Path Planning

15-494 Cognitive Robotics
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Outline

- Path planning as state space search
- RRTs: Rapidly-exploring Random Trees
- The RRT-Connect algorithm
- Collision detection
- Smoothing
- Path planning with constraints
- Path planning in Tekkotsu
Path Planning in Robotics

1. Navigation path planning
   - How to get from the robot's current location to a goal.
   - Avoid obstacles.
   - Provide for localization.

2. Manipulation path planning
   - Move an arm to grasp and manipulate an object.
   - Avoid obstacles.
   - Obey constraints (e.g., don't spill the coffee).
Navigation Planning

- **2D state space**: $(x,y)$ coordinates of the robot
  - Treat the robot as a point or a circle.

- **3D state space**: $(x,y,\theta)$ pose of the robot
  - Heading matters when the robot is asymmetric
  - Heading matters when the robot's motion is constrained
Cspace Transform

- The area around an obstacle that would cause a collision with the robot.

*Figure 4.4 - Mason, Mechanics Of Robotic Manipulation*
Arm Path Planning

- Cspace transform blocks out regions of joint space

*Figure 4.5 - Mason, Mechanics Of Robotic Manipulation*
State Space Search

The path planning problem:
Given an n-dimensional state space and

- a start state $S=[s_1,s_2,\ldots,s_n]$
- a goal state $G=[g_1,g_2,\ldots,g_n]$
- an admissibility predicate $P$ (collision test + constraints)

find a path from $S$ to $G$ such that every state on the path satisfies $P$. 
Best First Search

- Can get trapped in a cul de sac for a long time.
- Random search might be faster.
Rapidly-exploring Random Trees

• Described in LaValle (1998), Kuffner & LaValle (2000)

• Create a tree with start state S as the root.

• Repeat up to K times:
  - Pick a point $q$ in configuration space:
    • Sometimes $q$ should be a random point
    • Sometimes $q$ should be the goal state $G$
  - Find $n$, the closest tree node to $q$
  - Add a new node $n'$ some distance $\Delta$ toward $q$; make it a child of $n$
  - If $n'$ is close enough to the goal state $G$, return.
RRT Algorithm

• Rapidly samples the state space.
• Cannot get trapped in local minima.
• Works well in high-dimensional spaces.
• Does not generate smooth paths.
• Can't tell when no solution exists; only quits when it exceeds the iteration limit K.

http://msl.cs.uiuc.edu/rrt/treemovie.gif
RRTs for Arm Path Planning

- Each node encodes an arm configuration in joint space.
- Only add nodes that don't cause collisions (with self or obstacles).
- Alternately (i) extend the tree in random directions and (ii) move toward the goal.

Slide courtesy of Glenn Nickens
Implementation Notes

• Finding $n$, the nearest node in the tree to $q$, is the most expensive part of the algorithm.
  – Use K-D trees to efficiently find $n$?
  – In practice, K-D trees are slower unless you have a huge number of nodes (several thousand).

• Why only go a distance $\Delta$ toward the goal state $G$? Why not go as far as we can, in steps of $\Delta$?
  – With no obstacles, this reaches the goal very quickly, but random search will get there nearly as quickly as we keep extending the nearest node to the goal.
  – But when obstacles are present, this can waste time filling out branches that will ultimately fail.
  – Generating lots of extra nodes bloats the tree, which slows down the algorithm.
RRT-Connect Algorithm

- Variant of RRT that grows two trees:
  - one from the start state toward the goal
  - one from the goal state toward the start

- When the two trees connect, a solution has been found.

- Not guaranteed to be better than RRT, but often helps.
RRTs in the VeeTags World
RRT-Connect For Arms

- Use IK to calculate the goal configuration.
- Use FK to calculate arm configurations for collision detection.
Collision Detection

- Represent the robot and the obstacles as **convex polygons**.

- In 2D, use the Separating Axis Theorem to check for collisions.
  - Easy to code
  - Fast to compute

- In 3D, things get more complex.
  - Tekkotsu uses the GJK (Gilbert-Johnson-Keerthi) algorithm, used in many physics engines for video games.
Separating Axis Theorem

"If two convex polygons don't overlap, then there exists a line, parallel to one of their edges that separates them."
Separating Axis Theorem
Algorithm to Apply the SAT

• For every edge of polygon A and of polygon B:
  - Project all the vertices onto the line normal to that edge.
  - Calculate the min and max coordinates for each polygon
  - If \( \min A < \min B \) and \( \max A > \min B \) OR
    if \( \min B < \min A \) and \( \max B > \min A \)
    then the polygons collide.

• If you find any edge projection in which the ranges don't overlap, the polygons do not collide.
Arm Collision Detection

- Represent each link as a separate polygon.
- Check for:
  - Self-collisions other than link $n$ with link $n+1$
  - Collisions of a link with an obstacle
Path Smoothing

- The random component of RRT-Connect search often results in a jerky and meandering solution.
- Solution: apply a path smoothing algorithm.
- Repeat N times:
  - Pick two points on the path at random
  - See if we can linearly interpolate between those points without collisions.
  - If so, then snip out that segment of the path.
Smoothing An Arm Trajectory

- Start state
- Intermed. states
- End state
Path Planning With Constraints

- With no closeable fingers, arm motion is constrained to be within about 60° of finger direction or we'll lose the object.

http://www.youtube.com/watch?v=9oDQ754YVoc
Implementing Constraints

- Each time we generate a new state $n'$:
  - Check to see if $n'$ obeys the constraint
  - For finger motion constraint, check if the direction of motion from parent state $n$ to new state $n'$ is within $60^\circ$ of the finger direction.

- What if $n'$ doesn't obey the constraint?
  - Reject it and generate a new random $q$.
  - Or try to “fix” it by perturbing its value slightly so as to satisfy the constraint.
RRTs in Tekkotsu

- Tekkotsu/Planners/RRT/GenericRRT.h
- Works for any state space

- class RRTNodeBase
  - Subclass this to create a NodeValue_t to describe $q$
  - Define a CollisionChecker class

- class GenericRRT<typename NODE, size_t N>
  - Instantiate this template to create an RRT planner
  - NODE must be a subclass of RRTNodeBase
  - Define an AdmissibilityPredicate class
  - Define the extend(...) method to extend the tree
Planners in Tekkotsu

- Navigation/ShapeSpacePlannerXY
  - 2D navigation planner

- Navigation/ShapeSpacePlannerXYTheta
  - 2D + heading navigation planner

- Manipulation/ShapeSpacePlanner2DR
  - 2D planner for N-joint planar arm with revolute joints

- Manipulation/ShapeSpacePlanner3DR
  - 3D planner for N-joint planar arm with revolute joints
The Grasper

• Does arm path planning
  - Currently only for planar arms
  - 3D arm path planning coming soon

• Does manipulation planning
  - How to grasp an object
  - How to move an object without losing it
  - How to release an object

• Many other manipulation operations are possible.

• Use a GrasperNode to submit a GrasperRequest.