Homework 1
due Tuesday, Feb 25th just before class

70 points

Please show your work.
Write down your assumptions and steps so that I can give you partial credit.
Please list all sources that you use other than the course slides and notes.

1. (10 points) Homogeneous coordinates.
   a. How do we represent a point in 3D space using homogeneous coordinates?
   b. How do we represent a vector (a direction) in 3D space using homogeneous coordinates?
   c. Why do we use this representation?

2. (10 points) Implicit form of a line. An implicit equation for a line through the 2D points 
   \((x_0, y_0)\) and \((x_1, y_1)\) is given below. Prove that this equation is correct. To do this, you must 
   show that it interpolates the two points and is indeed an equation for a line.

   \[ f(x, y) = (y_1 - y_0)x + (x_0 - x_1)y + x_1y_0 - x_0y_1 \]

3. (5 points) Parametric form of a line. Write the parametric form for the line through the 2D 
   points \((x_0, y_0)\) and \((x_1, y_1)\).

4. (15 points) Direct illumination. The Phong Illumination equation is given below. (It has 
   also been referred to as Blinn-Phong in class.)

   \[ I = k_a L_a + k_d (l \cdot n) L_d + k_s (r \cdot v) \alpha L_s \]
   a. Sketch the \(l\), \(n\), \(r\), and \(v\) vectors and briefly define each one.
   b. What terms in this equation are properties of the surface material of objects in the 
      scene?
   c. What terms in this equation are properties of the light source(s)?
   d. What is the role of parameter \(\alpha\)?
   e. Why do we include an ambient light term?

5. (10 points) Transforms. Derive the transformation matrix that brings points in the world 
   frame into the camera frame. The camera is located at eye point \(e\), with look direction \(l\) and 
   up vector \(p\). Please start by sketching the scene to indicate your understanding of the 
   problem.
6. **(10 points) Barycentric coordinates.** Draw a triangle and label vertices a, b, and c. Sketch in points at the following barycentric coordinate locations. If a point should not be plotted because it does not have valid barycentric coordinates, please indicate this is the case.

   a. [0.2, 1.0, -0.2]
   b. [0.2, 0.3, 0.5]
   c. [.1, .1, .1]
   d. [-1, 1, 1]
   e. [.5, .4, .3]

7. **(10 points) Curves and splines.** Derive from scratch the equations for the Bezier spline using the following 4 facts:
   - the spline is a cubic spline in parameter u,
   - the spline interpolates control points P0 and P3,
   - the first derivative of the curve at P0 is 3(P1-P0), and
   - the first derivative of the curve at P3 is 3(P3-P2)