Alternate Final Exam

105 points
Closed book, closed notes, no calculators. Use the back of the sheets for extra space.
Write down your assumptions and steps so that I can give you partial credit.

1. (10 points) Homogeneous coordinates.
   a. How do we represent a point in 3D space using homogeneous coordinates?

   b. How do we represent a vector (a direction) in 3D space using homogeneous coordinates?

   c. Why do we use this representation?

2. (10 points) Implicit form of a line. An implicit equation for a line through the 2D points
   \((x_0, y_0)\) and \((x_1, y_1)\) is given below. Prove that this equation is correct.
   \[
   f(x, y) = (y_1 - y_0)x + (x_0 - x_1)y + x_1y_0 - x_0y_1
   \]
3. **(5 points) Parametric form of a line.** Write the parametric form for the line through the 2D points \((x_0, y_0)\) and \((x_1, y_1)\).

4. **(25 points) Direct illumination and ray tracing.** We have used an equation similar to the following to compute an image using ray tracing.

\[
I = k_a L_a + k_d (l \cdot n)L_d + k_s (r \cdot v)\alpha L_s + k_r I_r + k_l I_l
\]

a. Sketch the \(l, n, r,\) and \(v\) vectors and briefly define each one.

b. What terms in this equation are properties of the surface material of objects in the scene?

c. What terms in this equation are properties of the light source(s)?

d. What is the role of parameter \(\alpha\)?
e. Why do we include an ambient light term?

f. Give an argument for why \( k_s \) and \( k_r \) should be equal.

g. Give an argument for why \( k_s \) and \( k_r \) should not be equal.

5. (10 points) Spatial data structures.

a. What is a quad tree data structure?
b. Referring to the image above, use a quad tree data structure to build a bounding box hierarchy for the objects in the scene. Draw the tree representing this hierarchy, clearly showing where each of the objects belongs in this tree. Continue subdividing until further subdivisions do not provide any benefit in discriminating between objects.

6. (15 points) Ray tracing and radiosity. Given the choice of a ray tracing or a radiosity algorithm, which would you use in each of the following situations? Explain your answer in each case. If you believe that either algorithm (or neither algorithm) would be fine, say so and explain why.
   a. to render a glass of water
   
   b. for an architectural walkthrough
   
   c. to illustrate color bleeding
   
   d. for a scene that contains several mirrors
   
   e. to render a patch of sunlight where rays from the sun come directly through the window and hit the floor
7. **(20 points) Physically based simulation**
   a. Draw a system block diagram for a physically based simulation of a single particle acting under external forces. Your diagram should illustrate user input to set the initial state of the system, integration forward in time, graphical display of outputs, and additional input in the form of external forces, which may vary with time and state of the system.
   
   b. What variables are required to represent the state of the particle at any timestep?
   
   c. Give equations to advance the state of the system at each timestep using Euler integration. Please define all of your terms.
   
   d. What limitations of Euler integration may be improved by using higher order integrators such as RK4? Explain your answers.
8. (10 points) Animation. There are three techniques typically used for animation: keyframing, procedural animation (including physically based simulation), and motion capture. List the pros and cons of these three techniques for the purpose of animating human characters. Give at least one pro and one con for each.