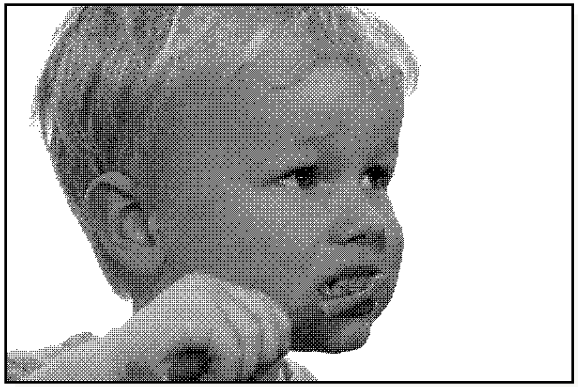
Image Processing

Overview

Images

Pixel Filters

Neighborhood Filters



Dithering

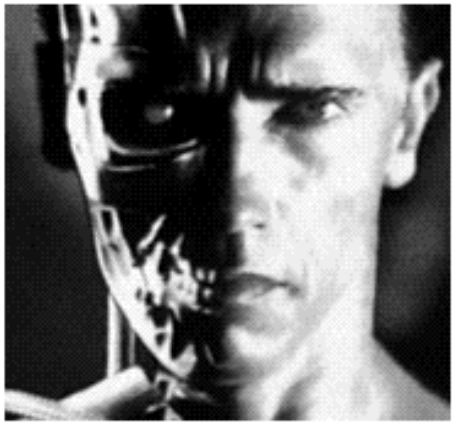
Image as a Function

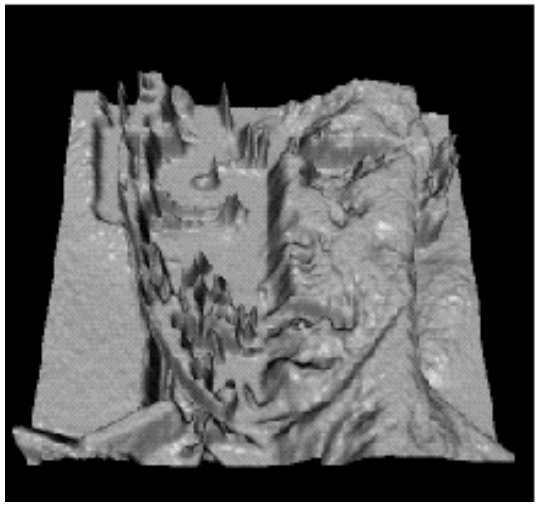
- We can think of an **image** as a function, f,
- f: $R^2 \rightarrow R$
  - $f(x, y)$ gives the **intensity** at position (x, y)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - f: $[a,b]$x$[c,d] \rightarrow [0,1]$

- A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

Image as a Function





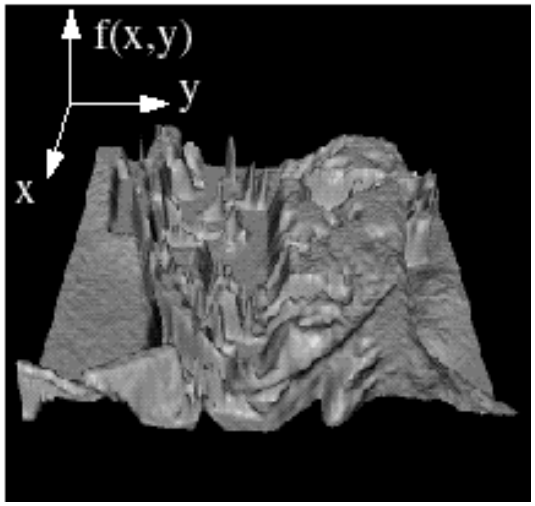


Image Processing

- Define a new image *g* in terms of an existing image *f*
  - We can transform either the domain or the range of *f*
- Range transformation:

$$g(x, y) = t(f(x, y))$$

What kinds of operations can this perform?

Image Processing

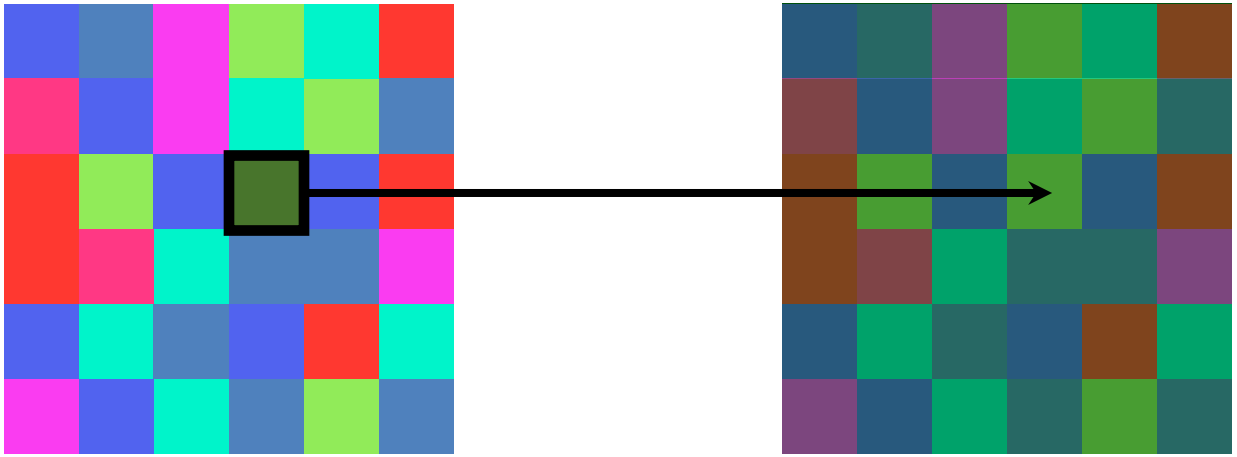
- Some operations preserve the range but change the domain of *f* :

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

- Still other operations operate on both the domain and the range of f .

Point Operations



Point Processing

Original

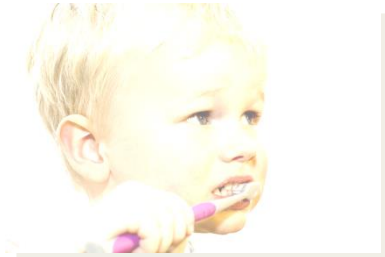
Darken

Lower Contrast

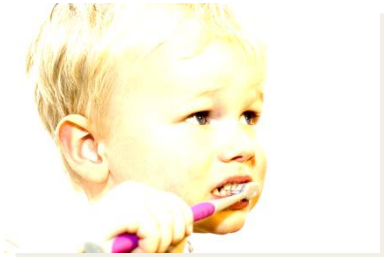
Nonlinear Lower Contrast

Invert

Lighten



Raise Contrast



Nonlinear Raise Contrast

Point Processing

Original

x

Darken

x - 128

Lower Contrast

x / 2

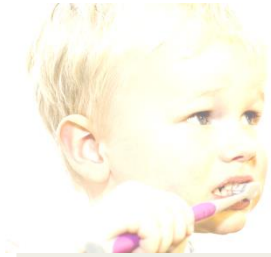
Nonlinear Lower Contrast

((x / 255.0) ^ 0.33) * 255.0

Invert

255 - x

Lighten



x + 128

Raise Contrast

x * 2

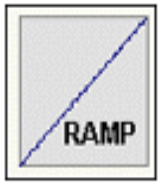
Nonlinear Raise Contrast

((x / 255.0) ^2) * 255.0

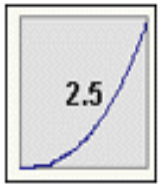
Gamma correction

Monitors have a intensity to voltage response curve which is roughly a 2.5 power function

Send v → actually display a pixel which has intensity equal to $v^{2.5}$



Graph of Input



Graph of Output L = V ^ 2.5

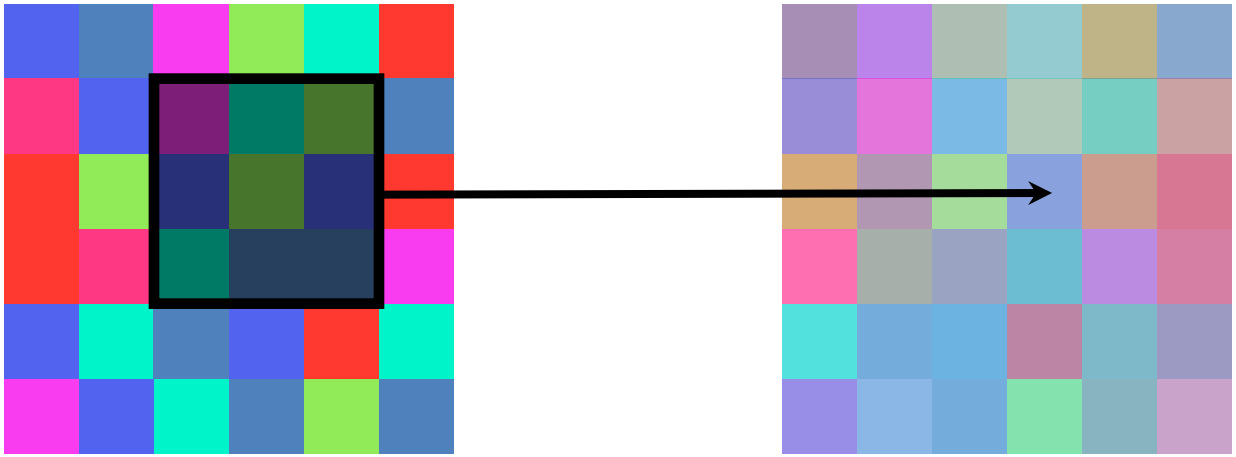
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

$\wp = 1.0$; $f(v) = v$

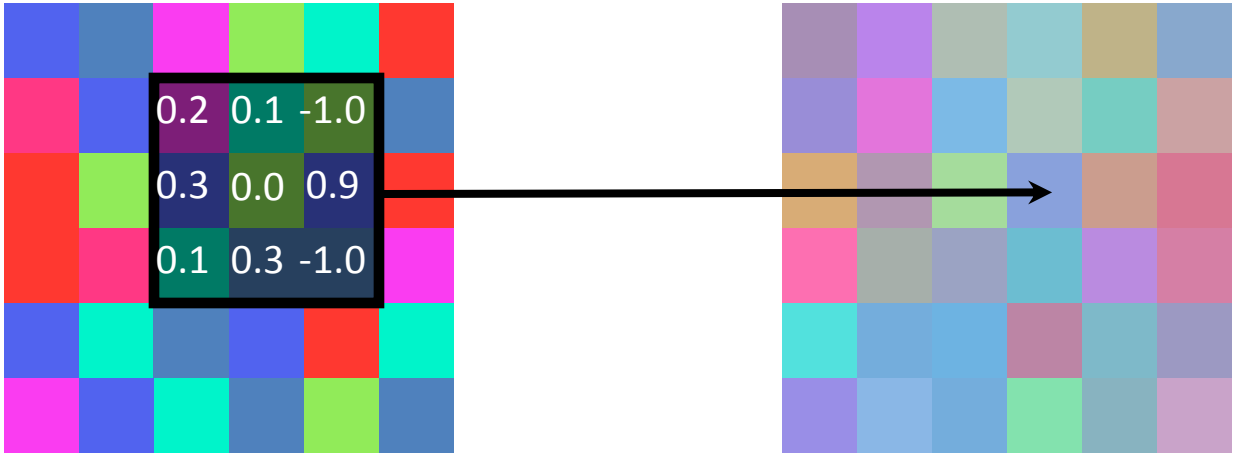
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

$\wp = 2.5$; $f(v) = v^{1/2.5} = v^{0.4}$

Neighborhood Operations



Convolution



$$F = \begin{bmatrix} 0.2 & 0.1 & -1.0 \\ 0.3 & 0.0 & 0.9 \\ 0.1 & 0.3 & -1.0 \end{bmatrix}$$

$$I' = F * I$$

Properties of Convolution

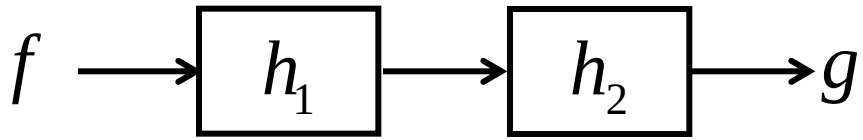
- Commutative

$$a * b = b * a$$

- Associative

$$(a * b) * c = a * (b * c)$$

- Cascade system



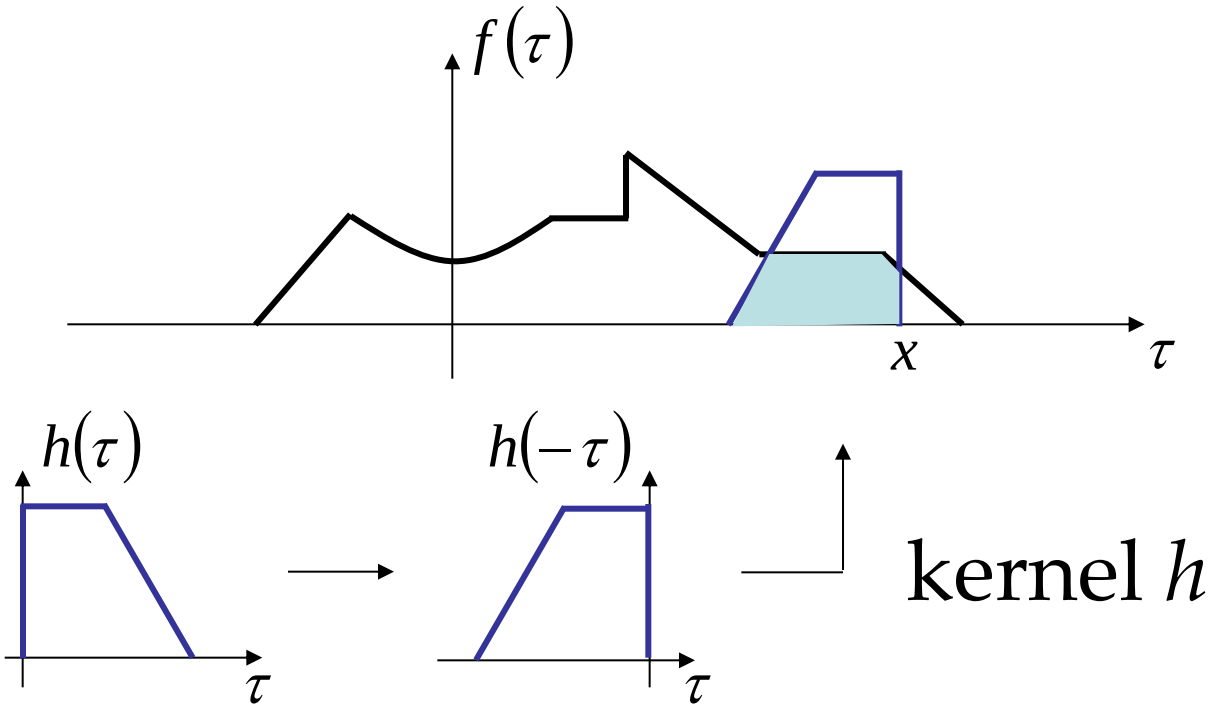
$= f \rightarrow [\, h_1 * h_2 \,] \rightarrow g$

$= f \rightarrow [\, h_2 * h_1 \,] \rightarrow g$

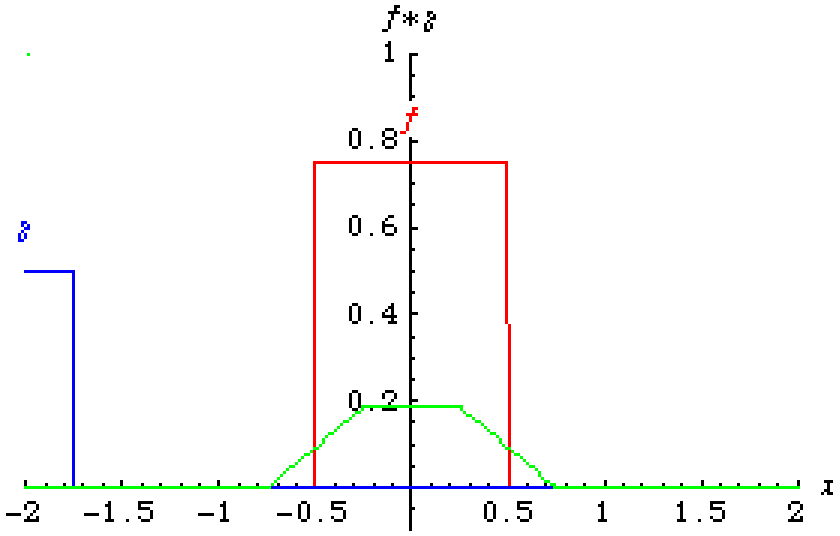
Convolution

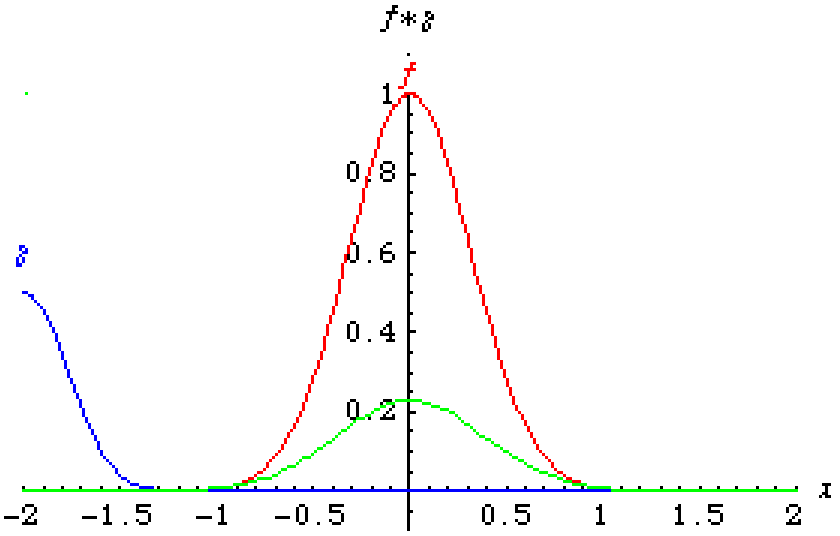
Convolution is linear and shift invariant

$$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)d\tau \qquad g = f * h$$



Convolution - Example





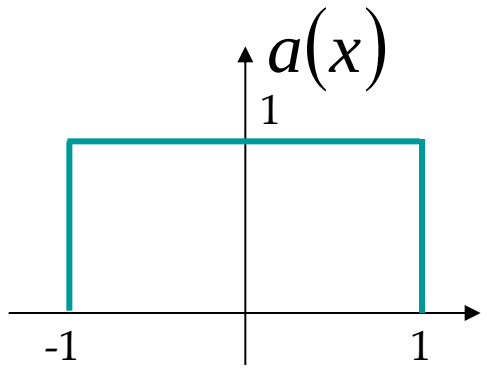
f

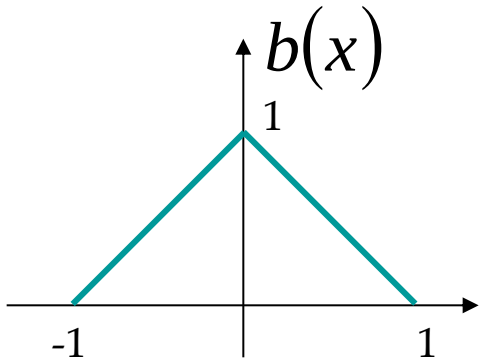
g

$f * g$

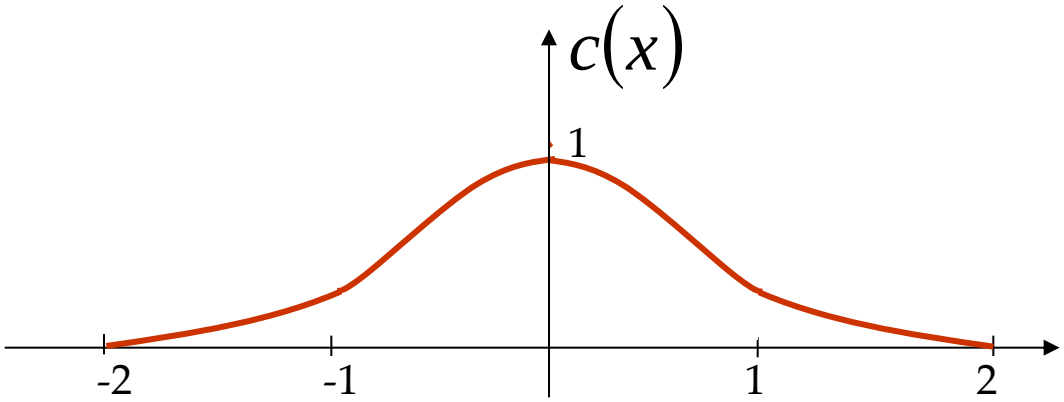
Eric Weinstein's Math World

Convolution - Example

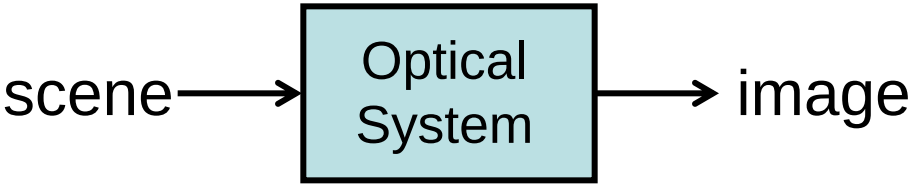




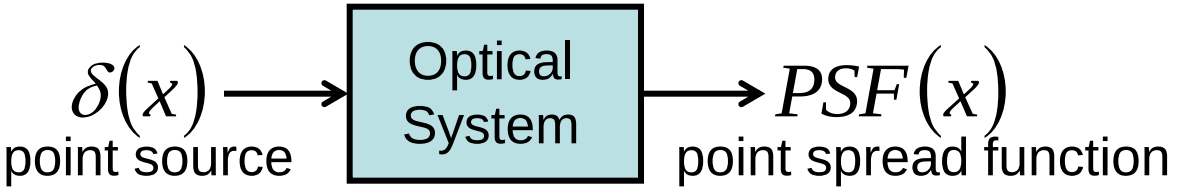
$$\downarrow \quad c = a * b$$



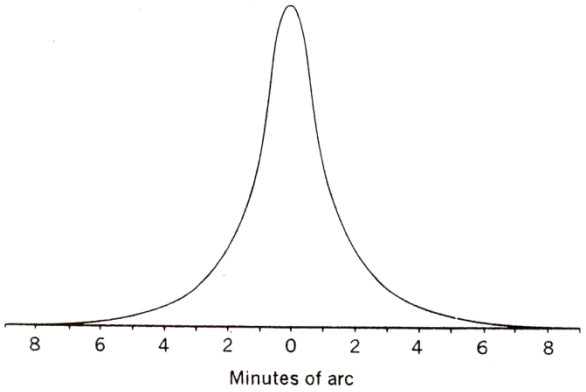
Point Spread Function



- Ideally, the optical system should be a Dirac delta function.
- However, optical systems are never ideal.



- Point spread function of Human Eyes



Point Spread Function

normal vision

myopia

hyperopia

astigmatism

Images by Richmond Eye Ass

Original Image

Blurred Image

Gaussian Smoothing

by Charles Allen Gillbert

by Harmon & Julesz

http://www.michaelbach.de/ot/cog_blureffects

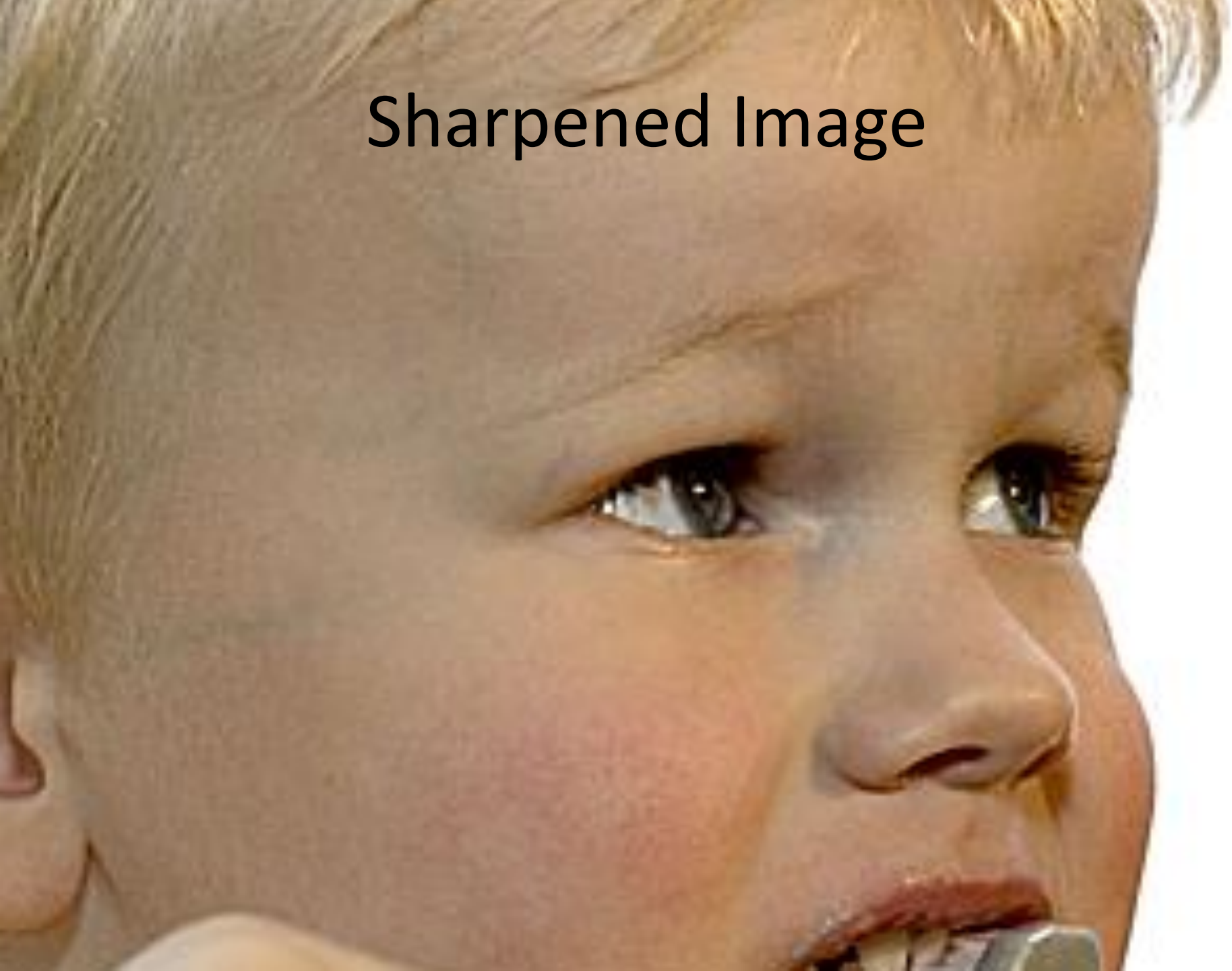
Gaussian Smoothing

http://www.michaelbach.de/ot/cog_blureffects

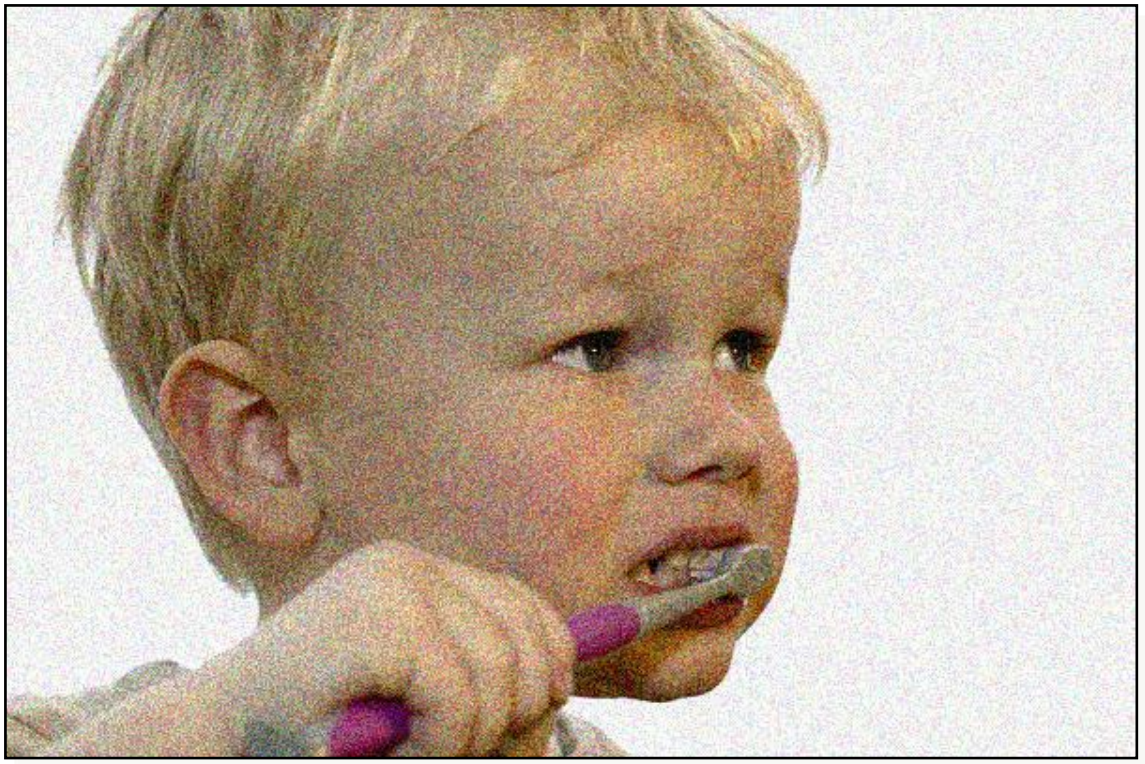
Original Image

Sharpened Image

Sharpened Image



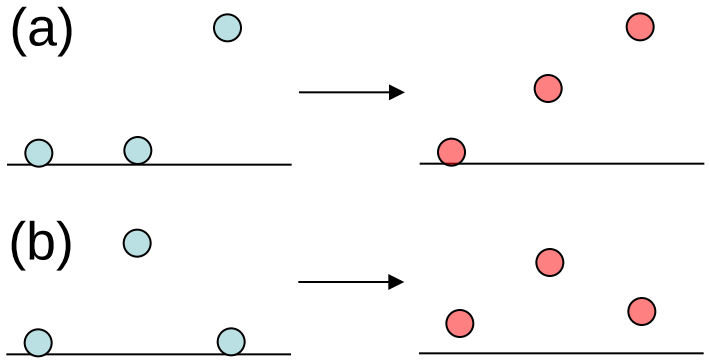
Noise



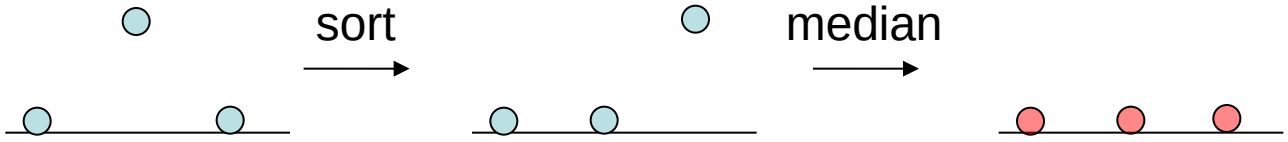
Blurred Noise

Median Filter

- Smoothing is averaging
  - (a) Blurs edges
  - (b) Sensitive to outliers



- Median filtering
  - Sort $N^2 - 1$ values around the pixel
  - Select middle value (median)



  - Non-linear (Cannot be implemented with convolution)

Median Filter

Can this be described as a convolution?

Original Image

Example: Noise Reduction

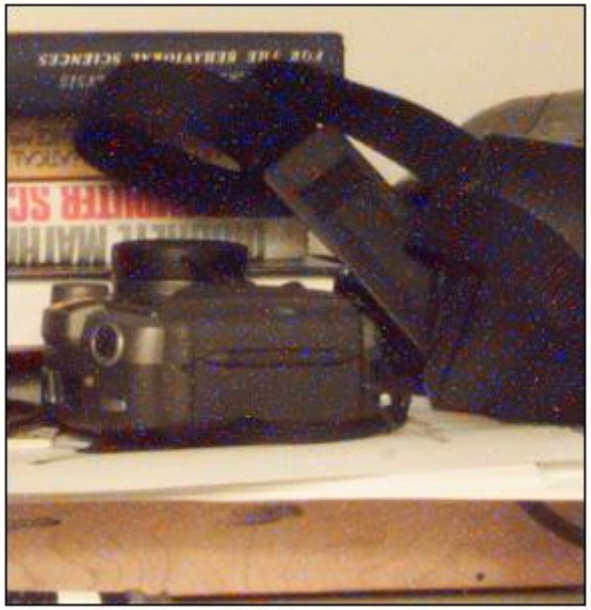
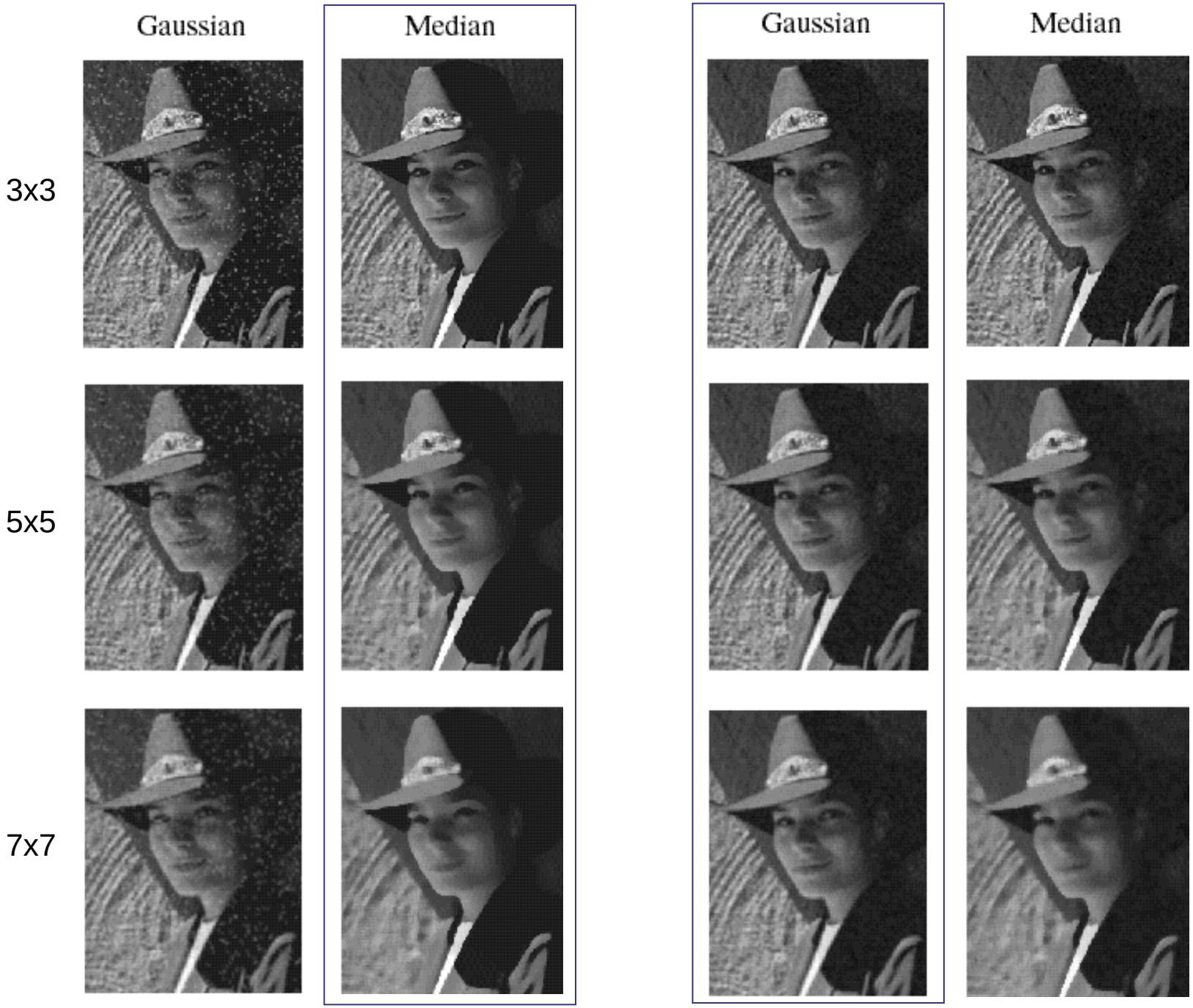


Image with noise

Median filter (5x5)


Salt and pepper noise
Gaussian noise
Gaussian
Median
Gaussian
Median
3x3
5x5
7x7

Example: Noise Reduction

Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Original image

Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Image with noise

Median filter (5x5)

Original Image

X-Edge Detection

Y-Edge Detection

General Edge Detection

Can this be described as a convolution?

Image Processing

- Some operations preserve the range but change the domain of *f* :

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

- Still other operations operate on both the domain and the range of f .

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

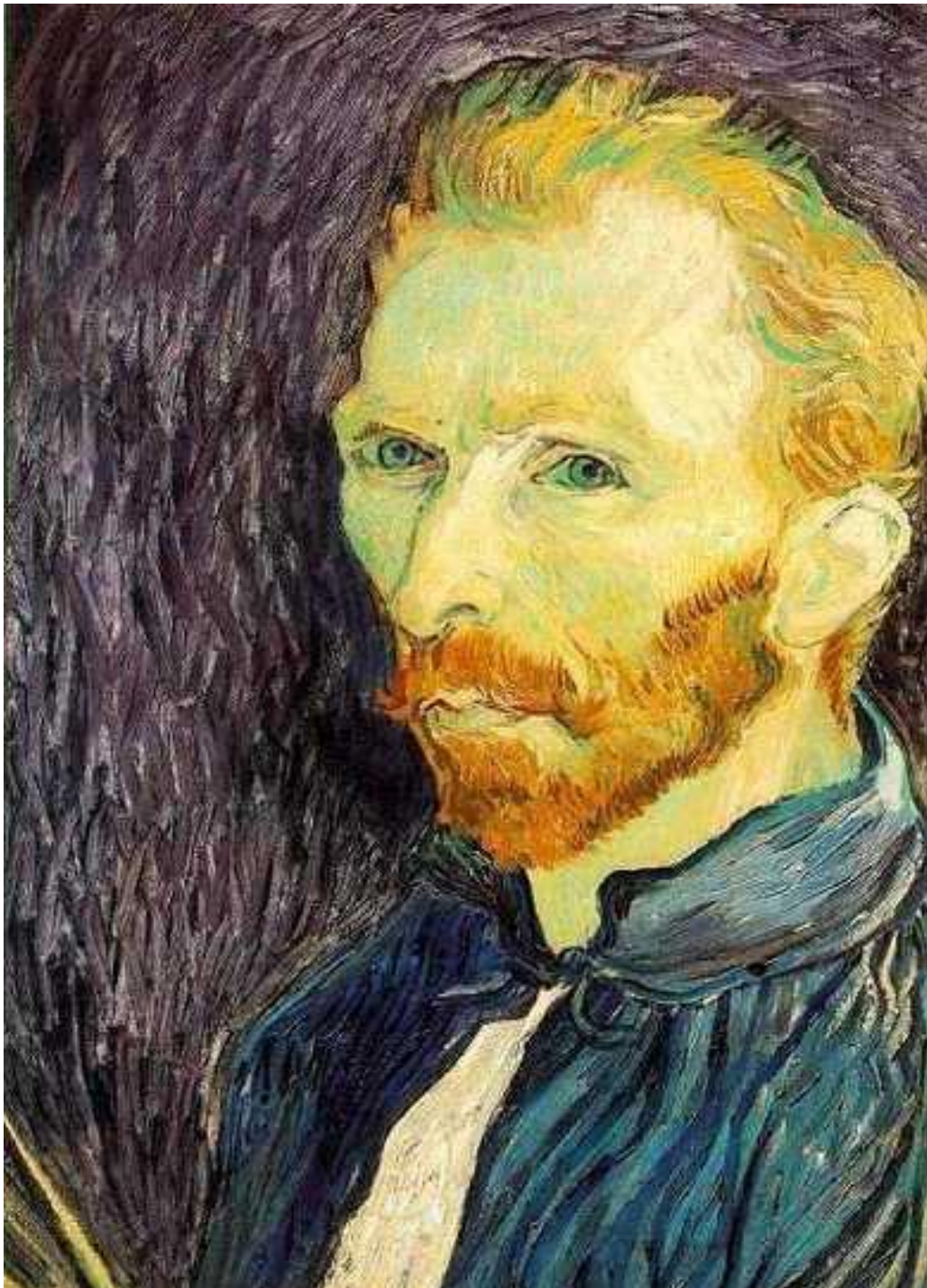
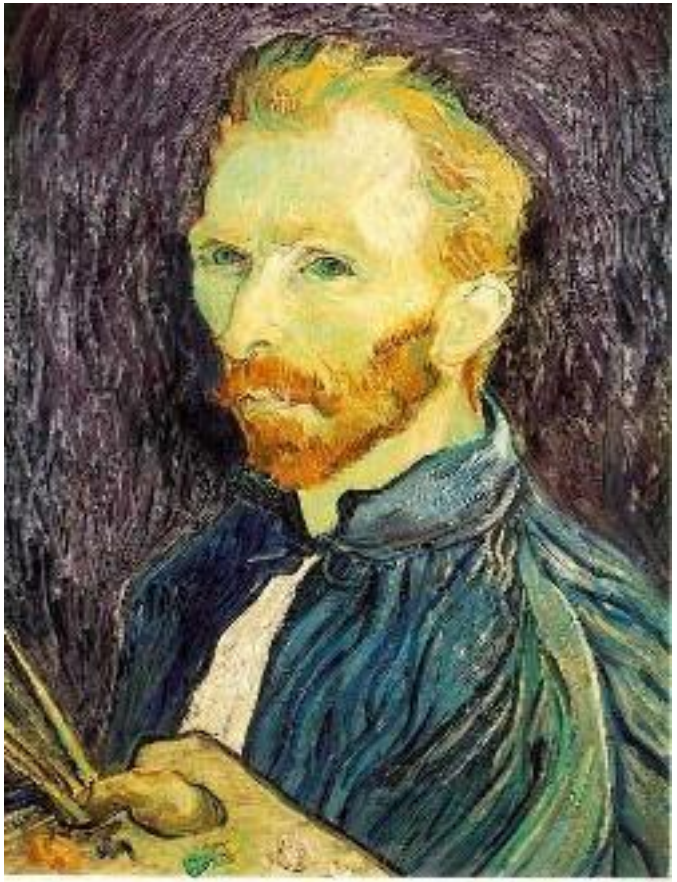


Image Sub-Sampling



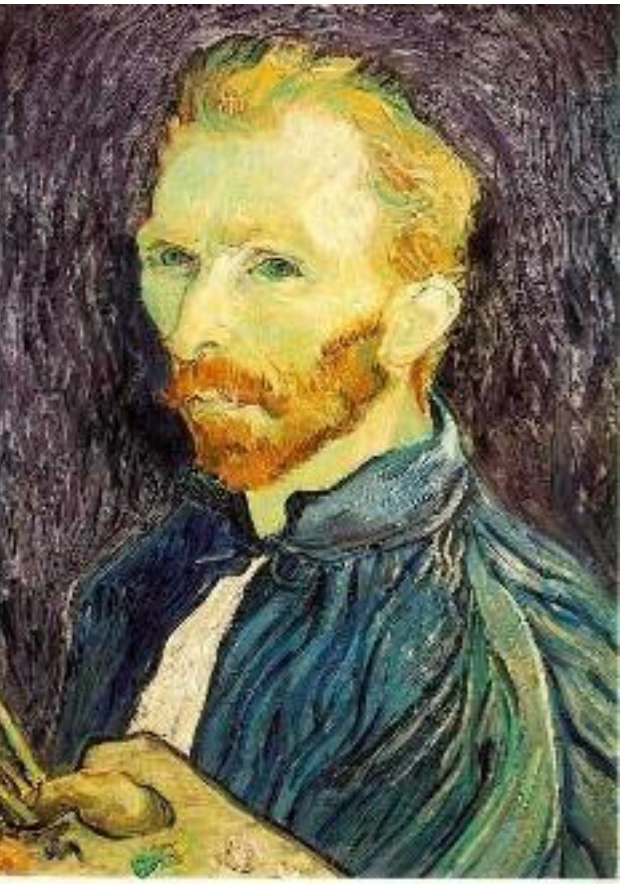
1/4

1/8

Throw away every other row and column to create a 1/2 size image

- called *image sub-sampling*

Image Sub-Sampling

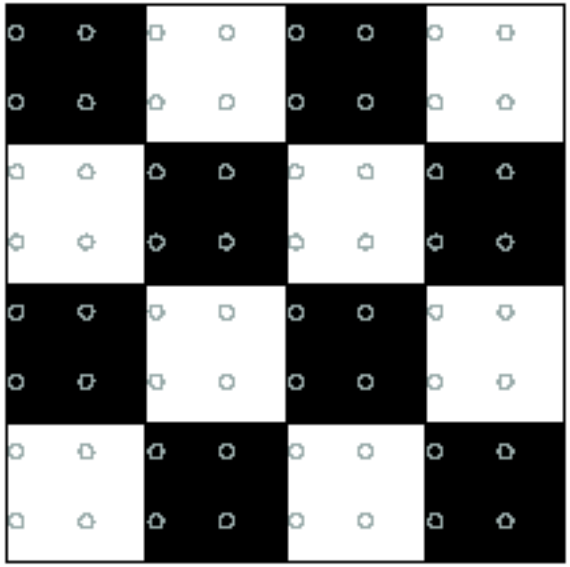


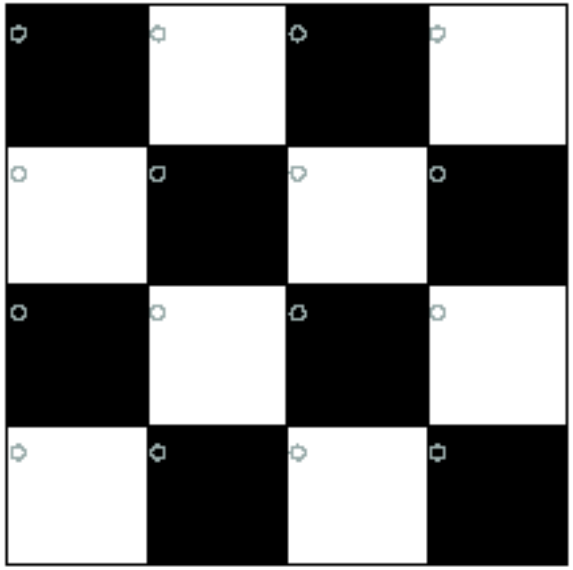
1/2

1/4 (2x zoom)

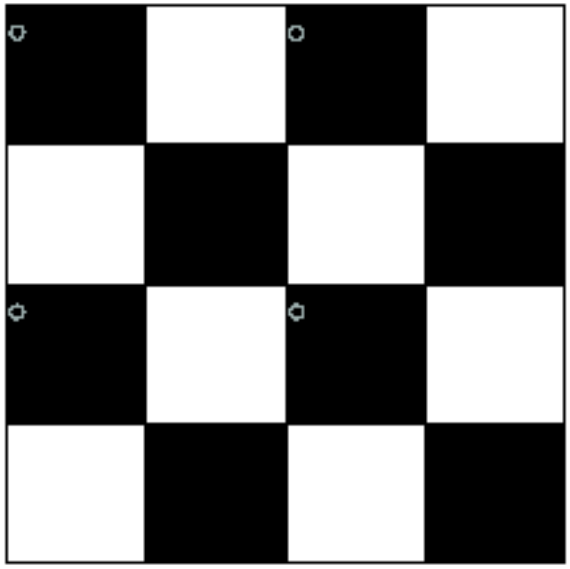
1/8 (4x zoom)

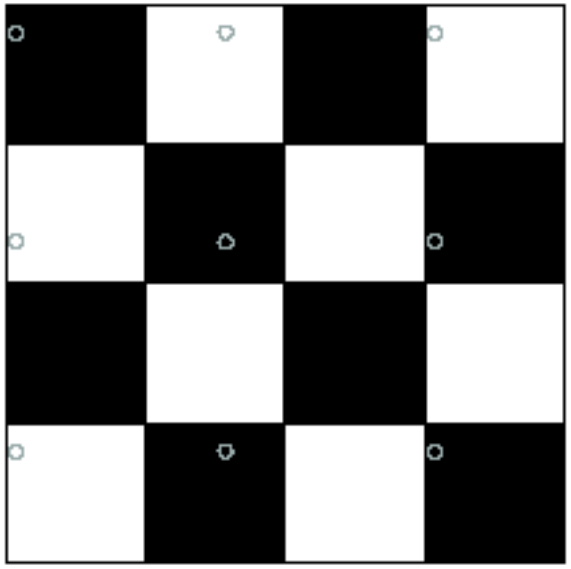
Good and Bad Sampling





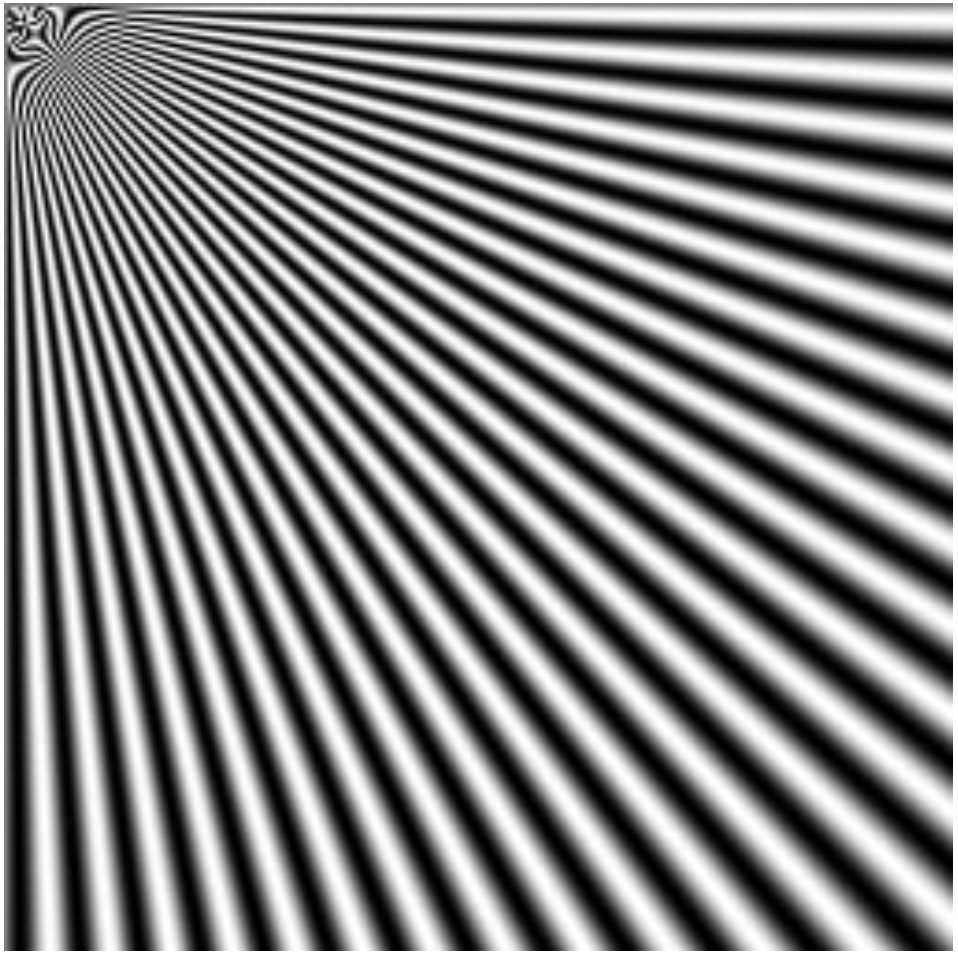
Good sampling:
- Sample often or,
- Sample wisely



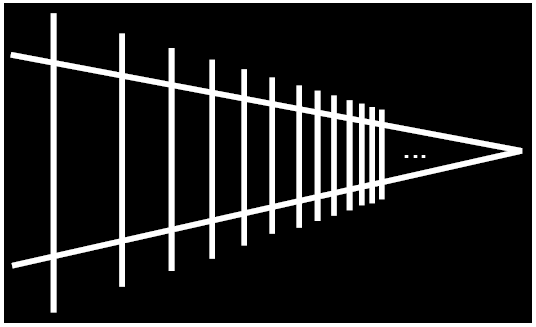


Bad sampling:
- see aliasing in action!

Aliasing

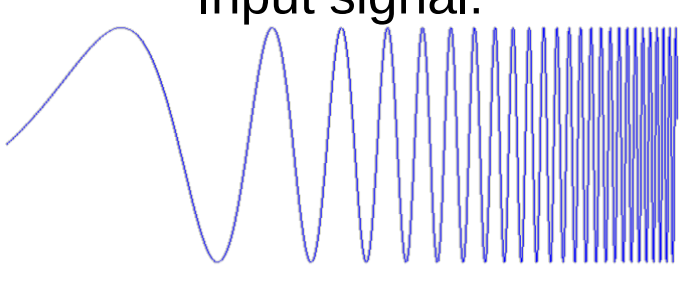


Alias: n., an assumed name

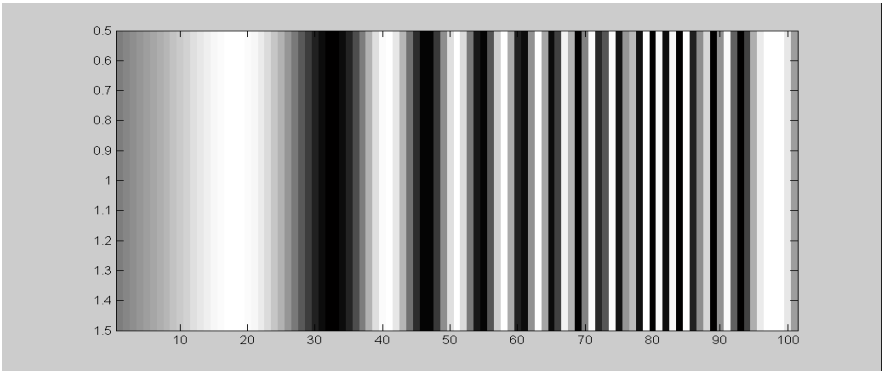


Picket fence receding into the distance will produce aliasing…

Input signal:

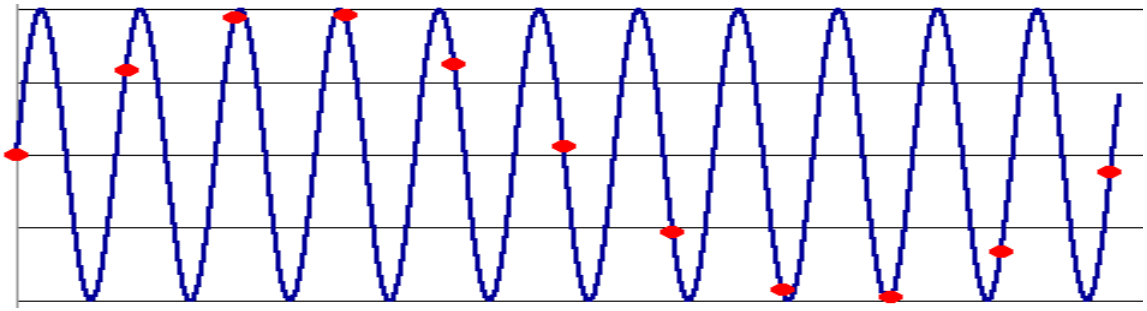


Matlab output:



x = 0:.05:5; imagesc(sin((2.^x).*x))

WHY?



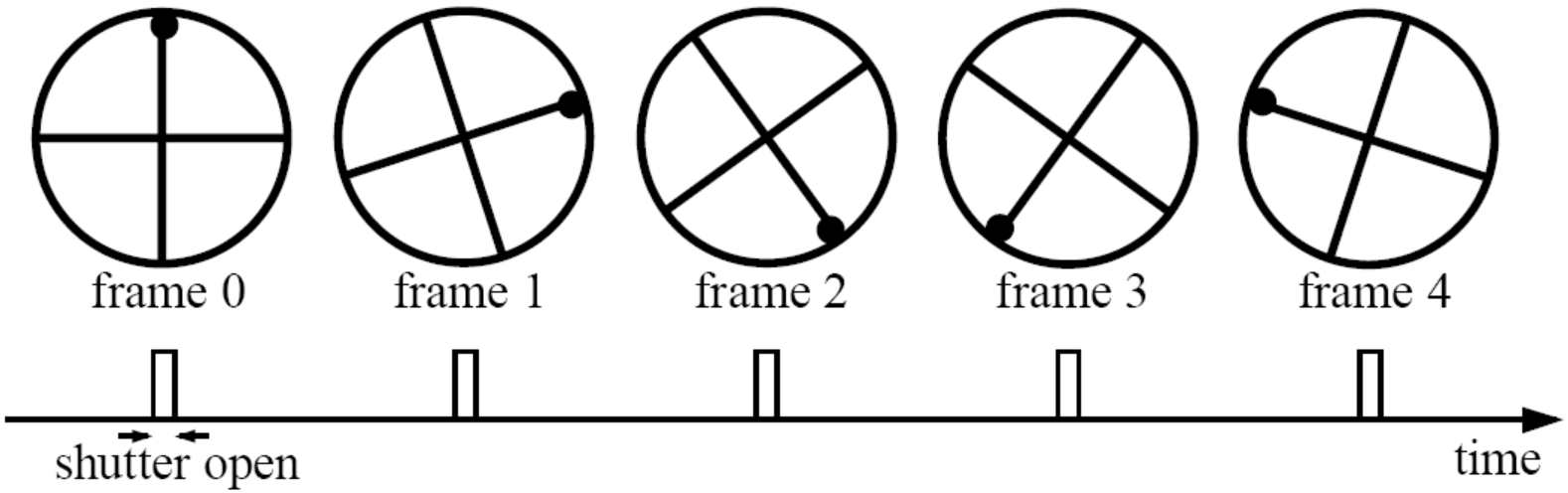
Alias!

Not enough samples

Really bad in video

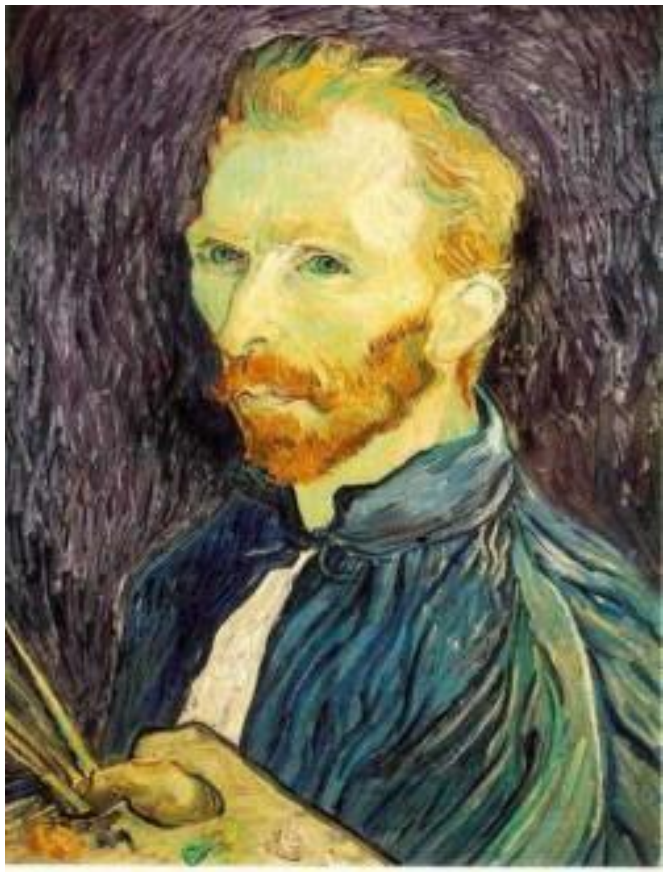
Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Sub-Sampling with Gaussian Pre-Filtering



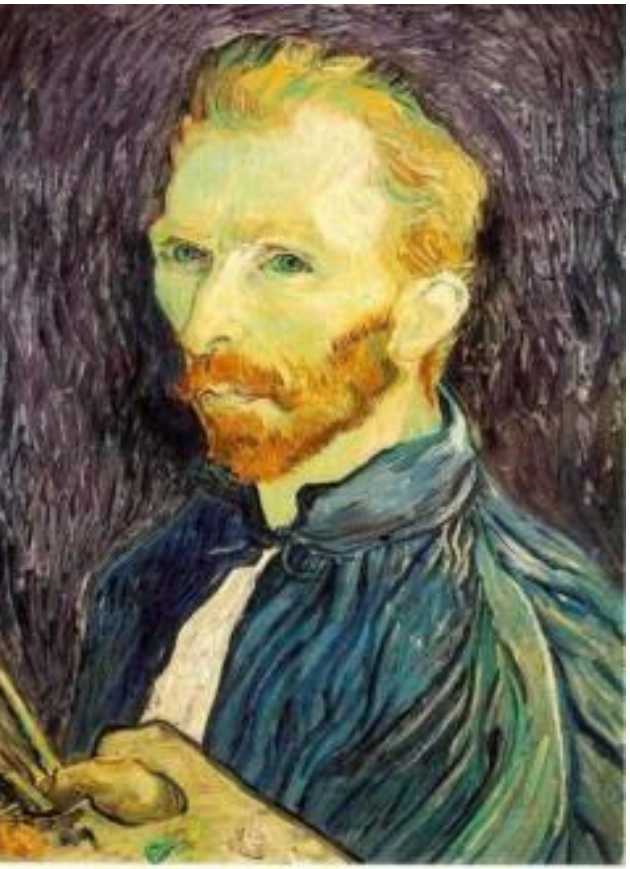
Gaussian 1/2

G 1/4

G 1/8

- Solution: filter the image, *then* subsample
  - Filter size should double for each ½ size reduction. Why?

Sub-Sampling with Gaussian Pre-Filtering

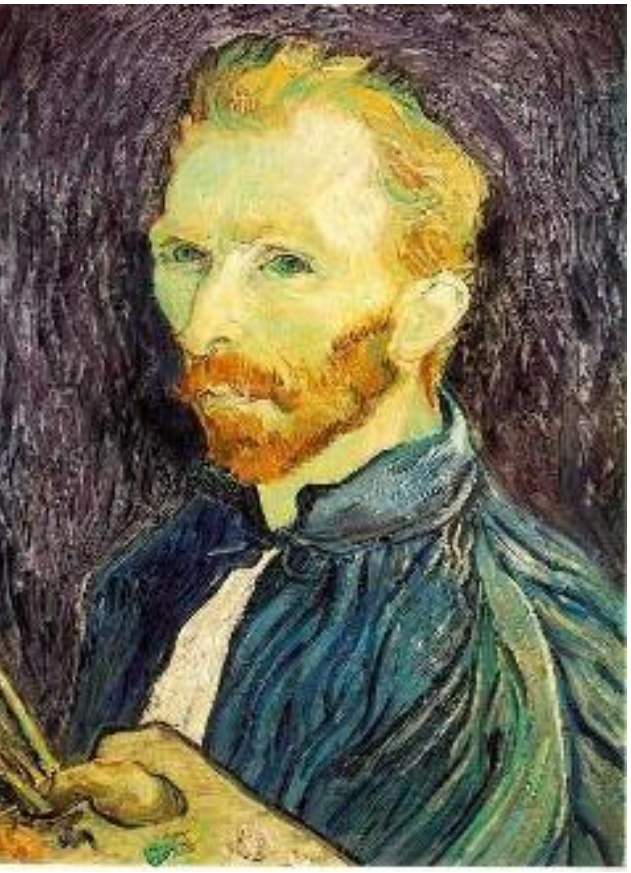


Gaussian 1/2

G 1/4

G 1/8

Compare with...



1/2

1/4 (2x zoom)

1/8 (4x zoom)

Canon D60 (w/ anti-alias filter)

Sigma SD9 (w/o anti-alias filter)

From Rick Matthews website, images by Dave Etchells

512 256 128 64 32 16 8

Figure from David Forsyth

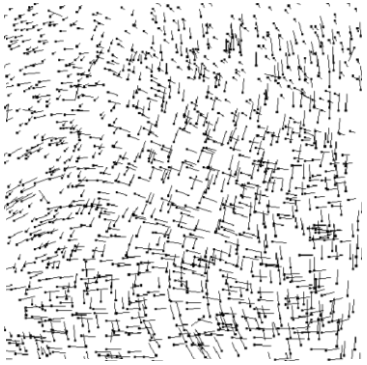
Original Image

Warped Image

Warped Image

orig

+



vector field

=

warped

how?

Advection (just like a fluid)