The Rendering Equation

Jim Kajiya · SIGGRAPH 1986

Goal: model physical phenomenon of light scattering off various types of surfaces

History: radiative heat transfer studies

Kajiya presents in a form well suited for CG

$$I(x, x') = g(x, x') \left[\epsilon(x, x') + \int_S \rho(x, x', x'') I(x', x'') \, dx'' \right]$$

briefly:

$I(x, x')$ is related to intensity of light passing from point x' to point x

... the value we ultimately want to know everywher

... how many dimensions?

x' is 3D

x is 3D

6D? 5D? 4D?

⇒ We assume no scattering by the atmosphere

$g(x, x')$ is a "geometry" term

$\epsilon(x, x')$ is related to the intensity of emitted light from x' to x

$\rho(x, x', x'')$ is related to the intensity of light scattered from x'' to x by a patch of surface at x'

The integral is taken over all surfaces S in the scene .... plus global background surface S_0

Parameters may differ for every wavelength (parameterization by wavelength implicit)

$I(x, x')$ unoccluded two-point transport intensity

energy of radiation per unit time per unit area of source dx' per unit area dx of target

$$dE = I(x, x')\, dt\, dx\, dx'$$

units of $I(x, x')$ are $J/m^4 s$

$g(x, x')$ geometry term

if x and x' are not mutually visible, $g = 0$

.. even for a transparent surface

→ intercepts radiation
emits it out the other side

if x and x' are mutually visible, $g = 1/r^2$

where r = distance between x & x'

$\epsilon(x, x')$ unoccluded two-point transport emittance

energy emitted per unit time per unit area of source, per unit area of target

$$dE = \frac{1}{r^2}\epsilon(x, x')\, dt\, dx\, dx'$$

units of ϵ are $J/m^2 s$

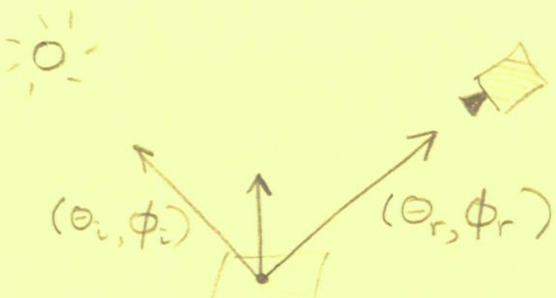
$\rho(x, x', x'')$ unoccluded three point transport reflectance from x'' to x through x'

⇒ also covers transmission through surfaces

$$dE = \frac{1}{r^2}\, \rho(x, x', x'')\, I(x', x'')\; dt\, dx\, dx'\, dx''$$

(ρ is dimensionless)

How does ρ relate to the BRDF?



BRDF: $$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

<u>radiance</u> of surface

<u>irradiance</u> at surface

$$\rho(x, x', x'') = f(\underbrace{\theta_i, \phi_i}_{\text{from } x'' \text{ to } x'}, \underbrace{\theta_r, \phi_r}_{\text{from } x' \text{ to } x}) \cos\theta \cos\theta_r$$

We have: