SOLUTIONS HW1

1 a. 3 $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

b. 3 $\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$

c. 4 We can combine translations, rotations, scale, & shear in one matrix operation.

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2. $\boxed{\text{Test } (x_0, y_0) \ \& \ (x_1, y_1)}$ to make sure both points are on the line.

3 $f(x_0, y_0) = (y_1 - y_0)x_0 + (x_0 - x_1)y_0 + x_1 y_0 - x_0 y_1$

$= 0$ $\boxed{\text{OK}}$

3 $f(x_1, y_1) = (y_1 - y_0)x_1 + (x_0 - x_1)y_1 + x_1 y_0 - x_0 y_1$

$= 0$ $\boxed{\text{OK}}$

4 $\boxed{\text{The function } f(x, y) \text{ is linear in both } x \ \& \ y.}$

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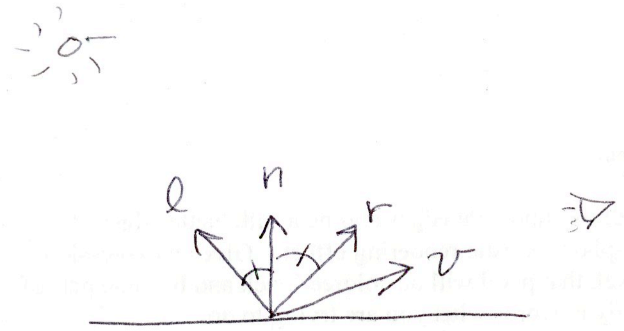
3. $$\boxed{P(t) = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t \begin{bmatrix} x_1 - x_0 \\ y_1 - x_0 \end{bmatrix}}$$

4. a.



l = unit vector to light source

n = surface normal (unit magnitude)

r = reflection of l about the normal

v = unit direction towards the viewer

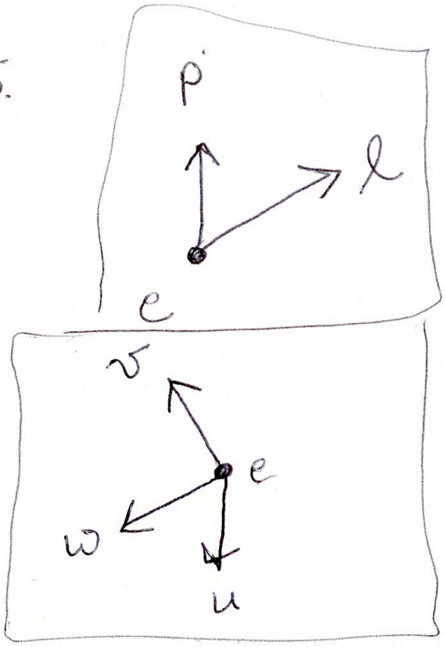
b. k_a, k_d, k_s, α

c. L_a, L_d, L_s

d. Control spread of the specular highlight

e. to capture global illumination effects
(e.g. light reaching a surface after bouncing off or passing through) other objects

5.



Let $w = -l$

$u = p \times w$

$v = w \times u$

or $w = -l$
$u = l \times p$
$v = p$

Goal: convert from world frame to uvw frame

$$\Rightarrow \quad u \xrightarrow{\text{maps to}} (1,0,0)^T$$

$$e \xrightarrow{\text{maps to}} (0,0,0)^T \qquad v \longrightarrow (0,1,0)^T$$

$$w \longrightarrow (0,0,1)^T$$

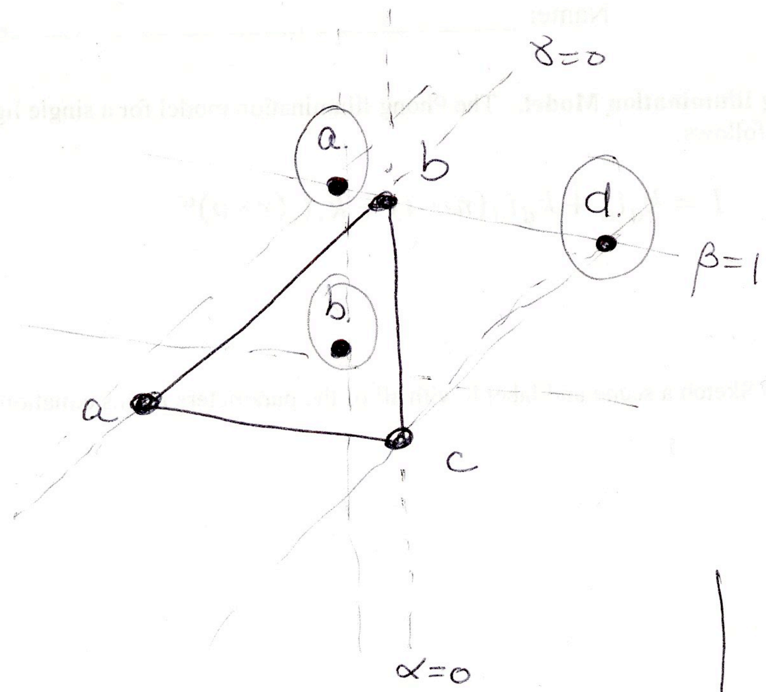
Result matrix is

$$\boxed{\begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

or... combining these matrices

$$\boxed{\begin{bmatrix} u_x & u_y & u_z & -(u \cdot e) \\ v_x & v_y & v_z & -(v \cdot e) \\ w_x & w_y & w_z & -(w \cdot e) \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

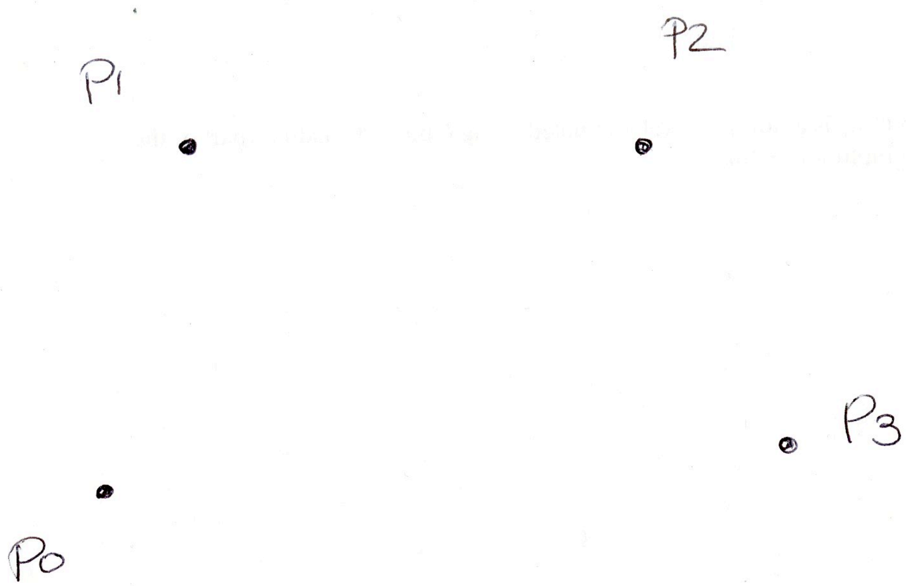
6.



c. no good $\Rightarrow$ barycentric coords don't sum to 1

e. no good

7.



$P(u) = au^3 + bu^2 + cu + d$

$p(0) = \underline{d = Po}$

$p(1) = \underline{a+b+c+d = P3}$

$p'(u) = 3au^2 + 2bu + c$

$p'(0) = \underline{c = 3(P1 - Po)}$

$p'(1) = \underline{3a + 2b + c = 3(P3 - P2)}$

Substituting for c & d

$a + b + 3(P1 - Po) + Po = P3 \Rightarrow a + b = 2Po - 3P1 + P3$

$3a + 2b + 3(P1 - Po) = 3(P3 - P2) \Rightarrow 3a + 2b = 3Po - 3P1 - 3P2 + 3P3$

$(-2)\quad -2a - 2b = -4Po + 6P1 - 2P3$

$a = -Po + 3P1 - 3P2 + P3$

$(-Po + 3P1 - 3P2 + P3) + b = 2Po - 3P1 + P3$

$b = 3Po - 6P1 + 3P2$

$c = -3Po + 3P1$

$d = Po$

In matrix form $[u^3\ u^2\ u\ 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Po \\ P_1 \\ P2 \\ P3 \end{bmatrix}$