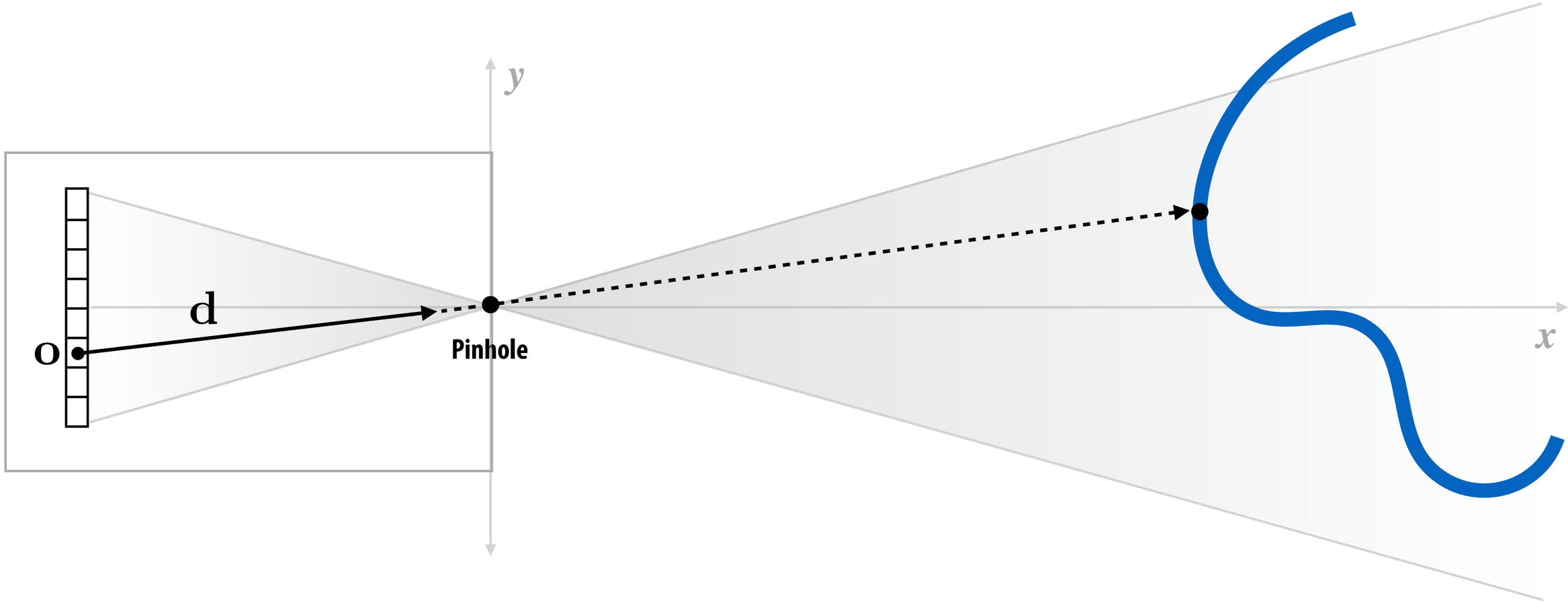
Lecture 17:

Accelerating Geometric Queries

Computer Graphics
CMU 15-462/15-662, Spring 2016

Ray tracer measures radiance along a ray



How do we efficiently detect what a ray hits?

Review: ray-triangle intersection

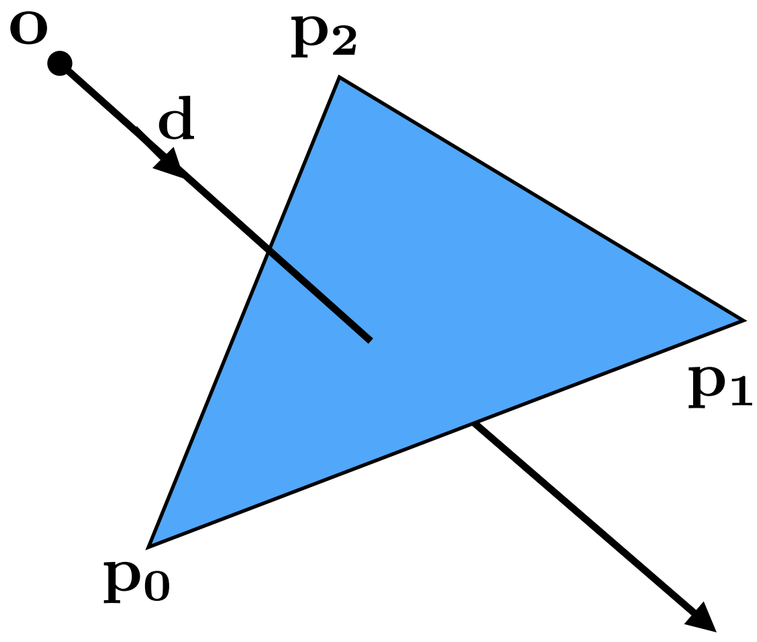
- **Find ray-plane intersection**

Parametric equation of a ray:

$$\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$$

ray origin

normalized ray direction



Plug equation for ray into implicit plane equation:

$$\mathbf{N}^\mathbf{T}\mathbf{x} = c$$

$$\mathbf{N}^\mathbf{T}(\mathbf{o} + t\mathbf{d}) = c$$

Solve for t corresponding to intersection point:

$$t = \frac{c - \mathbf{N}^\mathbf{T}\mathbf{o}}{\mathbf{N}^\mathbf{T}\mathbf{d}}$$

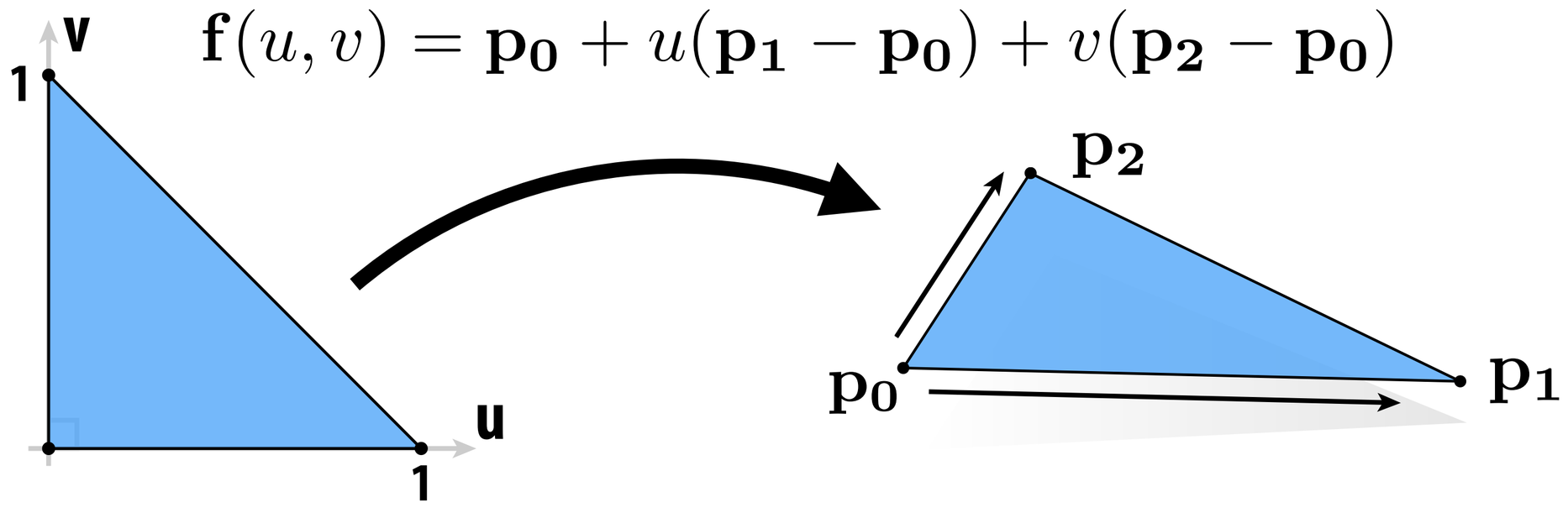
- **Determine if point of intersection is within triangle**

Review: ray-triangle intersection

- **Parameterize triangle given by vertices $\mathbf{p_0}, \mathbf{p_1}, \mathbf{p_2}$ using barycentric coordinates**

$$f(u,v) = (1-u-v)\mathbf{p_0} + u\mathbf{p_1} + v\mathbf{p_2}$$

- **Can think of a triangle as an affine map of the unit triangle**



Ray-triangle intersection

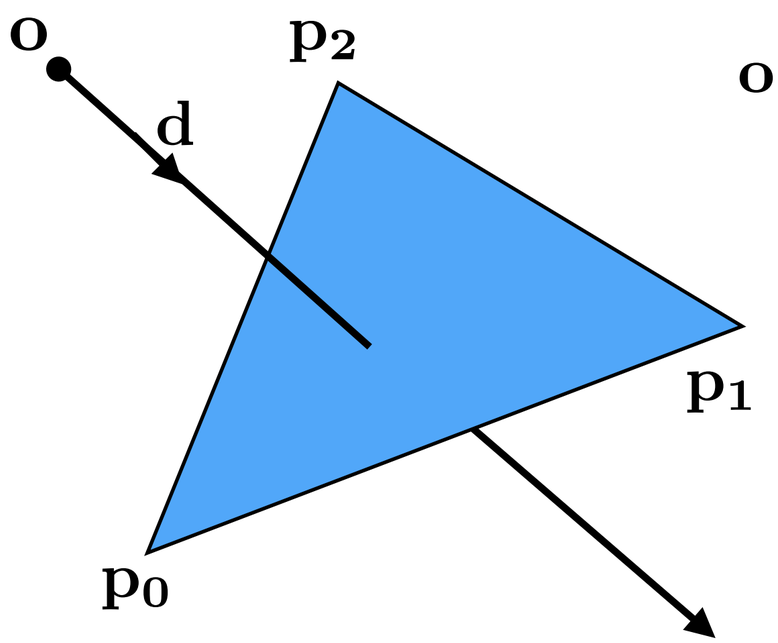
Plug parametric ray equation directly into equation for points on triangle:

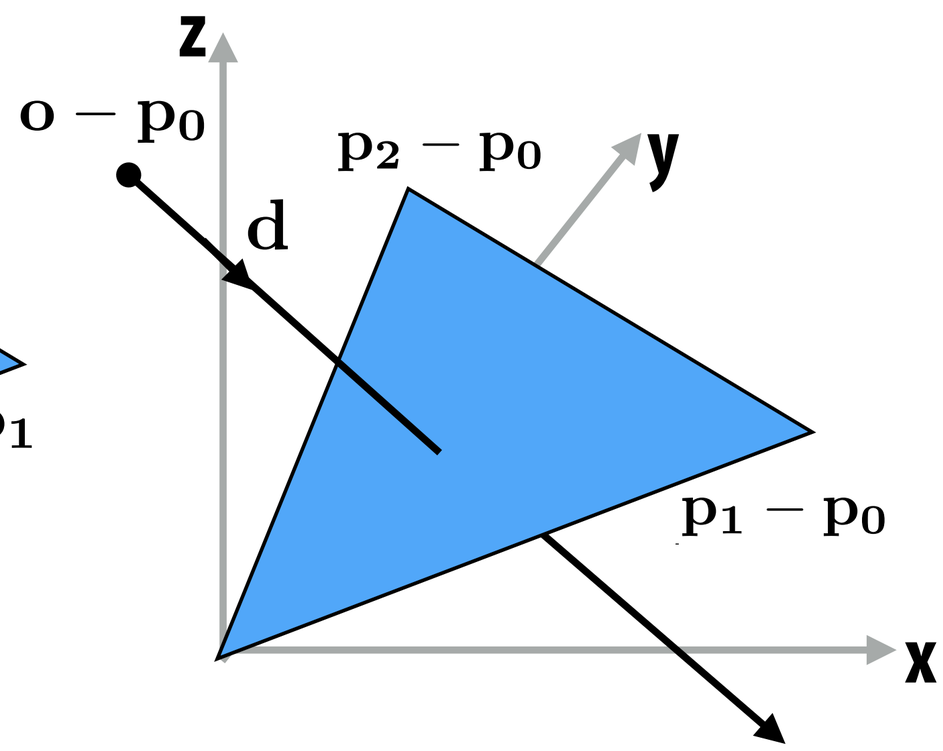
$$\mathbf{p_0} + u(\mathbf{p_1} - \mathbf{p_0}) + v(\mathbf{p_2} - \mathbf{p_0}) = \mathbf{o} + t\mathbf{d}$$

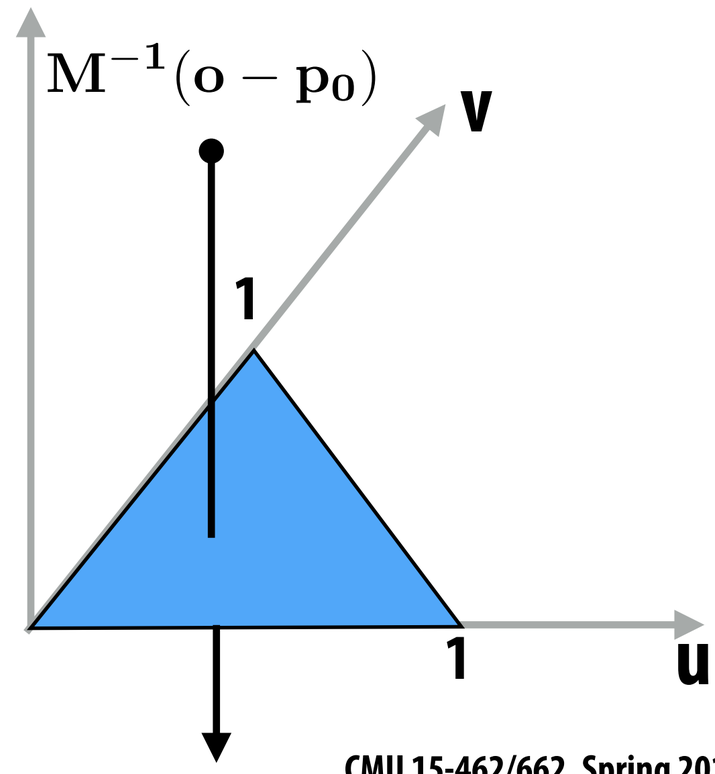
Solve for u, v, t:

$$\underbrace{\begin{bmatrix} \mathbf{p_1} - \mathbf{p_0} & \mathbf{p_2} - \mathbf{p_0} & -\mathbf{d} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} u \\ v \\ t \end{bmatrix} = \mathbf{o} - \mathbf{p_0}$$

$\mathbf{M}^{-1}$ **transforms triangle back to unit triangle in u,v plane, and transforms ray's direction to be orthogonal to plane**







Ray-primitive queries

Given primitive p:

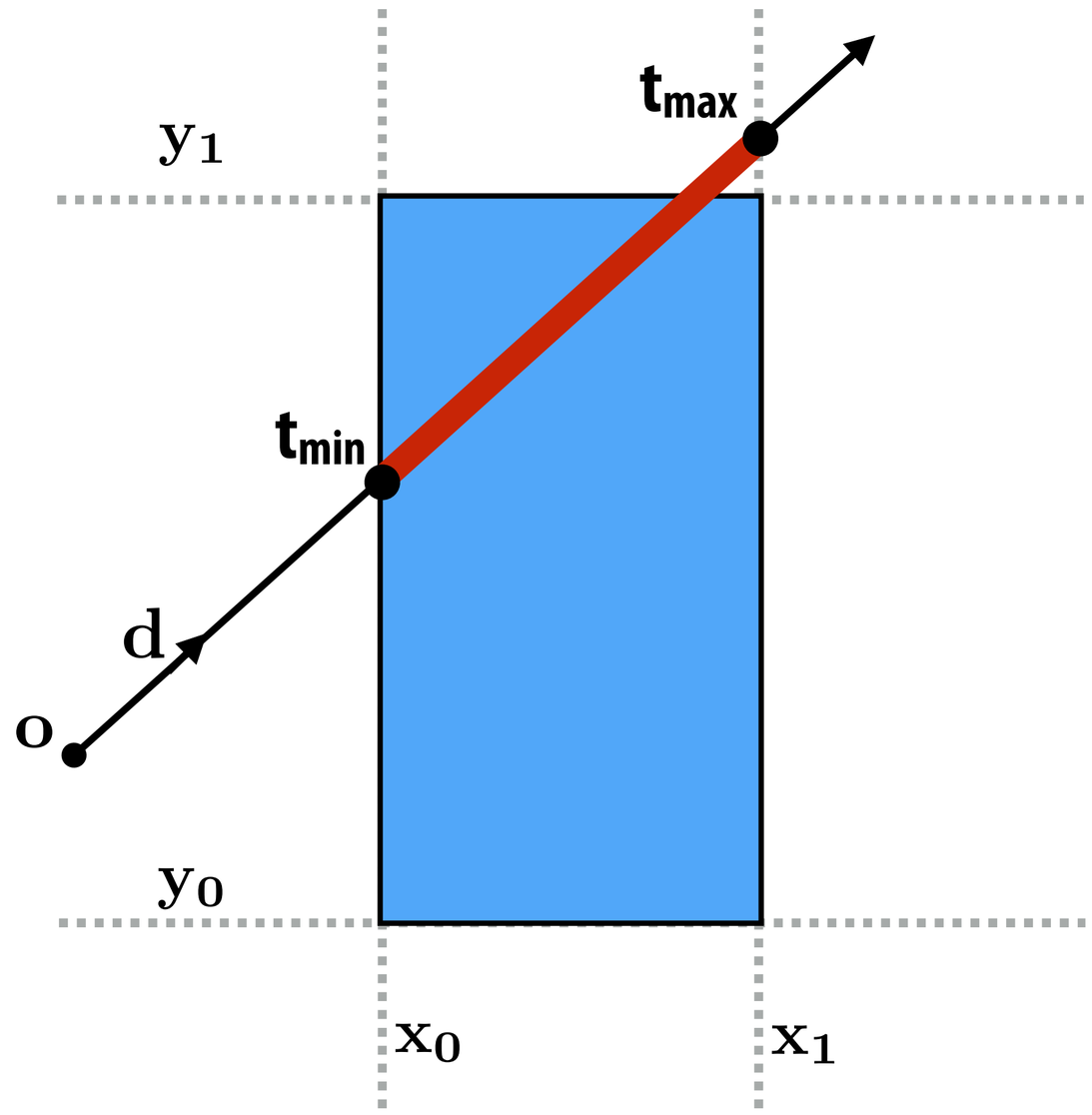
p.intersect(r) returns value of t corresponding to the point of intersection with ray r

p.bbox() returns axis-aligned bounding box of the primitive

```
tri.bbox():
   tri_min = min(p0, min(p1, p2))
   tri_max = max(p0, max(p1, p2))
   return bbox(tri_min, tri_max)
```

Ray-axis-aligned-box intersection

What is ray's closest/farthest intersection with axis-aligned box?



Find intersection of ray with all planes of box:

$$\mathbf{N^T}(\mathbf{o} + t\mathbf{d}) = c$$

Math simplifies greatly since plane is axis aligned (consider $x=x_0$ plane in 2D):

$$\mathbf{N^T} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

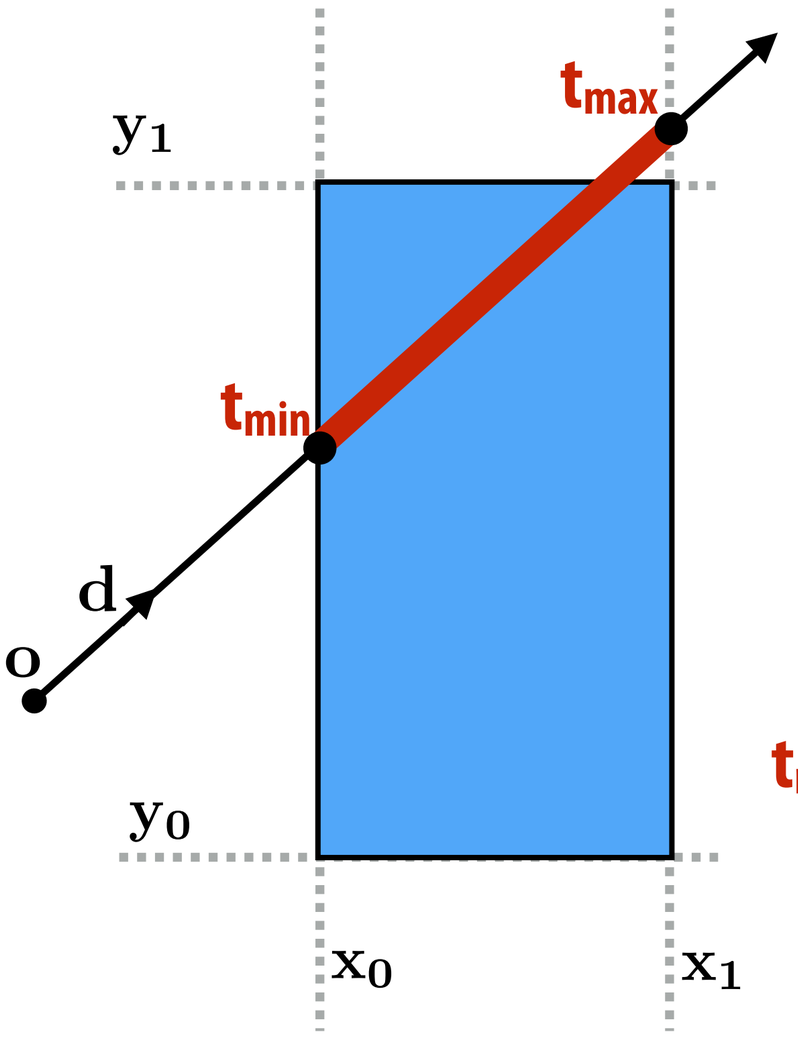
$$c = x_0$$

$$t = \frac{x_0 - \mathbf{o_x}}{\mathbf{d_x}}$$

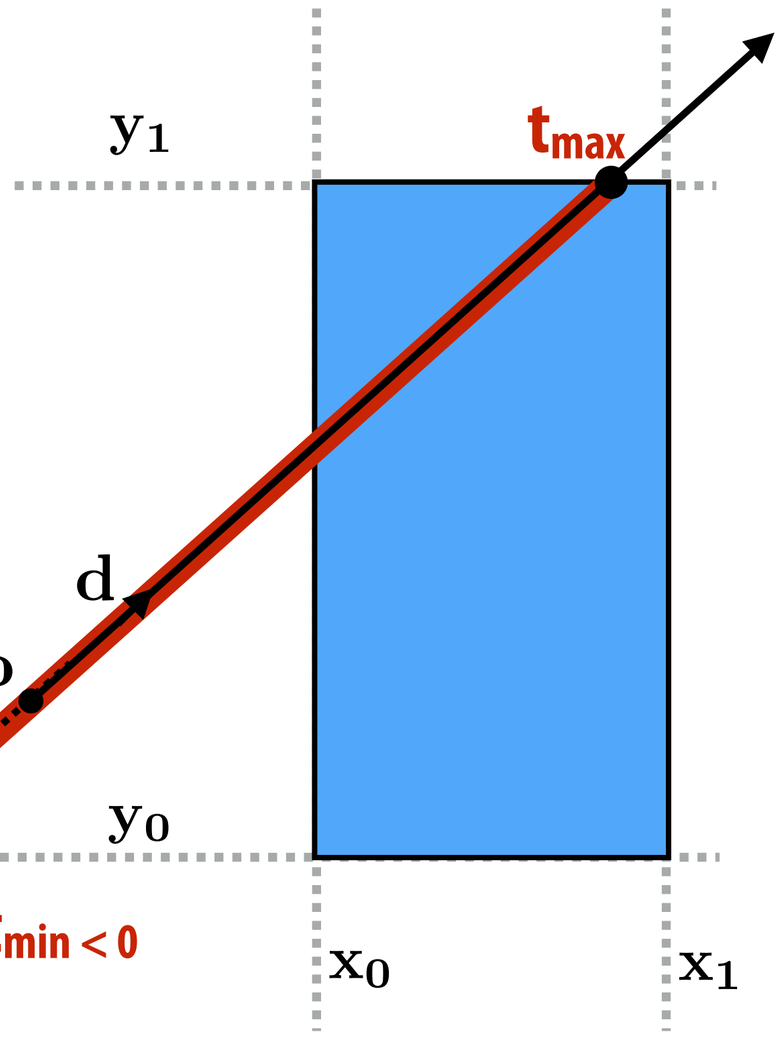
Figure shows intersections with $x=x_0$ and $x=x_1$ planes.

Ray-axis-aligned-box intersection

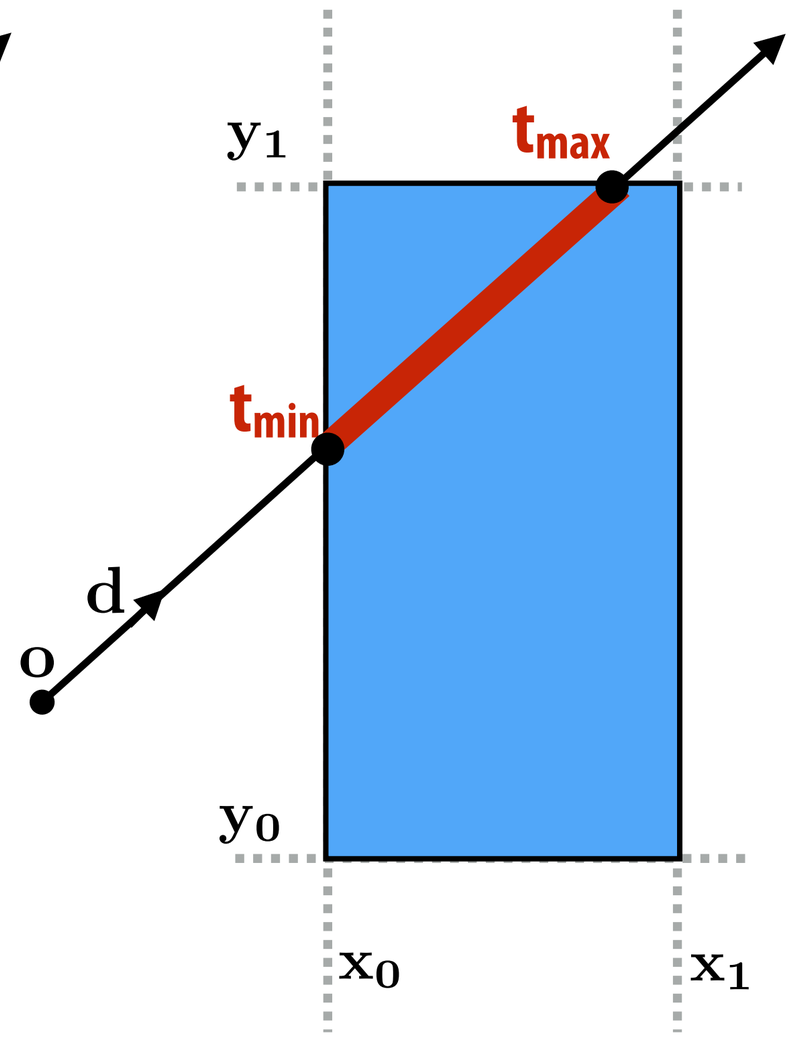
Compute intersections with all planes, take intersection of t_{min}/t_{max} intervals



Intersections with x planes



Intersections with y planes



Final intersection result

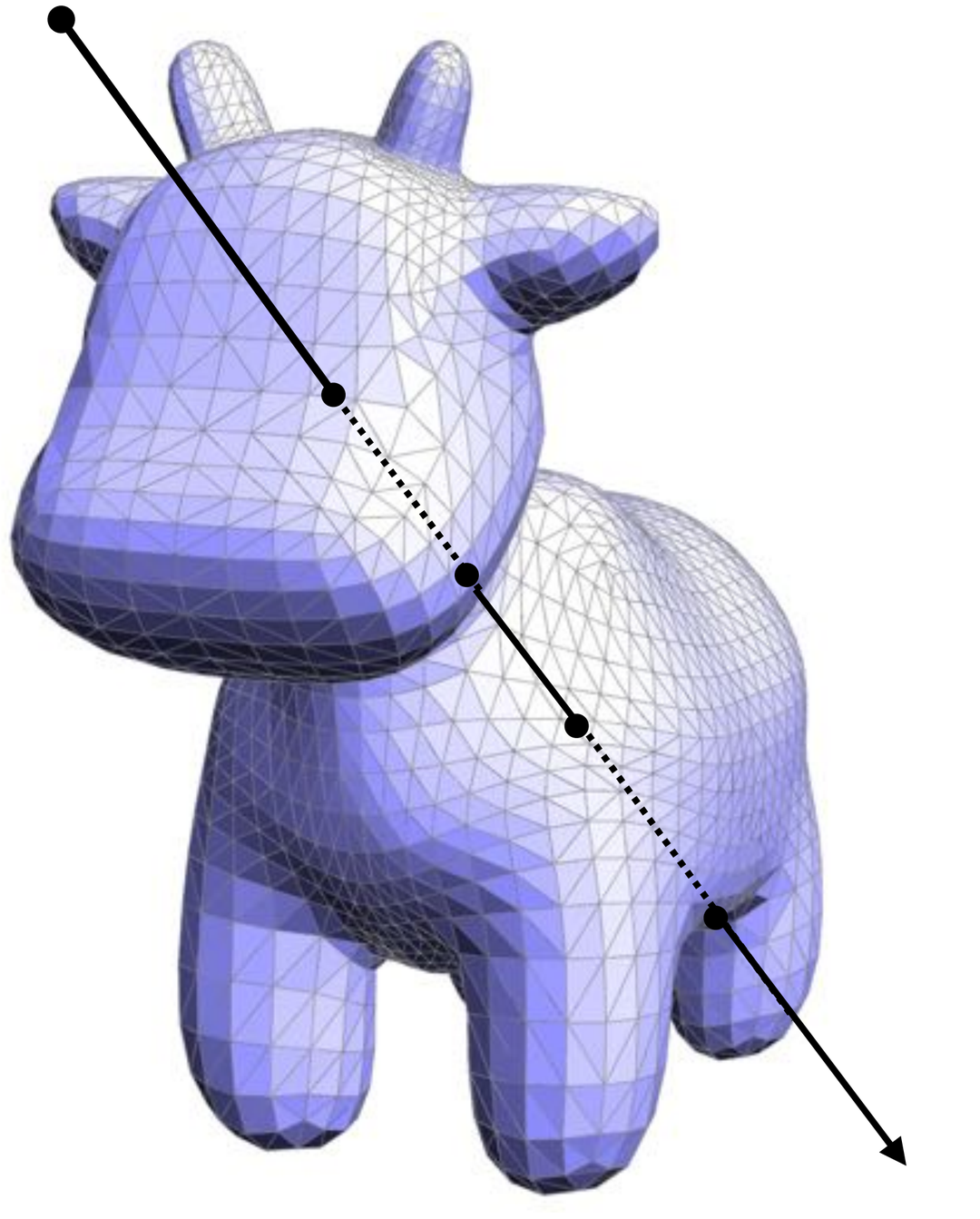
How do we know when the ray misses the box?

Ray-scene intersection

Given a scene defined by a set of N primitives and a ray r, find the closest point of intersection of r with the scene

"Find the first primitive the ray hits"

```
p_closest = NULL
t_closest = inf
for each primitive p in scene:
   t = p.intersect(r)
   if t >= 0 && t < t_closest:
      t_closest = t
      p_closest = p
```



Complexity: $O(N)$

A simpler problem

- Imagine I have a set of integers S
- Given a new integer k, find the element in S that is closest to k:

```
10    123    20    100    6    25    64    11    200    30
```

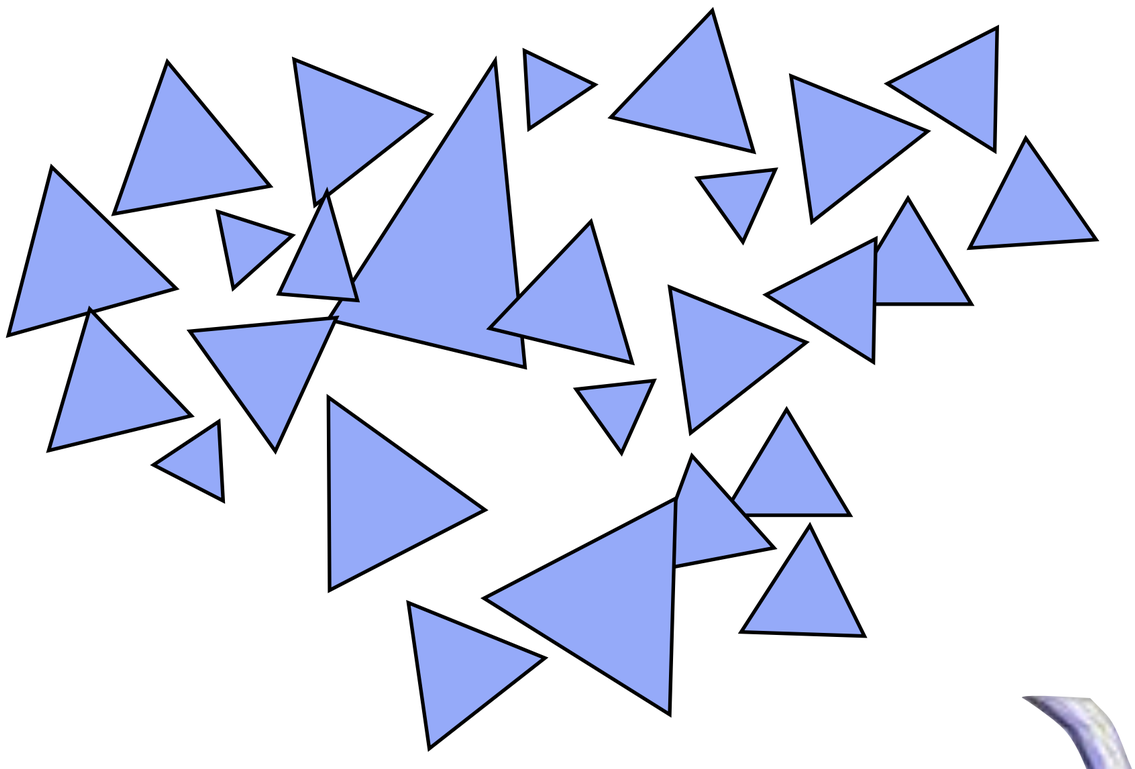
Example: k=18

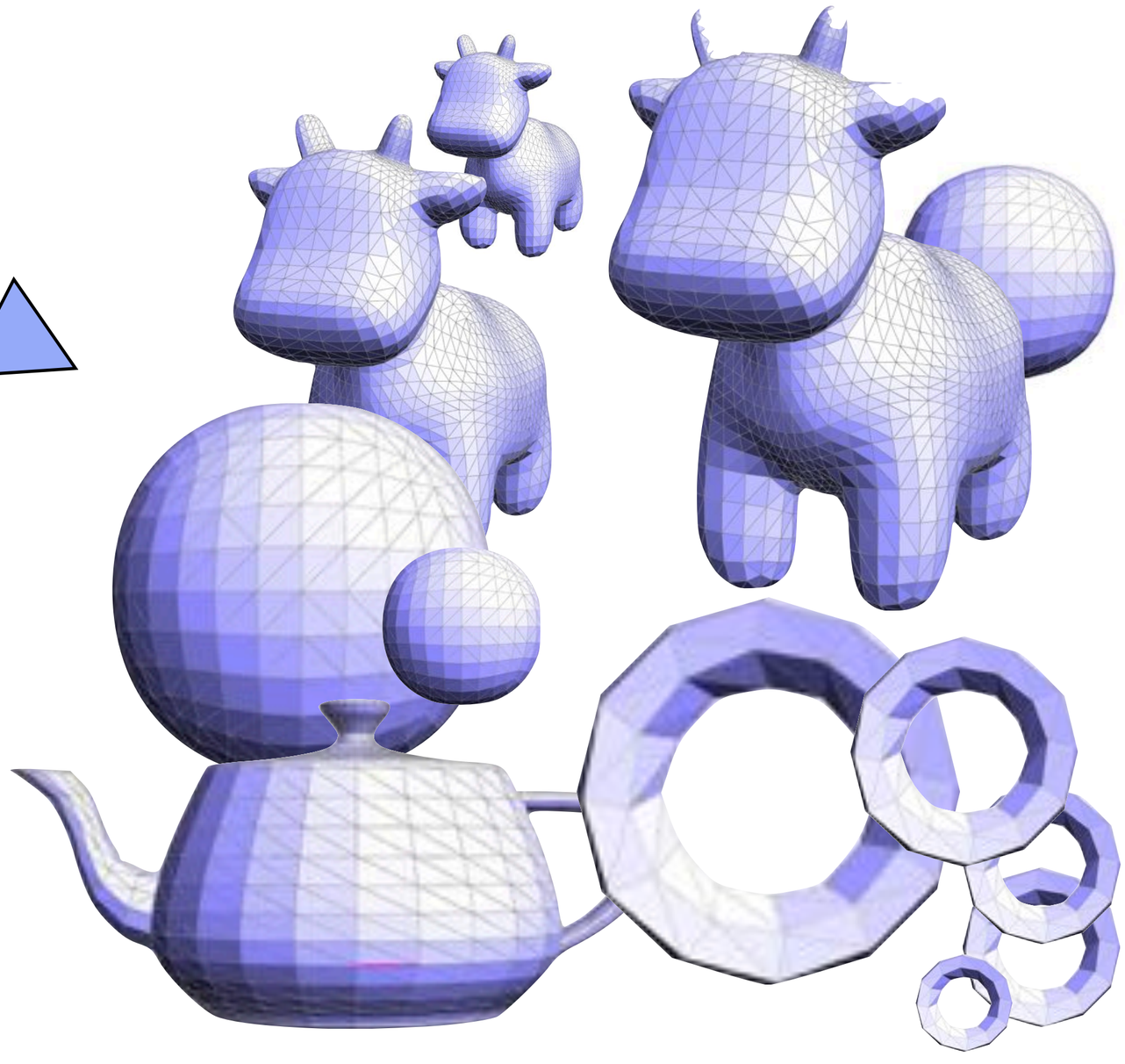
Sort integers:

```
6    10    11    20    25    30    64    100    123    200
```

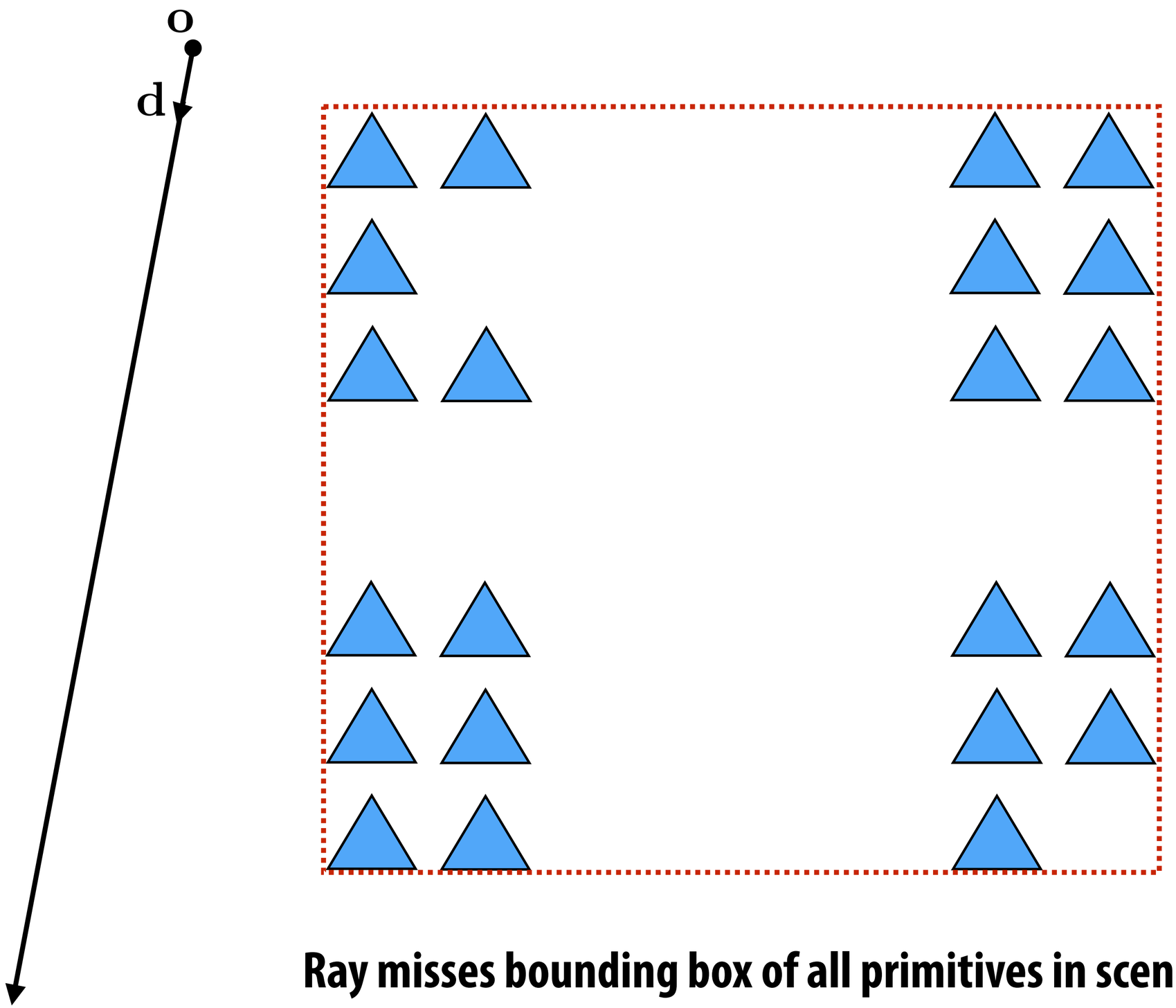
How would you perform a modified binary search?

How do we organize scene primitives to enable fast ray-scene intersection queries?





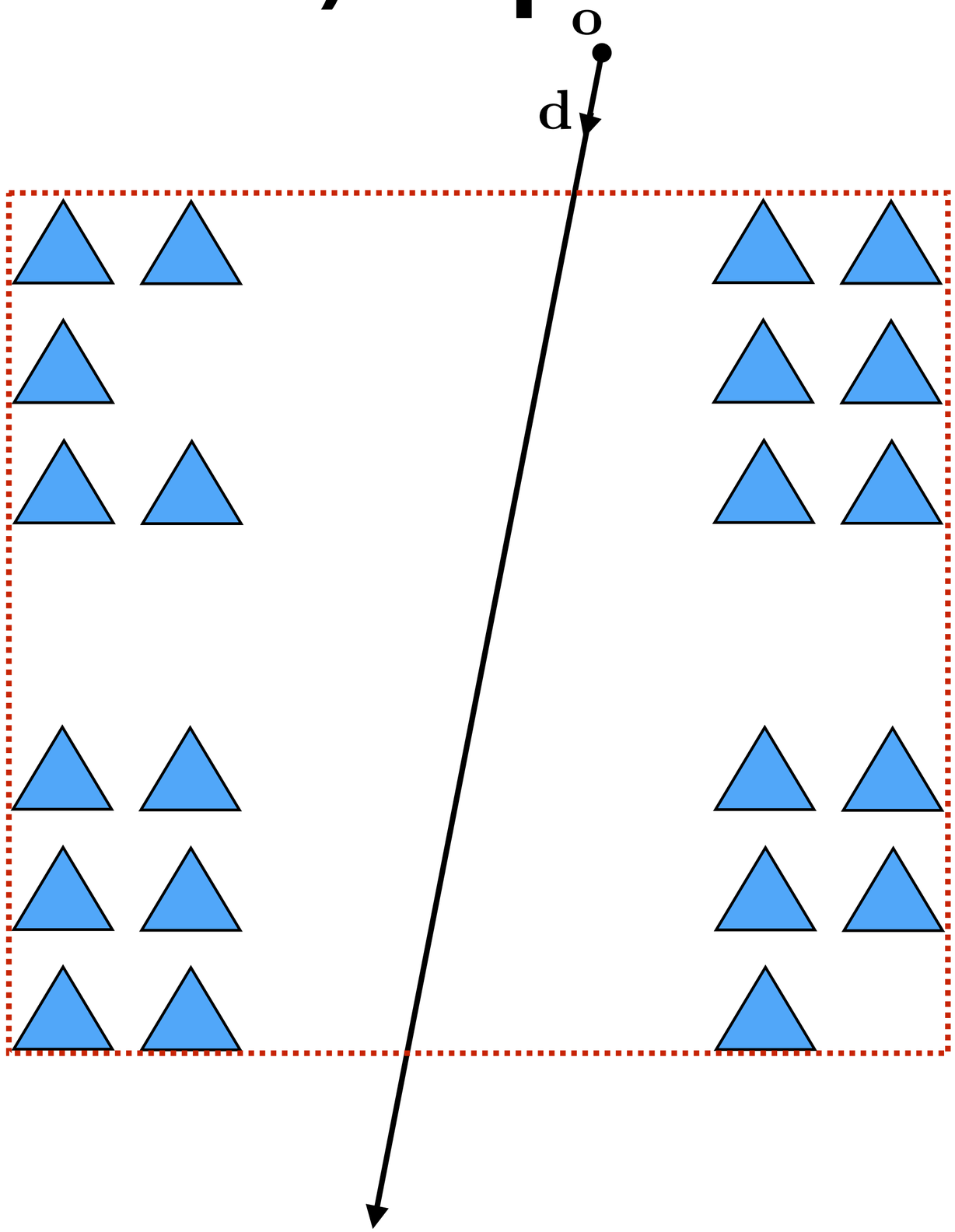
Simple case



Ray misses bounding box of all primitives in scene

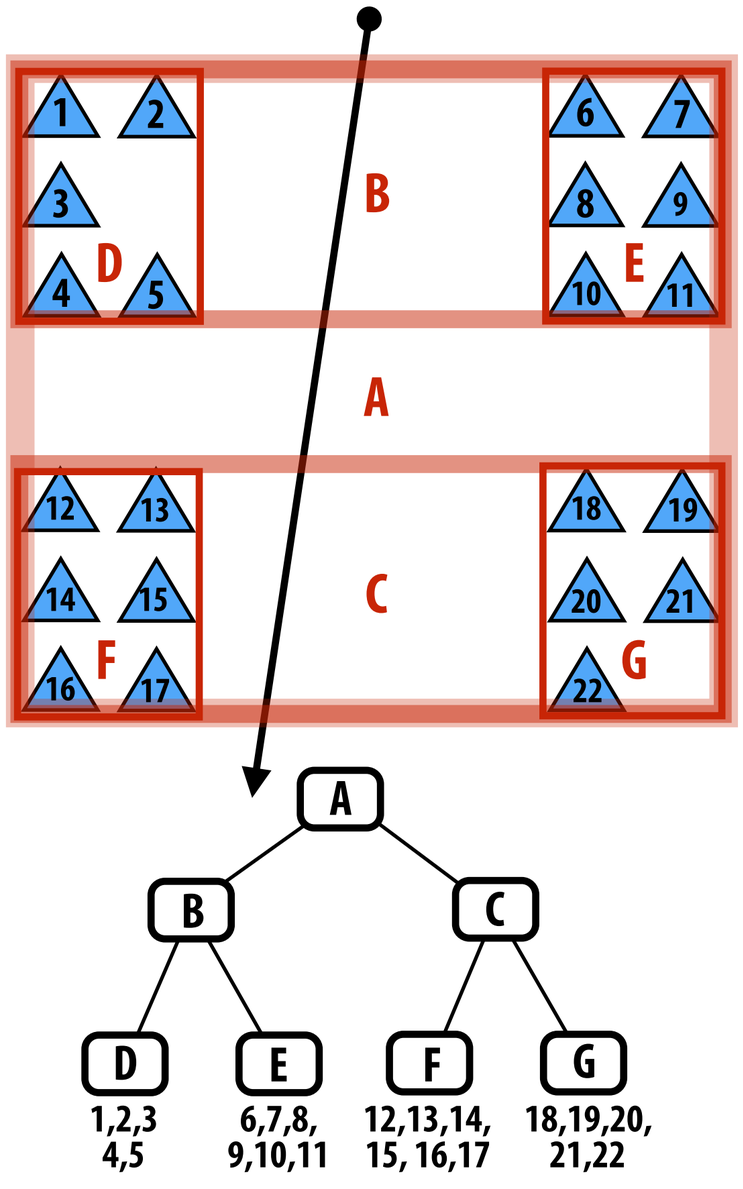
O(1) cost: requires 1 ray-box test

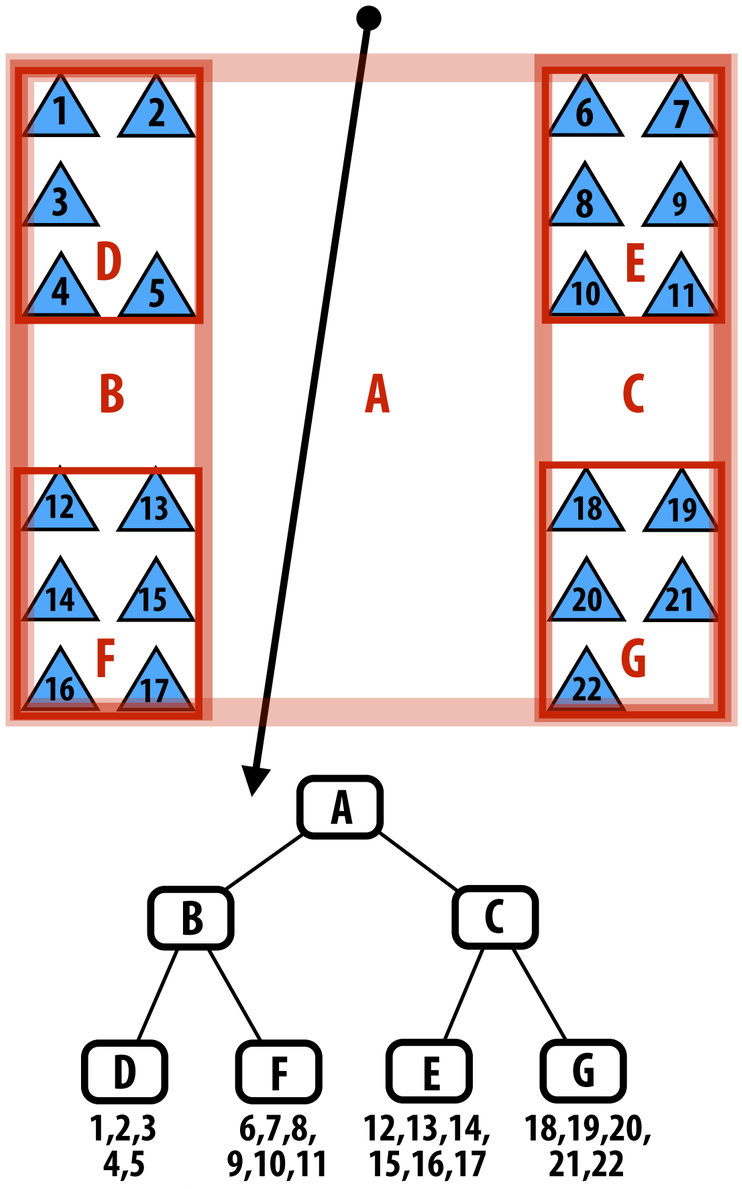
Another (should be) simple case



Bounding volume hierarchy (BVH)

- Interior nodes:
  - Represents subset of primitives in scene
  - Stores aggregate bounding box for all primitives in subtree
- Leaf nodes:
  - Contain list of primitives



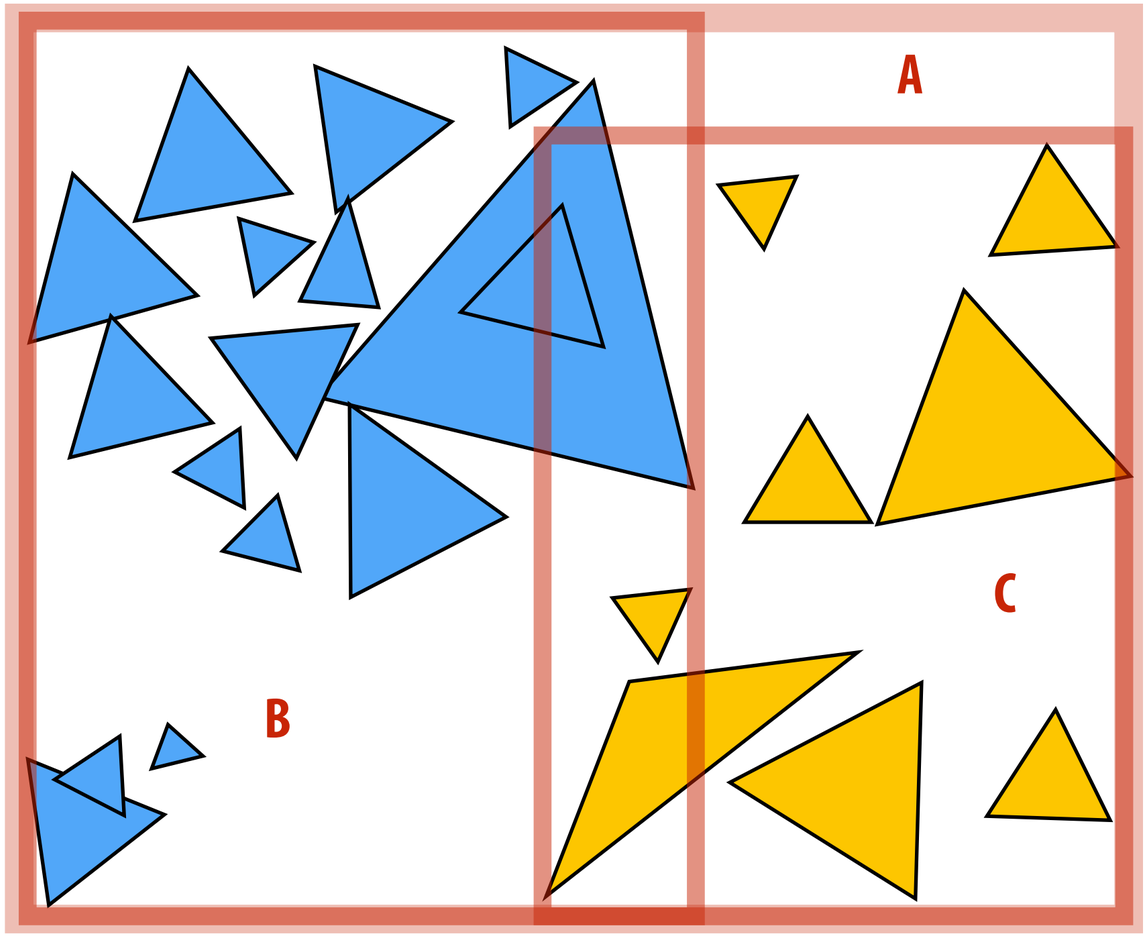


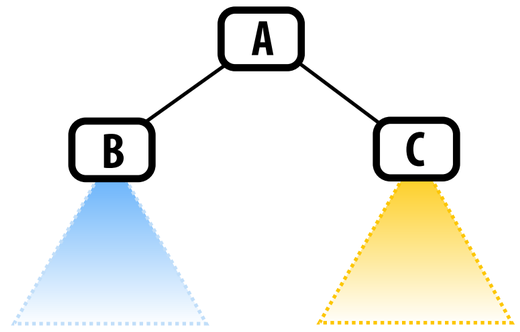
Left: two different BVH organizations of the same scene containing 22 primitives.

Is one BVH better than the other?

Another BVH example

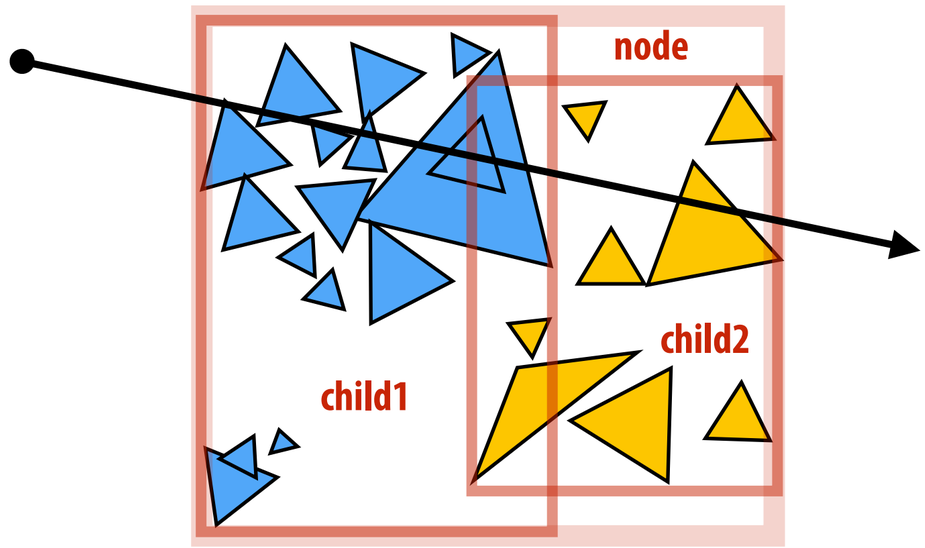
- BVH partitions each node's primitives into disjoints sets
  - Note: The sets can still be overlapping in space (below: child bounding boxes may overlap in space)





Ray-scene intersection using a BVH

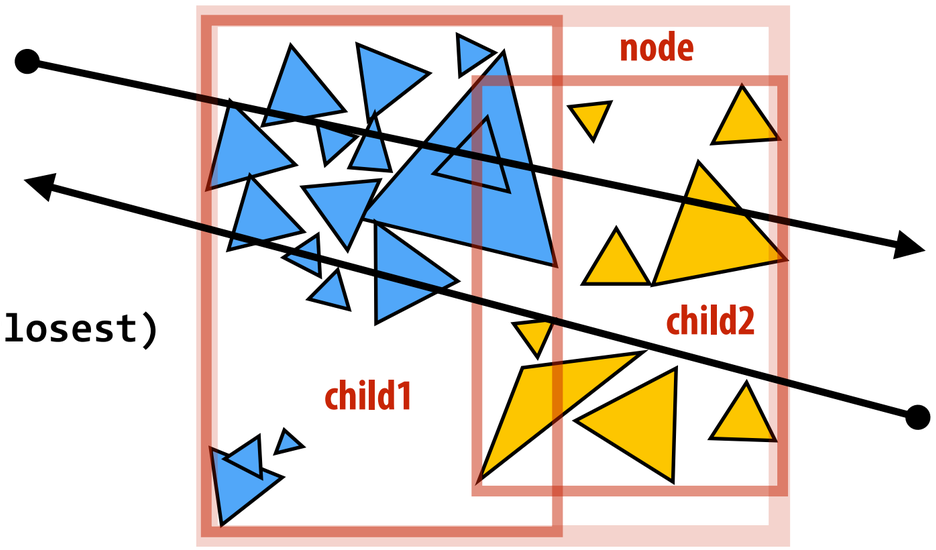
```
struct BVHNode {
   bool leaf;
   BBox bbox;
   BVHNode* child1;
   BVHNode* child2;
   Primitive* primList;
};

struct ClosestHitInfo {
   Primitive prim;
   float min_t;
};

void find_closest_hit(Ray* ray, BVHNode* node, ClosestHitInfo* closest) {

   if (!intersect(ray, node->bbox) || (closest point on box is farther than closest.min_t))
      return;

   if (node->leaf) {
      for (each primitive p in node->primList) {
         (hit, t) = intersect(ray, p);
         if (hit && t < closest.min_t) {
            closest.prim = p;
            closest.min_t = t;
         }
      }
   } else {
      find_closest_hit(ray, node->child1, closest);
      find_closest_hit(ray, node->child2, closest);
   }
}
```



How could this occur?

Improvement: "front-to-back" traversal

Invariant: only call find_closest_hit() if ray intersects bbox of node.



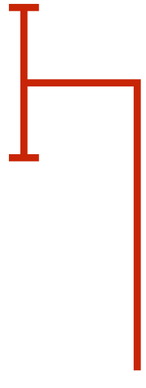

```
void find_closest_hit(Ray* ray, BVHNode* node, ClosestHitInfo* closest)
{

   if (node->leaf) {
      for (each primitive p in node->primList) {
         (hit, t) = intersect(ray, p);
         if (hit && t < closest.min_t) {
            closest.prim = p;
            closest.min_t = t;
         }
      }
   } else {
      (hit1, min_t1) = intersect(ray, node->child1->bbox);
      (hit2, min_t2) = intersect(ray, node->child2->bbox);

      NVHNode* first = (min_t1 <= min_t2) ? child1 : child2;
      NVHNode* second = (min_t1 <= min_t2) ? child2 : child1;

      find_closest_hit(ray, first, closest);
      if (second child's min_t is closer than closest.min_t)
         find_closest_hit(ray, second, closest);
   }
}
```

"Front to back" traversal. Traverse to closest child node first. Why?

Another type of query: any hit

Sometimes it's useful to know if the ray hits ANY primitive in the scene at all (don't care about distance to first hit)

```
bool find_any_hit(Ray* ray, BVHNode* node) {

   if (!intersect(ray, node->bbox))
      return false;

   if (node->leaf) {
      for (each primitive p in node->primList) {
         (hit, t) = intersect(ray, p);
         if (hit)
            return true;
   } else {
      return ( find_closest_hit(ray, node->child1, closest) ||
               find_closest_hit(ray, node->child2, closest) );
   }
}
```



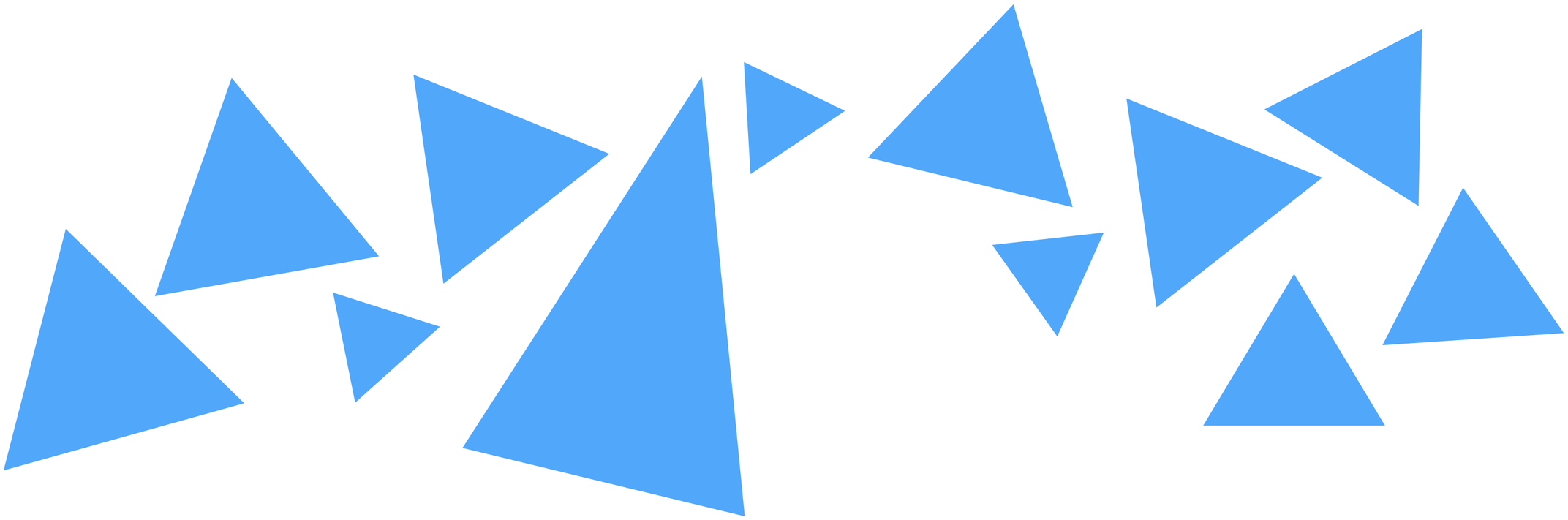
Interesting question of which child to enter first. How might you make a good decision?

For a given set of primitives, there are many possible BVHs

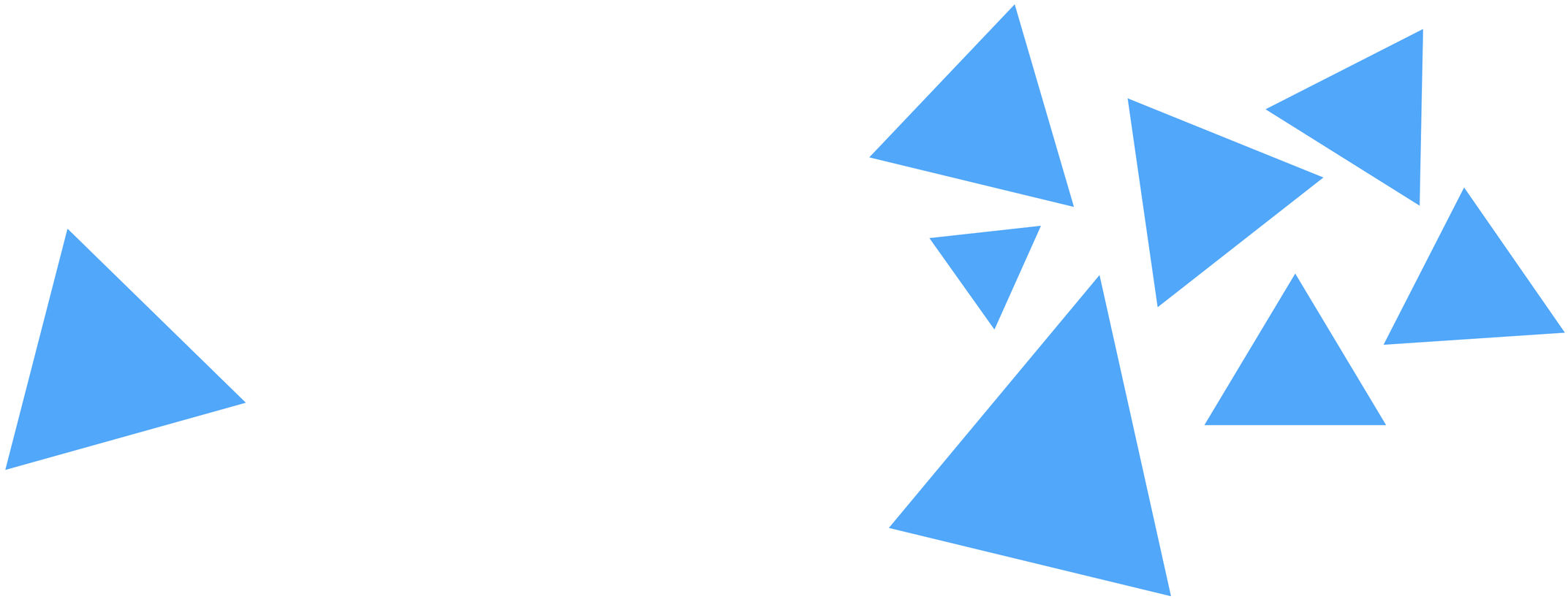
(2^N-2 ways to partition N primitives into two groups)

How do we build a high-quality BVH?

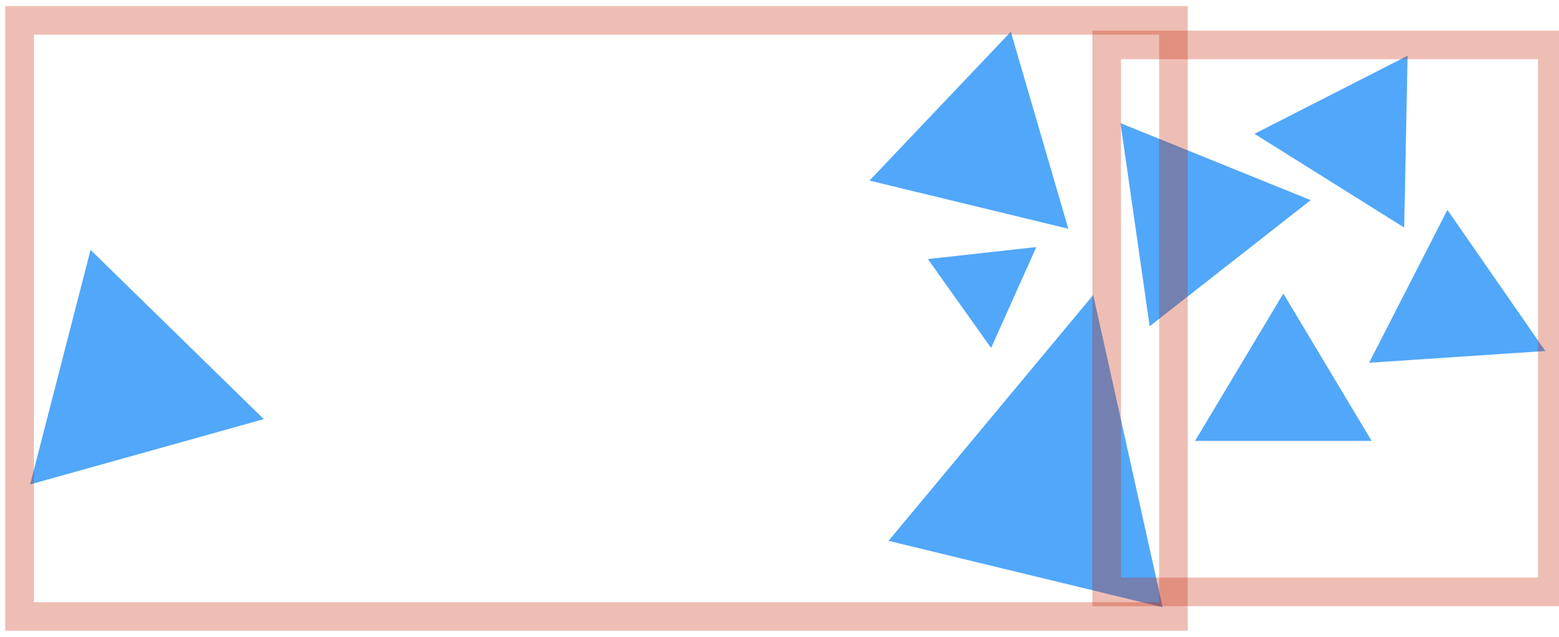
How would you partition these triangles into two groups?



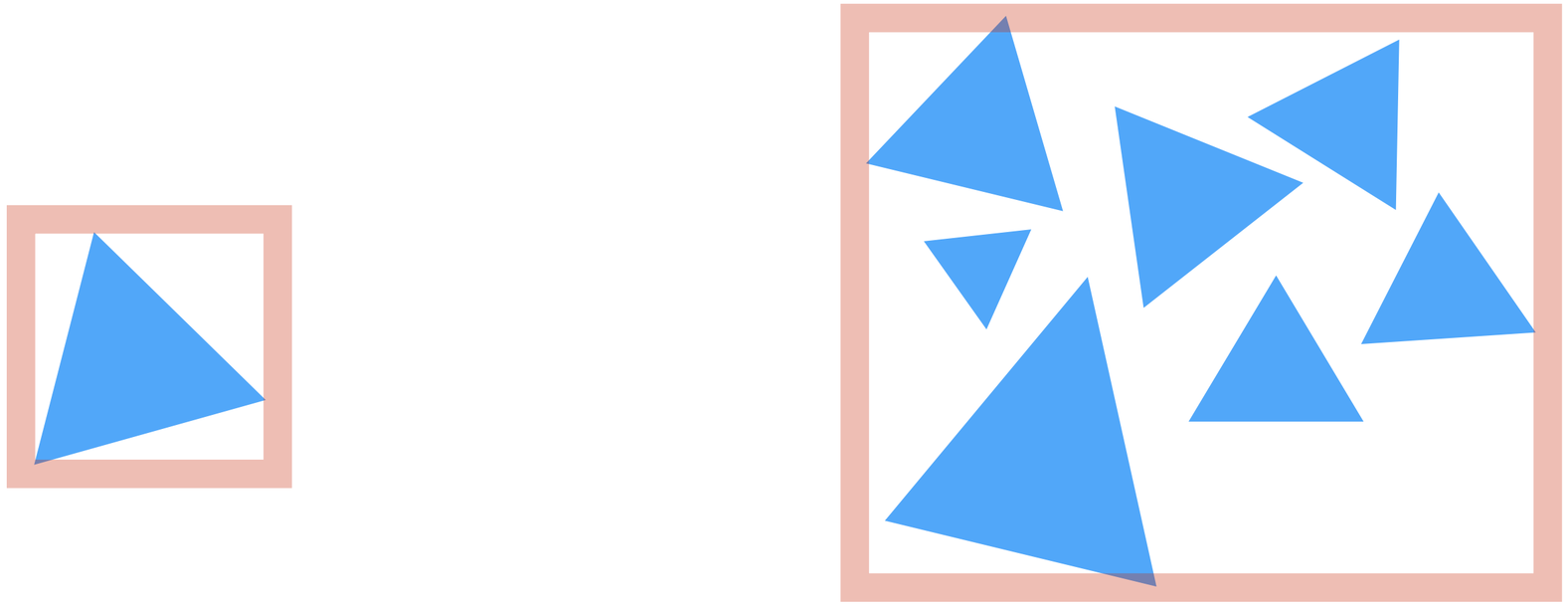
What about these?



Intuition about a "good" partition?



Partition into child nodes with equal numbers of primitives



Better partition

Intuition: want small bounding boxes (minimize overlap between children, avoid empty space)

What are we really trying to do?

A good partitioning minimizes the cost of finding the closest intersection of a ray with primitives in the node.

If a node is a leaf node (no partitioning):

$$C = \sum_{i=1}^{N} C_{\text{isect}}(i)$$

$$= NC_{\text{isect}}$$

Where $C_{\text{isect}}(i)$ is the cost of ray-primitive intersection for primitive i in the node.

(Common to assume all primitives have the same cost)

Cost of making a partition

The expected cost of ray-node intersection, given that the node's primitives are partitioned into child sets A and B is:

$$C = C_{\text{trav}} + p_A C_A + p_B C_B$$

C_{trav} **is the cost of traversing an interior node (e.g., load data, bbox check)**

C_A **and** C_B **are the costs of intersection with the resultant child subtrees**

p_A **and** p_B **are the probability a ray intersects the bbox of the child nodes A and B**

Primitive count is common approximation for child node costs:

$$C = C_{\text{trav}} + p_A N_A C_{\text{isect}} + p_B N_B C_{\text{isect}}$$

Where: $N_A = |A|, N_B = |B|$

Estimating probabilities

- For convex object A inside convex object B, the probability that a random ray that hits B also hits A is given by the ratio of the surface areas S_A and S_B of these objects.

$$P(\text{hit}A|\text{hit}B) = \frac{S_A}{S_B}$$

Surface area heuristic (SAH):

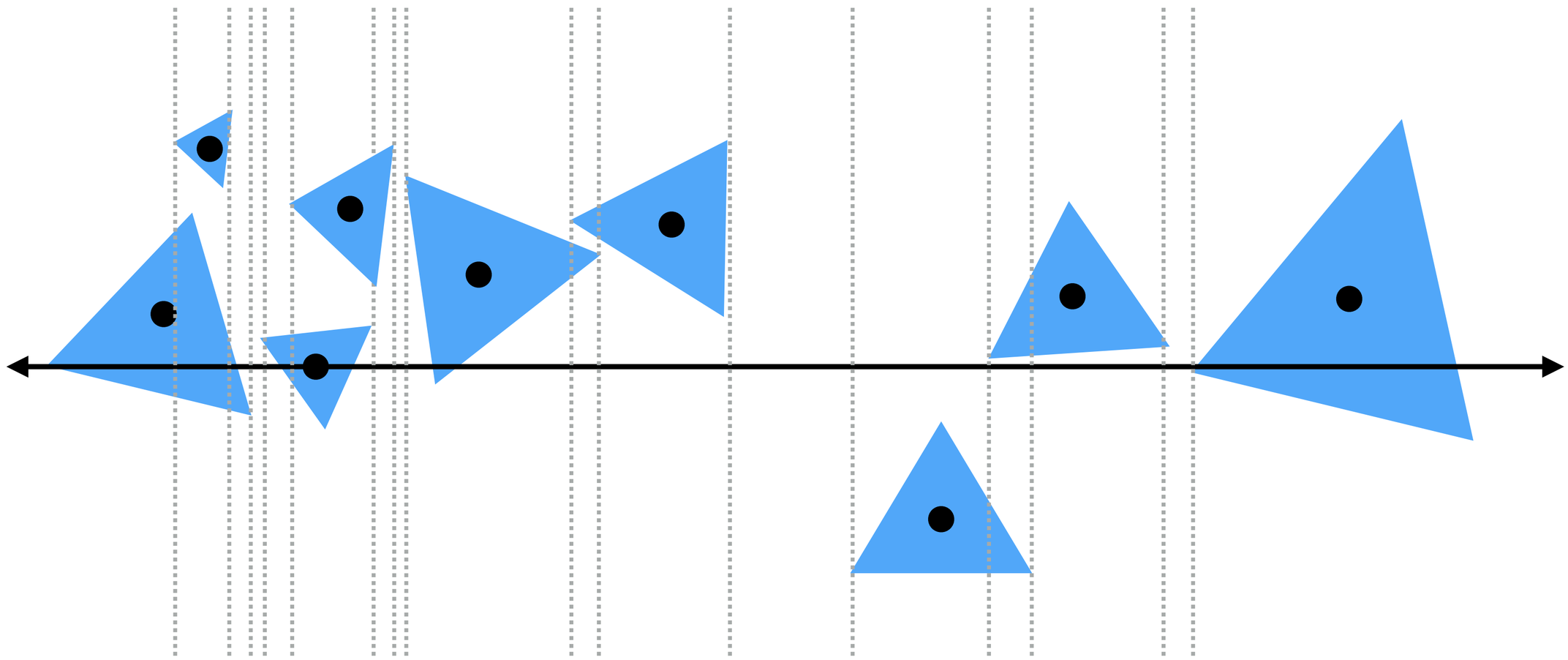
$$C = C_{\text{trav}} + \frac{S_A}{S_N} N_A C_{\text{isect}} + \frac{S_B}{S_N} N_B C_{\text{isect}}$$

Assumptions of the SAH (may not hold in practice):

- Rays are randomly distributed
- Rays are not occluded

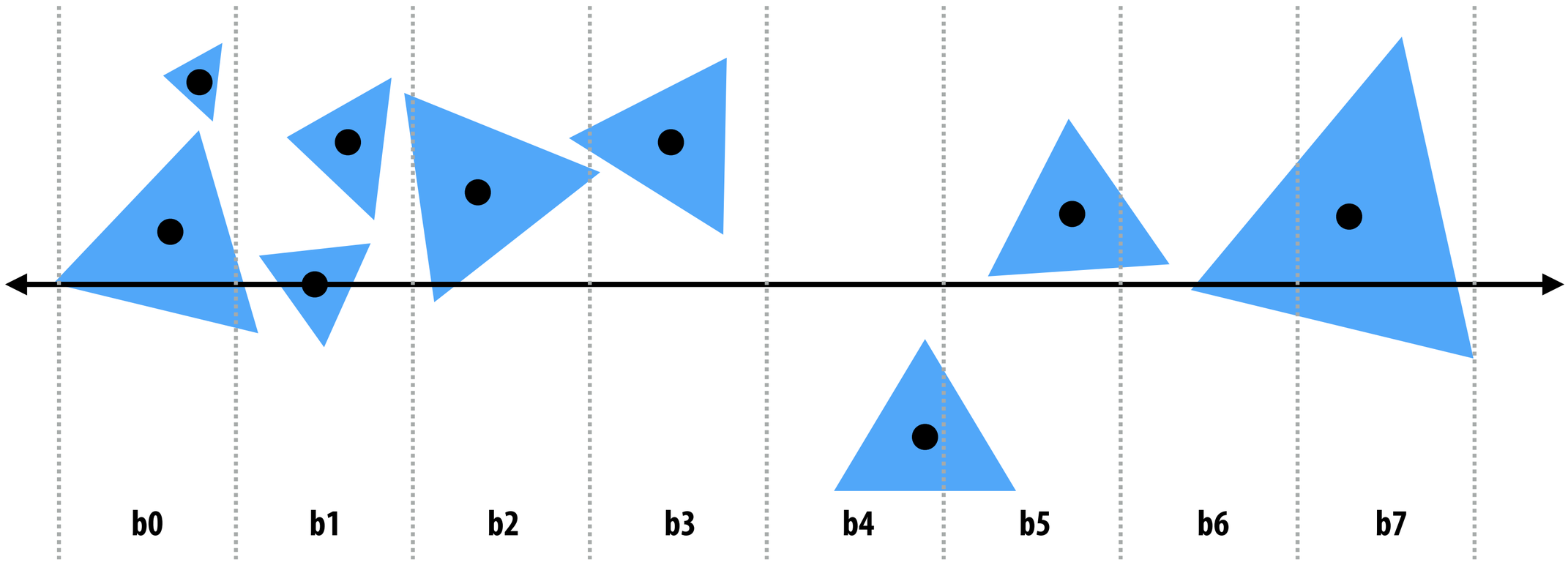
Implementing partitions

- Constrain search for good partitions to axis-aligned spatial partitions
  - Choose an axis
  - Choose a split plane on that axis
  - Partition primitives by the side of splitting plane their centroid lies
  - 2N-2 possible splitting positions for node with N primitives. (Why?)



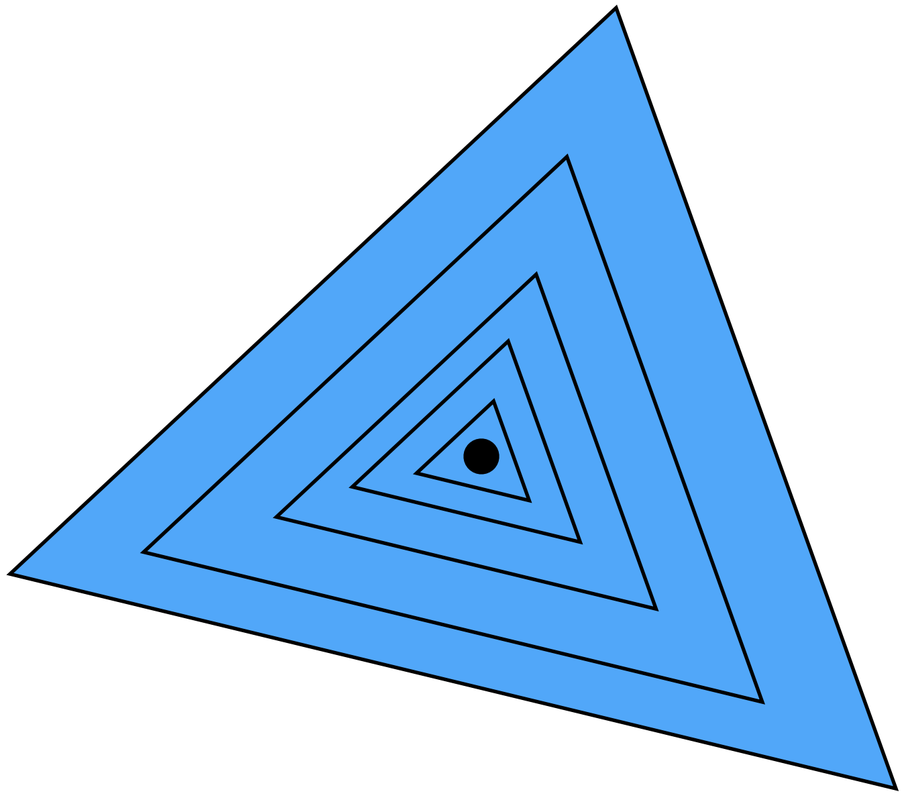
Efficiently implementing partitioning

- Efficient modern approximation: split spatial extent of primitives into B buckets (B is typically small: B < 32)

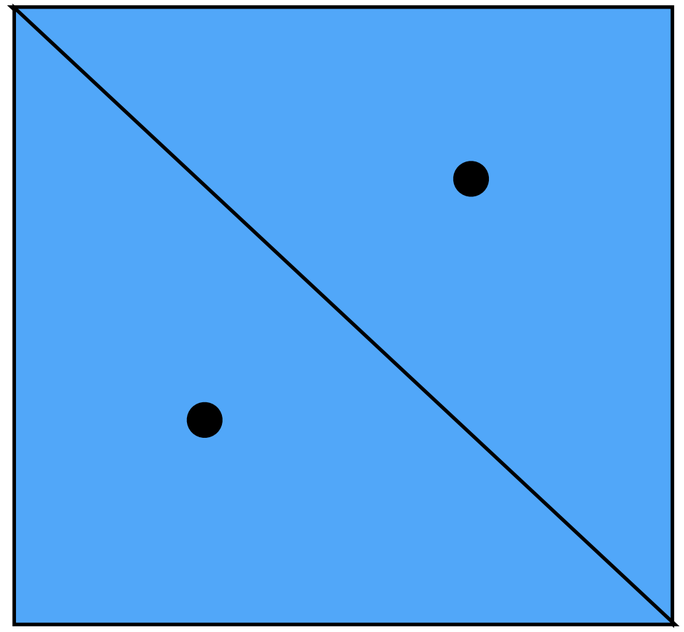



```
For each axis: x,y,z:
   initialize buckets
   For each primitive p in node:
      b = compute_bucket(p.centroid)
      b.bbox.union(p.bbox);
      b.prim_count++;
   For each of the B-1 possible partitioning planes evaluate SAH
Execute lowest cost partitioning found (or make node a leaf)
```

Troublesome cases



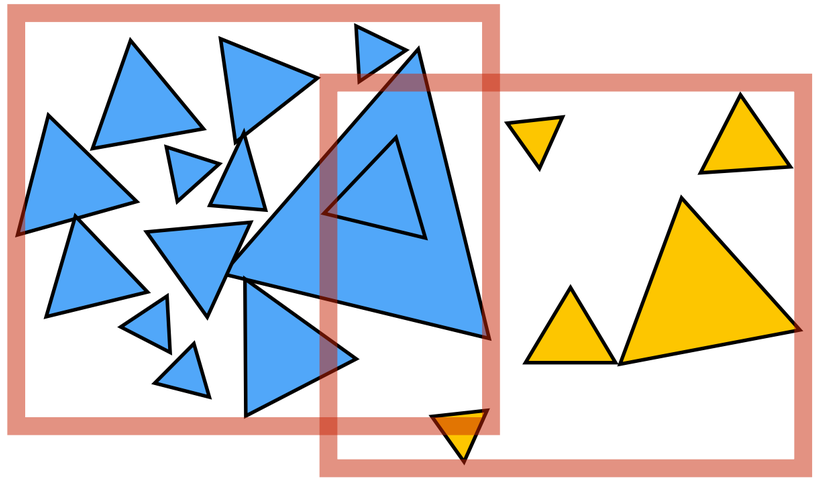
All primitives with same centroid (all primitives end up in same partition)



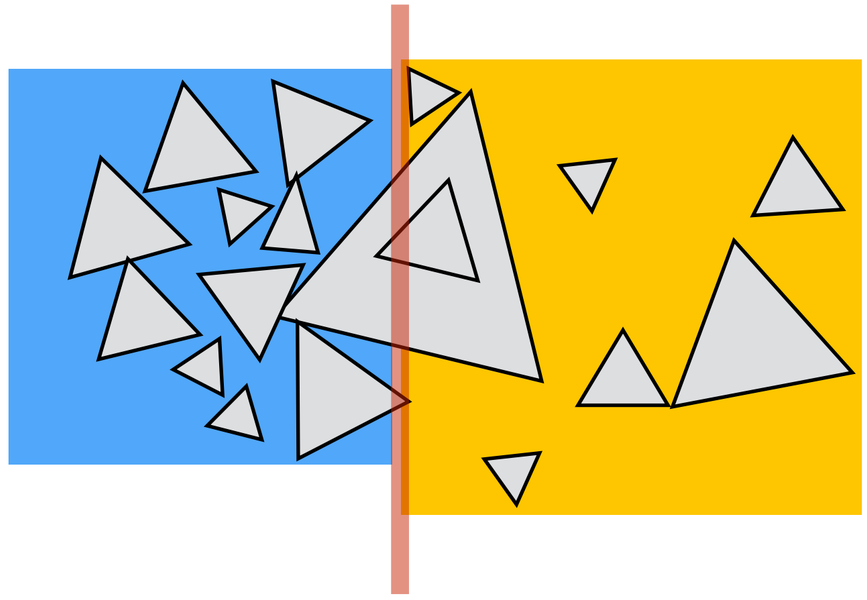
All primitives with same bbox (ray often ends up visiting both partitions)

Primitive-partitioning acceleration structures vs. space-partitioning structures

- Primitive partitioning (bounding volume hierarchy): partitions node's primitives into disjoint sets (but sets may overlap in space)

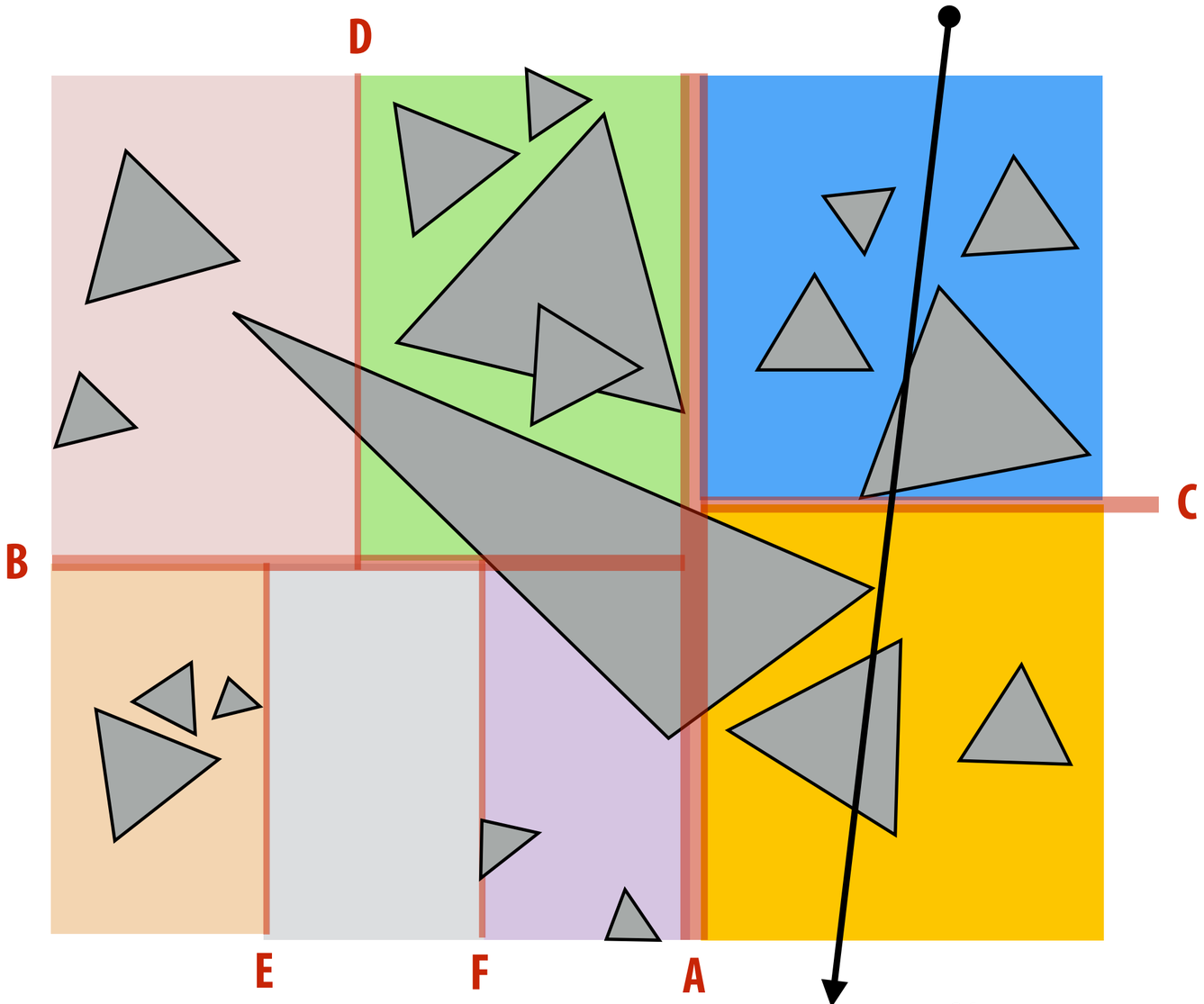


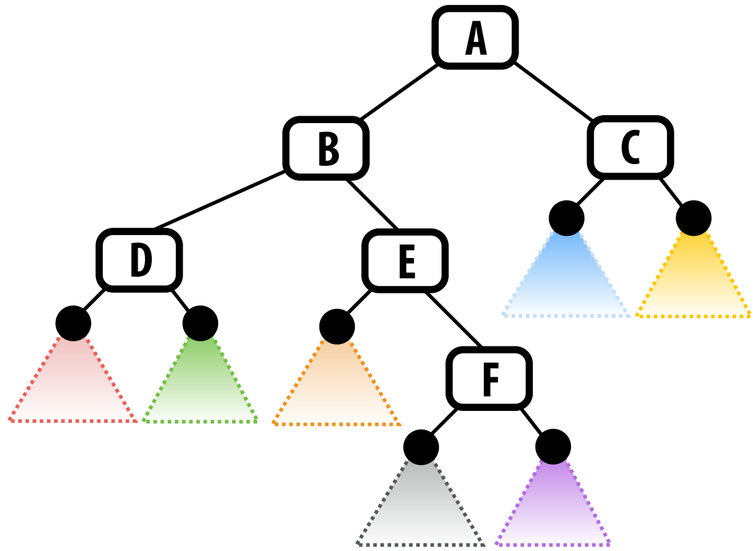
- Space-partitioning (grid, K-D tree) partitions space into disjoint regions (primitives may be contained in multiple regions of space)



K-D tree

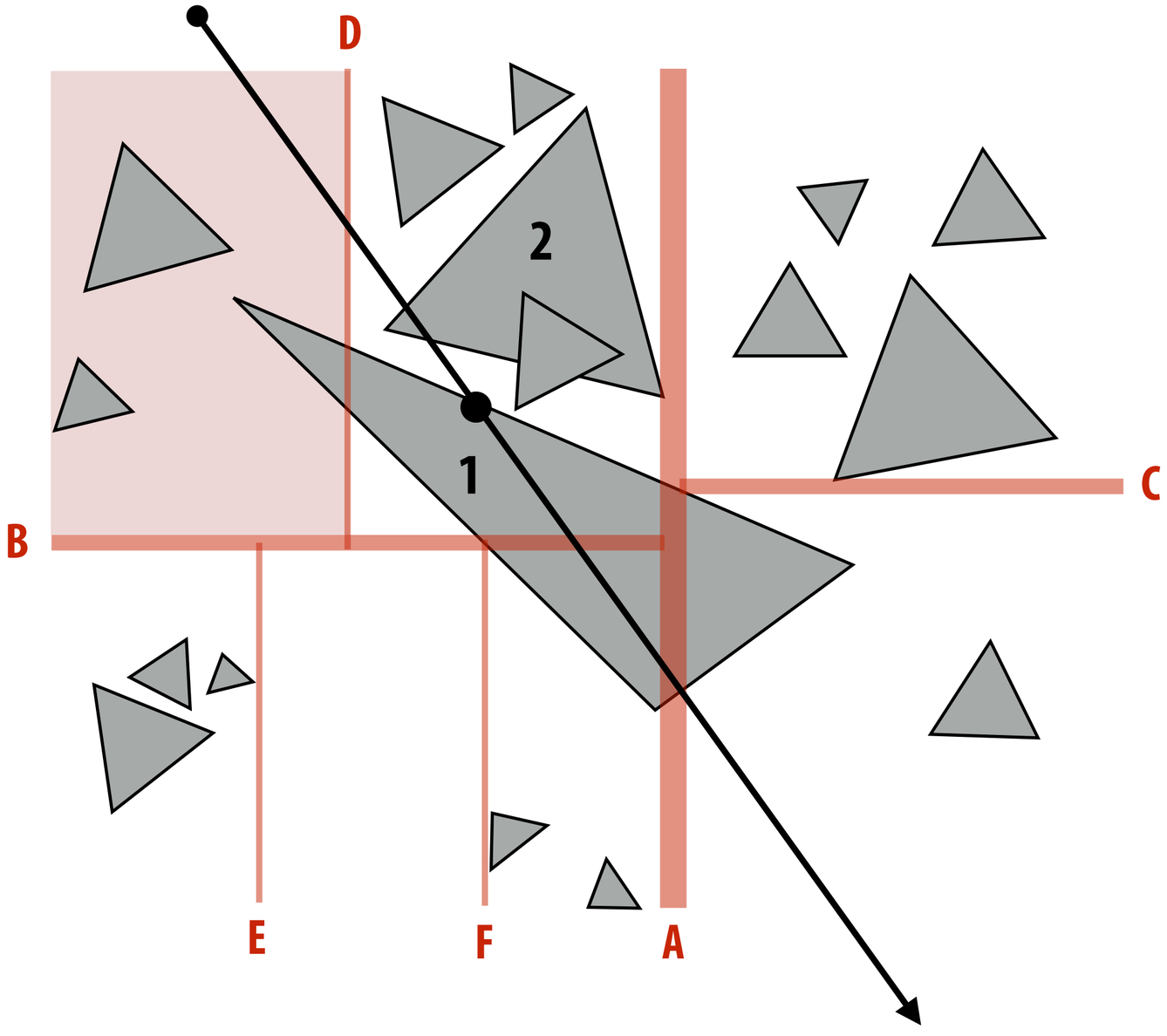
- Recursively partition <u>space</u> via axis-aligned partitioning planes
  - Interior nodes correspond to spatial splits (still correspond to spatial volume)
  - Node traversal can proceed in front-to-back order (unlike BVH, can terminate search after first hit is found).
  - Intuition: partitions curve out empty space (construction of K-D tree may produce more tree nodes than primitives depending on ratio of C_{trav} and C_{isect})

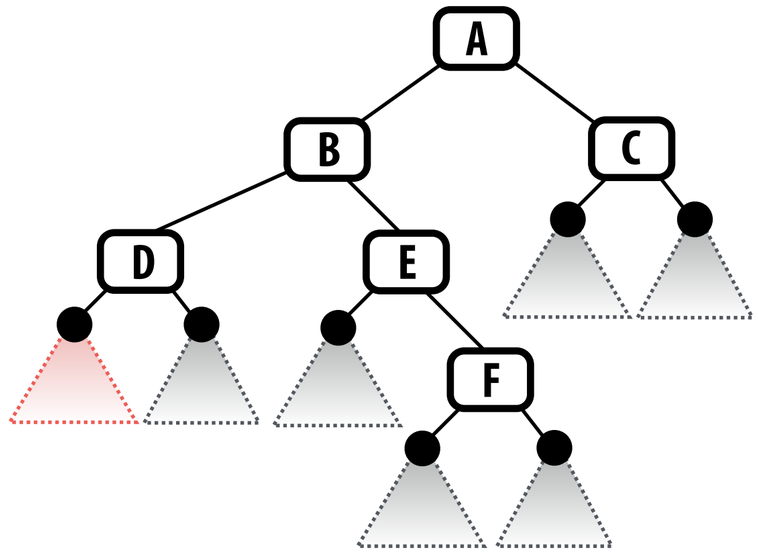




Challenge: objects overlap multiple nodes

- Want node traversal to proceed in front-to-back order so traversal can terminate search after first hit found





Triangle 1 overlaps multiple nodes.

Ray hits triangle 1 when in highlighted leaf cell.

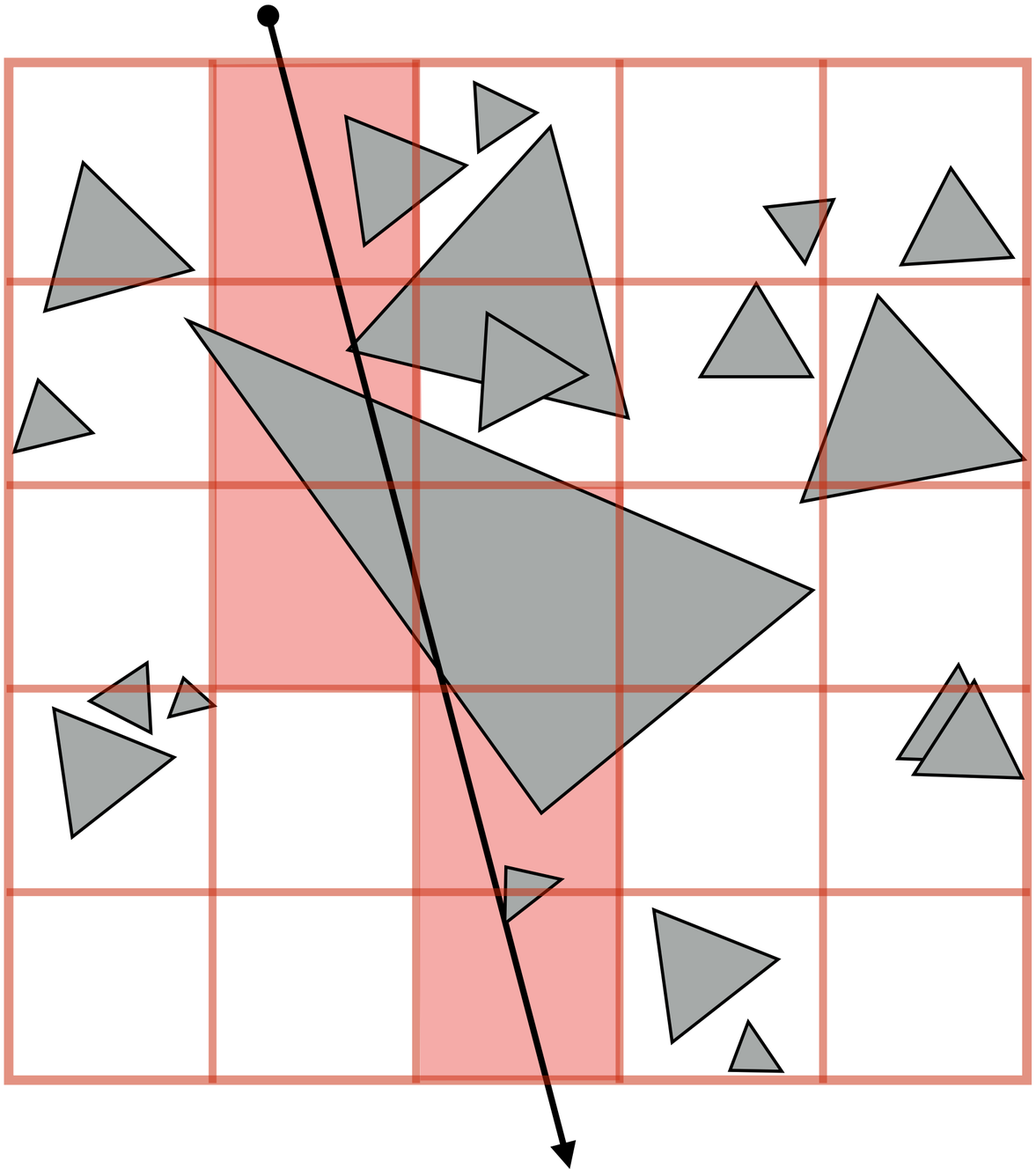
But intersection with triangle 2 is closer! (Haven't traversed to that node yet)

Solution: require primitive intersection point to be within current leaf node.

(primitives may be intersected multiple times by same ray *)

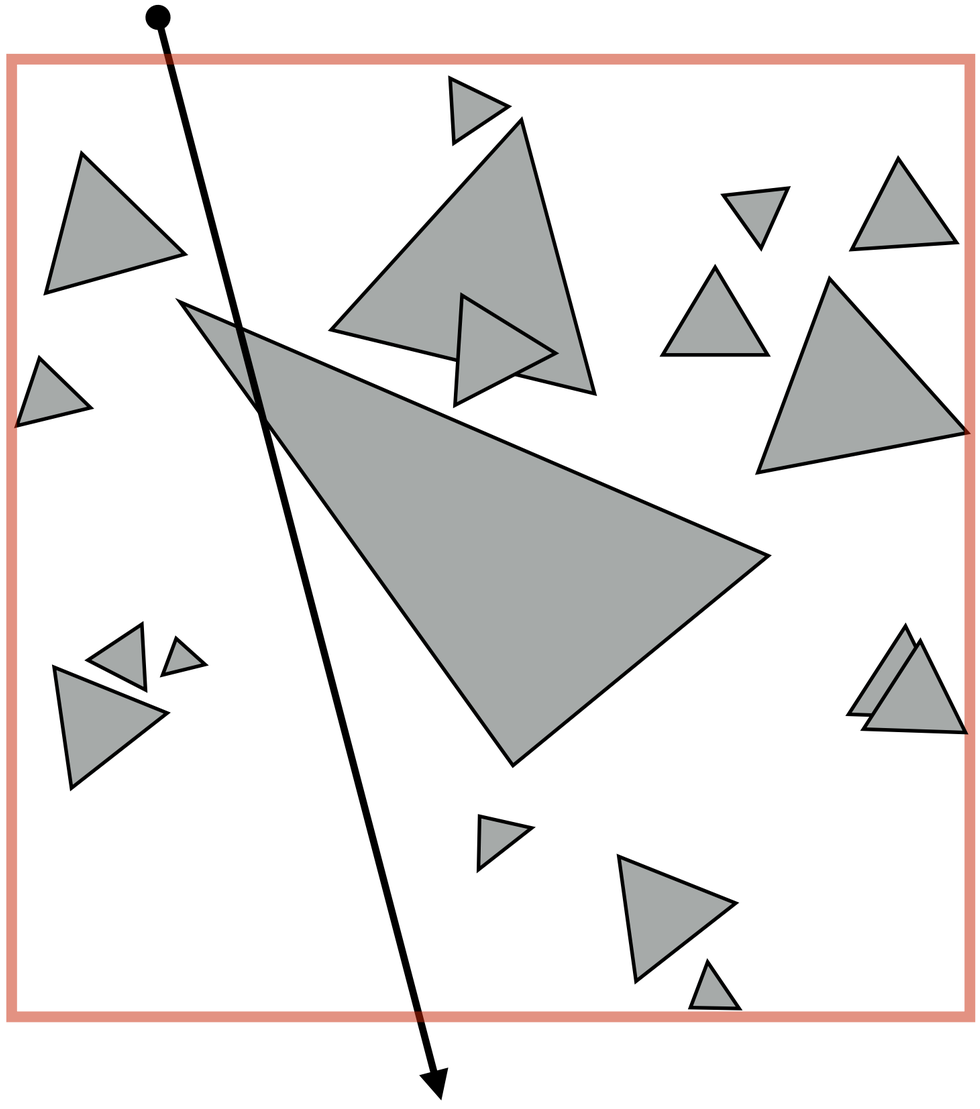
* Mailboxing can be used to avoid repeated intersections

Uniform grid

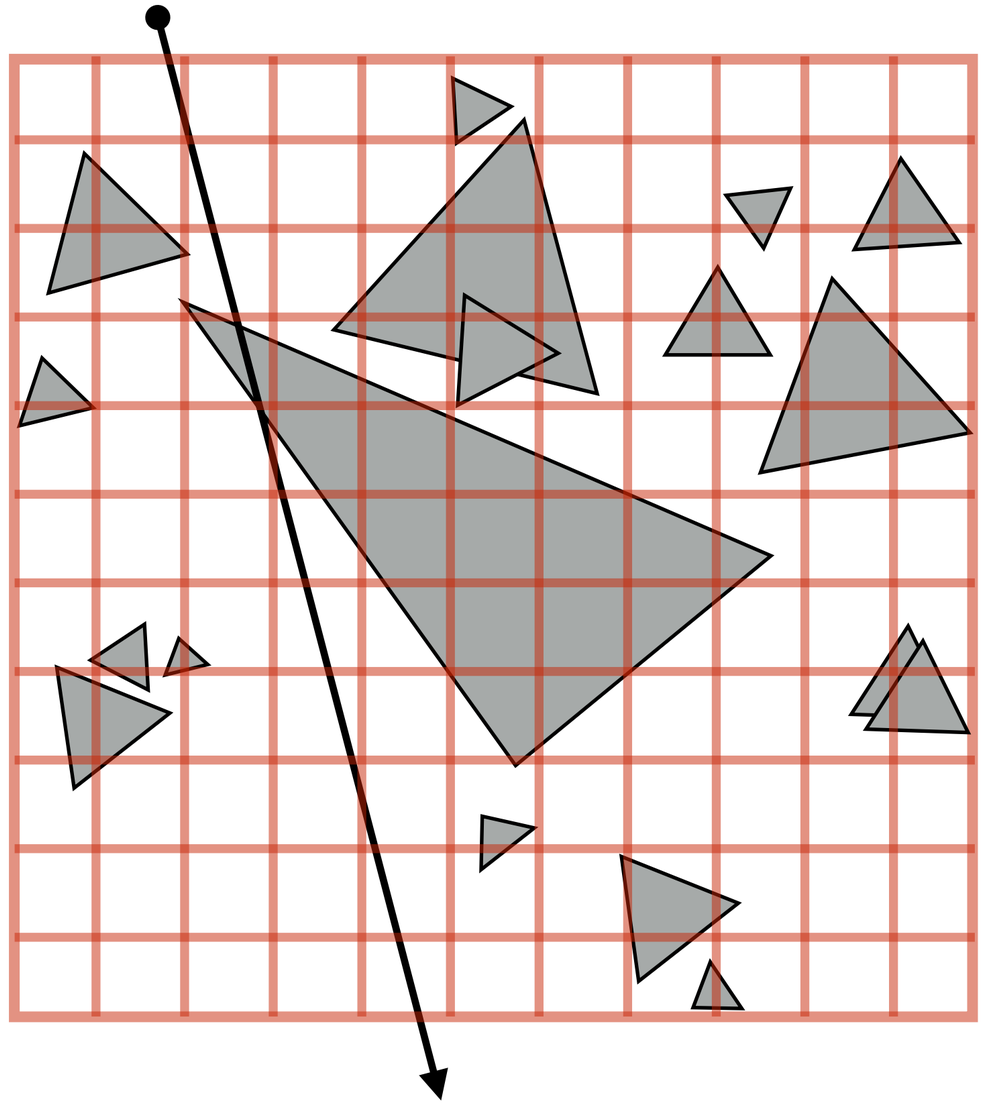


- Partition space into equal sized volumes (“voxels”)
- Each grid cell contains primitives that overlap voxel. (very cheap to construct acceleration structure)
- Walk ray through volume in order
  - Very efficient implementation possible (think: 3D line rasterization)
  - Only consider intersection with primitives in voxels the ray intersects

What should the grid resolution be?



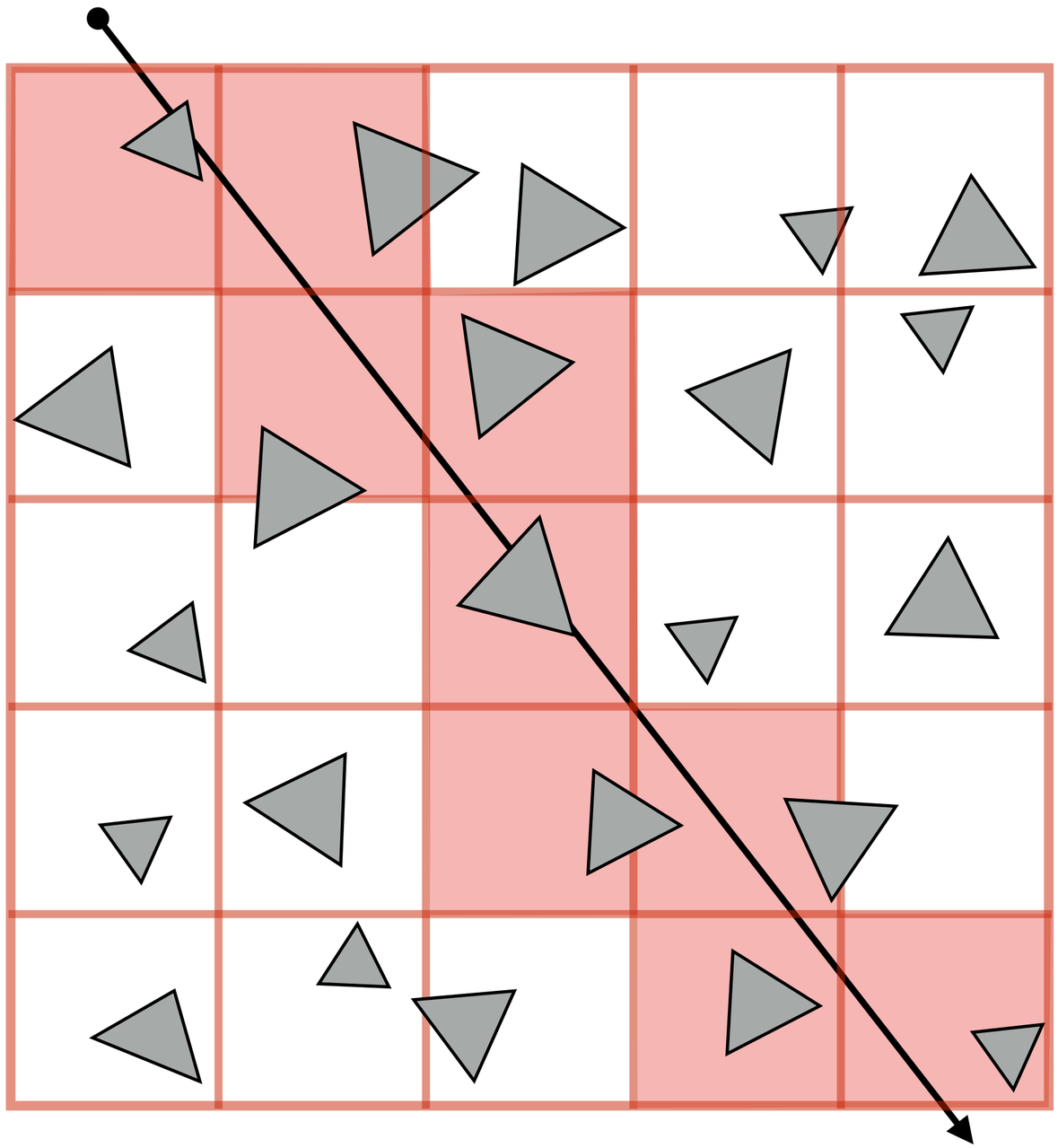
Too few grids cell: degenerates to brute-force approach



Too many grid cells: incur significant cost traversing through cells with empty space

Heuristic

- **Choose number of voxels ~ total number of primitives**
  (constant prims per voxel — assuming uniform distribution of primitives)



Intersection cost: $O(\sqrt[3]{N})$

Uniform distribution of primitives

Terrain / height fields:

[Image credit: Misuba Renderer]

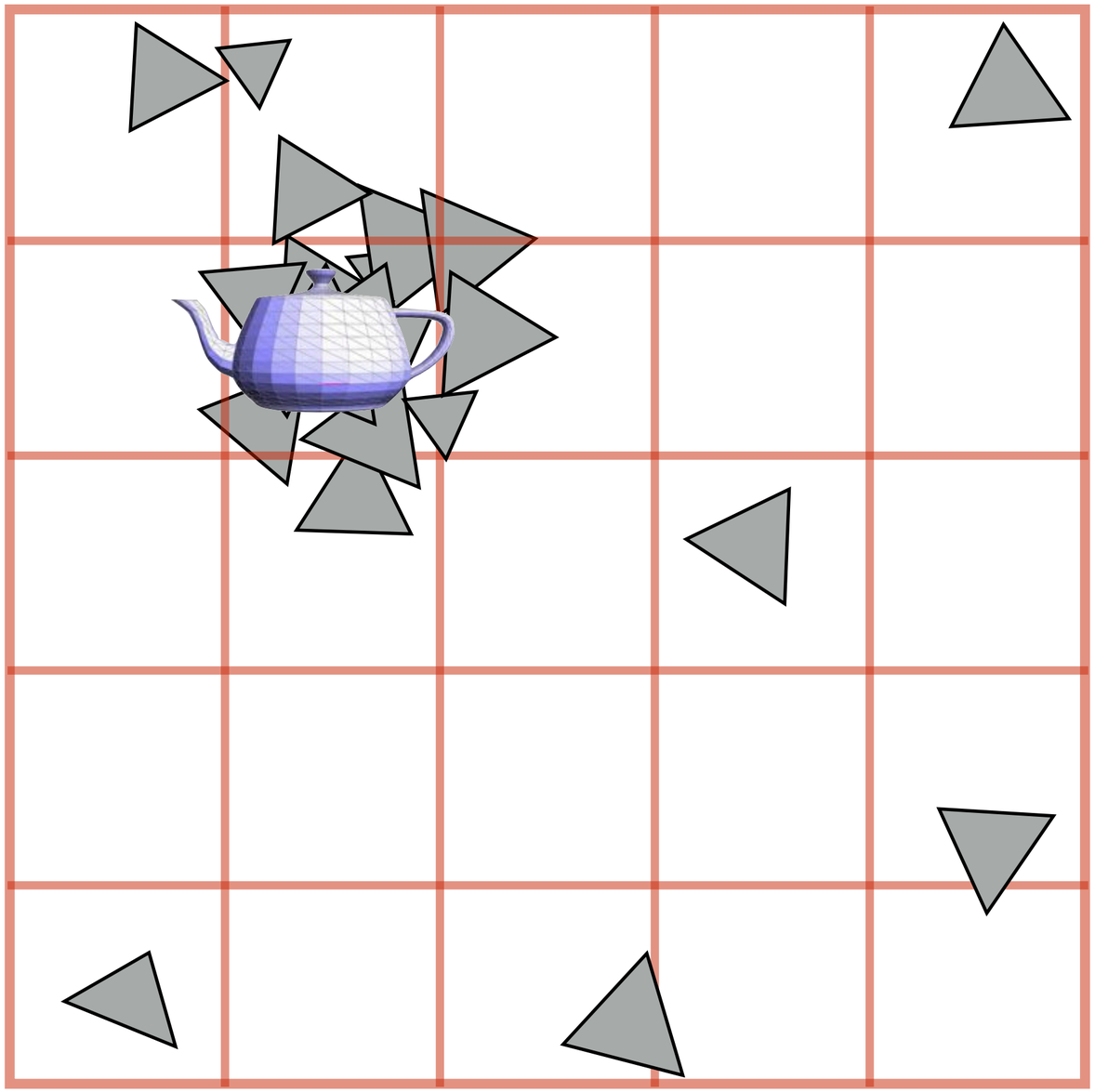
Grass:

[Image credit: www.kevinboulanger.net/grass.html]

Example credit: Pat Hanrahan

Uniform grid cannot adapt to non-uniform distribution of geometry in scene

(Unlike K-D tree, location of spatial partitions is not dependent on scene geometry)



"Teapot in a stadium problem"

- **Scene has large spatial extent.**
- **Contains a high-resolution object that has small spatial extent (ends up in one grid cell)**

Non-uniform distribution of geometric detail

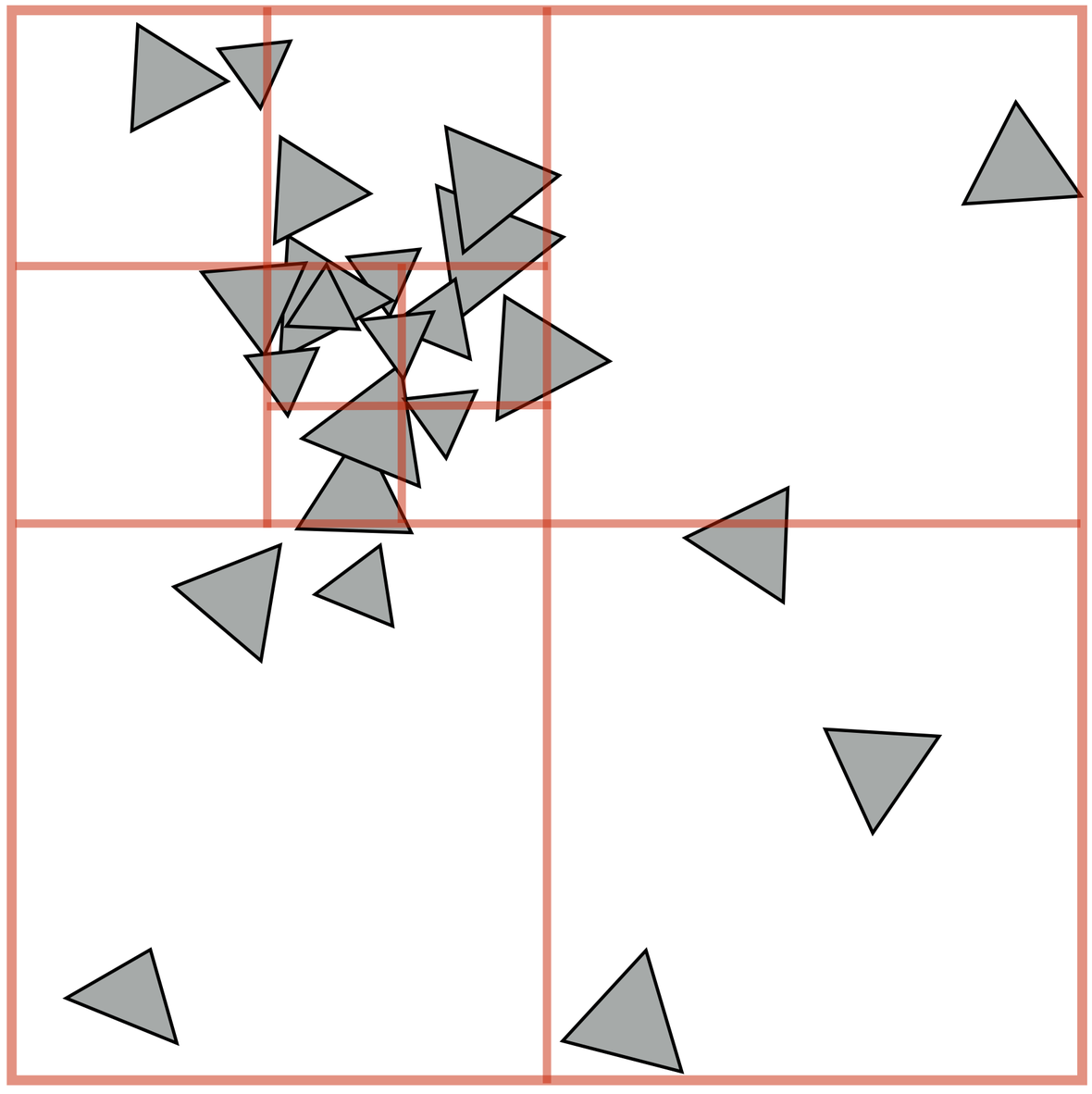
[Image credit: Pixar]

Quad-tree / octree

Like uniform grid: easy to build (don't have to choose partition planes)

Has greater ability to adapt to location of scene geometry than uniform grid.

But lower intersection performance than K-D tree (only limited ability to adapt)



Quad-tree: nodes have 4 children (partitions 2D space)

Octree: nodes have 8 children (partitions 3D space)

Summary of accelerating geometric queries: choose the right structure for the job

- Primitive vs. spatial partitioning:
  - Primitive partitioning: partition sets of objects
    - Bounded number of BVH nodes, simpler to update if primitives in scene change position
  - Spatial partitioning: partition space
    - Traverse space in order (first intersection is closest intersection), may intersect primitive multiple times

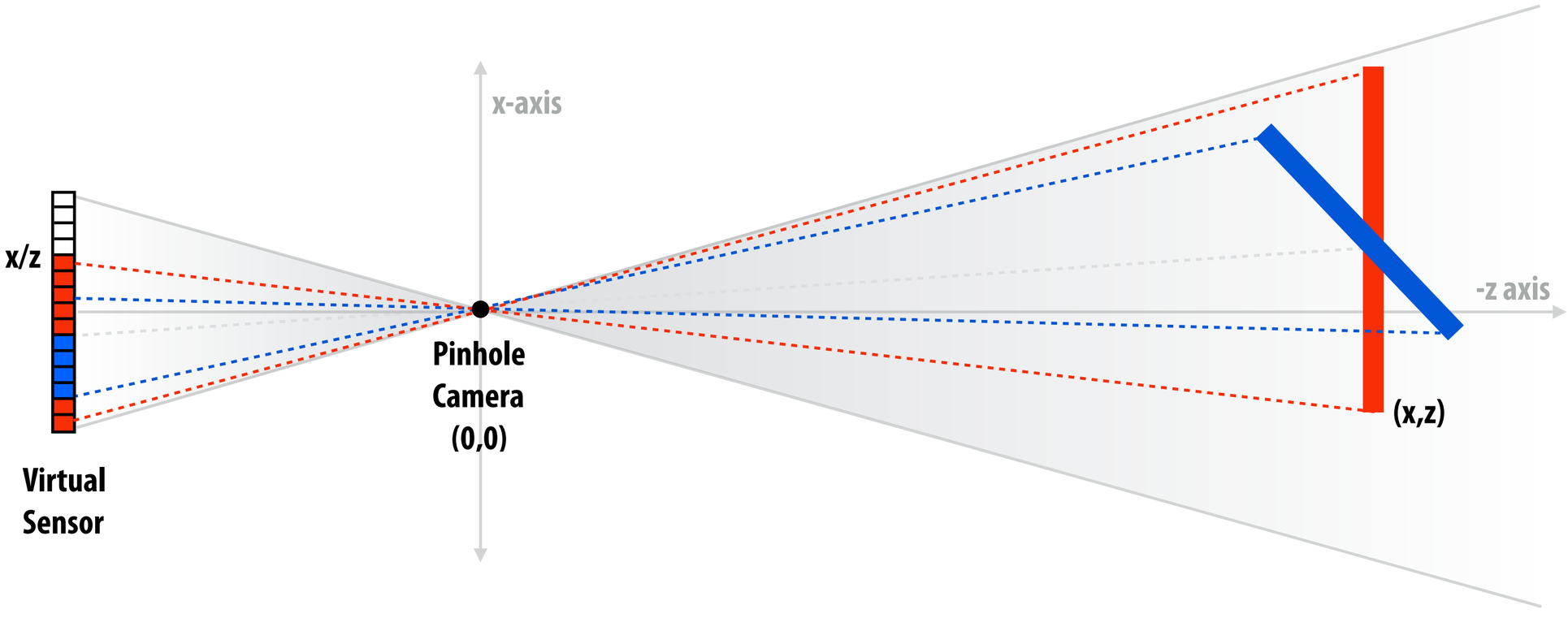
- Adaptive structures (BVH, K-D tree)
  - More costly to construct (must be able to amortize construction over many geometric queries)
  - Better intersection performance under non-uniform distribution of primitives

- Non-adaptive accelerations structures (uniform grids)
  - Simple, cheap to construct
  - Good intersection performance if scene primitives are uniformly distributed

- Many, many combinations thereof

The visibility problem

- **What scene geometry is visible at each screen sample?**
  - What scene geometry projects into a screen pixel? (coverage)
  - Which geometry is visible from the camera at that pixel? (occlusion)



Basic rasterization algorithm

Sample = 2D point

Coverage: 2D triangle/sample tests (does projected triangle cover 2D sample point)

Occlusion: depth buffer

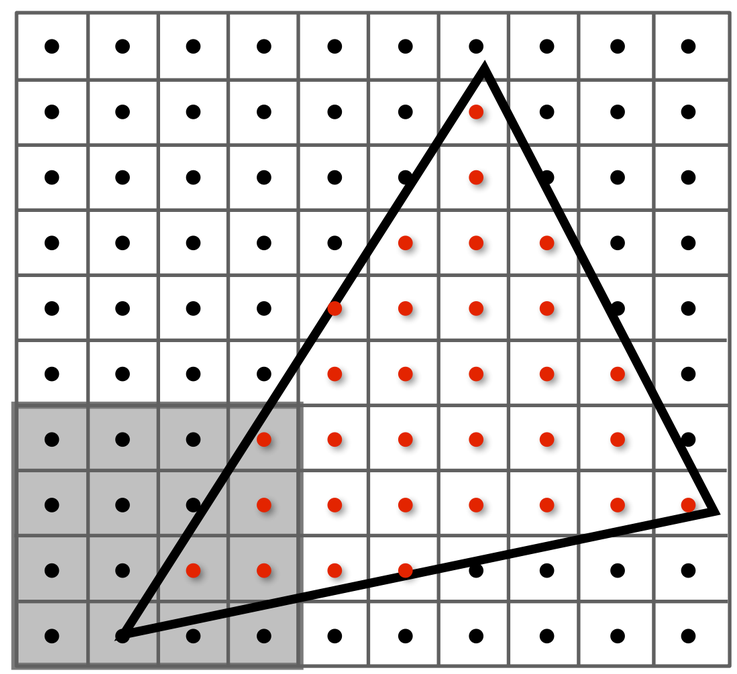
```
initialize z_closest[] to INFINITY              // store closest-surface-so-far for all samples
initialize color[]                              // store scene color for all samples
for each triangle t in scene:                   // loop 1: triangles
    t_proj = project_triangle(t)
    for each 2D sample s in frame buffer:       // loop 2: visibility samples
        if (t_proj covers s)
            compute color of triangle at sample
            if (depth of t at s is closer than z_closest[s])
                update z_closest[s] and color[s]
```

"Given a triangle, find the samples it covers"

(finding the samples is relatively easy since they are distributed uniformly on screen)

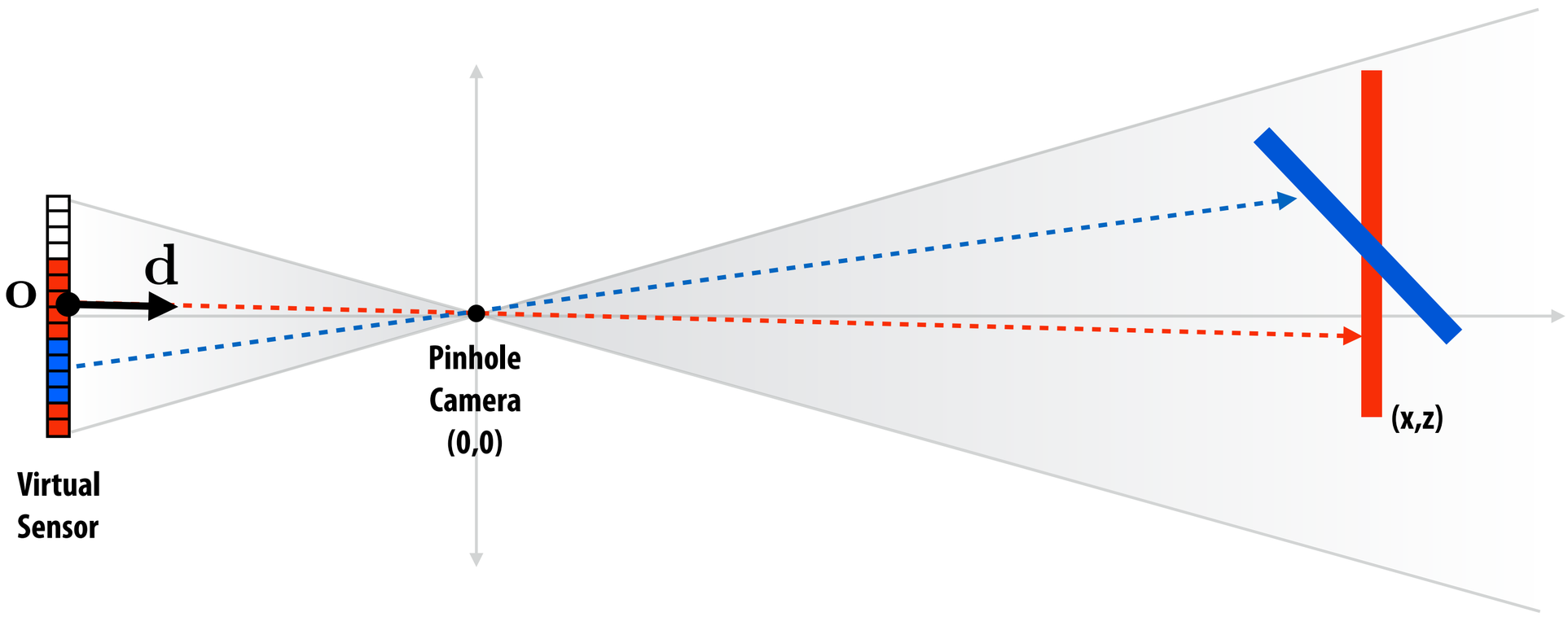
But what from this lecture do modern hierarchical rasterization algorithms remind you of?

(for each tile of image, if triangle overlaps tile, check all samples in tile)



The visibility problem (described differently)

- **In terms of casting rays from the camera:**
  - **What scene primitive is hit by a ray originating from a point on the virtual sensor and traveling through the aperture of the pinhole camera? (coverage)**
  - **What primitive is the first hit along that ray? (occlusion)**



Basic ray casting algorithm

Sample = a ray in 3D

Coverage: 3D ray-triangle intersection tests (does ray "hit" triangle)

Occlusion: closest intersection along ray

```
initialize color[]                                  // store scene color for all samples
for each sample s in frame buffer:                  // loop 1: visibility samples (rays)
    r = ray from s on sensor through pinhole aperture
    r.min_t = INFINITY                              // only store closest-so-far for current ray
    r.tri = NULL;
    for each triangle tri in scene:                   // loop 2: triangles
        if (intersects(r, tri)) {                     // 3D ray-triangle intersection test
            if (intersection distance along ray is closer than r.min_t)
                update r.min_t and r.tri = tri;
        }
    color[s] = compute surface color of triangle r.tri at hit point
```

Compared to rasterization approach: just a reordering of the loops! (+ math in 3D)

"Given a ray, find the closest triangle it hits"

As we saw today, the brute force "for each triangle" loop is typically accelerated using an acceleration structure. (A rasterizer's "for each sample" inner loop is not just a loop over all screen samples either.)

Basic rasterization vs. ray casting

- **Rasterization:**
  - Proceeds in triangle order (never have to store in entire scene, naturally supports unbounded size scenes)
  - Store depth buffer (random access to regular structure of fixed size)
- **Ray casting:**
  - Proceeds in screen sample order
    - Never have to store closest depth so far for the entire screen (just current ray)
    - Natural order for rendering transparent surfaces (process surfaces in the order the are encountered along the ray: front-to-back or back-to-front)
  - Must store entire scene (random access to irregular structure of variable size: depends on complexity and distribution of scene)
- **Modern high-performance implementations of rasterization and ray-casting embody very similar techniques**
  - Hierarchies of rays/samples
  - Hierarchies of geometry

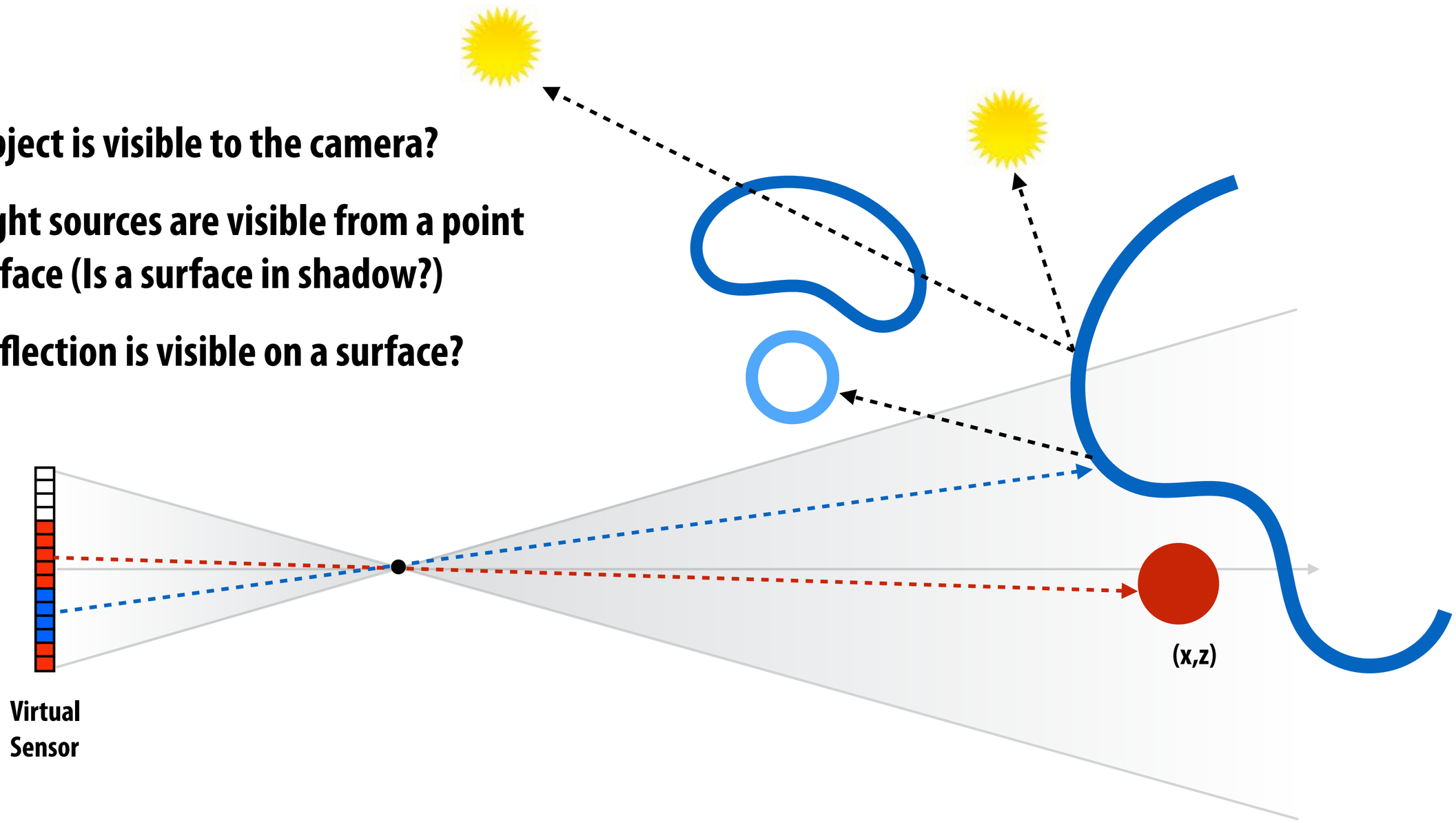
Ray-scene intersection is a general visibility primitive:

What object is visible along this ray?

What object is visible to the camera?

What light sources are visible from a point on a surface (Is a surface in shadow?)

What reflection is visible on a surface?



(In contrast, rasterization is a highly-specialized solution for computing visibility for a set of uniformly distributed rays originating from the same point.)

What you should know:

- Compute ray-triangle intersection, including checking whether the ray passed through the inside of the triangle.
- Compute ray - bounding box intersection
- Construct a bounding box hierarchy for a given collection of objects.
- Calculate traversal order of a bounding box hierarchy for a given ray.
- What is the Surface Area Heuristic (SAH) and what goals is it trying to achieve?
- Explain how to choose a bounding box partition using the SAH
- Be able to distinguish between object-centric (primitive partitioning) acceleration structures and space-centric (space-partitioning) acceleration structures
- Know the difference between these acceleration structures, how to build them, how to traverse them, and when to use each type:
  - bounding box and bounding sphere hierarchies
  - KD-trees
  - octrees
  - grids