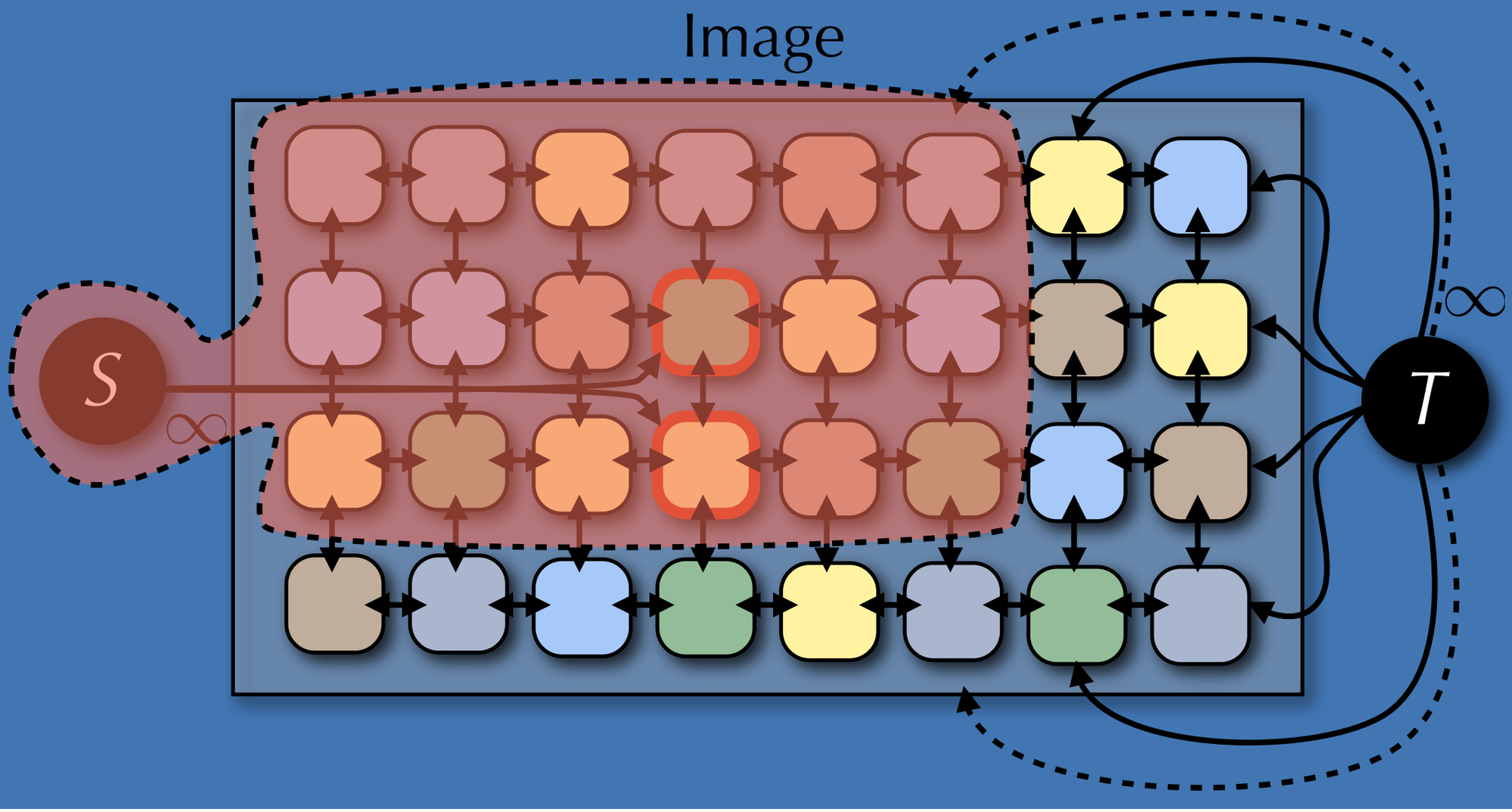

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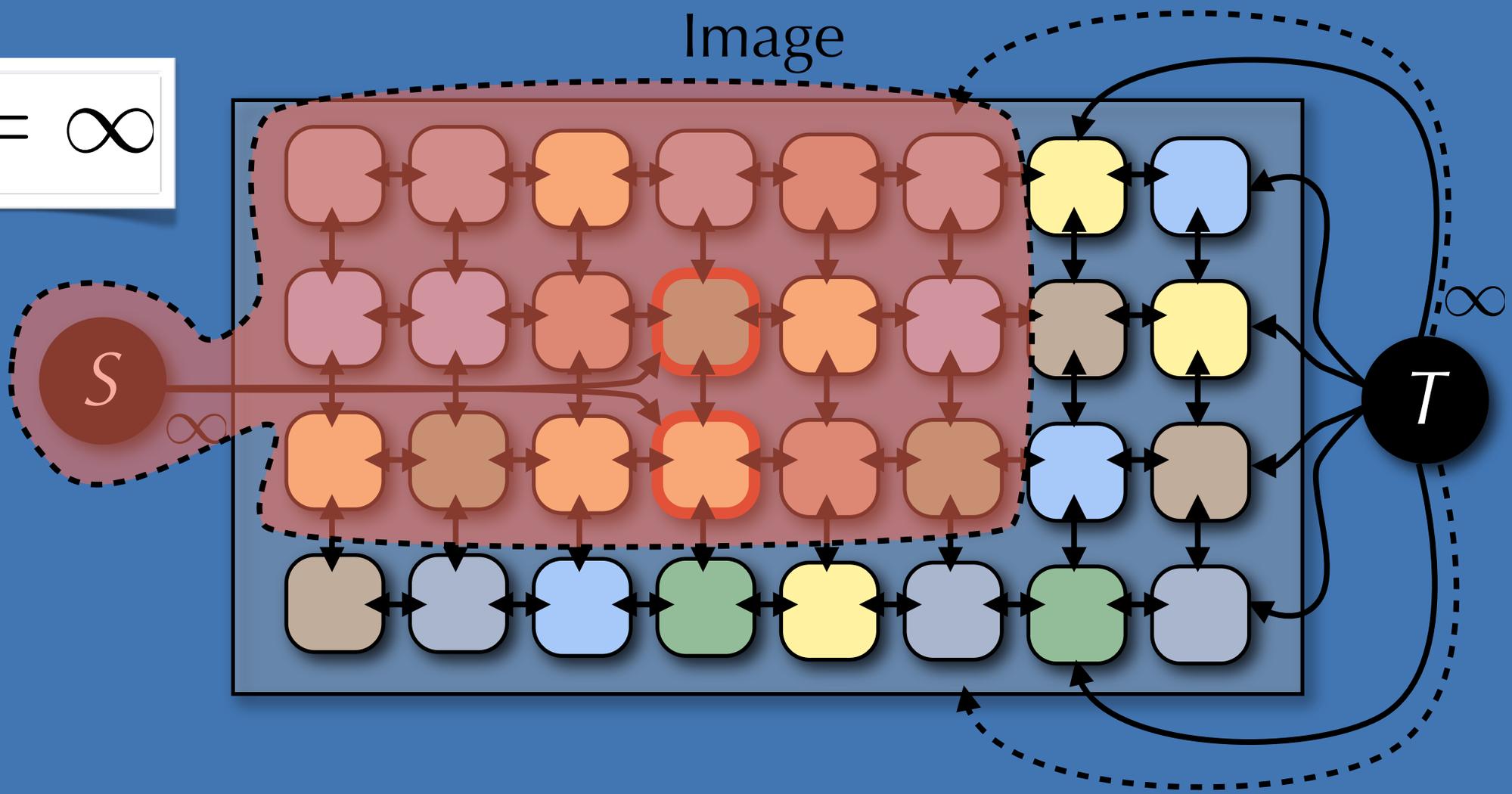

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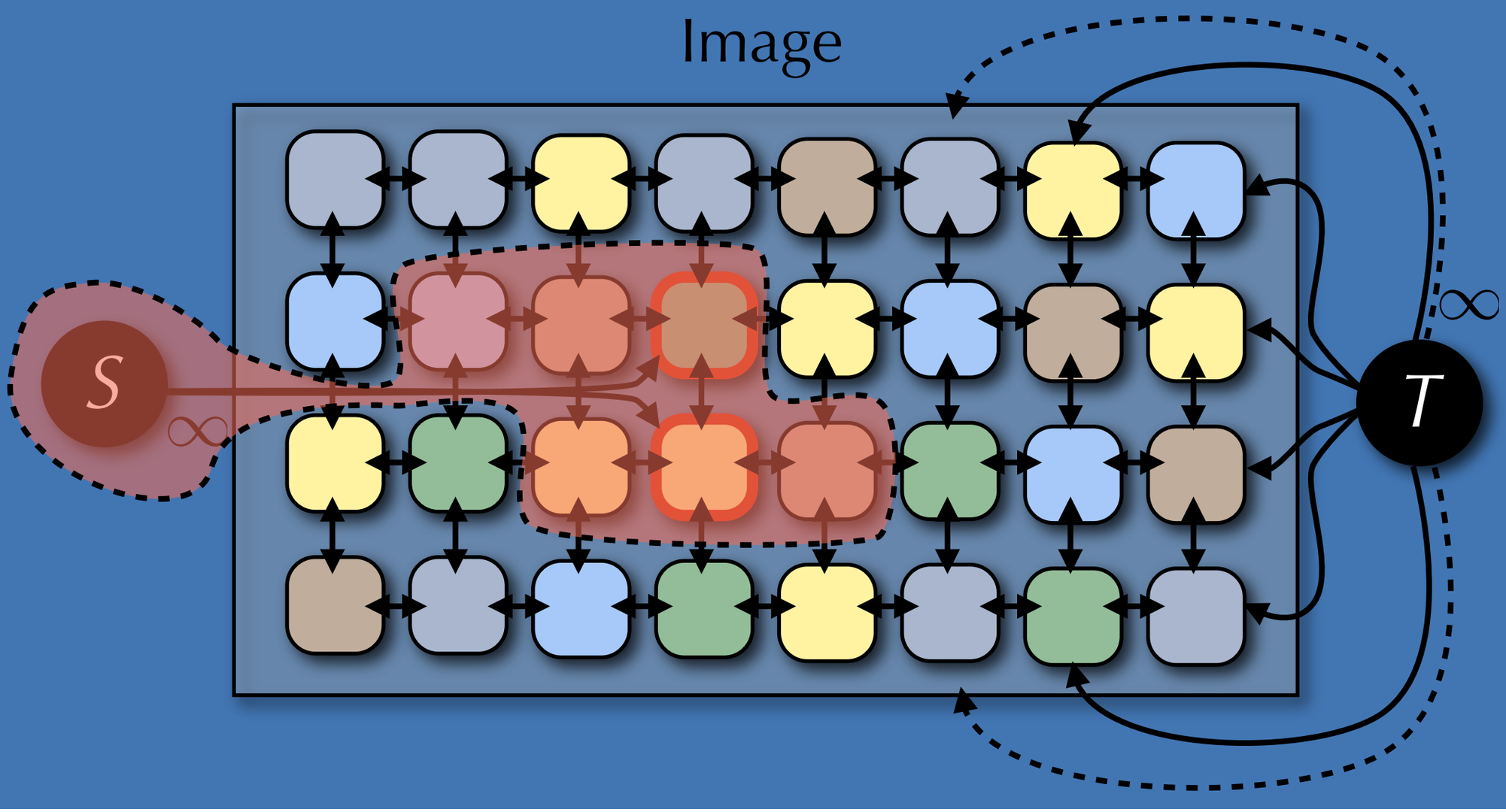

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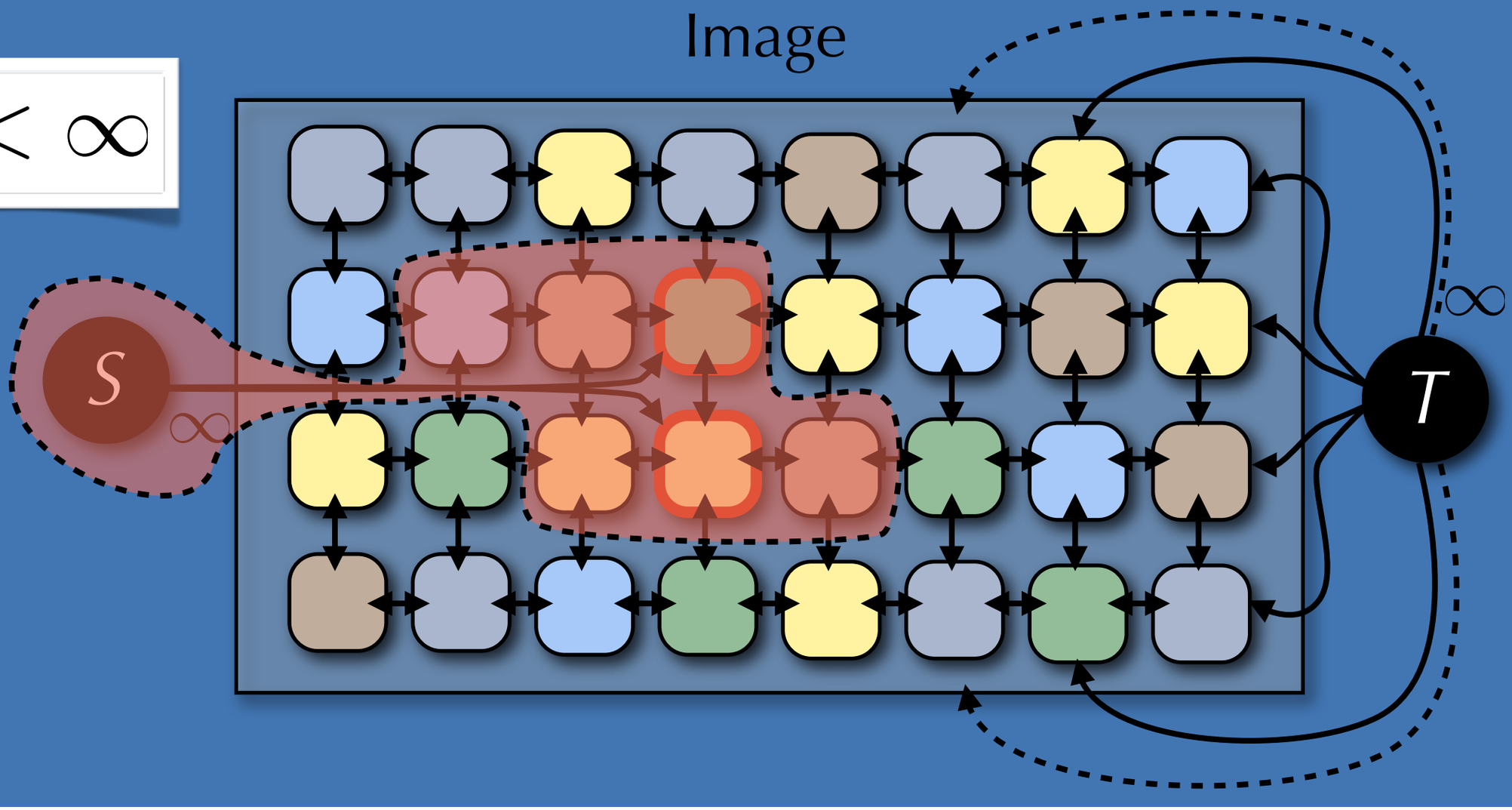
$\phi < \infty$



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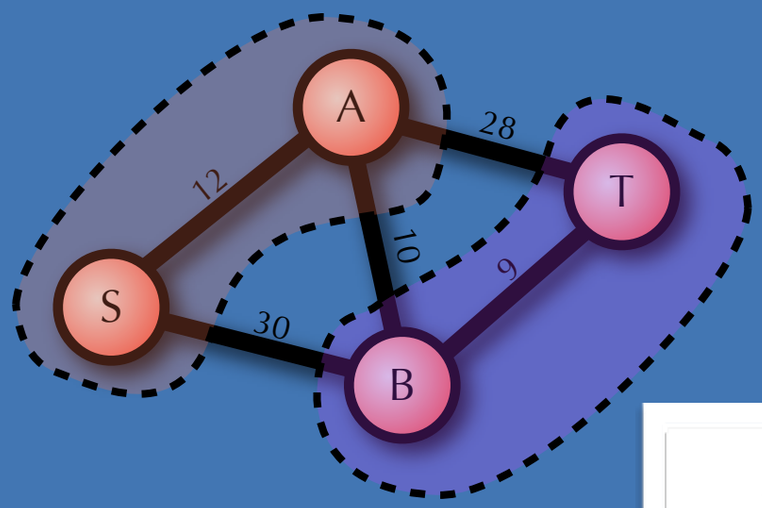

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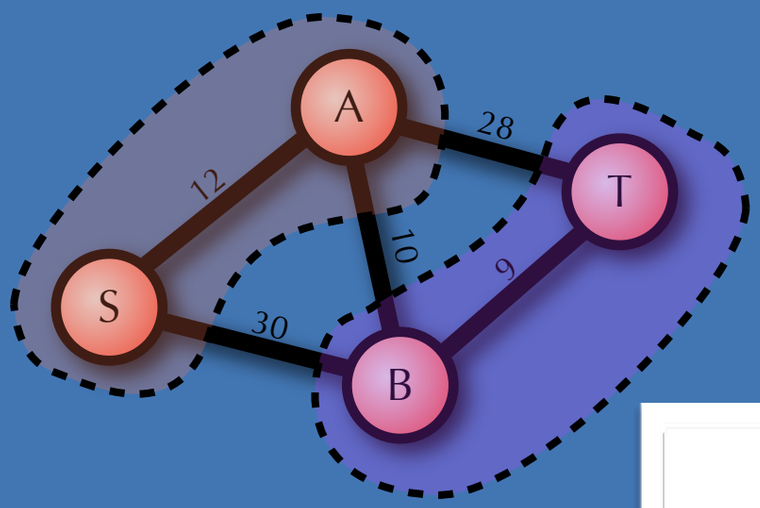

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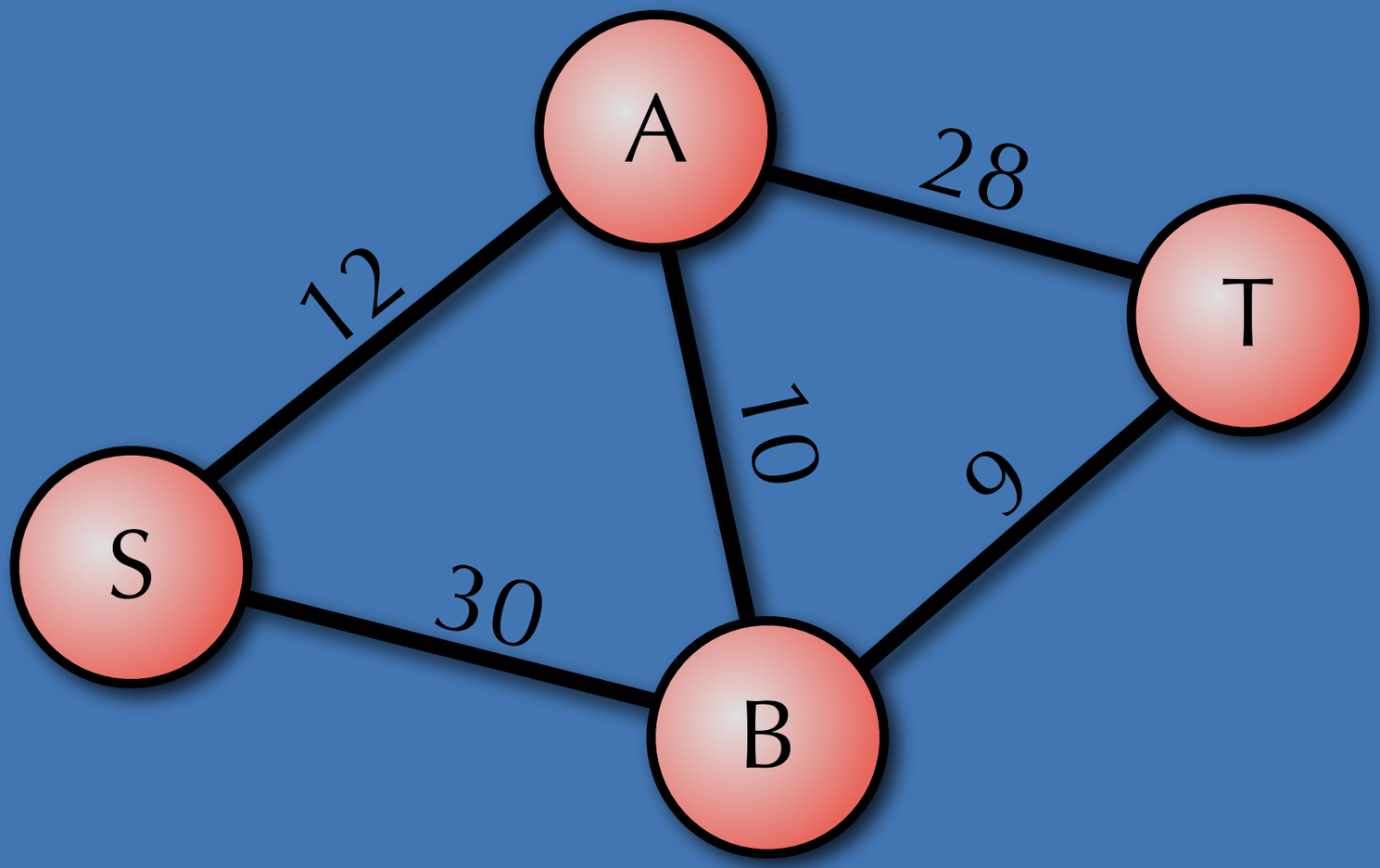
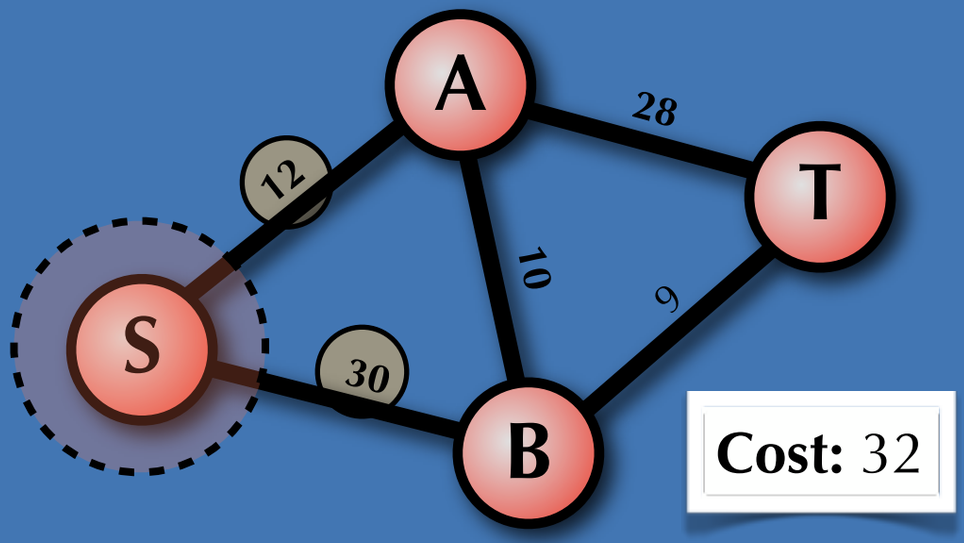


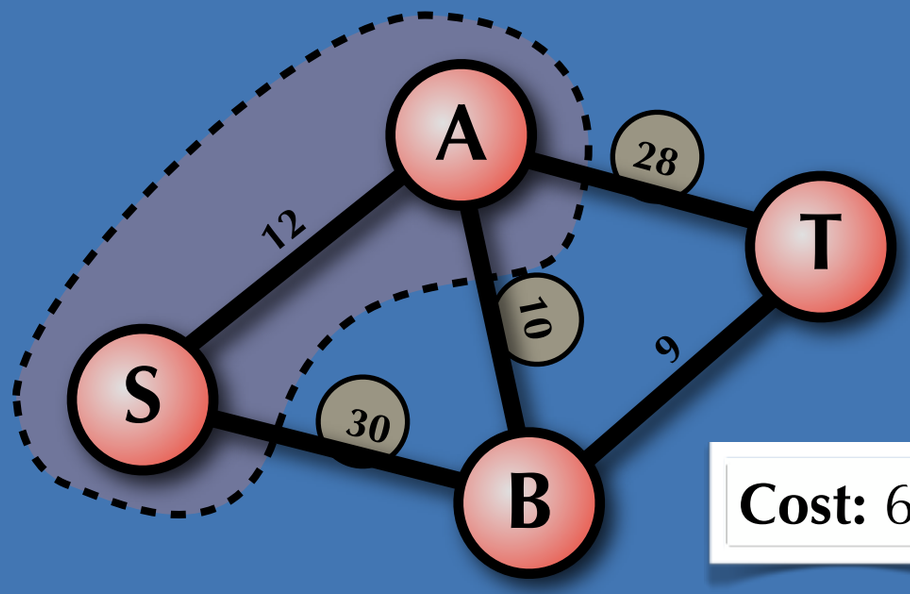
Properties

- Graph with **positive** edge weights.
- Special S (source) and T (sink)

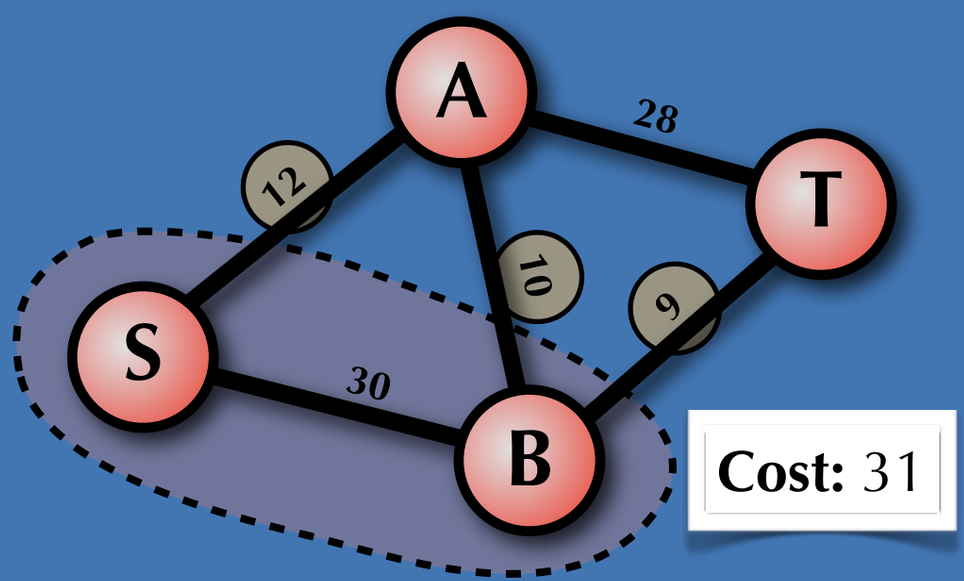
Graph Cuts



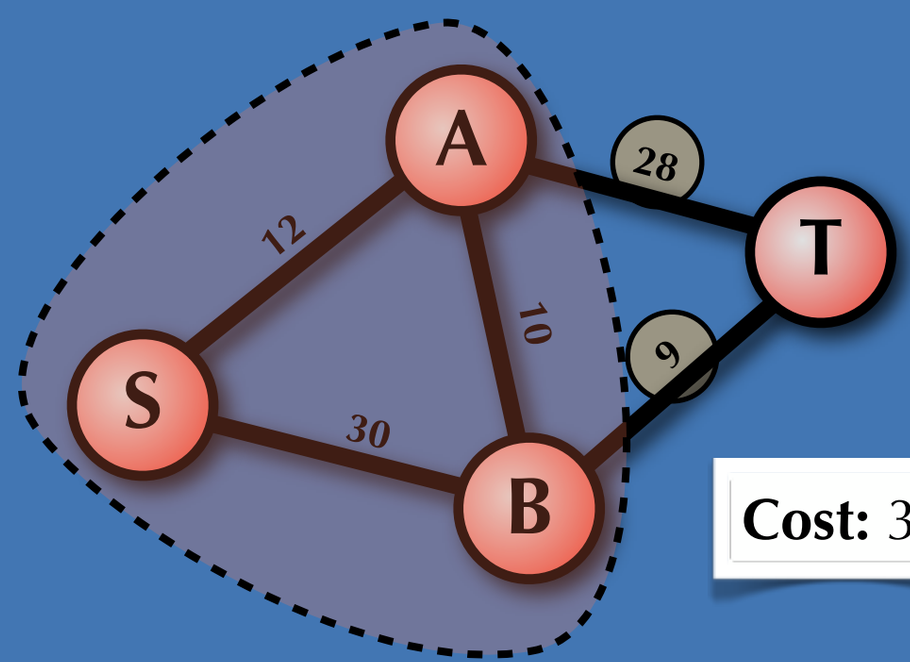
Cost: 32



Cost: 68

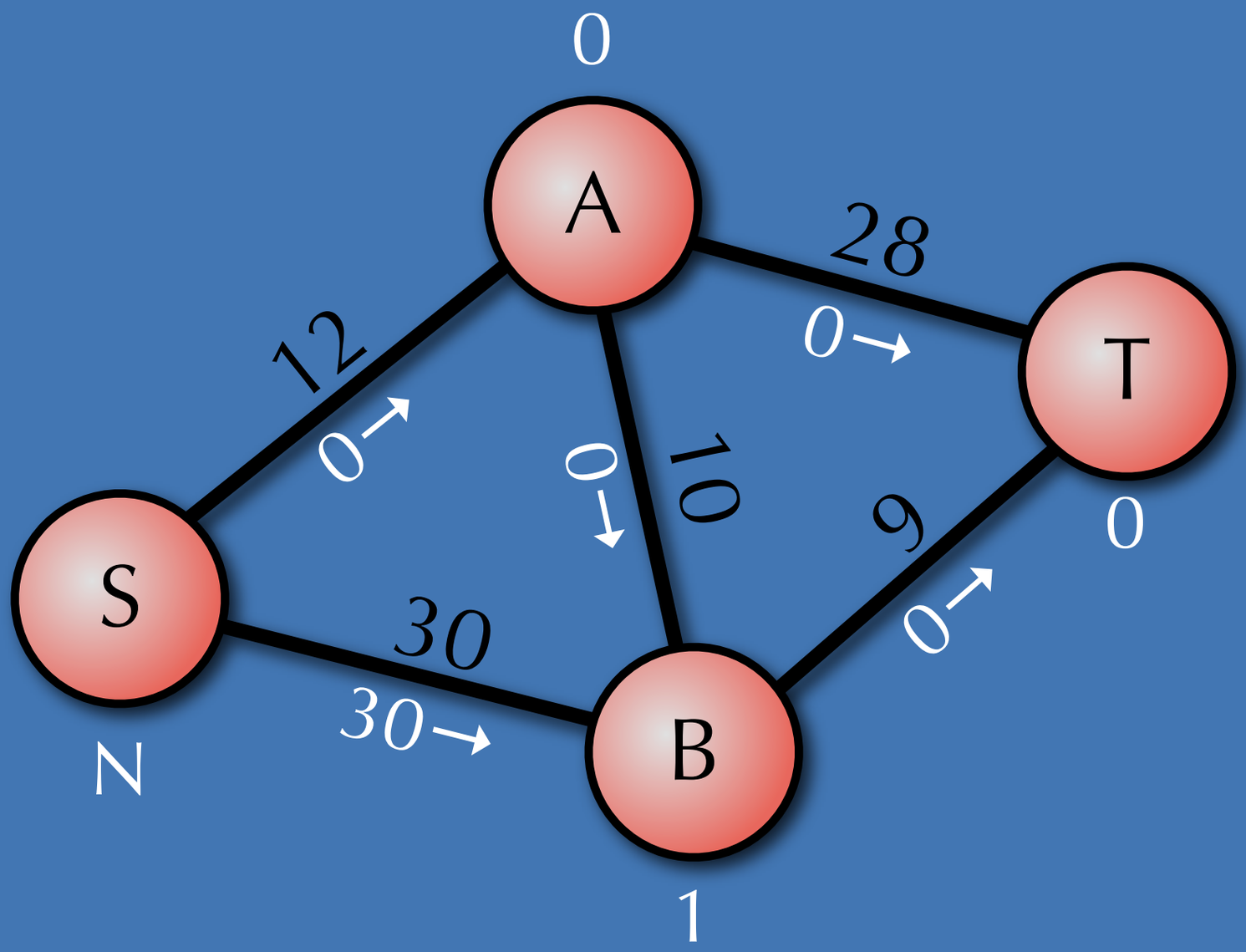


Cost: 31



Cost: 37

Goldberg-Tarjan



Invariants

Valid Preflow

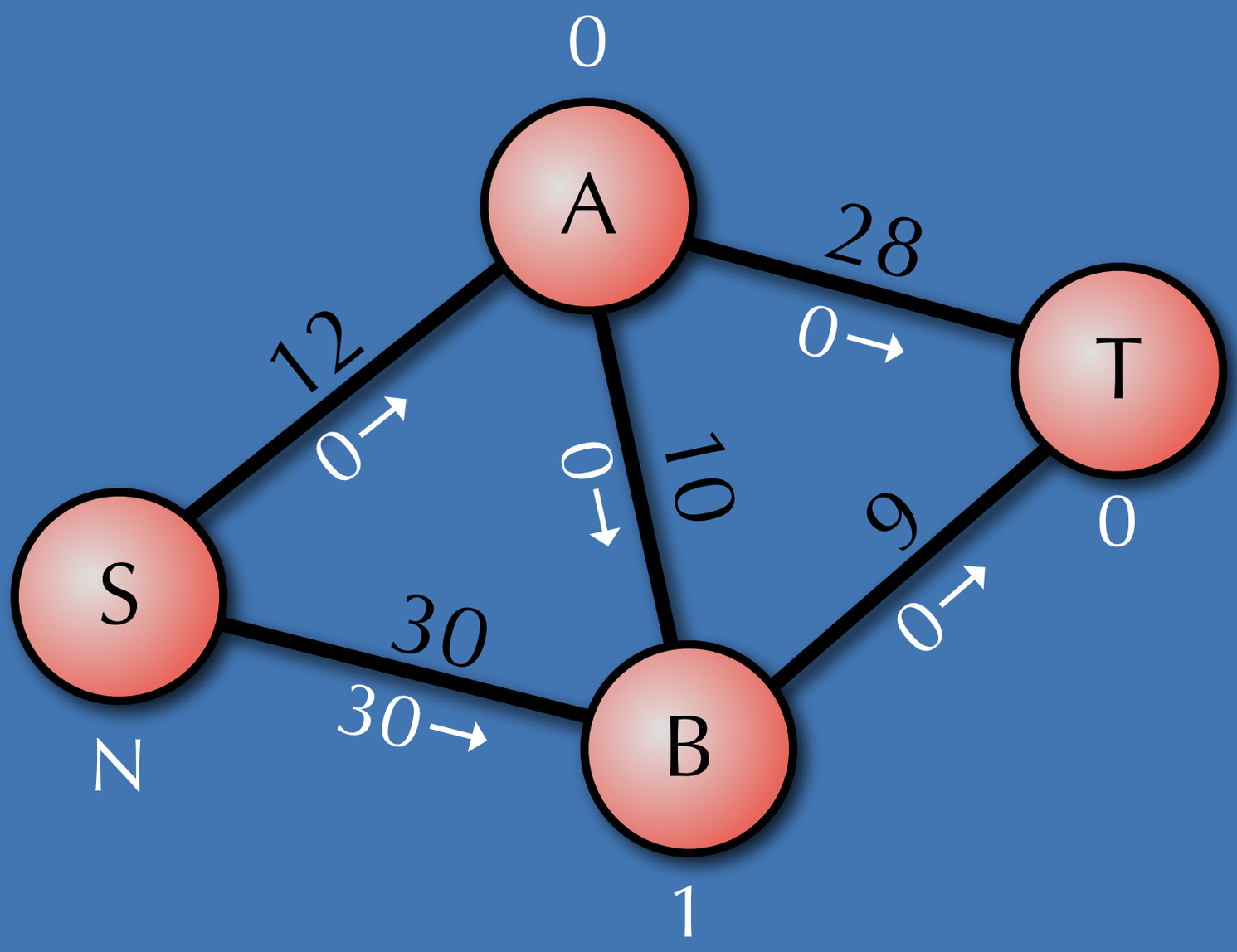
- **For each edge**: flow does not exceed capacity.
- **For each vertex**: outgoing flow does not exceed incoming flow.

Valid Labeling

- **Along each *residual* edge**: Label does not increase by more than 1.
- **The *source* (S) vertex:** has label N.
- **The *sink* (T) vertex:** has label 0.

Goldberg-Tarjan

Operations



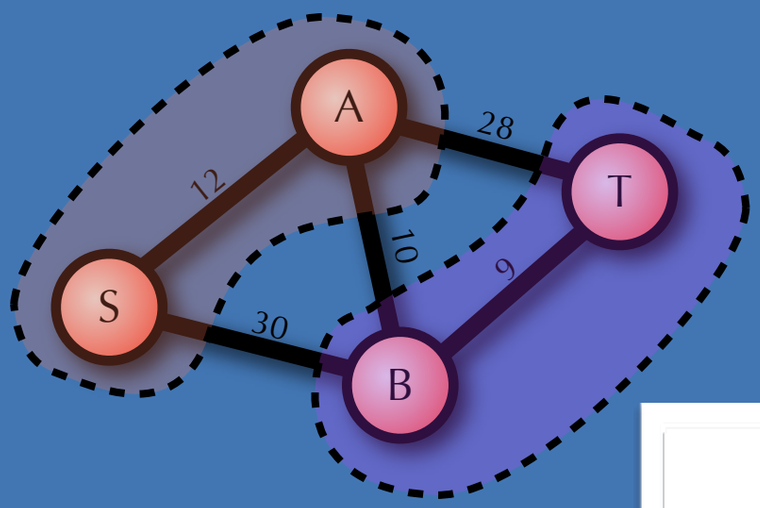
Push

- **Send maximum possible flow across edge** if the *from* vertex has a higher label than the *to* vertex.

Relabel

- **Increase a vertex's label** to the *lowest* possible value that is 1 greater than it's neighbors over edges with excess capacity.

Overview



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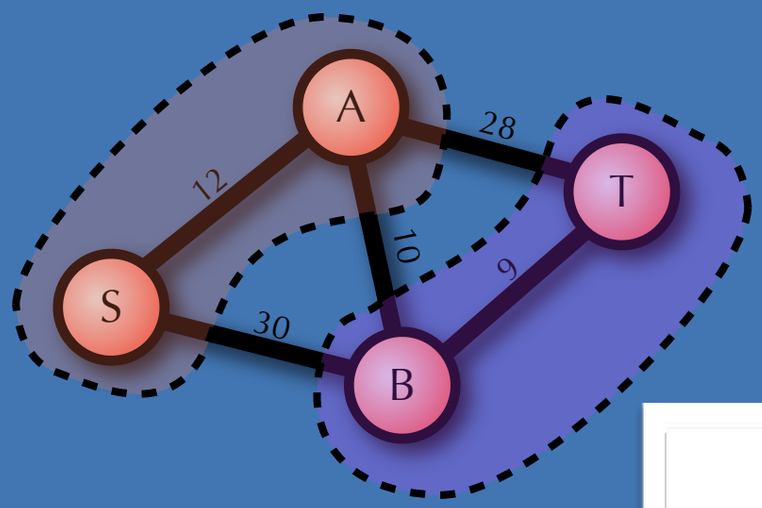

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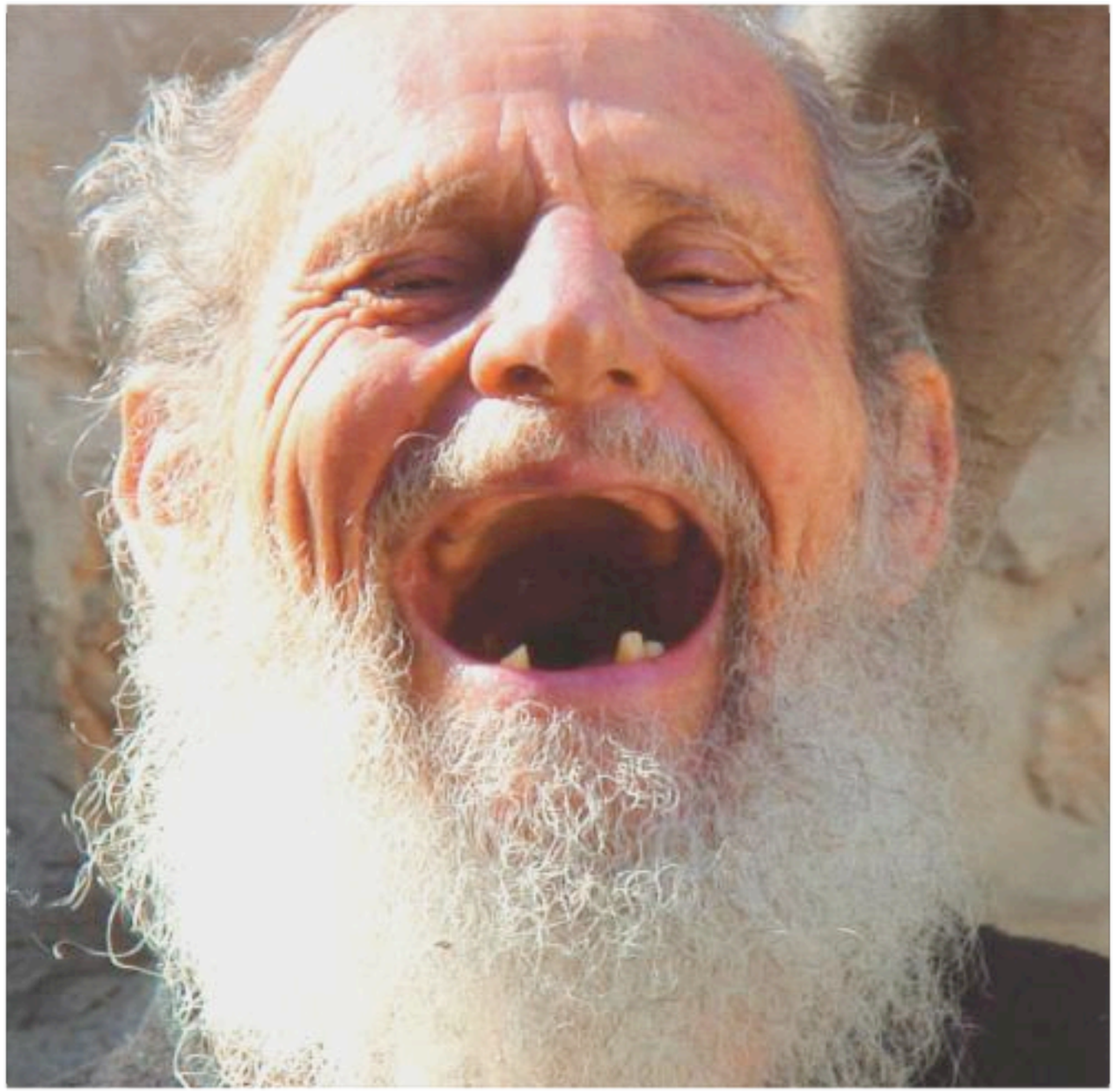
No Project 3



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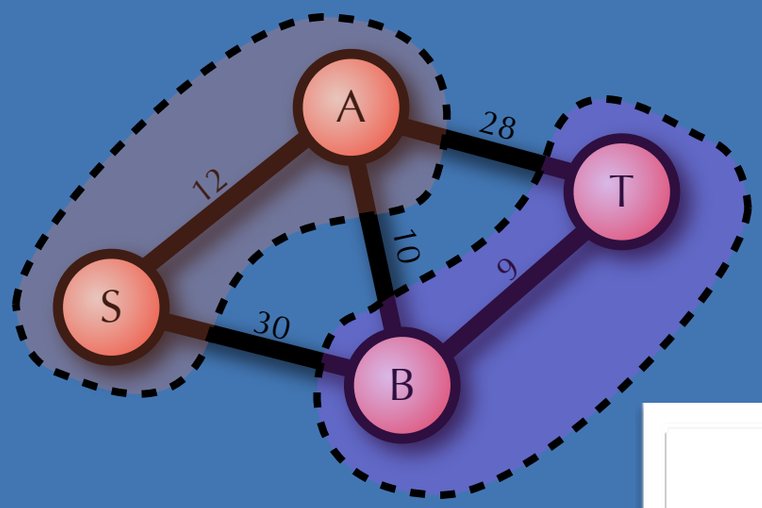

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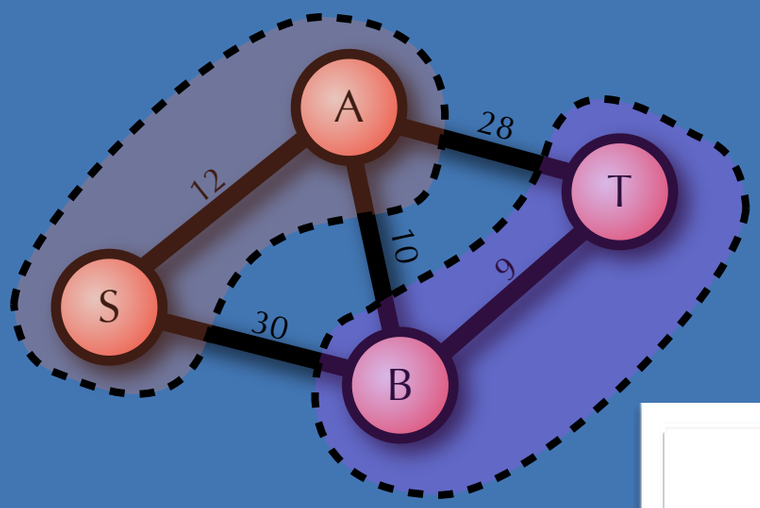

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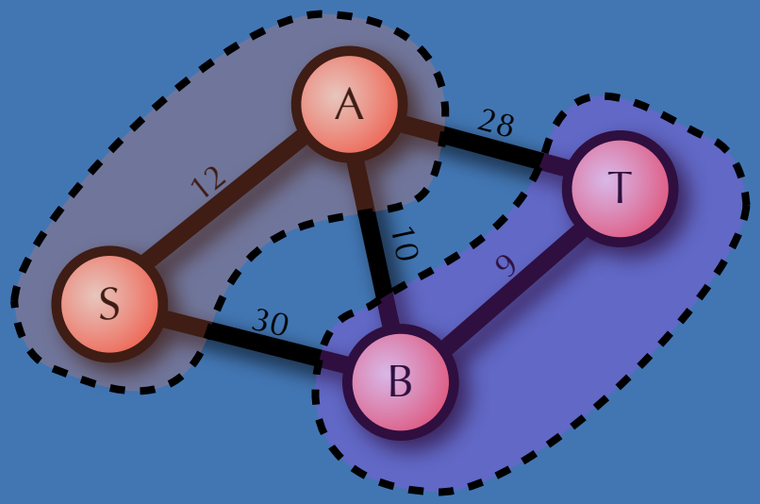

Graph Cuts

...and Images

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