Lecture 16:
Reflection

Computer Graphics
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Slides credit: a majority of these slides were created by Matt Pharr and Pat Hanrahan
Ray tracer measures radiance along a ray

In the ray tracing algorithms we’ve discussed so far: want to measure radiance traveling in the direction opposite the ray direction.
Ray tracer measures radiance along a ray

Radiance entering camera in direction $d =$ light from scene light sources that is reflected off surface in direction $d$. 
Reflection models

- Definition: reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident (same) side without change in frequency.
Categories of reflection functions

- **Ideal specular**
  Perfect mirror

- **Ideal diffuse**
  Uniform reflection in all directions

- **Glossy specular**
  Majority of light distributed in reflection direction

- **Retro-reflective**
  Reflects light back toward source

Diagrams illustrate how incoming light energy from given direction is reflected in various directions.
Materials: diffuse
Materials: plastic
Materials: red semi-gloss paint
Materials: Ford mystic lacquer paint
Materials: mirror
Materials: gold
Materials
Bidirectional reflectance distribution function (BRDF)

\[
dL_r(x, \omega_r) \propto dE_i(\omega_i)
\]

Differential irradiance landing on surface from differential cone of directions \(\omega_i\)
\[
dE(\omega_i) = dL(\omega_i) \cos \theta_i
\]

Differential radiance reflected in direction \(\omega_r\) (due to differential irradiance from \(\omega_i\))
\[
dL_r(\omega_r)
\]
Bidirectional reflectance distribution function (BRDF)

$$f_r(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{dL_i(\omega_i) \cos \theta_i} \left[ \frac{1}{\text{sr}} \right]$$

BRDF defines the fraction of energy arriving from $\omega_i$ that is reflected in the direction $\omega_r$. 
The reflection equation

\[ L_r(x, \omega_r) = \int H^2 f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i \, d\omega_i \]

\[ dL_r(\omega_r) = f_r(\omega_i \rightarrow \omega_r) dL_i(\omega_i) \cos \theta_i \]
Solving the reflection equation

\[ L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i \, d\omega_i \]

- **Basic Monte Carlo estimate:**
  - Generate directions \( \omega_j \) sampled from some distribution \( p(\omega) \)
  - To reduce variance \( p(\omega) \) should match BRDF or incident radiance function
  - Compute the estimator

\[
\frac{1}{N} \sum_{j=1}^{N} f_r(p, \omega_j \rightarrow \omega_r) \frac{L_i(p, \omega_j) \cos \theta_j}{p(\omega_j)}
\]
Estimating reflected light

// Assume:
//   Ray ray hits surface at point hit_p
//   Normal of surface at hit point is hit_n

Vector3D wr = -ray.d;   // outgoing direction
Spectrum Lr = 0.;
for (int i = 0; i < N; ++i) {
    Vector3D wi;        // sample incident light from this direction
    float pdf;          // p(wi)

    generate_sample(brdf, &wi, &pdf);       // generate sample according to brdf

    Spectrum f = brdf->f(wr, wi);
    Spectrum Li = trace_ray(Ray(hit_p, wi));  // compute incoming Li
    Lr += f * Li * fabs(dot(wi, hit_n)) / pdf;
}
return Lr / N;
Properties of BRDFs

- Linearity

- Reciprocity principle

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]
Properties of BRDFs

- Isotropic vs. anisotropic
  - If isotropic, \( f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i) \)
  - Then, from reciprocity,
    \[
    f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)
    \]
Energy conservation

\[ f_r(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{\text{sr}} \right] \]

Outgoing energy cannot exceed incoming energy
(reflection does not create energy)

\[
\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r \ d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i \ d\omega_i}
\]

\[
= \int_{\Omega_r} \int_{\Omega_i} f_r(\omega_i \rightarrow \omega_r) \ L_i(\omega_i) \ \cos \theta_i \ d\omega_i \ \cos \theta_r \ d\omega_r
\]

\[
\leq 1
\]
Energy conservation

Overall fraction of light reflected by surface
(assuming constant incident light from all directions)

\[
\rho = \frac{\int_{H^2} \int_{H^2} f_r(\omega_i \rightarrow \omega_r) C \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{H^2} C \cos \theta_i d\omega_i}
= \frac{1}{\pi} \int_{H^2} \int_{H^2} f_r(\omega_i \rightarrow \omega_r) C \cos \theta_i d\omega_i \cos \theta_r d\omega_r
\leq 1
\]
Hemispherical incident radiance

Consider view of hemisphere from this point
Hemispherical incident radiance

At any point on any surface in the scene, there’s an incident radiance field that gives the directional distribution of illumination at the point

\[ L_i(\omega) = L_i(x, y, \sqrt{1 - x^2 - y^2}) \]
Ideal specular reflection

Incident radiance

Exitant radiance
Diffuse reflection

Incident radiance

Exitant radiance

Exitant radiance is the same in all directions
Plastic

Incident radiance

Exitant radiance
Copper

Incident radiance

Exitant radiance
Perfect specular reflection
Perfect specular reflection

\[ \theta = \theta_o = \theta_i \]

\[ \omega_o + \omega_i = 2 \cos \theta \vec{n} = 2(\omega_i \cdot \vec{n})\vec{n} \]

\[ \omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n} \]
Specular reflection BRDF

\[ L_i(\theta_i, \phi_i) \quad \rightarrow \quad L_o(\theta_o, \phi_o) \]

\[ \theta_i \quad \theta_o \]

\[ L_o(\theta_o, \phi_o) = L_i(\theta_o, \phi_o \pm \pi) \]

\[ f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) \]
Specular reflection and the reflection equation

\[ L_o(\theta_o, \phi_o) = \int f_r(\theta_i, \phi_i; \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos \theta_i \, d\cos \theta_i \, d\phi_i \]
\[ = \int \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) L_i(\theta_i, \phi_i) \cos \theta_i \, d\cos \theta_i \, d\phi_i \]
\[ = L_i(\theta_r, \phi_r \pm \pi) \]

Whitted’s ray tracing method!

![Diagram showing the specular reflection and the reflection equation](image-url)
Transmission

In addition to reflecting off surface, light may be transmitted through surface.

Light refracts when it enters a new medium.
Snell’s Law

Transmitted angle depends on index of refraction of medium incident ray is in and index of refraction of medium light is entering.

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

<table>
<thead>
<tr>
<th>Medium</th>
<th>( \eta ) *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>Air (sea level)</td>
<td>1.00029</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>1.333</td>
</tr>
<tr>
<td>Glass</td>
<td>1.5-1.6</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
</tbody>
</table>

* index of refraction is wavelength dependent (these are averages)
Law of refraction

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

\[ \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} \]

\[ = \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 \sin^2 \theta_i} \]

\[ = \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 (1 - \cos^2 \theta_i)} \]

Total internal reflection:
When light is moving from a more optically dense medium to a less optically dense medium: \( \frac{\eta_i}{\eta_t} > 1 \)

Light incident on boundary from large enough angle will not exit medium.
Optical manhole

Total internal reflection

[Livingston and Lynch]
Fresnel reflection

Reflectance depends on angle of incidence and polarization of light

This example: reflectance increases with grazing angle

[Lafortune et al. 1997]
Fresnel reflection (dielectric, $\eta=1.5$)
Fresnel reflectance (conductor)
Without Fresnel (fixed reflectance/transmission)
Glass with Fresnel reflection/transmission
Lambertian reflection

Assume light is equally likely to be reflected in each output direction

\[ L_o(\omega_o) = \int_{H^2} f_r L_i(\omega_i) \cos \theta_i \, d\omega_i \]

\[ = f_r \int_{H^2} L_i(\omega_i) \cos \theta_i \, d\omega_i \]

\[ = f_r E \]

\[ f_r = \frac{\rho}{\pi} \]
Anisotropic reflection

Reflection depends on azimuthal angle $\phi$

Results from oriented microstructure of surface e.g., brushed metal
Measuring BRDFs
Measuring BRDFs: motivation

- Avoid need to develop / derive models
  - Automatically includes all of the scattering effects present
- Can accurately render with real-world materials
  - Useful for product design, special effects, ...
- Theory vs. practice:

[Bagher et al. 2012]
Measuring BRDFs

- General approach:

  ```
  foreach outgoing direction wo
  move light to illuminate surface with a thin beam from wo
  for each incoming direction wi
  move sensor to be at direction wi from surface
  measure incident radiance
  ```

- Improving efficiency:
  - Isotropic surfaces reduce dimensionality from 4D to 3D
  - Reciprocity reduces # of measurements by half
  - Clever optical systems...
Measuring BRDFs: gonioreflectometer

[Li et al. 2005]
Image-based BRDF measurement

[Marschner et al. 1999]
Challenges in measuring BRDFs

- Accurate measurements at grazing angles
  - Important due to Fresnel effects
- Measuring with dense enough sampling to capture high frequency specularities
- Retro-reflection
- Spatially-varying reflectance, ...
Representing measured BRDFs

- Desirable qualities
  - Compact representation
  - Accurate representation of measured data
  - Efficient evaluation for arbitrary pairs of directions
  - Good distributions available for importance sampling
Tabular representation

- Store regularly-spaced samples in \((\theta_i, \theta_o, |\phi_i - \phi_o|)\)
  - Better: reparameterize angles to better match specularities

- Generally need to resample measured values to table

- Very high storage requirements

MERL BRDF Database
[Matusik et al. 2004]
90*90*180 measurements
Basis functions

- Can fit existing models, e.g. Cook-Torrance, 3 parameters per wavelength, $k_d, k_s, k_e$

$$f_r(\omega_i \rightarrow \omega_o) = k_d + k_s (\vec{h} \cdot \vec{n})^{k_e} \quad \vec{h} = \omega_i + \omega_o$$

- More sophisticated, e.g. [Bagher et al. 2012] 11 parameter model:
Simulation: velvet

[Westin et al. 1992]
Simulation: brushed aluminum

[Westin et al. 1992]
Simulation: nylon

[Westin et al. 1992]
Translucent materials: Jade
Translucent materials: skin
Subsurface scattering

- Visual characteristics of many surfaces caused by light entering at different points than it exits
  - Violates a fundamental assumption of the BRDF

[Jensen et al 2001]

[Donner et al 2008]
Scattering functions

- Generalization of BRDF; describes exitant radiance at one point due to incident differential irradiance at another point:

\[ S(x_i, \omega_i, x_o, \omega_o) \]

- Generalization of reflection equation integrates over all points on the surface and all directions(!)

\[ L(x_o, \omega_o) = \int_A \int_{H^2} S(x_i, \omega_i, x_o, \omega_o) L_i(x_i, \omega_i) \cos \theta_i \ d\omega_i \ dA \]
BRDF
Fiber model

[Marschner et al. 2003]
Hair appearance
Summary

- BRDF describes how light reflects off a surface.
- BRDF defines the fraction of energy incident on surface from direction $\omega_i$ that is reflected in the direction $\omega_r$.
- Light is also transmitted through surfaces.
  - Snell’s Law gives angle of transmitted ray.
  - Amount of light reflected/transmitted is computed via Fresnel equations.
  - Can think of BTDF (bidirectional transmission distribution function) describing directional distribution of transmission.

- Subsurface scattering
  - Light exits surface at different point than it entered (e.g., skin).
What you should know

- Draw diagrams to illustrate reflection for (1) a perfectly specular (mirror) surface, (2) a glossy surface, (3) a diffuse surface, (4) a retroreflective surface.
- What is a BRDF? What are the inputs and outputs of a BRDF function?
- How would you measure a BRDF? How many degrees of freedom would you need?
- Given a ray and a surface normal, calculate the direction of perfect reflection.
- Given a ray, a surface normal, and indices of refraction, calculate the direction of perfect transmission using Snell’s Law.
- What is Fresnel reflection? Sketch curves to illustrate the effect for dielectrics vs. conductors as we have seen in class. Label your axes. Informally, what does this effect show?
- What is subsurface scattering?
- How can we extend the idea of BRDF to subsurface scattering? What additional parameters must be sampled?