Spatial Data Structures

Hierarchical Bounding Volumes
Grids
Octrees
BSP Trees

Speeding Up Computations

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n^2 computations

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n^2 computations

Spatial data-structures

Spatial Data Structures

- We'll look at
  - Hierarchical bounding volumes
  - Grids
  - Octrees
  - K-d trees and BSP trees

- Good data structures can give speed up ray tracing by 10x or 100x

Bounding Volumes

- Wrap things that are hard to check for intersection in things that are easy to check
  - Example: wrap a complicated polygonal mesh in a box
  - Ray can't hit the real object unless it hits the box
  - Adds some overhead, but generally pays for itself.

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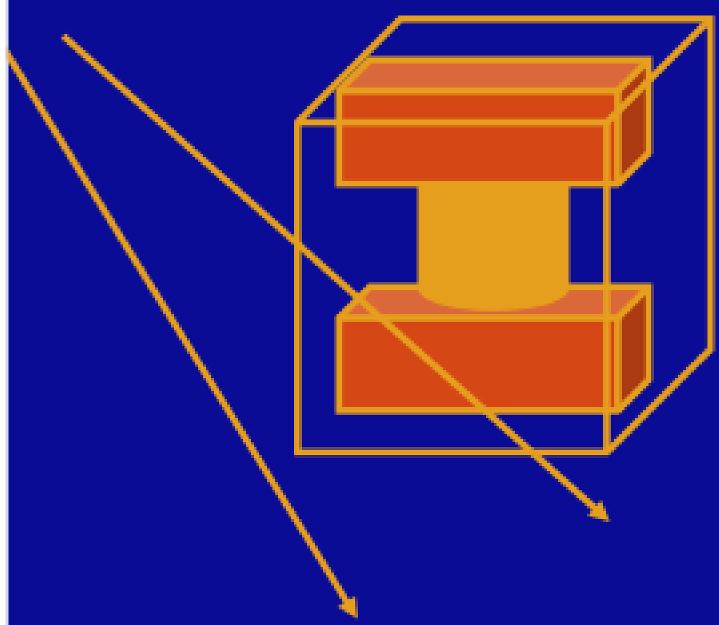
  - box can be axis-aligned or not



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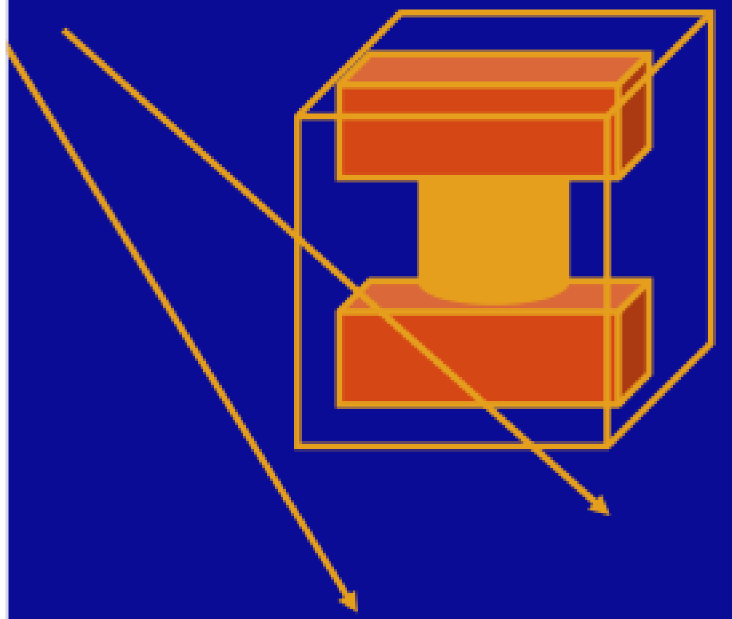
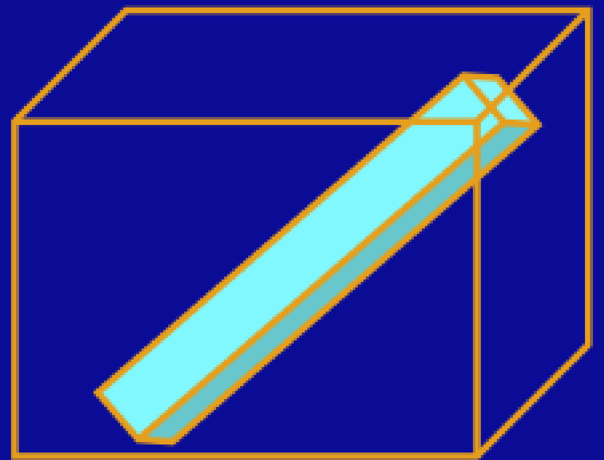
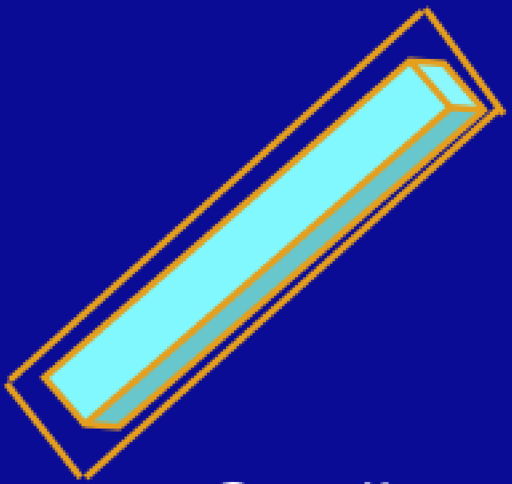
- But you don't want expensive intersection tests!





Bad!



Good!

Bounding Volumes

- You want a snug fit!
- But you don't want expensive intersection tests!
- Use the ratio of the object volume to the enclosed volume as a measure of fit.

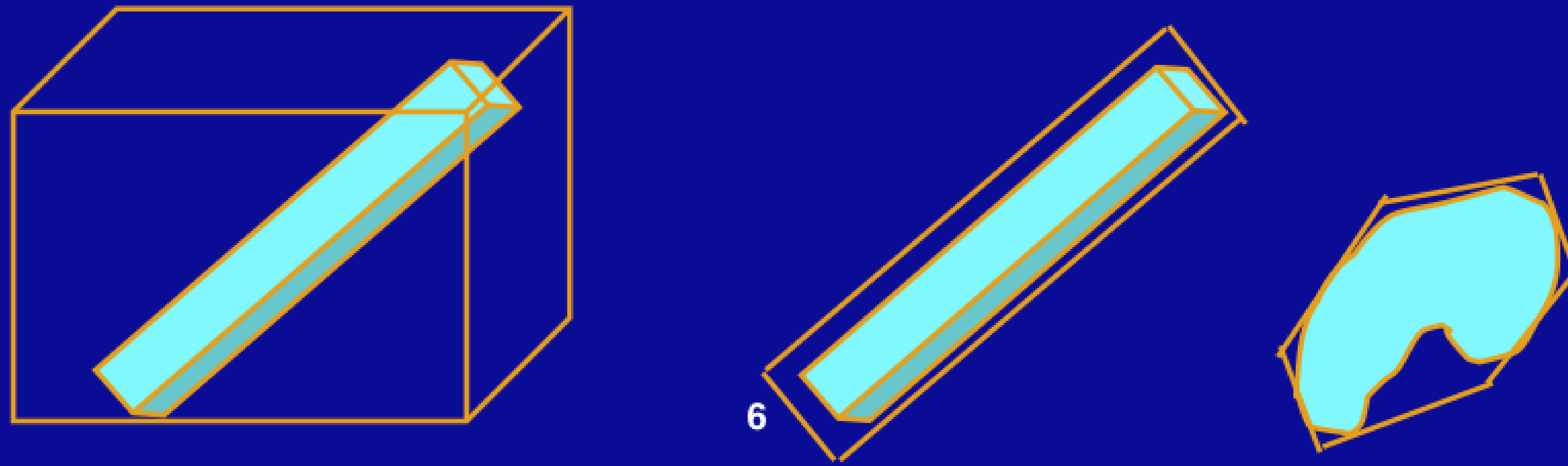
- Cost = n*B + m*I

  n - is the number of rays tested against the bounding volume

  B - is the cost of each test (Do not need to compute exact intersection!)

  m - is the number of rays which actually hit the bounding volume

  I - is the cost of intersecting the object within



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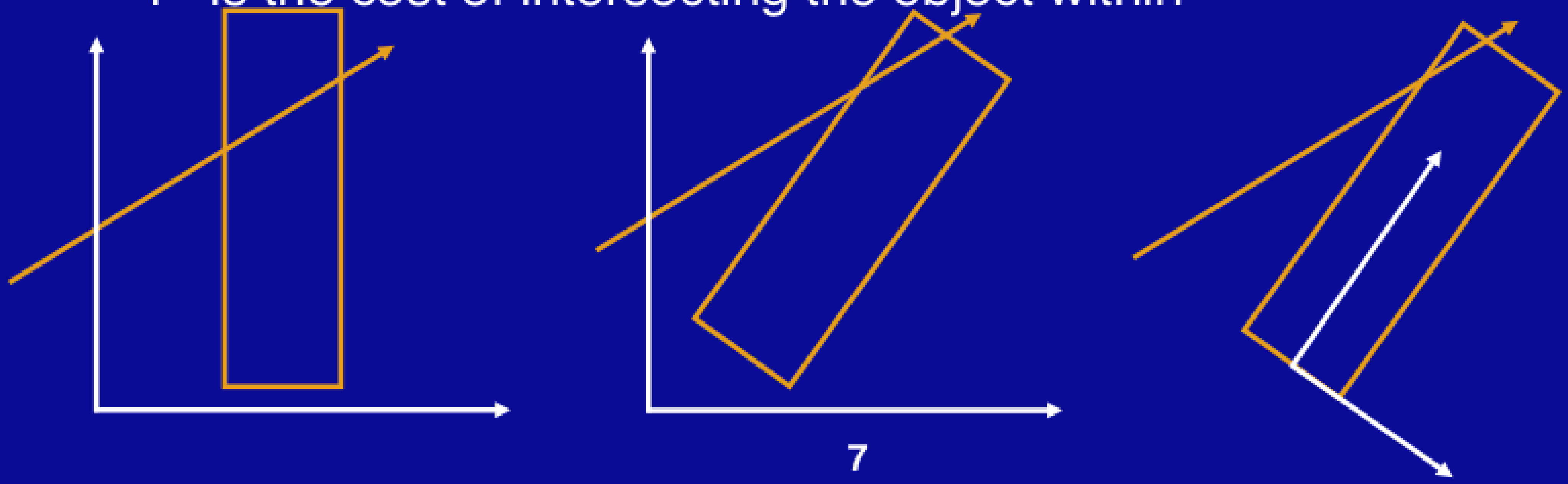
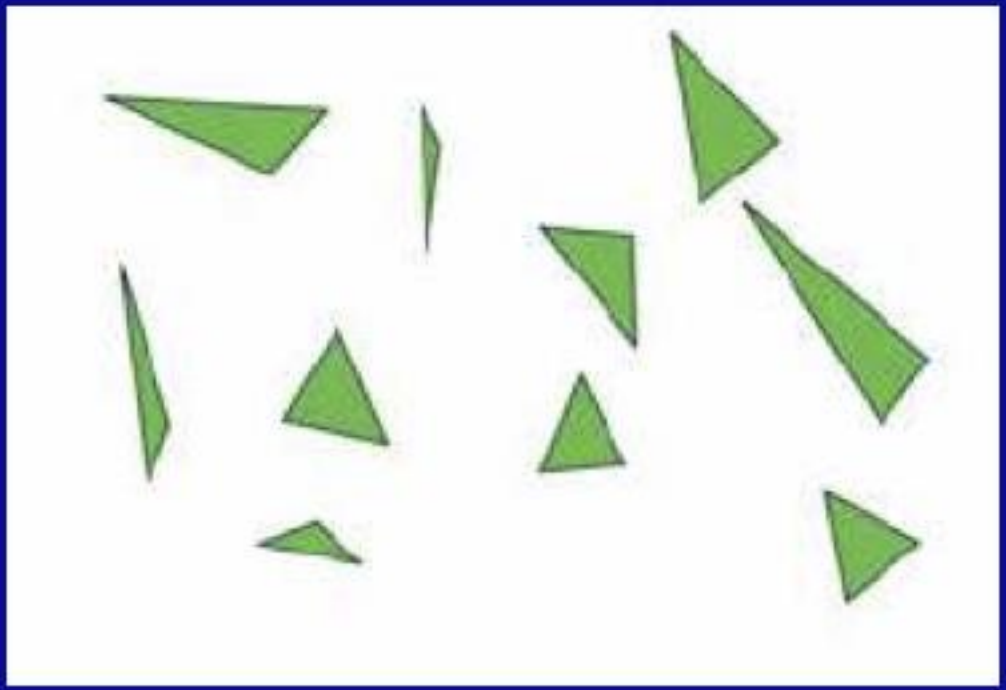


Hierarchical Bounding Volumes

- Still need to check ray against every object --- O(n)
- Use tree data structure
  - Larger bounding volumes contain smaller ones



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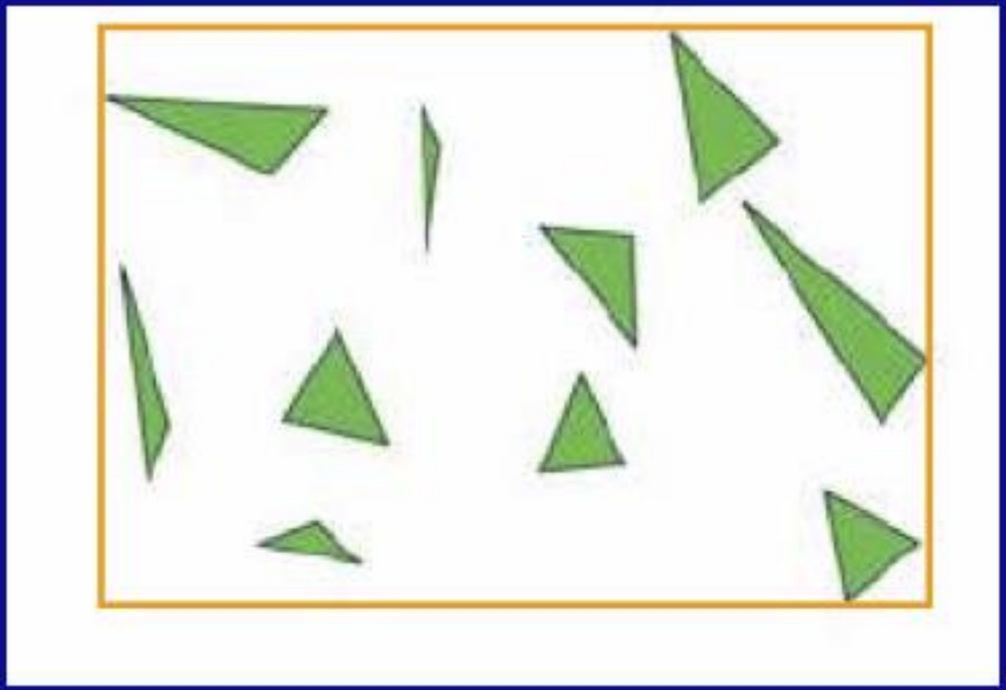

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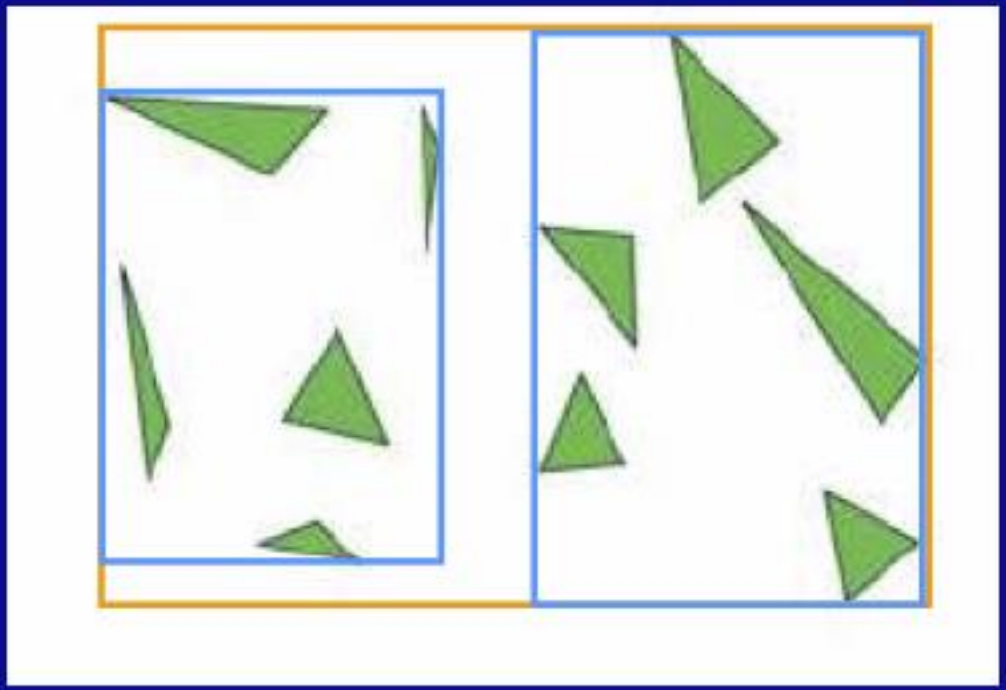

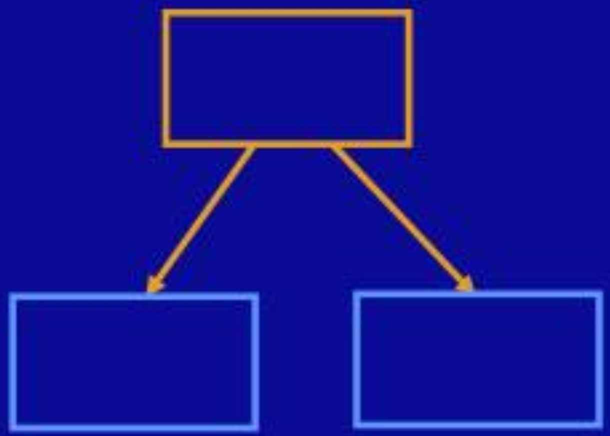

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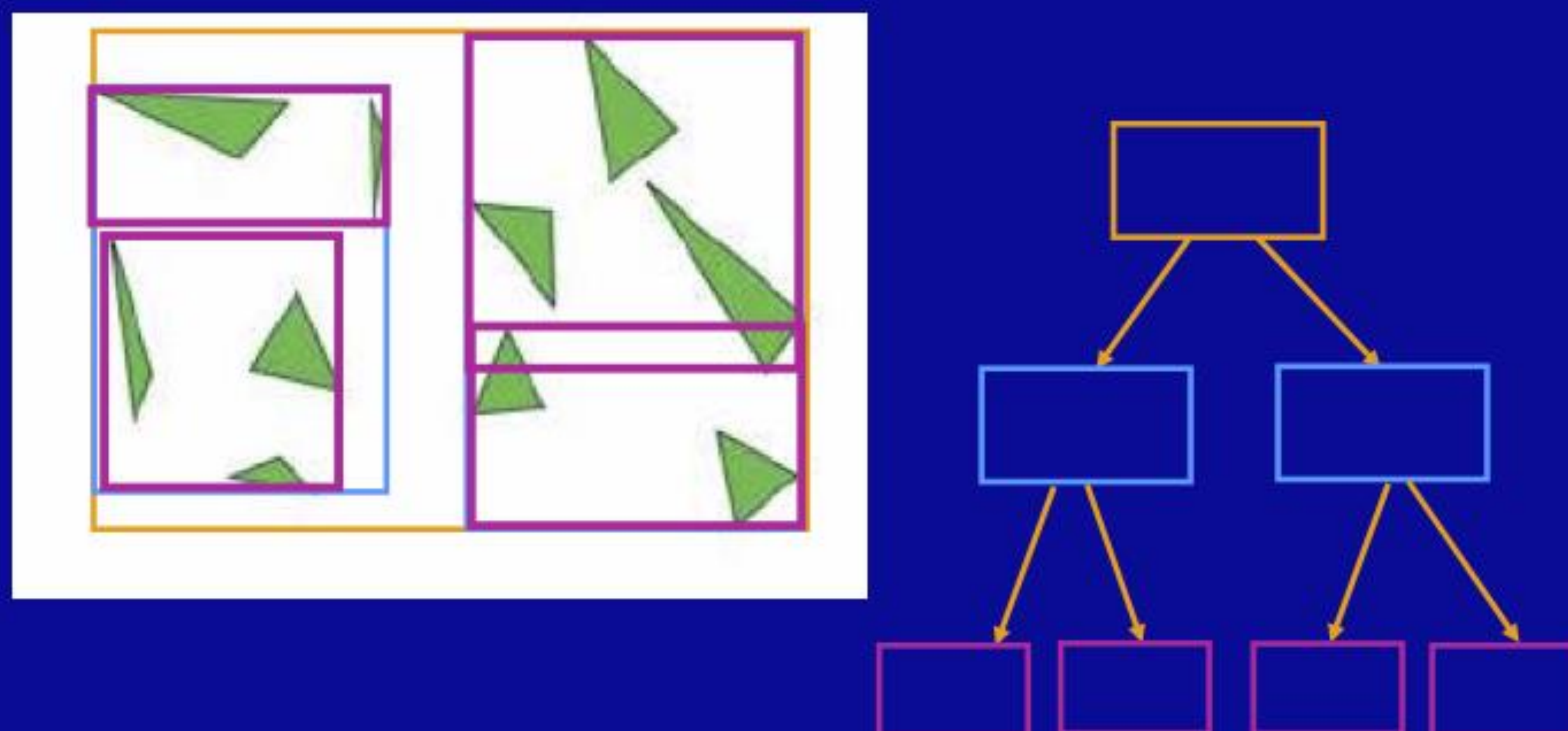

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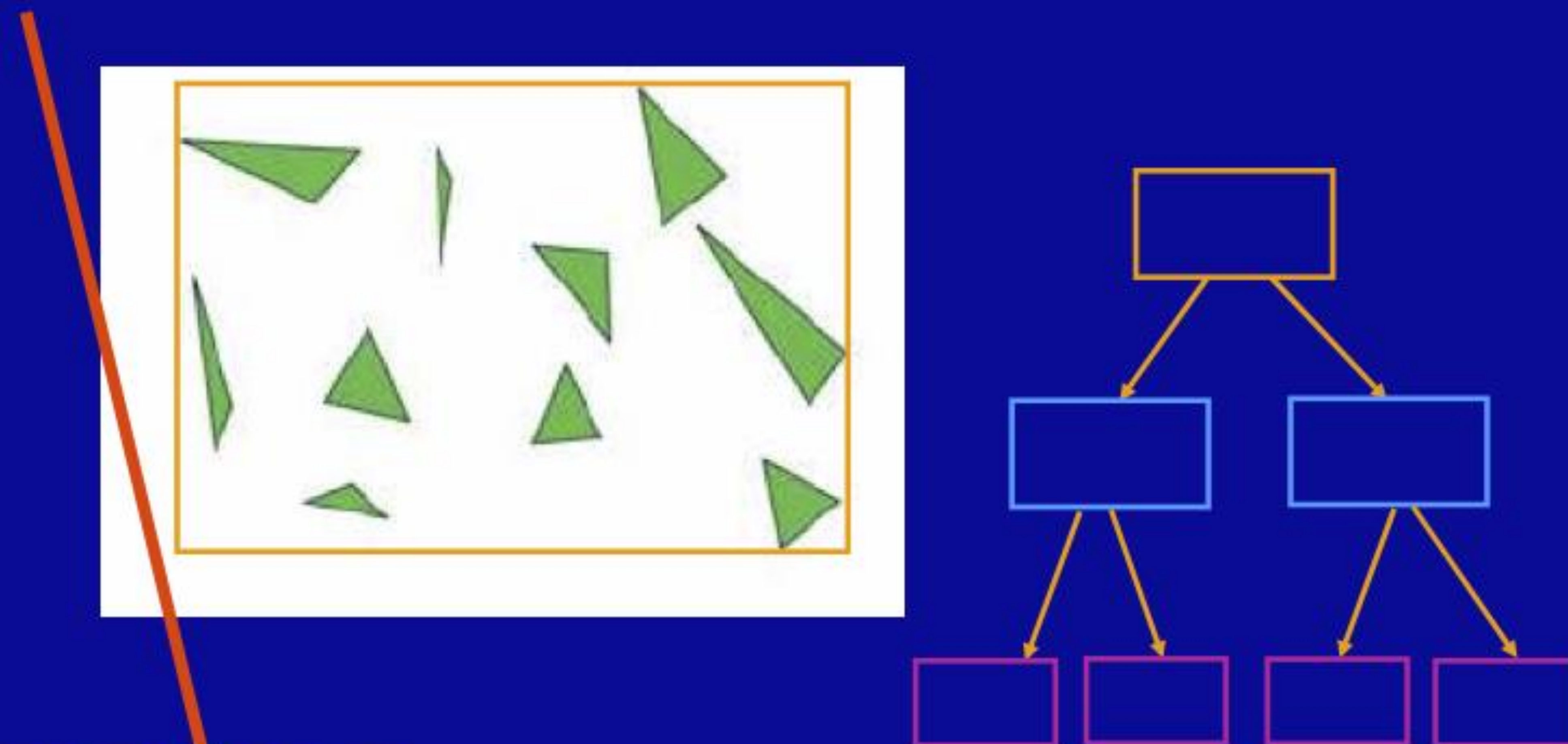

Check intersect root
If not return no intersections

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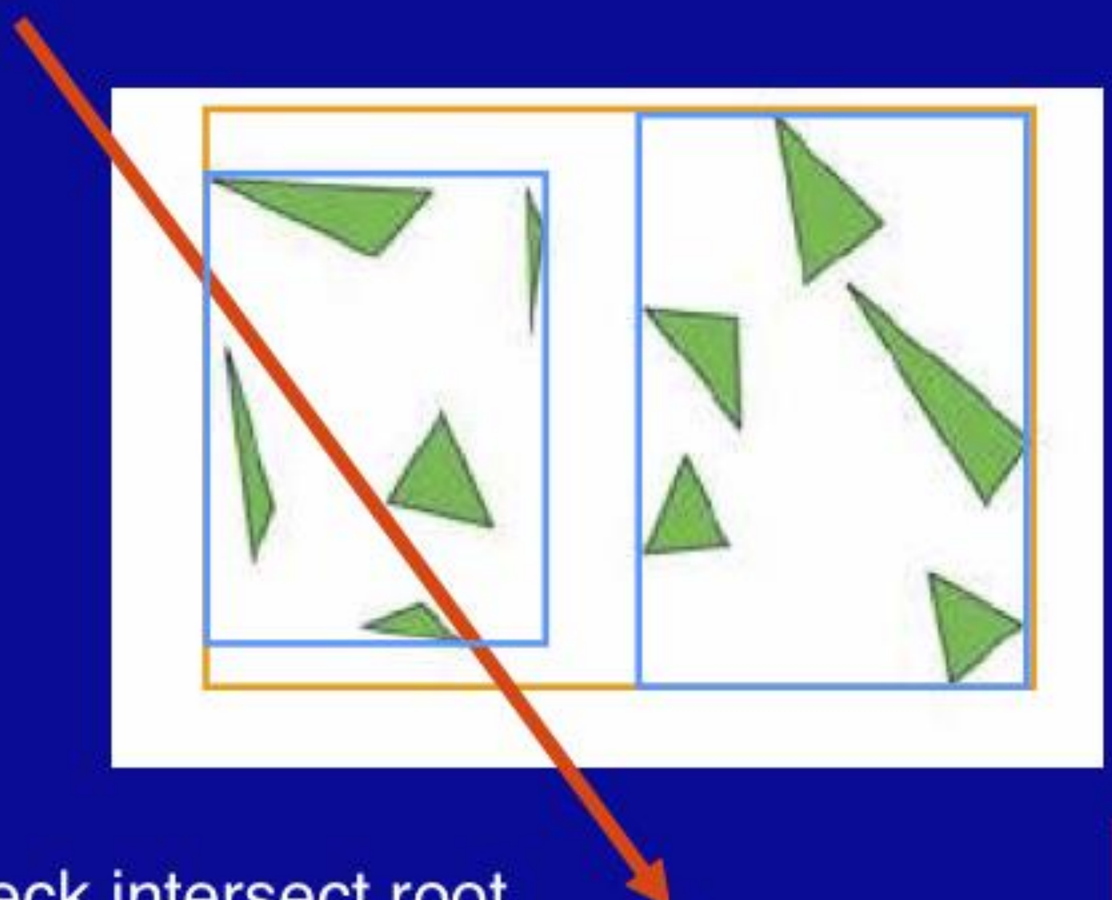

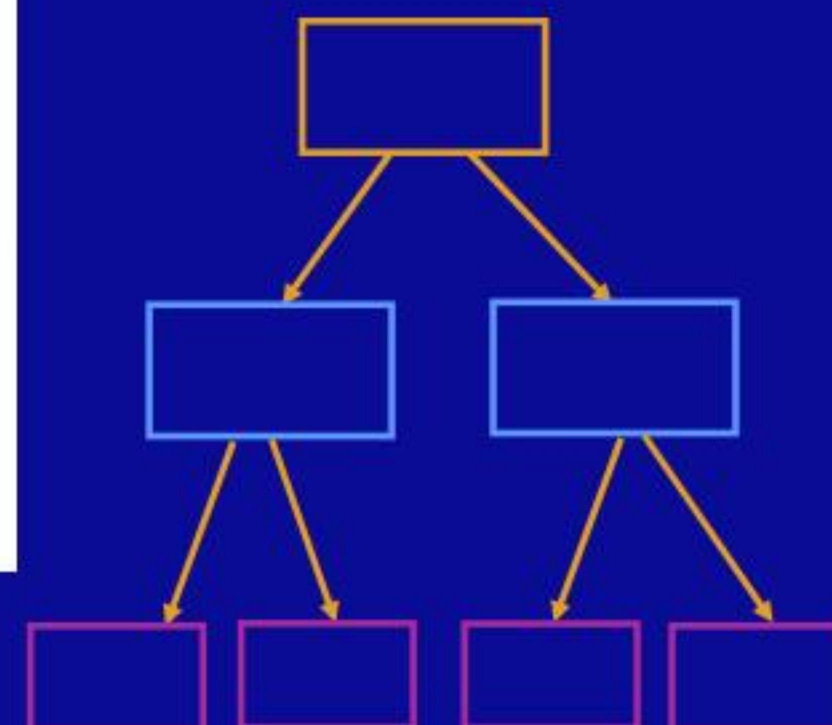

Check intersect root
If intersect
        check intersect left sub-tree
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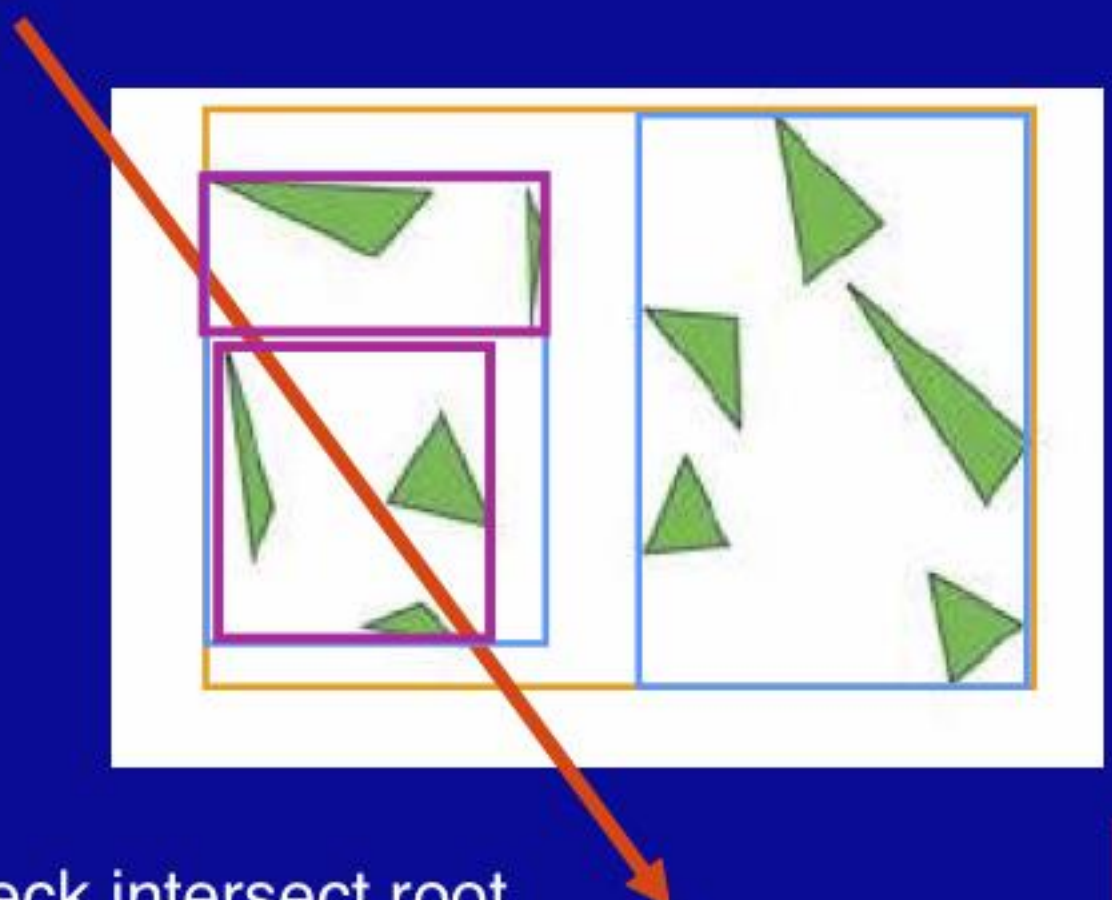

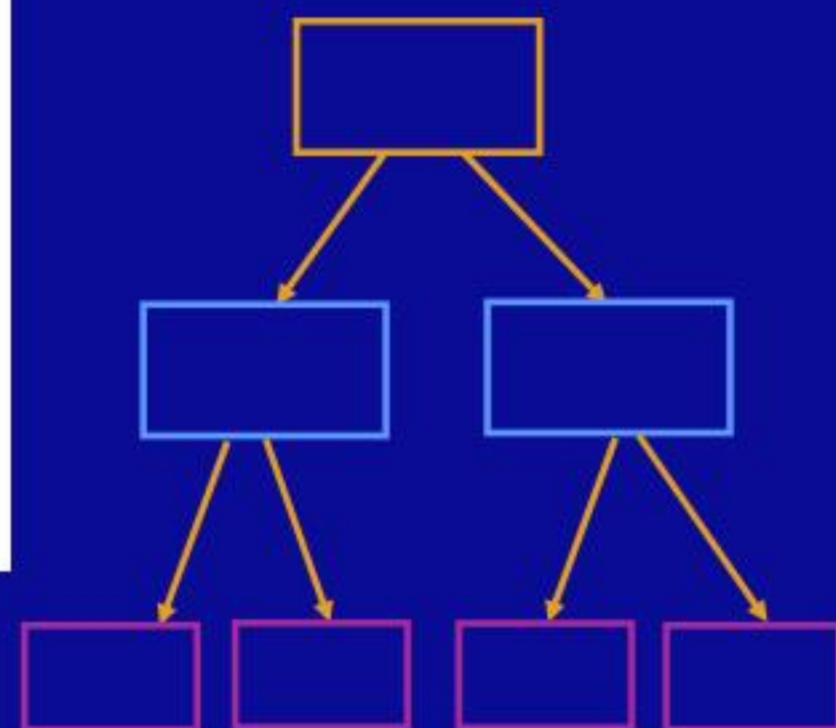

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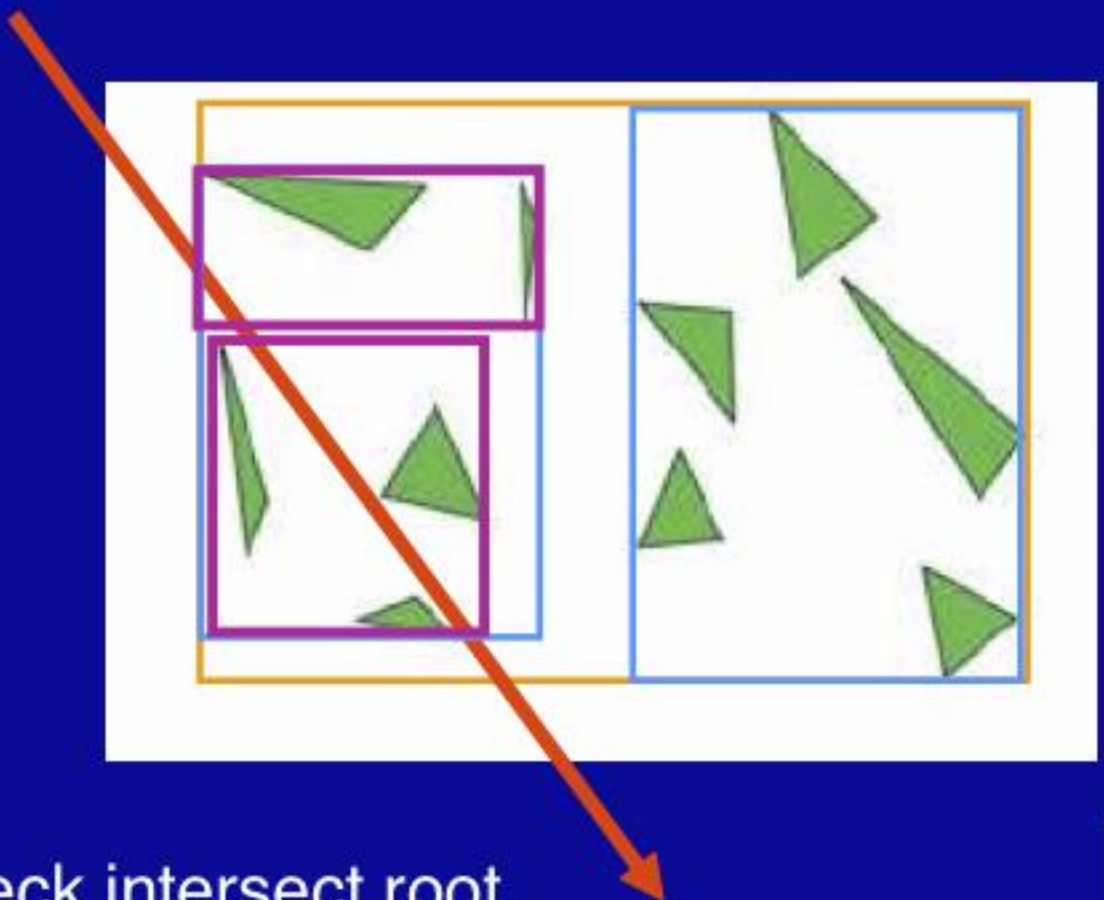

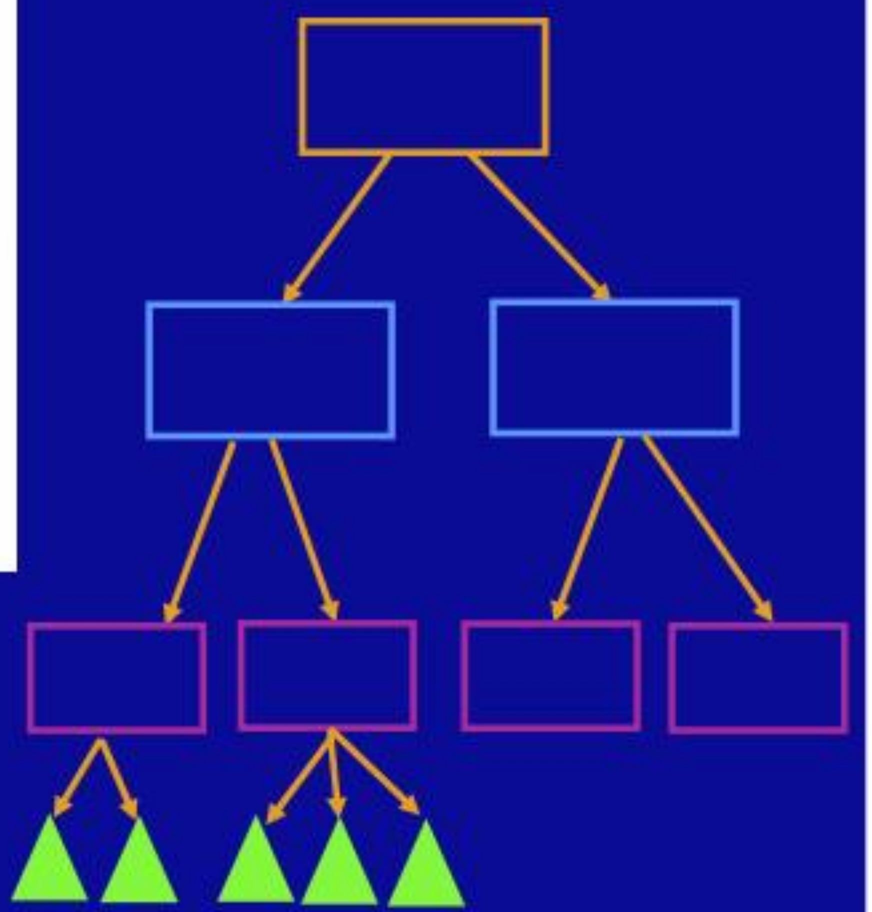

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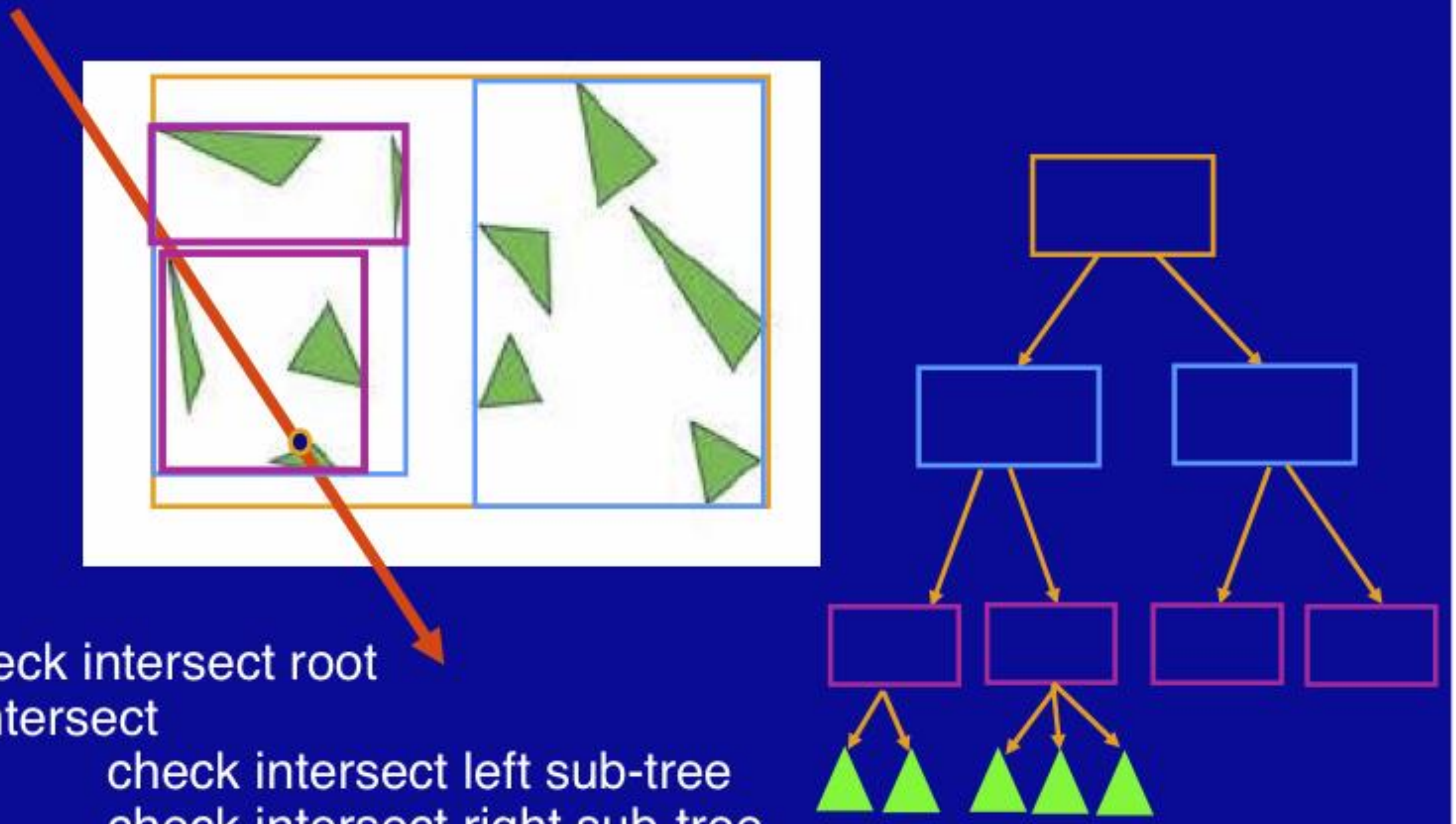

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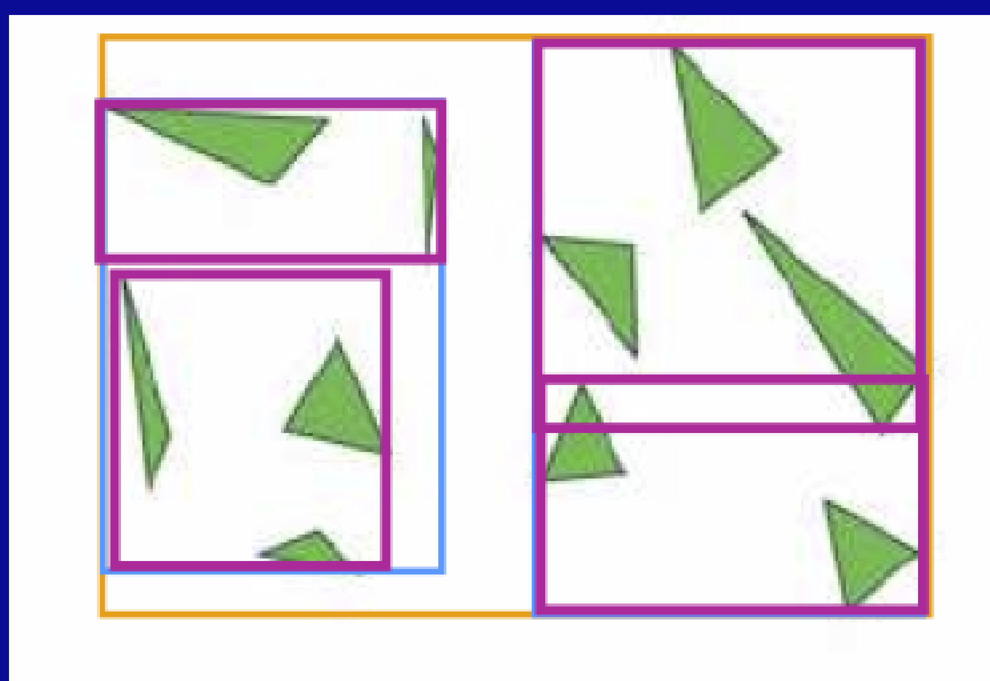
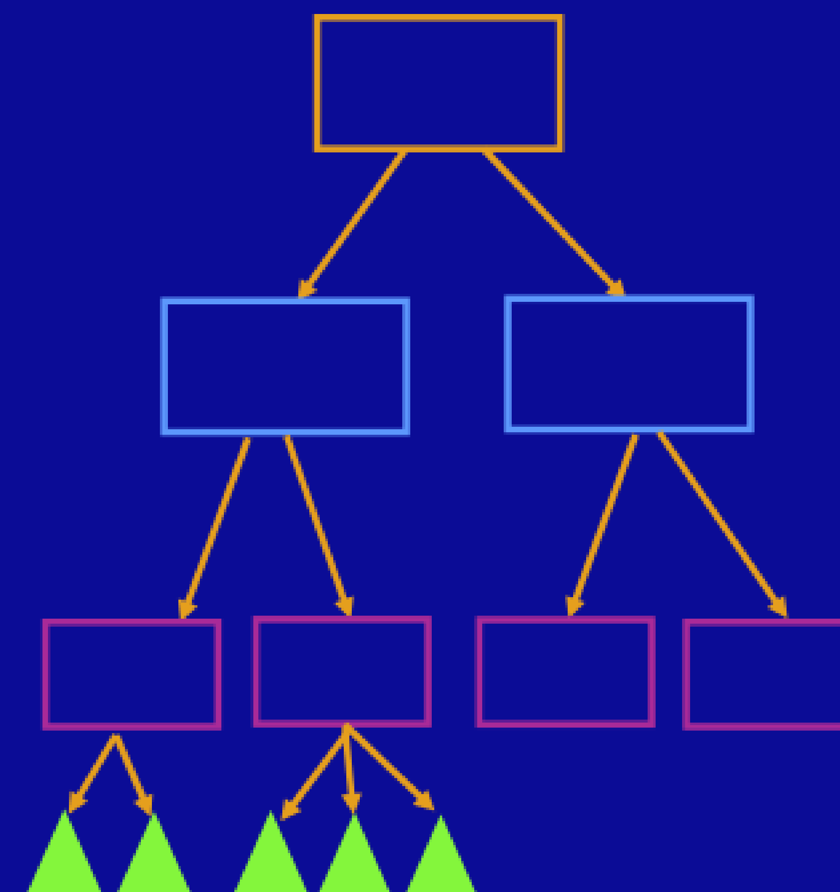
Hierarchical Bounding Volumes

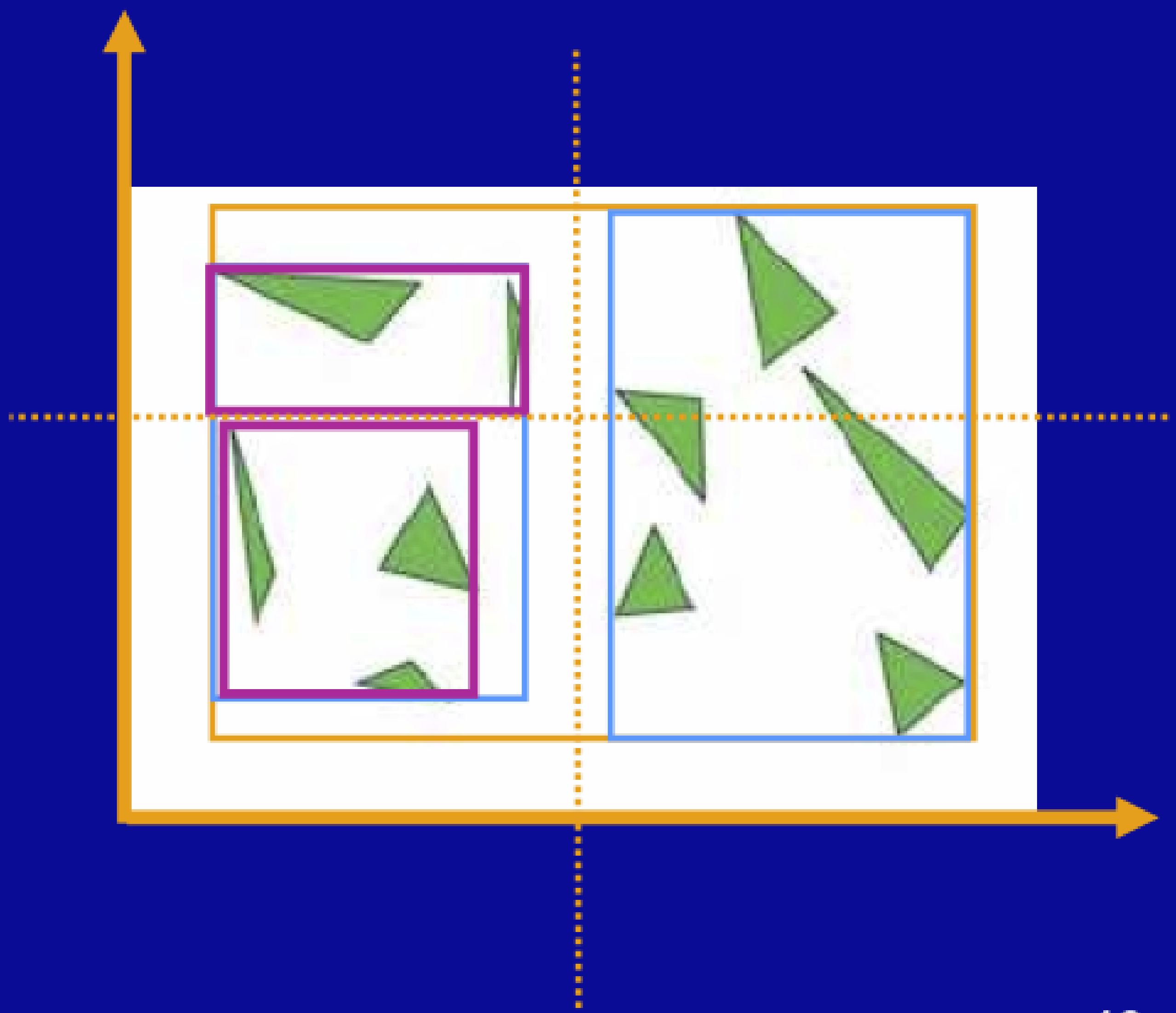
- Many ways to build a tree for the hierarchy
- Works well:
  - Binary
  - Roughly balanced
  - Boxes of sibling trees not overlap too much





Hierarchical Bounding Volumes

- Sort the surfaces along the axis before dividing into two boxes
- Carefully choose axis each time
- Choose axis that minimizes sum of volumes



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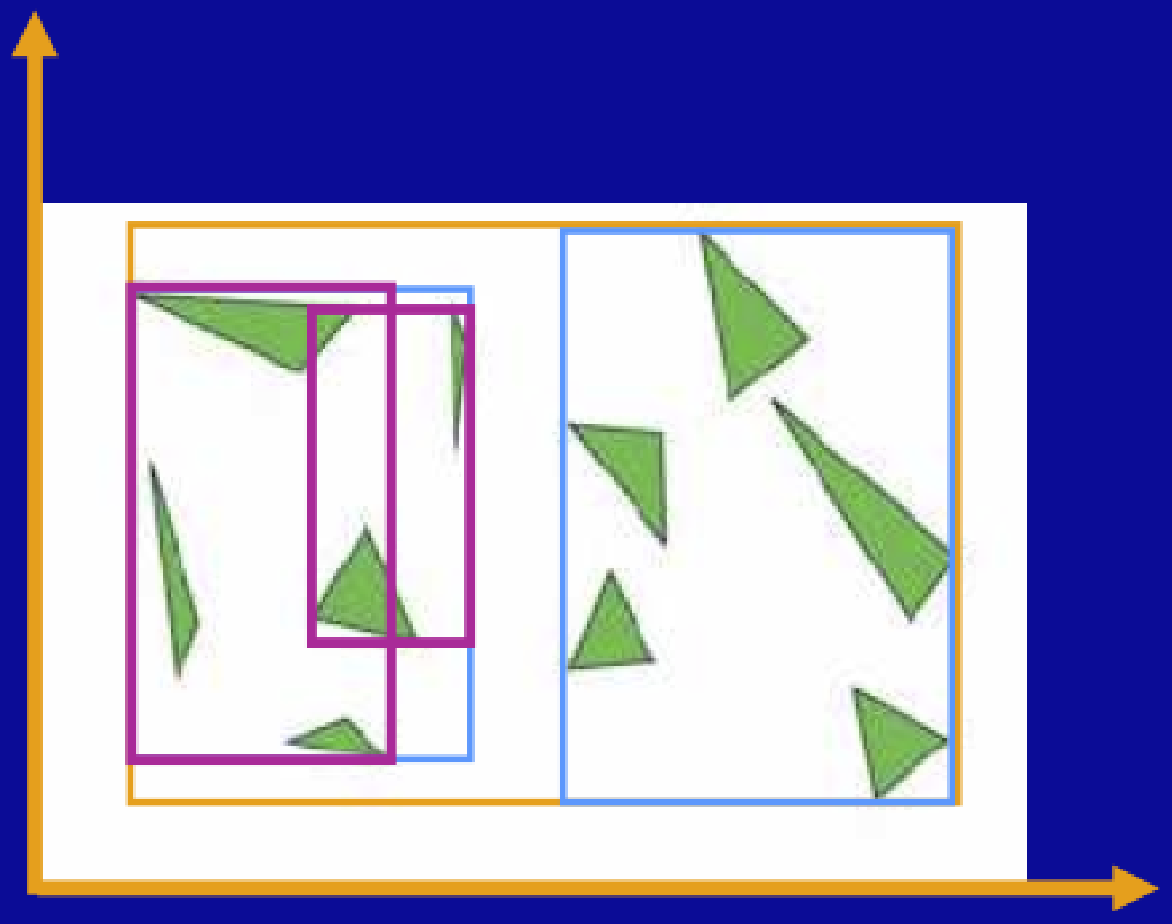
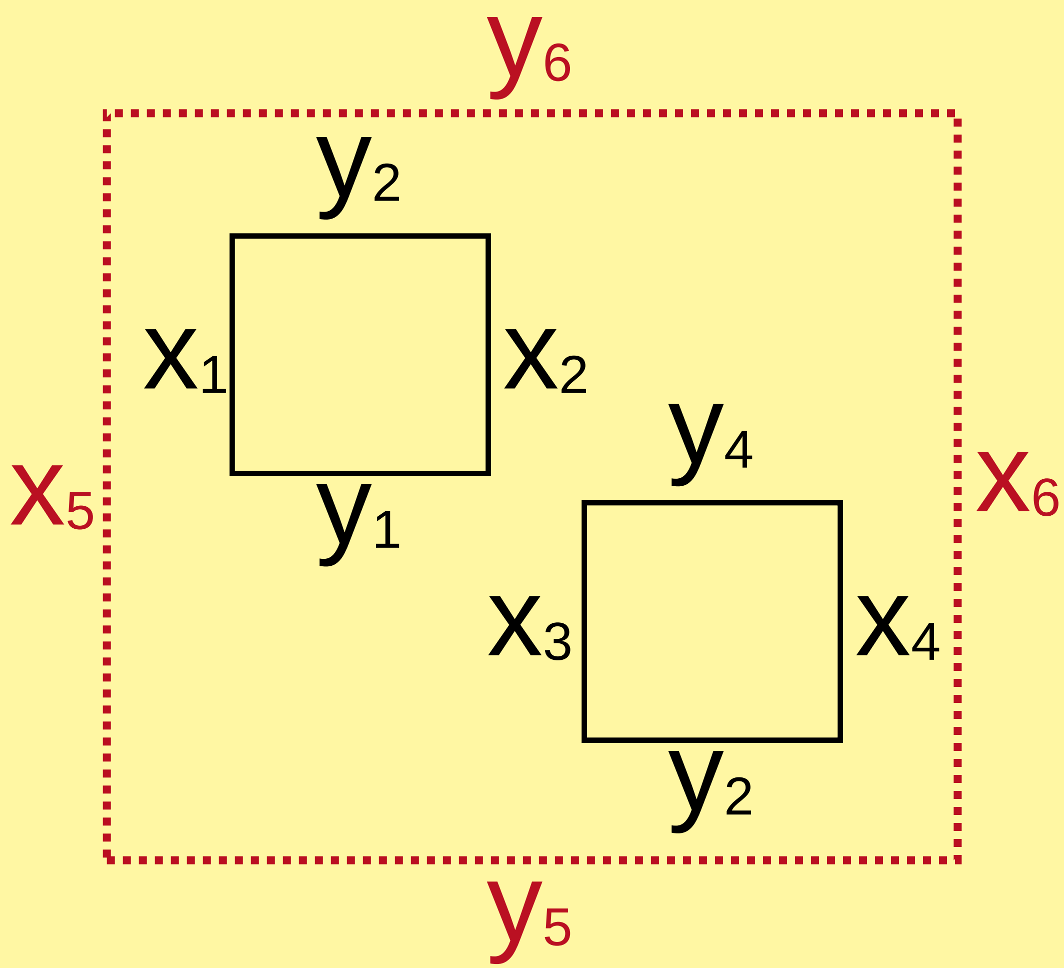


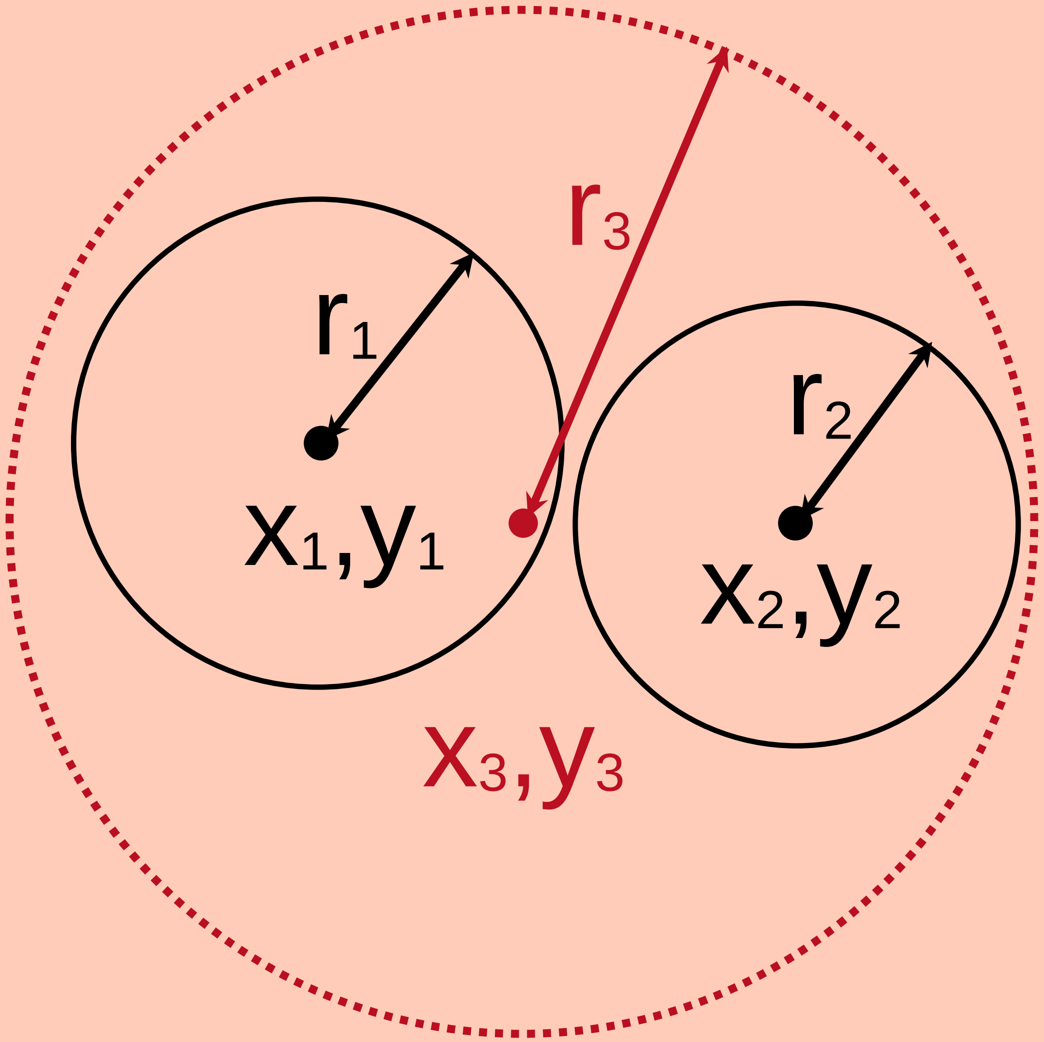
Hierarchical Bounding Volumes

- Works well if you use good (appropriate) bounding volumes and hierarchy
- Should give O(log n) rather than O(n) complexity
  (n=# of objects)
- Can have multiple classes of bounding volumes and pick the best for each enclosed object

Questions



Given two bounding boxes at one level of the hierarchy, how do you compute the boxes for the next level?



How about for bounding spheres?

Tight Bounding Spheres

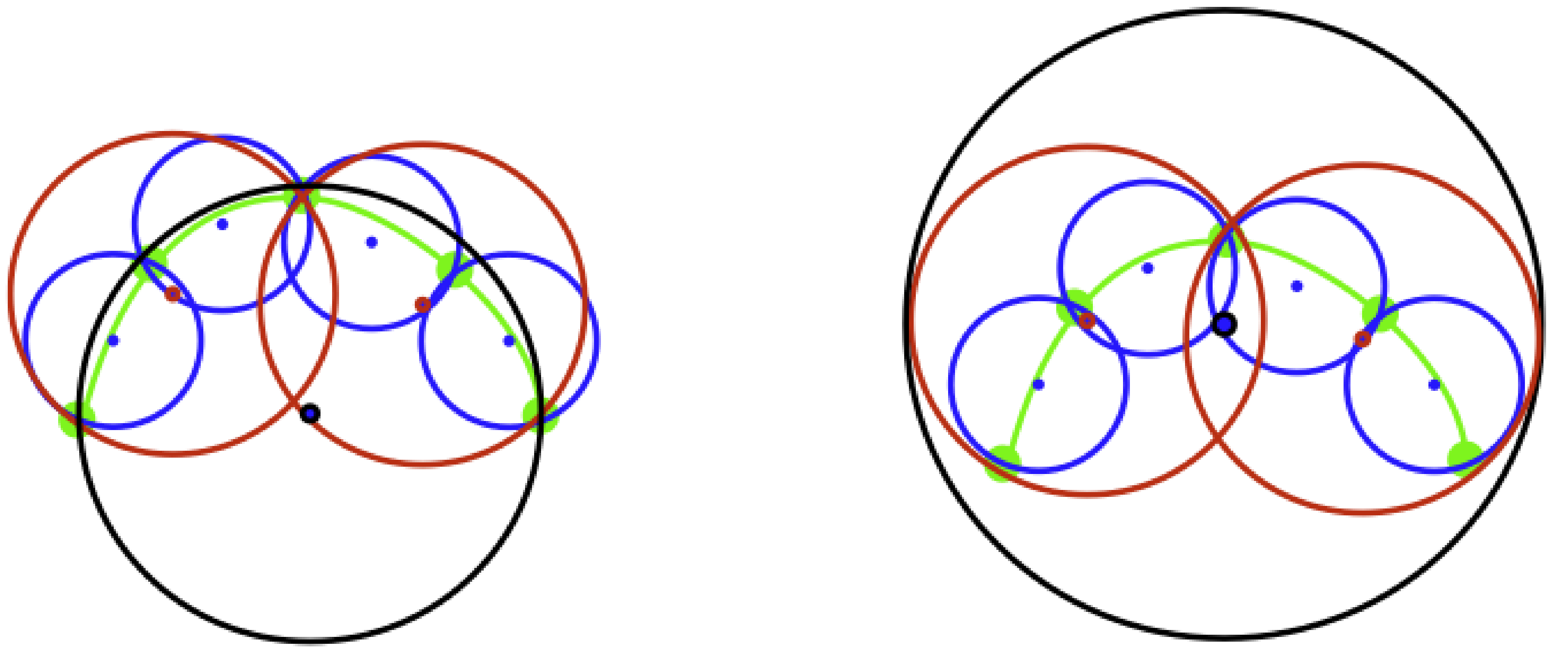


Figure 3: *The wrapped hierarchy* (left) has smaller spheres than the layered hierarchy (right). The base geometry is shown in green, with five vertices. Notice that in a wrapped hierarchy the bounding sphere of a node at one level need not contain the spheres of its descendents and so can be significantly smaller. However, since each sphere contains all the points in the base geometry, it is sufficient for collision detection.

Hierarchical bounding volumes

Spatial Subdivision

- Grids
- Octrees
- K-d trees and BSP trees

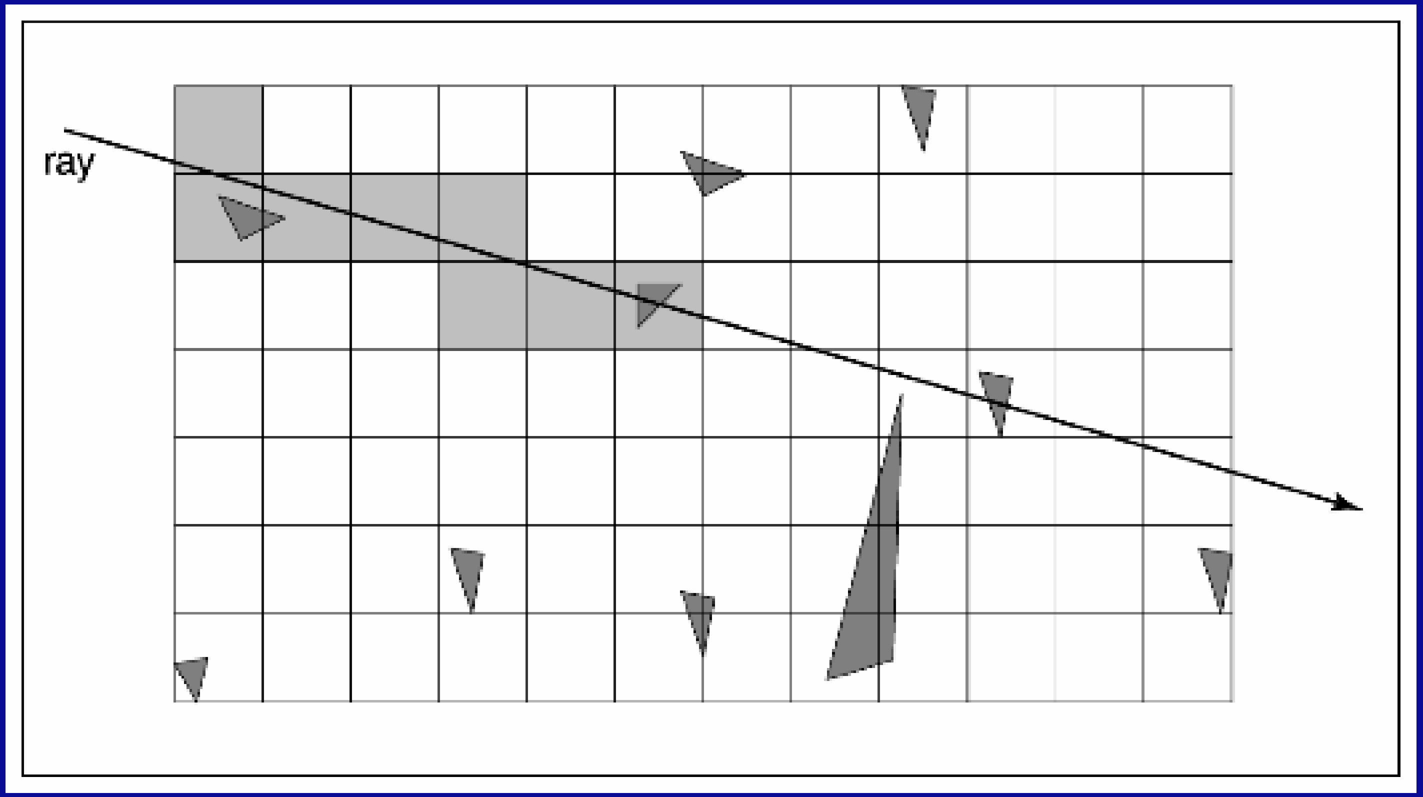
3D Spatial Subdivision

- Bounding volumes enclose the objects (object-centric)

- Instead could divide up the space—the further an object is from the ray the less time we want to spend checking it
  - Grids
  - Octrees
  - K-d trees and BSP trees

Grids

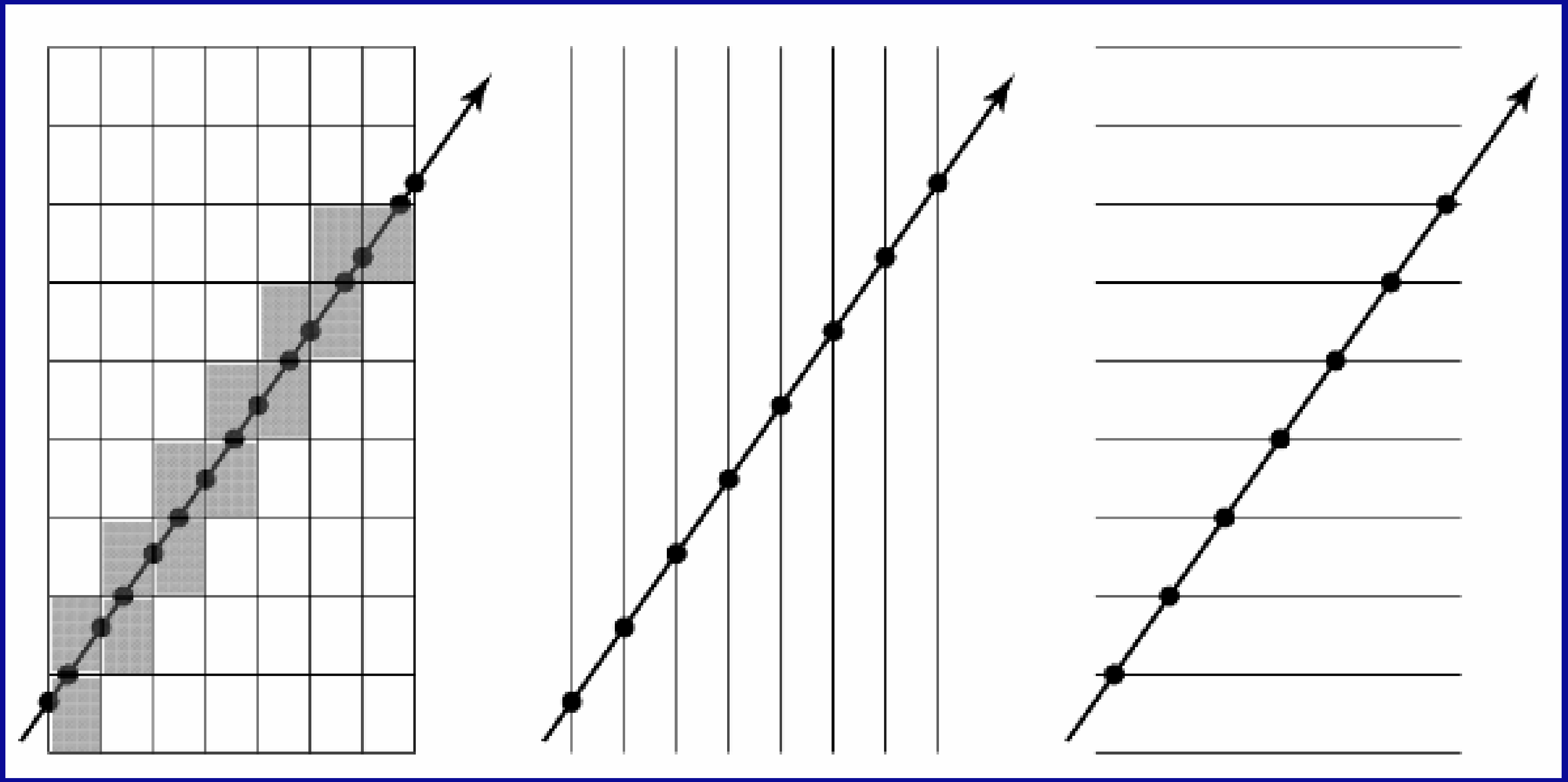
- Data structure: a 3-D array of cells (voxels) that tile space
  - Each cell points to list of all surfaces intersecting that cell



- Intersection testing:
  - Start tracing at cell where ray begins
  - Step from cell to cell, searching for the first intersection point
  - At each cell, test for intersection with all surfaces pointed to by that cell
  - If there is an intersection, return the closest one

Grids

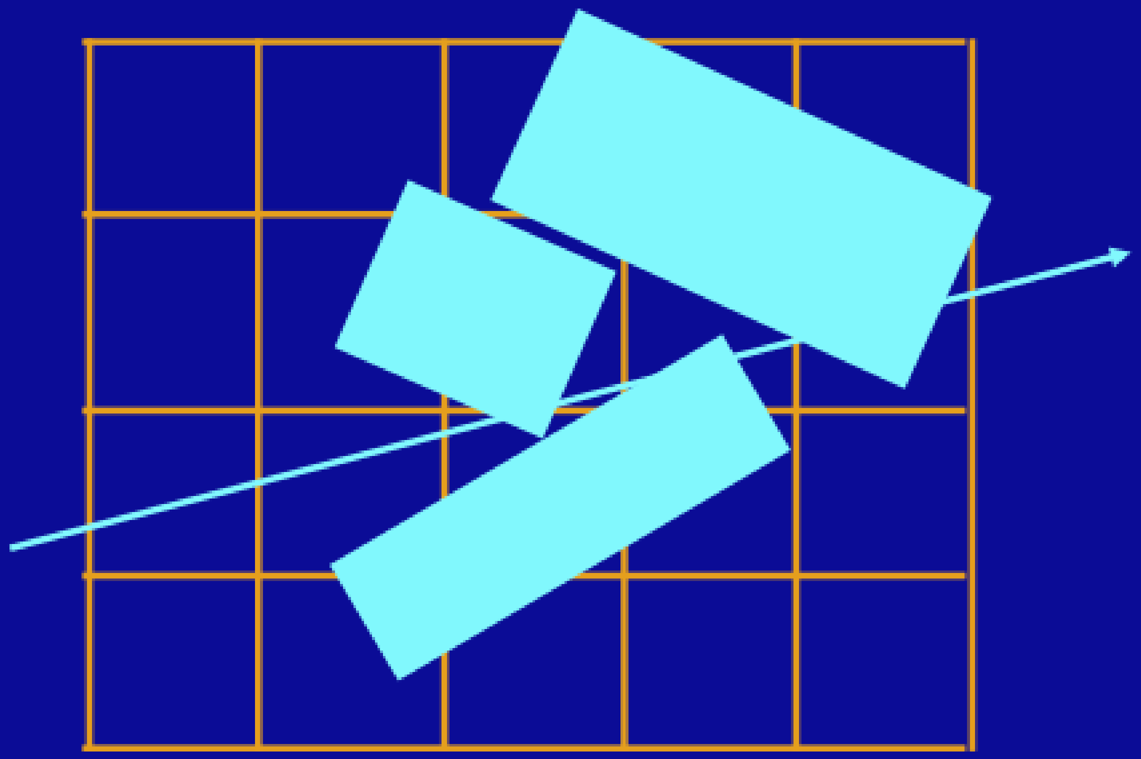
- Cells are traversed in an incremental fashion
- Hits of sets of parallel lines are very regular



More on Grids

- Be Careful! The fact that a ray passes through a cell and hits an object doesn't mean the ray hit that object in *that* cell

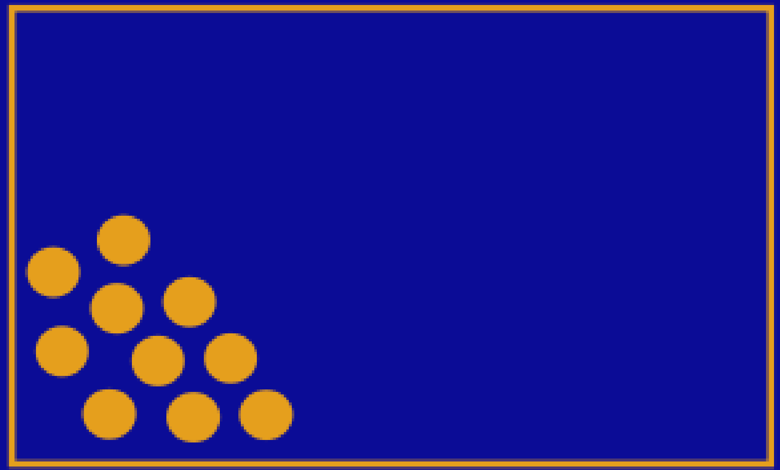
- Optimization: cache intersection point and ray id in "mailbox" associated with each object



- Step from cell to cell
- Get object intersecting cell
- Find closest intersection
- If found intersection --- done

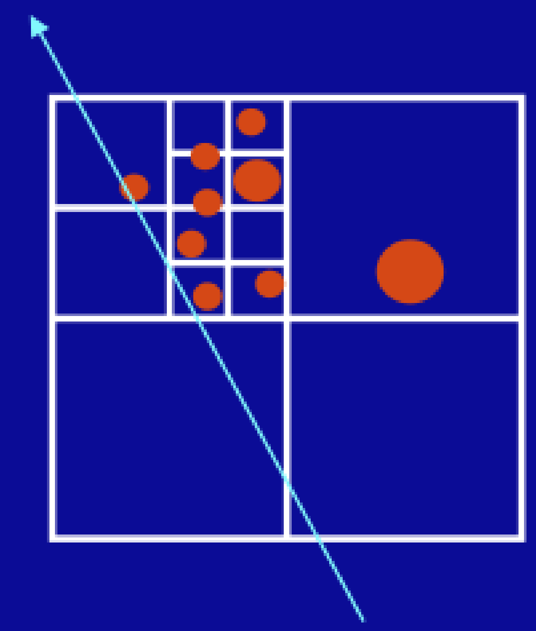
More on Grids

- Grids are a poor choice when the world is nonhomogeneous (clumpy)
  - many polygons clustered in a small space
- How many cells to use?
  - too few $\Rightarrow$ many objects per cell $\Rightarrow$ slow
  - too many $\Rightarrow$ many empty cells to step through $\Rightarrow$ slow
- Non-uniform spatial subdivision is better!



Octrees

- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get "simple enough"



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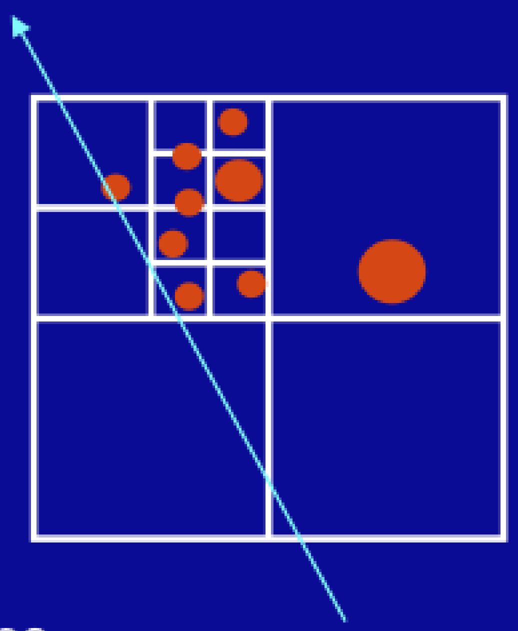


- Quadtree is the 2-D generalization of binary tree
  - node (cell) is a square
  - recursively split into four equal sub-squares
  - stop when leaves get "simple enough"
- Octree is the 3-D generalization of quadtree
  - node (cell) is a cube, recursively split into eight equal sub-cubes
  - for ray tracing:
    - stop subdivision based on number of objects
    - internal nodes store pointers to children, leaves store list of surfaces
  - more expensive to traverse than a grid
  - but an octree adapts to non-homogeneous scenes better

```
trace(cell, ray) {        // returns object hit or NONE
    if cell is leaf, return closest(objects_in_cell(cell))
    for child cells pierced by ray, in order     // 1 to 4 of these
        obj = trace(child, ray)
        if obj!=NONE return obj
    return NONE
}
```

Which Data Structure is Best for Ray Tracing?

Grids

- Easy to implement
- Require a lot of memory
- Poor results for inhomogeneous scenes

Octrees

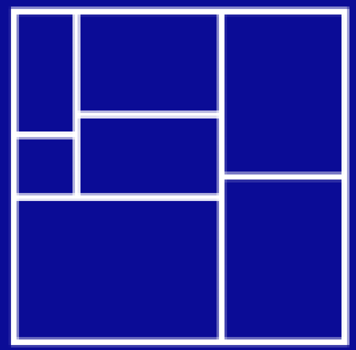
- Better on most scenes (more adaptive)

Spatial subdivision expensive for animations

- Hierarchical bounding volumes
- Better for dynamic scenes
- Natural for hierarchical objects

k-d Trees and BSP Trees

- Relax the rules for quadtrees and octrees:



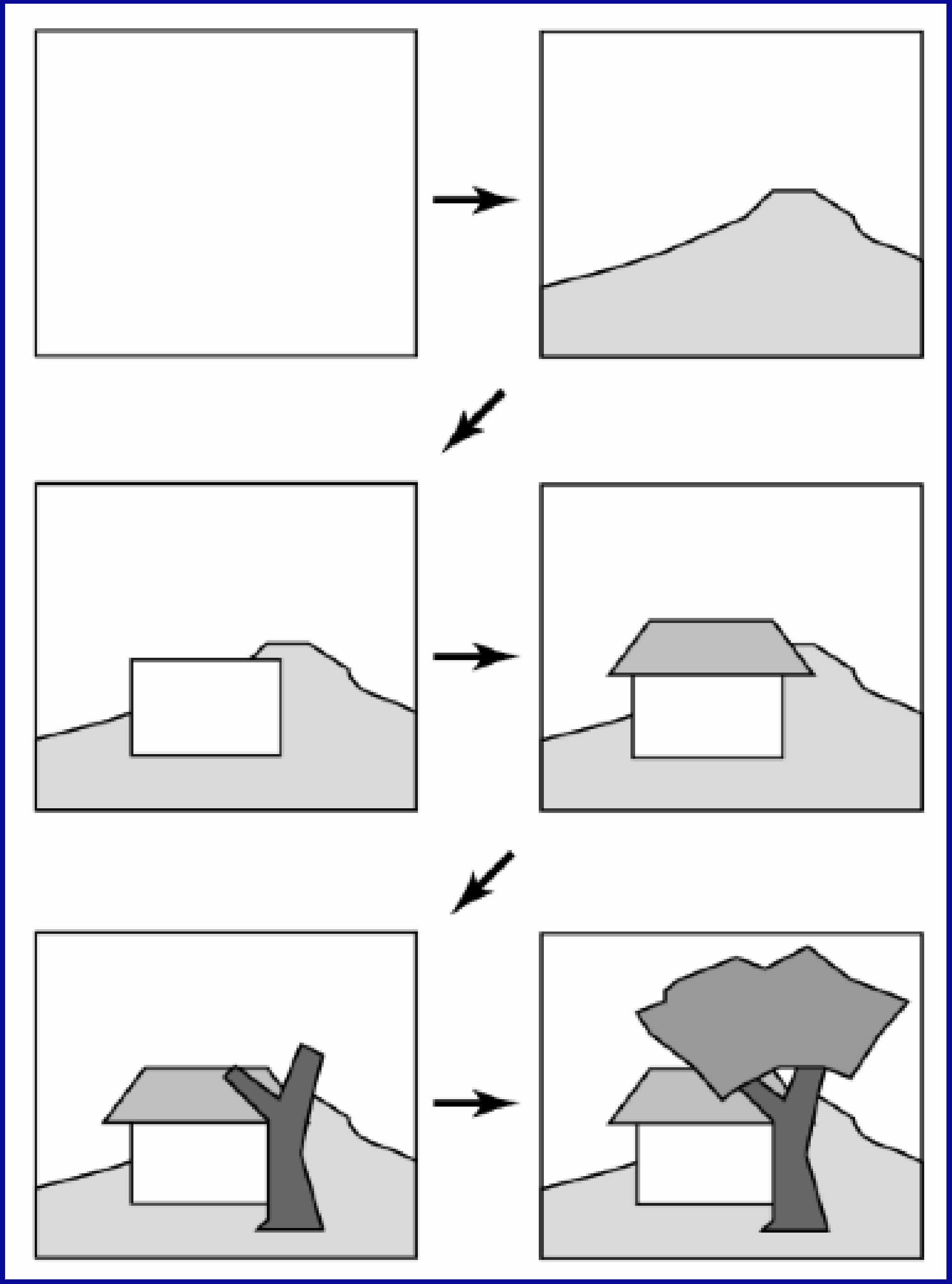
- k-dimensional (k-d) tree
  - don't always split at midpoint
  - split only one dimension at a time (i.e. *x* or *y* or *z*)

- binary space partitioning (BSP) tree
  - permit splits with any line
  - In 2-D space split with lines (most of our examples)
  - 3-D space split with planes
  - K-D space split with *k*-1 dimensional hyperplanes

- useful for Painter's algorithm (hidden surface removal)

Painters Algorithm

Hidden Surface Elimination

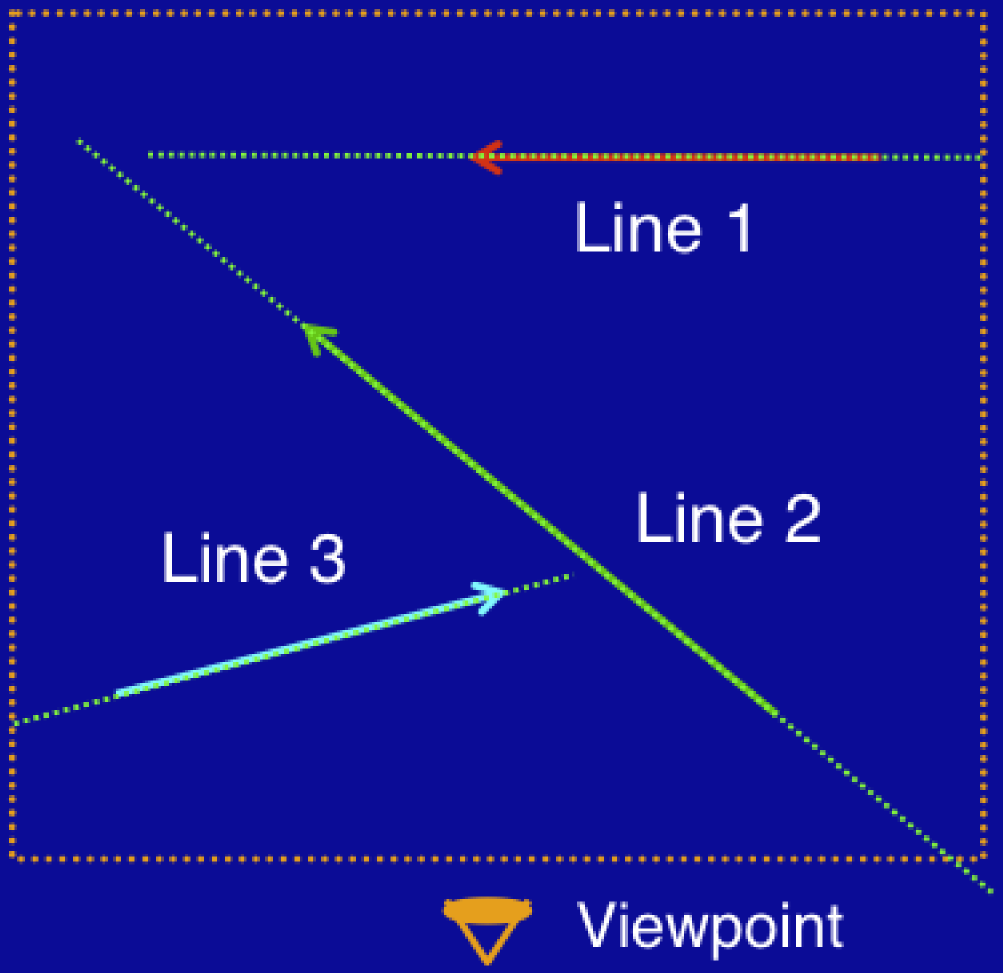


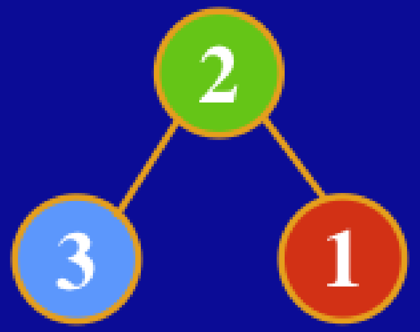
Painters Algorithm

- Need to sort objects back to front
- Order depends on the view point
- Partition objects using BSP tree
- View independent

Building a BSP Tree

- Let's look at simple example with 3 line segments
- Arrowheads are to show left and right sides of lines.
- Using line 1 or 2 as root is easy.
- (examples from http://www.geocities.com/SiliconValley/2151/bsp.html)

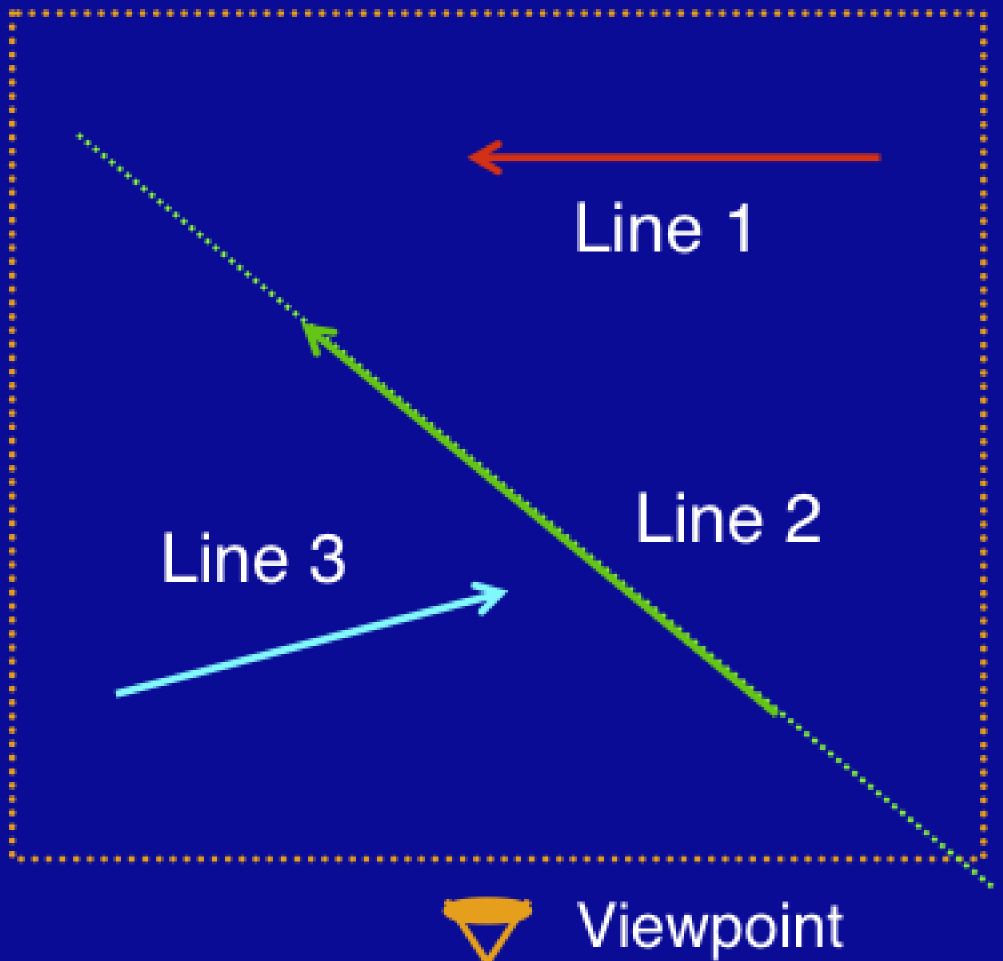


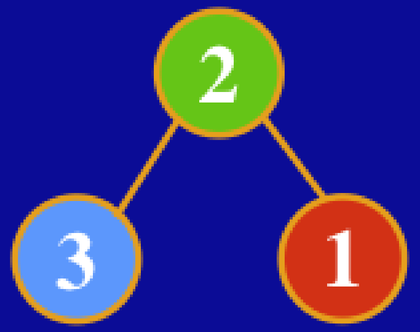


a BSP tree using 2 as root

Drawing Objects

- Traverse the tree from the root
- If view point is on the left of the line --- traverse right sub-tree first
- Draw the root
- Traverse left sub-tree

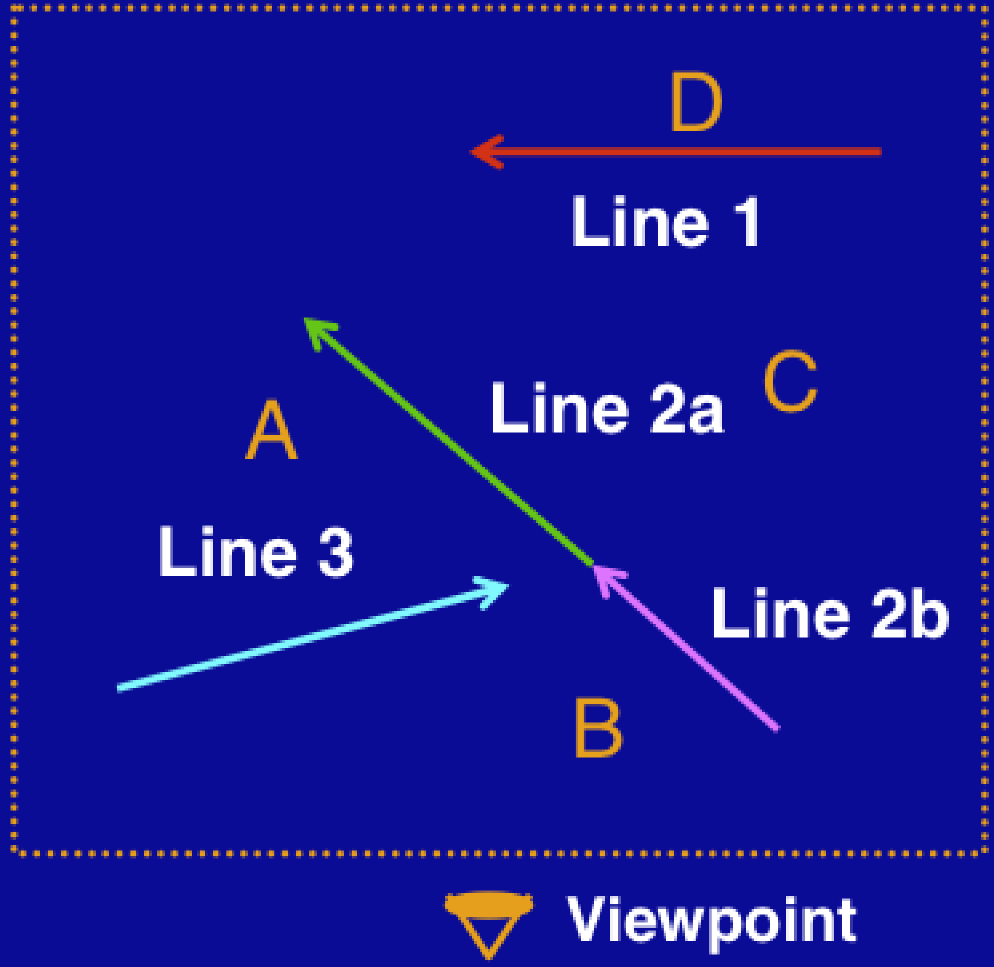


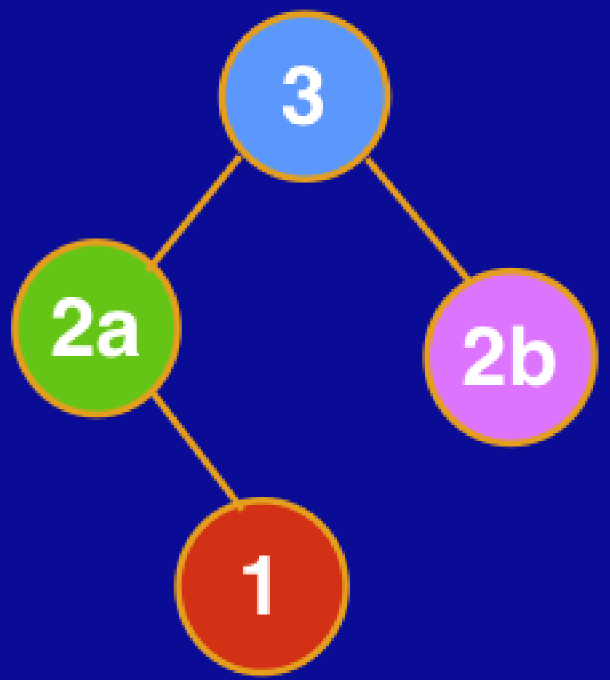


a BSP tree using 2 as root

Building the Tree 2

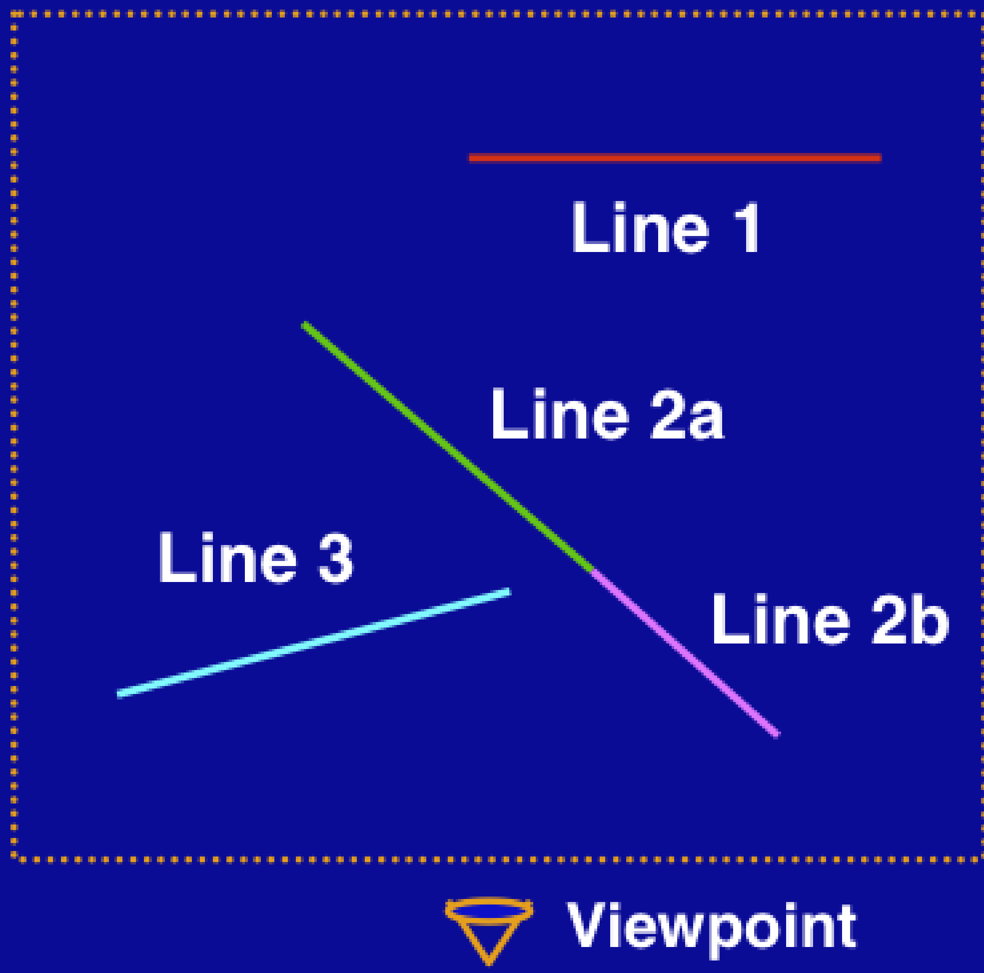
Using line 3 for the root requires a split





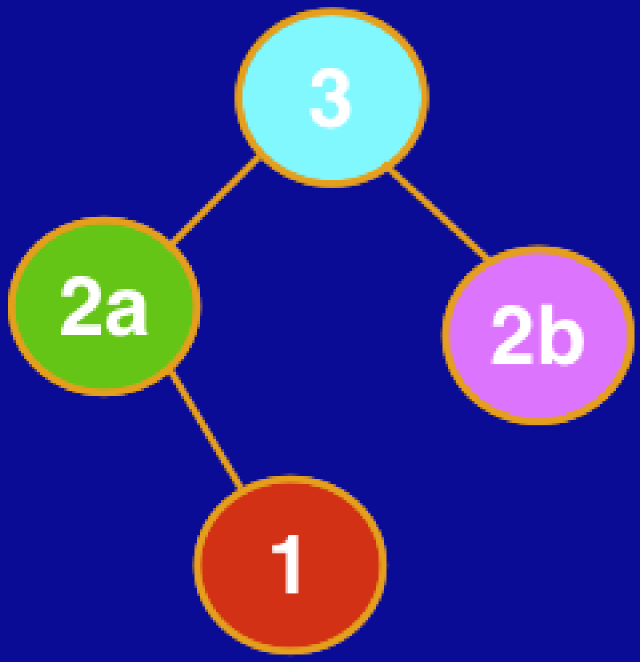
Drawing Back to Front

- Use Painter's Algorithm for hidden surface removal



Steps:

– *Draw objects on far side of line 3*
  » *Draw objects on far side of line 2a*
    – Draw line 1
  » Draw line 2a
– Draw line 3
– *Draw objects on near side of line 3*
  » Draw line 2b

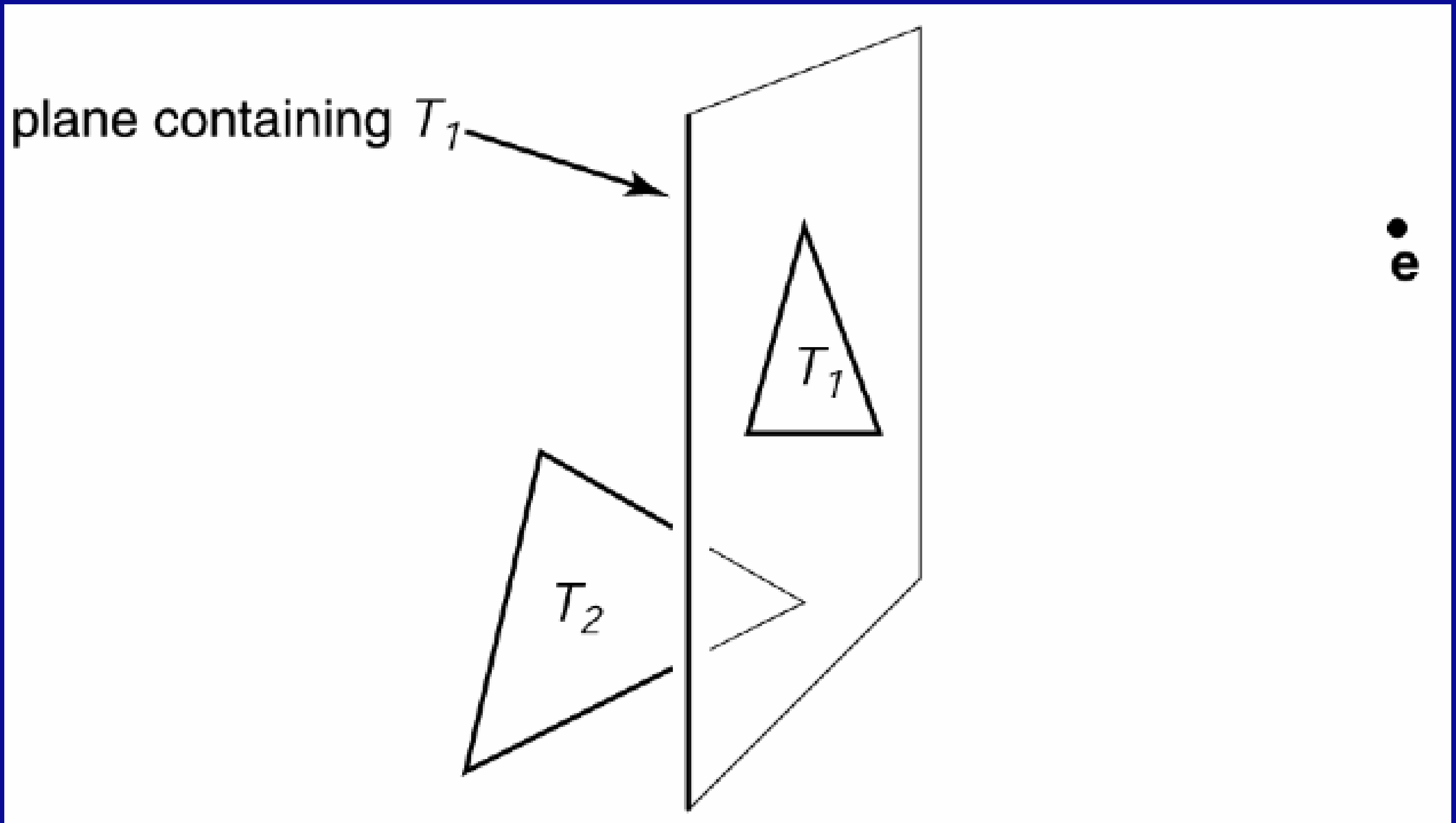


Triangles

Use plane containing triangle T_1 to split the space

If view point is on one side of the plane draw polygons on the other side first

T_2 does not intersect plane of T_1



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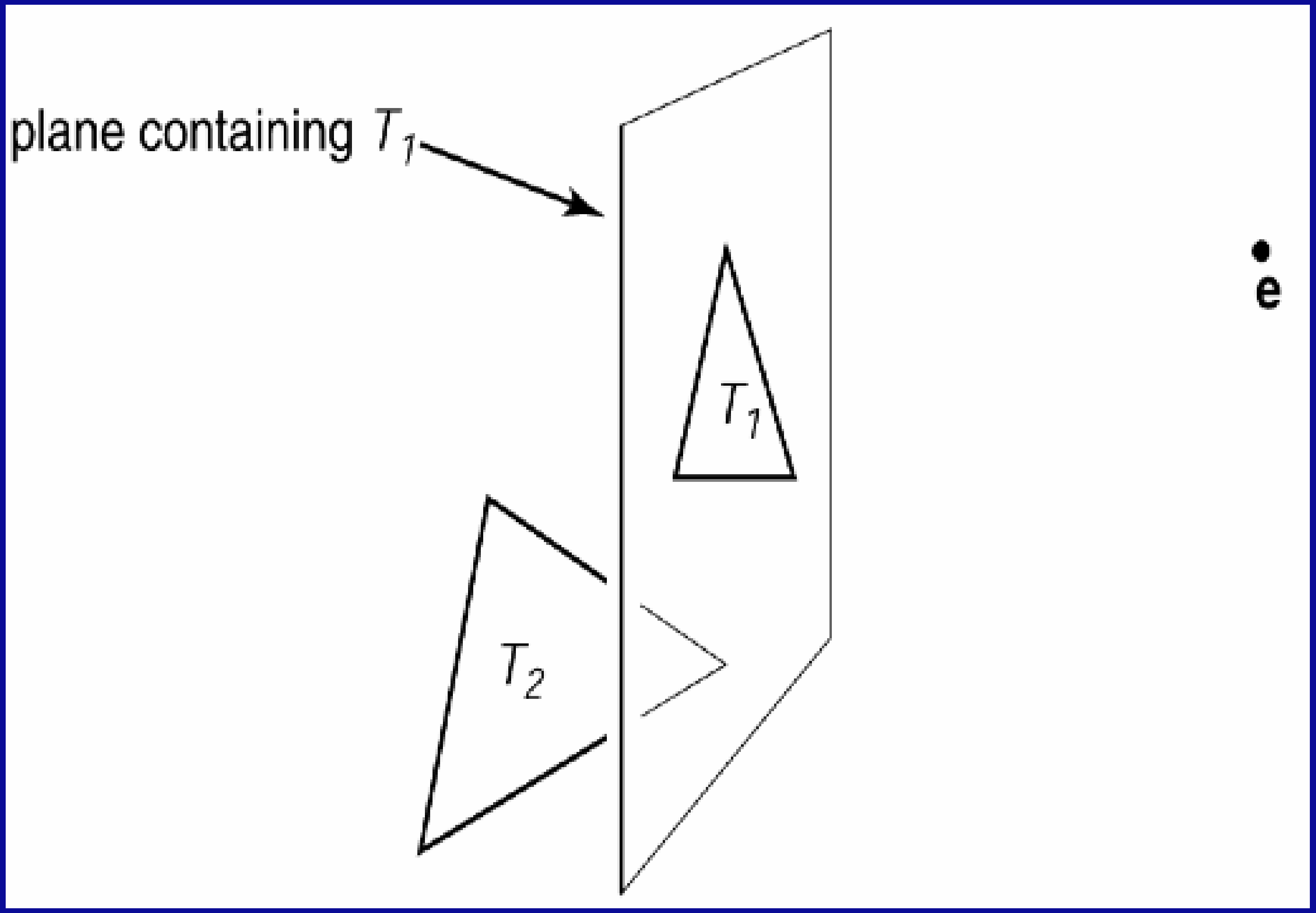
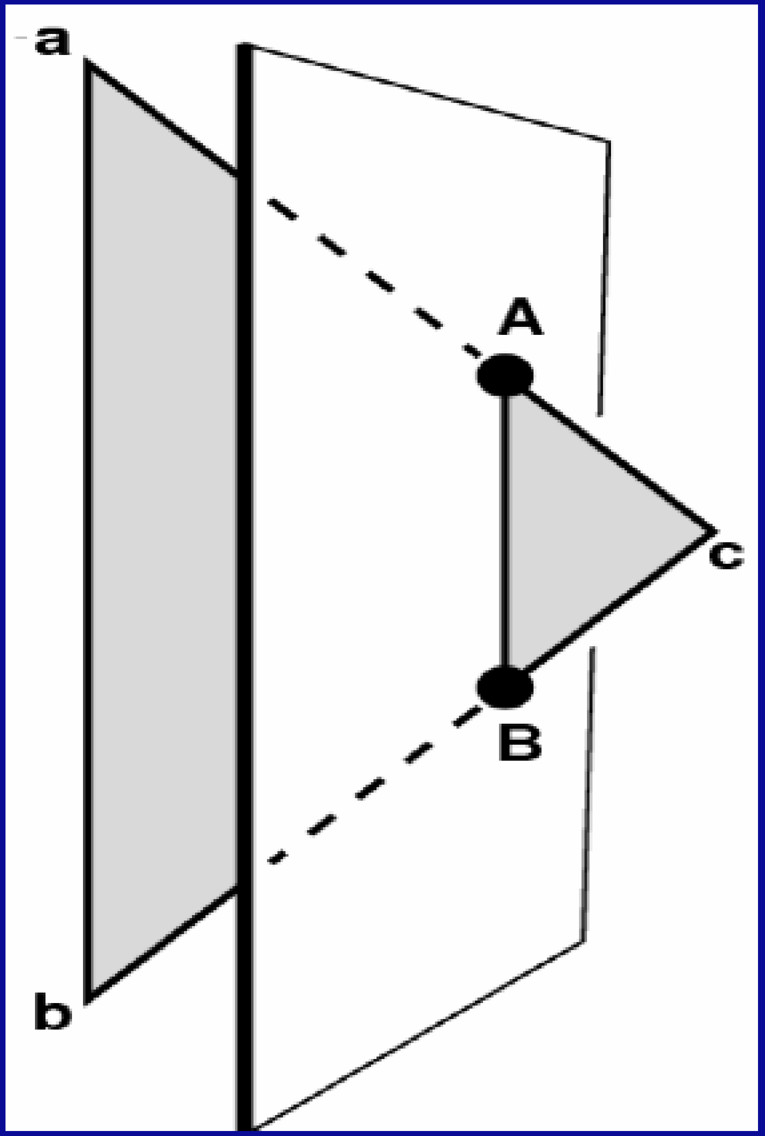
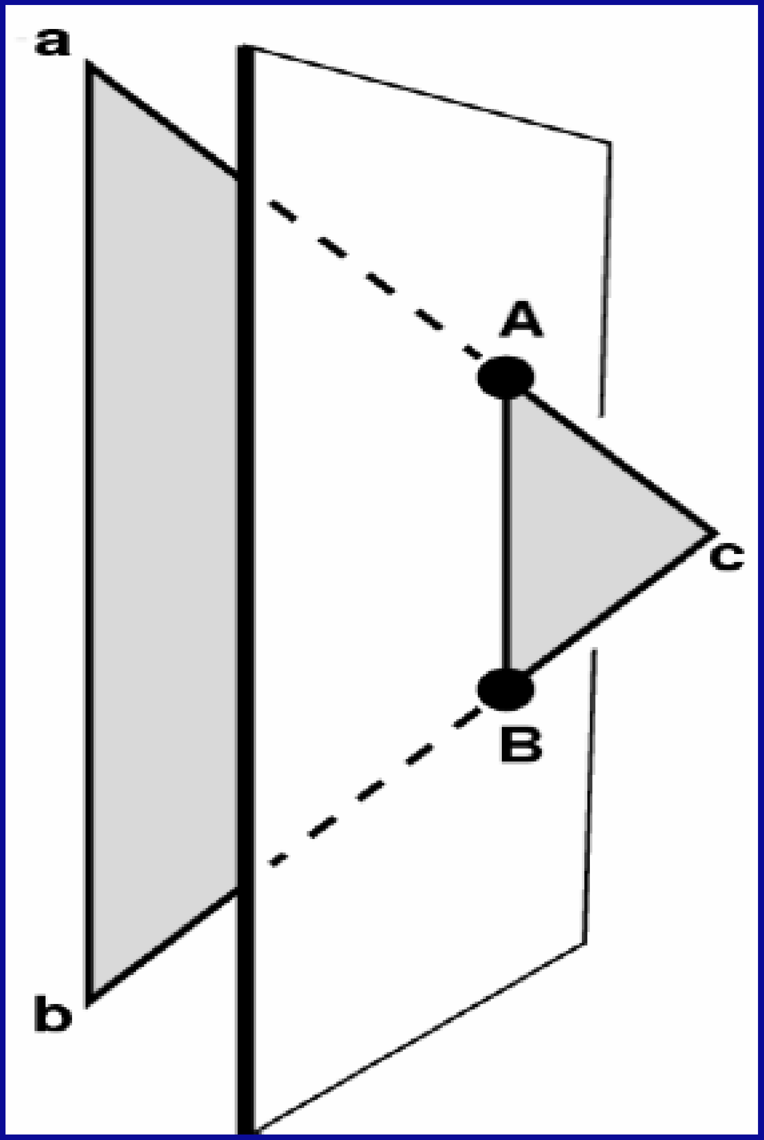
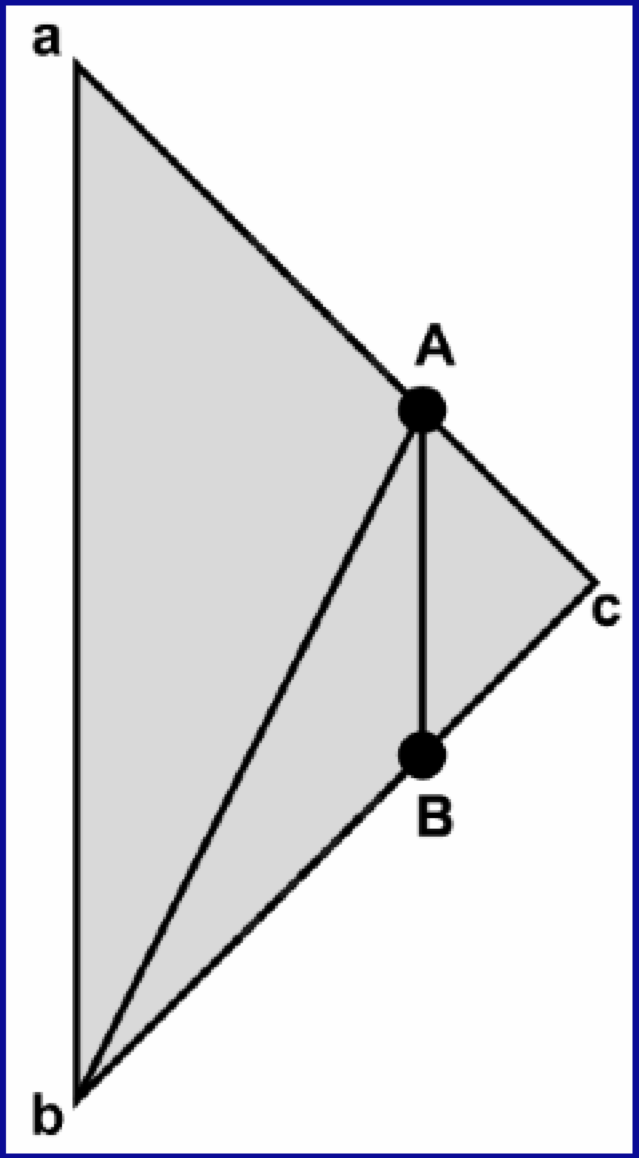




Triangles

Split Triangle





Painter's Algorithm with BSP trees

- Build the tree
  - Involves splitting some polygons
  - Slow, but done only once for static scene
- Correct traversal lets you draw in back-to-front or front-to-back order for any viewpoint
  - Order is view-dependent
  - Pre-compute tree once
  - Do the "sort" on the fly
- Will not work for changing scenes

Drawing a BSP Tree

- Each polygon has a set of coefficients:
  $Ax + By + Cz + D$
- Plug the coordinates of the viewpoint in and see:
  >0 : front side
  <0 : back facing
  =0 : on plane of polygon
- Back-to-front draw: inorder traversal, do farther child first
- Front-to-back draw: inorder traversal, do near child first

```
front_to_back(tree, viewpt) {
   if (tree == null)  return;
   if (positive_side_of(root(tree), viewpt)) {
      front_to_back(positive_branch(tree, viewpt);
      display_polygon(root(tree));
      front_to_back(negative_branch(tree, viewpt);
   }
   else { …draw negative branch first…}
}
```

Building a Good Tree - the tricky part

- A naïve partitioning of n polygons will yield $O(n^3)$ polygons because of splitting!

- Algorithms exist to find partitionings that produce $O(n^2)$.
  - For example, try all remaining polygons and add the one which causes the fewest splits
  - Fewer splits -> larger polygons -> better polygon fill efficiency

- Also, we want a balanced tree.

Demos

BSP Tree construction

http://symbolcraft.com/graphics/bsp/index.html

- KD Tree construction

http://www.cs.umd.edu/~brabec/quadtree/index.html

Practice Problems

Sketch a (2D) scene with 5 triangles and give a KD tree that divides up the scene.   Illustrate your KD tree in 2 ways: with bounding planes drawn into your scene and with a tree structure having triangles at the roots. Make sure you label triangles in your scene and your tree so that we can determine the mapping between the two.

Practice Problems

Describe how to construct a KD tree. Describe how to perform ray-object intersection using this data structure. Your ray-object intersection algorithm should check regions from front to back order, so that the search for an intersection may be halted when an intersection point is found.

In the KD tree scenario, what problem is introduced when the splitting planes intersect some of the objects? Assume that you do not want to use the plane to split the objects (e.g., splitting a sphere into two parts may not be desirable for ray tracing). How can you solve or work around this problem?