Lecture 8:
The Rasterization Pipeline
(and its implementation on GPUs)

Computer Graphics
CMU 15-462/15-662, Spring 2016
What you know how to do (at this point in the course)

Position objects and the camera in the world

Determine the position of objects relative to the camera

Project objects onto the screen

Sample triangle coverage

Compute triangle attribute values at covered sample points

Sample texture maps
What else do you need to know to render a picture like this?

**Surface representation**
How to represent complex surfaces?

**Occlusion**
Determining which surface is visible to the camera at each sample point

**Lighting/materials**
Describing lights in scene and how materials reflect light.
Course roadmap

Drawing Things

- Key concepts:
  - Sampling (and anti-aliasing)
  - Coordinate Spaces and Transforms

- Introduction
- Drawing a triangle (by sampling)
- Transforms and coordinate spaces
- Perspective projection and texture sampling
- Today: putting it all together: end-to-end rasterization pipeline

Geometry

Materials and Lighting
Occlusion
Occlusion: which triangle is visible at each covered sample point?

Opaque Triangles

50% transparent triangles
Review from last class

Assume we have a triangle defined by the screen-space 2D position and distance (“depth”) from the camera of each vertex.

\[
\begin{bmatrix}
\mathbf{p}_{0x} & \mathbf{p}_{0y}
\end{bmatrix}^T, \quad d_0
\]
\[
\begin{bmatrix}
\mathbf{p}_{1x} & \mathbf{p}_{1y}
\end{bmatrix}^T, \quad d_1
\]
\[
\begin{bmatrix}
\mathbf{p}_{2x} & \mathbf{p}_{2y}
\end{bmatrix}^T, \quad d_2
\]

How do we compute the depth of the triangle at covered sample point \((x, y)\)?

Interpolate it just like any other attribute that varies linearly over the surface of the triangle.
Occlusion using the depth-buffer (Z-buffer)

For each coverage sample point, depth-buffer stores depth of closest triangle at this sample point that has been processed by the renderer so far.

Closest triangle at sample point \((x,y)\) is triangle with minimum depth at \((x,y)\)

Initial state of depth buffer before rendering any triangles (all samples store farthest distance)

Grayscale value of sample point used to indicate distance

Black = small distance
White = large distance
Depth buffer example
Example: rendering three opaque triangles
Occlusion using the depth-buffer (Z-buffer)

Processing yellow triangle:
depth = 0.5

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test

Color buffer contents

Depth buffer contents
Occlusion using the depth-buffer (Z-buffer)

After processing yellow triangle:

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<tr>
<th>Color buffer contents</th>
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Grayscale value of sample point used to indicate distance:
White = large distance
Black = small distance
Red = sample passed depth test

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Occlusion using the depth-buffer (Z-buffer)

Processing blue triangle:
depth = 0.75

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test

Color buffer contents

Depth buffer contents
# Occlusion using the depth-buffer (Z-buffer)

After processing blue triangle:

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Occlusion using the depth-buffer (Z-buffer)

Processing red triangle:
depth = 0.25

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = sample passed depth test

Color buffer contents

Depth buffer contents
Occlusion using the depth-buffer (Z-buffer)

After processing red triangle:

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Grayscale value of sample point used to indicate distance:
White = large distance
Black = small distance
Red = sample passed depth test
Occlusion using the depth buffer

```cpp
bool pass_depth_test(d1, d2) {
  return d1 < d2;
}

depth_test(tri_d, tri_color, x, y) {
  if (pass_depth_test(tri_d, zbuffer[x][y])) {
    // triangle is closest object seen so far at this
    // sample point. Update depth and color buffers.
    zbuffer[x][y] = tri_d; // update zbuffer
    color[x][y] = tri_color; // update color buffer
  }
}```
Does depth-buffer algorithm handle interpenetrating surfaces?

Of course!

Occlusion test is based on depth of triangles at a given sample point. The relative depth of triangles may be different at different sample points.

Green triangle in front of yellow triangle

Yellow triangle in front of green triangle
Does depth-buffer algorithm handle interpenetrating surfaces?

Of course!

Occlusion test is based on depth of triangles at a given sample point. The relative depth of triangles may be different at different sample points.
Does depth buffer work with super sampling?

Of course! Occlusion test is per sample, not per pixel!

This example: green triangle occludes yellow triangle
Color buffer contents
Color buffer contents (4 samples per pixel)

```
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<tr>
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<tbody>
<tr>
<td><img src="image1.png" alt="Sample 1" /></td>
<td><img src="image2.png" alt="Sample 2" /></td>
<td><img src="image3.png" alt="Sample 3" /></td>
<td><img src="image4.png" alt="Sample 4" /></td>
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<tr>
<td><img src="image5.png" alt="Sample 5" /></td>
<td><img src="image6.png" alt="Sample 6" /></td>
<td><img src="image7.png" alt="Sample 7" /></td>
<td><img src="image8.png" alt="Sample 8" /></td>
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<td><img src="image9.png" alt="Sample 9" /></td>
<td><img src="image10.png" alt="Sample 10" /></td>
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<td><img src="image12.png" alt="Sample 12" /></td>
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<tr>
<td><img src="image13.png" alt="Sample 13" /></td>
<td><img src="image14.png" alt="Sample 14" /></td>
<td><img src="image15.png" alt="Sample 15" /></td>
<td><img src="image16.png" alt="Sample 16" /></td>
</tr>
</tbody>
</table>
```
Final resampled result

Note anti-aliasing of edge due to filtering of green and yellow samples.
Summary: occlusion using a depth buffer

- Store one depth value per coverage sample (not per pixel!)

- Constant space per sample
  - Implication: constant space for depth buffer

- Constant time occlusion test per covered sample
  - Read-modify write of depth buffer if “pass” depth test
  - Just a read if “fail”

- Not specific to triangles: only requires that surface depth can be evaluated at a screen sample point

But what about semi-transparent surfaces?
Compositing
Representing opacity as alpha

Alpha describes the opacity of an object
- Fully opaque surface: $\alpha = 1$
- 50% transparent surface: $\alpha = 0.5$
- Fully transparent surface: $\alpha = 0$

Red triangle with decreasing opacity
Alpha: additional channel of image (rgba)

\[ \alpha \text{ of foreground object} \]
Over operator:

Composite image B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

A over B $\neq$ B over A
“Over” is not commutative
Over operator: non-premultiplied alpha

Composite image B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

A first attempt:

\[
A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T \\
B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T
\]

Composited color:

\[
C = \alpha_B B + (1 - \alpha_B) \alpha_A A
\]

Appearance of semi-transparent A

A over B ≠ B over A

“Over” is not commutative
Over operator: premultiplied alpha

Composite image B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

Non-premultiplied alpha:

$$A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T$$
$$B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T$$
$$C' = \alpha_B B + (1 - \alpha_B)\alpha_A A$$

Premultiplied alpha:

$$A' = \begin{bmatrix} \alpha_A A_r & \alpha_A A_g & \alpha_A A_b & \alpha_A \end{bmatrix}^T$$
$$B' = \begin{bmatrix} \alpha_B B_r & \alpha_B B_g & \alpha_B B_b & \alpha_B \end{bmatrix}^T$$
$$C'' = B + (1 - \alpha_B)A$$

Composite alpha:

$$\alpha_C = \alpha_B + (1 - \alpha_B)\alpha_A$$

Notice premultiplied alpha composites alpha just like how it composites rgb.
Non-premultiplied alpha composites alpha differently than rgb.
Applying “over” repeatedly

Composite image C with opacity $\alpha_C$ over B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

Non-premultiplied alpha is not closed under composition:

$$A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T$$
$$B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T$$
$$C = \alpha_B B + (1 - \alpha_B)\alpha_A A$$
$$\alpha_C = \alpha_B + (1 - \alpha_B)\alpha_A$$

Consider result of compositing 50% red over 50% red:

$$C = \begin{bmatrix} 0.75 & 0 & 0 \end{bmatrix}^T$$
$$\alpha_C = 0.75$$

Wait… this result is the premultiplied color!

“Over” for non-premultiplied alpha takes non-premultiplied colors to premultiplied colors (“over” operation is not closed)

Cannot compose “over” operations on non-premultiplied values: $\overline{\overline{C, \overline{B, A}}}$

Closed form of non-premultiplied alpha:

$$C = \frac{1}{\alpha_C} (\alpha_B B + (1 - \alpha_B)\alpha_A A)$$
Another problem with non-premultiplied alpha

Consider pre-filtering a texture with an alpha matte

Desired filtered result

Downsampling non-premultiplied alpha image results in 50% opaque brown

0.25 * ((0, 1, 0, 1) + (0, 1, 0, 1) + (0, 0, 0, 0) + (0, 0, 0, 0)) = (0, 0.5, 0, 0.5)

Result of filtering premultiplied image

Composited over white

Input color

Filtered result

Input alpha

Filtered color

Filtered alpha

Filtered result
Summary: advantages of premultiplied alpha

- Simple: compositing operation treats all channels (rgb and a) the same
- More efficient than non-premultiplied representation: “over” requires fewer math ops
- Closed under composition
- Better representation for filtering textures with alpha channel
Color buffer update: semi-transparent surfaces

Color buffer values and tri_color are represented with premultiplied alpha

\[
\text{over}(c_1, c_2) \{
\text{return } c_1 + (1-c_1.a) * c_2;
\}
\]

\[
\text{update_color_buffer}(\text{tri}_d, \text{tri}_color, x, y) \{
\text{if } \text{pass_depth_test}(\text{tri}_d, \text{zbuffer}[x][y]) \{
\text{// update color buffer}
\text{// Note: no depth buffer update}
\text{color}[x][y] = \text{over}(\text{tri}_color, \text{color}[x][y]);
\}
\}
\]

What is the assumption made by this implementation?

**Triangles must be rendered in back to front order!**

What if triangles are rendered in front to back order?

**Modify code:** \( \text{over}(\text{color}[x][y], \text{tri}_color) \)
Putting it all together

Consider rendering a mixture of opaque and transparent triangles

Step 1: render opaque surfaces using depth-buffered occlusion
   If pass depth test, triangle overwrites value in color buffer at sample

Step 2: disable depth buffer update, render semi-transparent surfaces in back-to-front order.
   If pass depth test, triangle is composited OVER contents of color buffer at sample
End-to-end rasterization pipeline ("real-time graphics pipeline")
Command: draw these triangles!

Inputs:

\[
\text{list\_of\_positions} = \{ v0x, v0y, v0z, \\
v1x, v1y, v1z, \\
v2x, v2y, v2z, \\
v3x, v3y, v3z, \\
v4x, v4y, v4z, \\
v5x, v5y, v5z \};
\]

\[
\text{list\_of\_texcoords} = \{ v0u, v0v, \\
v1u, v1v, \\
v2u, v2v, \\
v3u, v3v, \\
v4u, v4v, \\
v5u, v5v \};
\]

Object-to-camera-space transform: \( T \)

Perspective projection transform \( P \)

Size of output image \( (W, H) \)

Use depth test / update depth buffer: YES!
Step 1:
Transform triangle vertices into camera space
Step 2:
Apply perspective projection transform to transform triangle vertices into normalized coordinate space

Camera-space positions: 3D

Normalized space positions

Note: I’m illustrating normalized 3D space after the homogeneous divide, it is more accurate to think of this volume in 3D-H space as defined by:
(-w, -w, -w, w) and (w, w, w, w)
Step 3: clipping

- Discard triangles that lie complete outside the unit cube (culling)
  - They are off screen, don’t bother processing them further
- Clip triangles that extend beyond the unit cube to the cube
  - Note: clipping may create more triangles

Triangles before clipping

Triangles after clipping
Step 4: transform to screen coordinates

Transform vertex xy positions from normalized coordinates into screen coordinates (based on screen w,h)
Step 5: setup triangle (triangle preprocessing)

Compute triangle edge equations
Compute triangle attribute equations

\[ E_{01}(x, y) \quad U(x, y) \]
\[ E_{12}(x, y) \quad V(x, y) \]
\[ E_{20}(x, y) \]
\[ \frac{1}{w}(x, y) \]
\[ Z(x, y) \]
Step 6: sample coverage

Evaluate attributes $z, u, v$ at all covered samples
Step 6: compute triangle color at sample point

e.g., sample texture map *

* So far, we’ve only described computing triangle’s color at a point by interpolating per-vertex colors, or by sampling a texture map. Later in the course, we’ll discuss more advanced algorithms for computing its color based on material properties and scene lighting conditions.
Step 7: perform depth test (if enabled)

Also update depth value at covered samples (if necessary)
Step 8: update color buffer (if depth test passed)
OpenGL/Direct3D graphics pipeline *

Structures rendering computation as a series of operations on vertices, primitives, fragments, and screen samples

Operations on vertices
- **Vertex Processing**
  - Vertex stream

Operations on primitives (triangles, lines, etc.)
- **Primitive Processing**
  - Primitive stream

Operations on fragments
- **Fragment Generation (Rasterization)**
  - Fragment stream
- **Fragment Processing**
  - Shaded fragment stream

Operations on screen samples
- **Screen sample operations (depth and color)**

* Several stages of the modern OpenGL pipeline are omitted
OpenGL/Direct3D graphics pipeline *

**Operations on vertices**
- **Vertex Processing**
  - Vertex stream

**Operations on primitives (triangles, lines, etc.)**
- **Primitive Processing**
  - Primitive stream

**Operations on fragments**
- **Fragment Generation (Rasterization)**
  - Fragment stream

**Operations on screen samples**
- **Fragment Processing**
  - Shaded fragment stream
- **Screen sample operations (depth and color)**

### Pipeline inputs:
- Input vertex data
- Parameters needed to compute position on vertices in normalized coordinates (e.g., transform matrices)
- Parameters needed to compute color of fragments (e.g., textures)
- “Shader” programs that define behavior of vertex and fragment stages

* several stages of the modern OpenGL pipeline are omitted
Shader programs

Define behavior of vertex processing and fragment processing stages
Describe operation on a single vertex (or single fragment)

Example GLSL fragment shader program

```glsl
uniform sampler2D myTexture;
uniform vec3 lightDir;
varying vec2 uv;
varying vec3 norm;

void diffuseShader()
{
  vec3 kd;
  kd = texture2d(myTexture, uv);
  kd *= clamp(dot(-lightDir, norm), 0.0, 1.0);
  gl_FragColor = vec4(kd, 1.0);
}
```

Shader function executes once per fragment.

Outputs color of surface at sample point corresponding to fragment.

(this shader performs a texture lookup to obtain the surface's material color at this point, then performs a simple lighting computation)
Goal: render very high complexity 3D scenes

- 100’s of thousands to millions of triangles in a scene
- Complex vertex and fragment shader computations
- High resolution screen outputs (2-4 Mpixel + supersampling)
- 30-60 fps
Graphics pipeline implementation: GPUs

Specialized processors for executing graphics pipeline computations

Discrete GPU card (NVIDIA GeForce Titan X)

Integrated GPU: part of modern Intel CPU die
Modern GPUs offer ~2-4 TFLOPs of performance for executing vertex and fragment shader programs.
Summary

- Occlusion resolved independently at each screen sample using the depth buffer

- Alpha compositing for semi-transparent surfaces
  - Premultiplied alpha forms simply repeated composition
  - “Over” compositing operations is not commutative: requires triangles to be processed in back-to-front (or front-to-back) order

- Graphics pipeline:
  - Structures rendering computation as a sequence of operations performed on vertices, primitives (e.g., triangles), fragments, and screen samples
  - Behavior of parts of the pipeline is application-defined using shader programs.
  - Pipeline operations implemented by highly, optimized parallel processors and fixed-function hardware (GPUs)