Graph Cuts

...and Images

Adrien Treuille
Carnegie Mellon University
Overview

• Overview of P3
• Optimal Masks
• Optimal Masks as Graph Cuts
• Parallel Solution
• An announcement!
Overview

- Overview of P3
  - Optimal Masks
  - Optimal Masks as Graph Cuts
  - Parallel Solution
  - An announcement!

Wednesday, February 23, 2011
Start Here...
Move the Foreground
The “Minimum” Mask
Graph Cuts Starting
More Graph Cuts...
More Graph Cuts...
More Graph Cuts...
More Graph Cuts...
Graph Cuts Finished!
Paste in the Foreground
More Poisson Blending
More Poisson Blending
More Poisson Blending

…it’s never really done.
Overview of P3

**Graph Cuts**
- Background
- Foreground
- Minimum Mask
  - Iteration 0
  - More Iterations
  - Optimal Mask

**Poisson Blending**
- Pasted Pixels
  - Iteration 0
  - More Iterations
  - More Iterations

Wednesday, February 23, 2011
Overview

- Overview of P3
  - Optimal Masks
  - Optimal Masks as Graph Cuts
  - Parallel Solution
  - An announcement!
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution
- An announcement!
The Question

How can we pick the optimal mask?
Mask Objective

\[ S_{MIN} \subseteq S_{OPT} \subseteq S_{MAX} \]

\[ \phi(S) = \{ \} \]
Intermediate Masks

Let’s just look at pixels adjacent to the “cut.”

- **Low values** mean that it’s a **good** place to cut.
- **High values** mean that it’s a **bad** place to cut.

Where do we put good and bad values?

Wednesday, February 23, 2011
Mask Objective

\[ S_{MIN} \subseteq S_{OPT} \subseteq S_{MAX} \]

\[ \phi(S') = \begin{cases} 
\infty & \text{if } S_{MIN} \nsubseteq S \\
\infty & \text{if } S \nsubseteq S_{MAX} \\
\sum w_{ij} \forall (i \in S') \sim (j \notin S') & \text{otherwise}
\end{cases} \]
Pairwise Objective

Color Disparity

\[ w_{ij} = |f_i - b_i| + |f_j - b_j| \]

Gradient Disparity

\[ w_{ij} = |(f_i - f_j) - (b_i - b_j)| \]

What others might there be?
Image Saliency
Overview

• Overview of P3

• Optimal Masks

• Optimal Masks as Graph Cuts

• Parallel Solution

• An announcement!

Wednesday, February 23, 2011
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution
- An announcement!
Image Graph Cuts

\[ \phi(S) = \begin{cases} 
\infty & \text{if } S_{\text{MIN}} \nsubseteq S \\
\infty & \text{if } S \nsubseteq S_{\text{OPT}} \\
\sum w_{ij} \forall (i \in S) \sim (j \notin S) & \text{if } S_{\text{OPT}} \subseteq S \subseteq S_{\text{MAX}} \\
\infty & \text{otherwise}
\end{cases} \]
Image Graph Cuts

\[ \phi(S) = \begin{cases} 
\infty & \text{if } S_{\text{MIN}} \not\subset S \\
\infty & \text{if } S \not\subset S_{\text{MAX}} \\
\sum w_{ij} \forall (i \in S) \sim (j \notin S) & \text{otherwise}
\end{cases} \]
Image Graph Cuts

\[ \phi(S) = \begin{cases} 
\infty & \text{if } S_{\text{MIN}} \not\subseteq S \\
\infty & \text{if } S \not\subseteq S_{\text{MAX}} \\
\sum w_{ij} & \forall (i \in S) \sim (j \notin S) 
\end{cases} \]

\[ S_{\text{MIN}} \subseteq S_{\text{OPT}} \subseteq S_{\text{MAX}} \]
Image Graph Cuts

\[ \phi(S) = \begin{cases} 
\infty & \text{if } S_{\text{MIN}} \not\subseteq S \\
\infty & \text{if } S \not\subseteq S_{\text{MAX}} \\
\sum w_{ij} \forall (i \in S) \sim (j \notin S) & \text{otherwise}
\end{cases} \]
Image Graph Cuts

\[ \phi(S) = \begin{cases} 
\infty & \text{if } S_{\text{MIN}} \not\subseteq S \\
\infty & \text{if } S \not\subseteq S_{\text{MAX}} \\
\sum w_{ij} \quad \forall (i \in S) \sim (j \notin S) & \text{otherwise}
\end{cases} \]
Image Graph Cuts

\[
\phi(S) = \begin{cases} 
\infty & \text{if } S \notin S_{MIN} \\
\infty & \text{if } S \notin S_{OPT} \\
\sum w_{ij} \forall (i \in S) \sim (j \notin S) & \text{otherwise} 
\end{cases}
\]
$\phi(S) = \begin{cases} 
\infty & \text{if } S_{\text{MIN}} \notin S \\
\infty & \text{if } S \notin S_{\text{MAX}} \\
\sum w_{ij} & \forall (i \in S) \sim (j \notin S) 
\end{cases}$

$S_{\text{MIN}} \subseteq S_{\text{OPT}} \subseteq S_{\text{MAX}}$

$\phi < \infty$
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution
- An announcement!
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution
- An announcement!
Graph Cuts

Properties

- Graph with positive edge weights.
- Special S (source) and T (sink)
Invariants

**Valid Preflow**
- **For each edge**: flow does not exceed capacity.
- **For each vertex**: outgoing flow does not exceed incoming flow.

**Valid Labeling**
- **Along each residual edge**: Label does not increase by more than 1.
- **The source (S) vertex**: has label N.
- **The sink (T) vertex**: has label 0.
Goldberg-Tarjan

Operations

- **Push**
  - Send maximum possible flow across edge if the *from* vertex has a higher label than the *to* vertex.

- **Relabel**
  - Increase a vertex’s label to the *lowest* possible value that is 1 greater than its neighbors over edges with excess capacity.
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution
- An announcement!
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution
- An announcement!
No Project 3
Overview

- Overview of P3
- Optimal Masks
- Optimal Masks as Graph Cuts
- Parallel Solution

- An announcement!
Overview

• Overview of P3
• Optimal Masks
• Optimal Masks as Graph Cuts
• Parallel Solution
• An announcement!
Graph Cuts

...and Images

Adrien Treuille
Carnegie Mellon University

Wednesday, February 23, 2011