

# Image Processing



# Overview



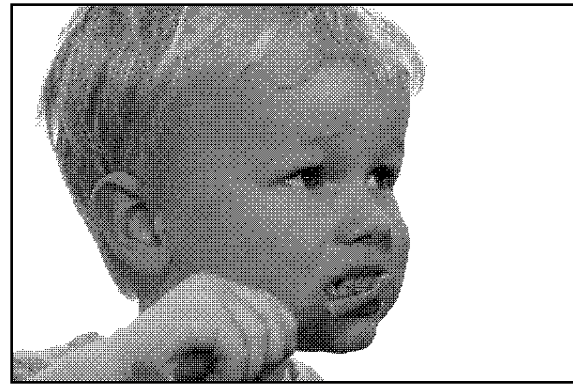
Images



Pixel Filters



Neighborhood Filters



Dithering

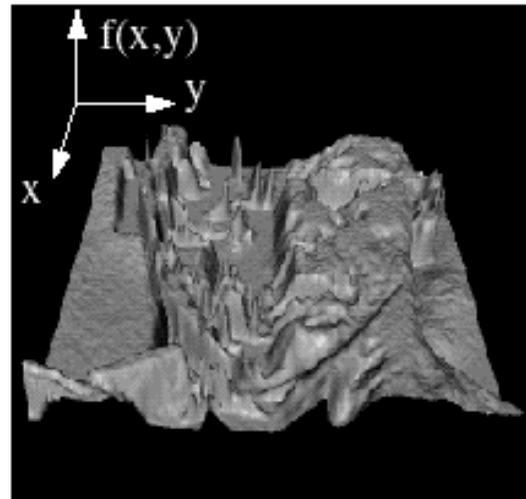
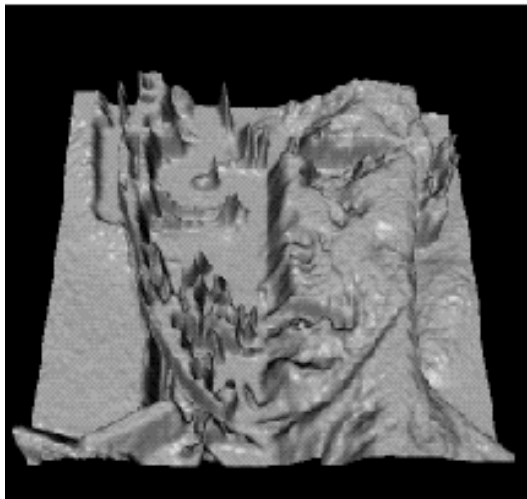
# Image as a Function

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- We can think of an **image** as a function,  $f$ ,
- $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ 
  - $f(x, y)$  gives the **intensity** at position  $(x, y)$
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - $f: [a,b] \times [c,d] \rightarrow [0,1]$
- A color image is just three functions pasted together. We can write this as a “vector-valued” function:
$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

# Image as a Function

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# Image Processing

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- Define a new image  $g$  in terms of an existing image  $f$ 
  - We can transform either the domain or the range of  $f$
- Range transformation:

$$g(x, y) = t(f(x, y))$$

What kinds of operations can this perform?

# Image Processing

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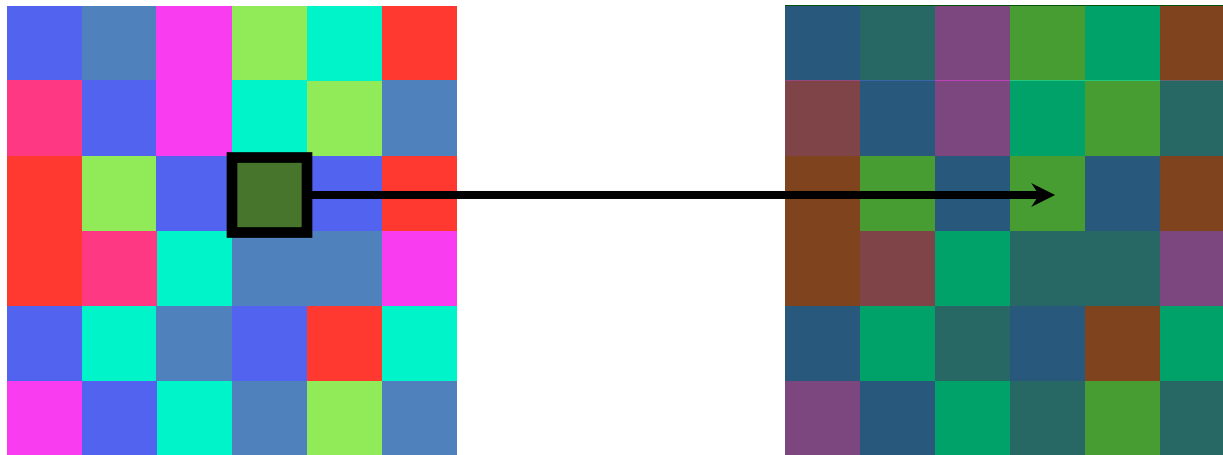
- Some operations preserve the range but change the domain of  $f$ :

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

- Still other operations operate on both the domain and the range of  $f$ .

# Point Operations



# Point Processing

**Original**



**Darken**



**Lower Contrast**



**Nonlinear Lower Contrast**



**Invert**



**Lighten**



**Raise Contrast**



**Nonlinear Raise Contrast**



# Point Processing

Original



$$x$$

Darken



$$x - 128$$

Lower Contrast



$$x / 2$$

Nonlinear Lower Contrast



$$((x / 255.0) ^{0.33}) * 255.0$$

Invert



$$255 - x$$

Lighten



$$x + 128$$

Raise Contrast



$$x * 2$$

Nonlinear Raise Contrast

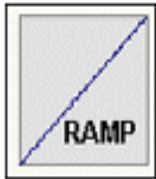


$$((x / 255.0) ^{2}) * 255.0$$

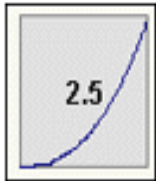
# Gamma correction

Monitors have a intensity to voltage response curve which is roughly a 2.5 power function

Send  $v \rightarrow$  actually display a pixel which has intensity equal to  $v^{2.5}$



Graph of Input

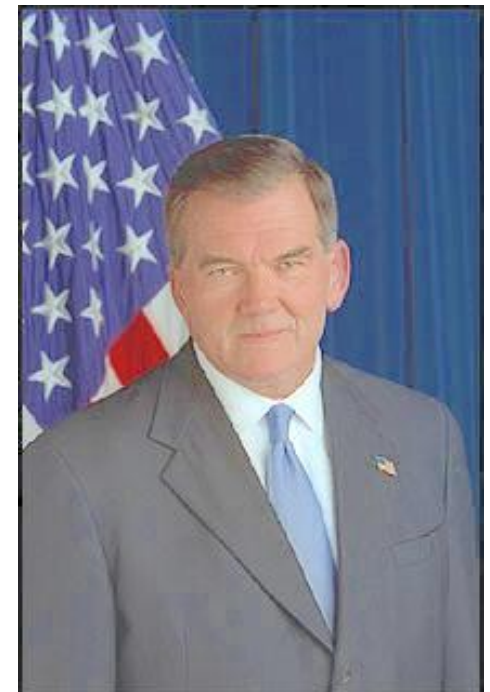


Graph of Output  $L = V ^ 2.5$



Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

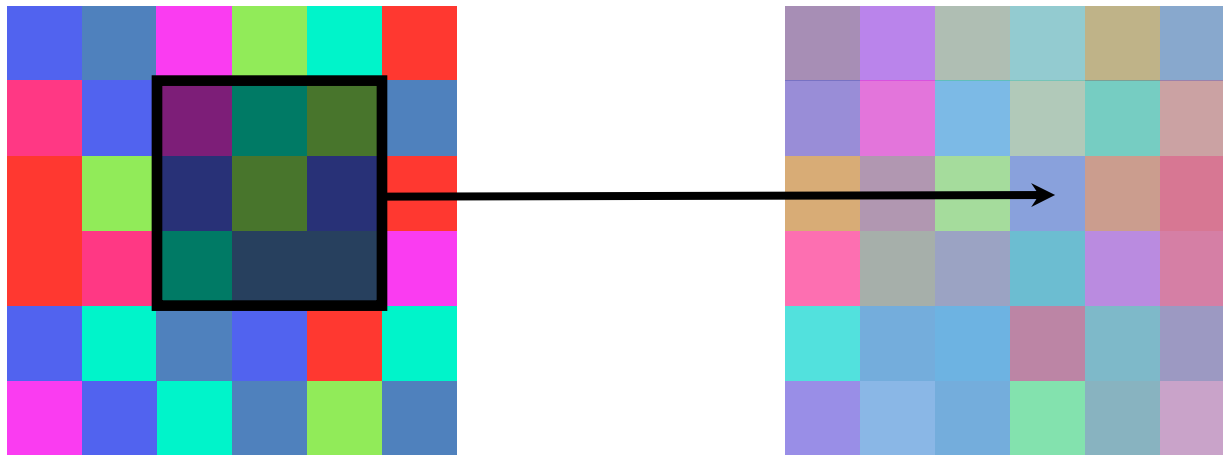
$$\beta = 1.0; f(v) = v$$



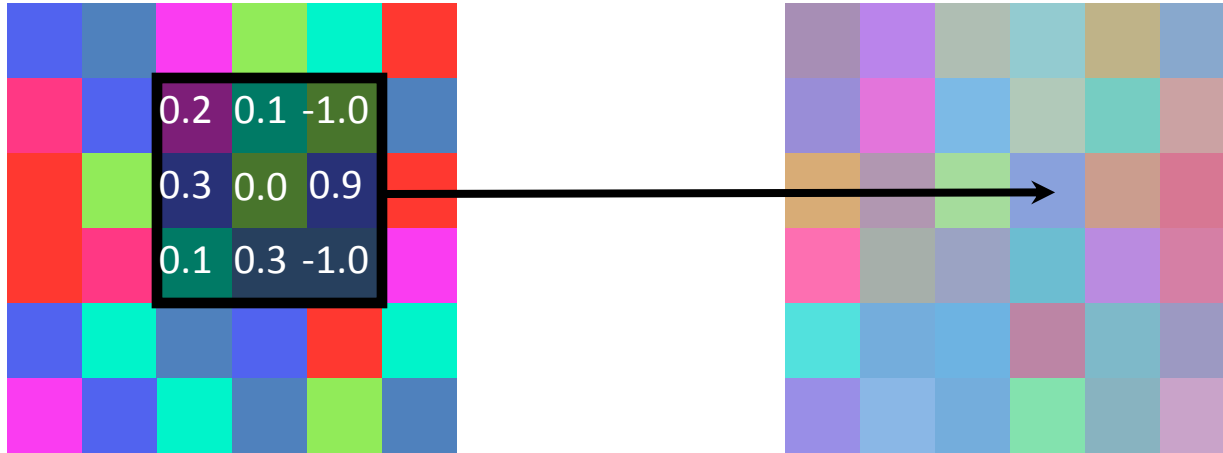
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

$$\beta = 2.5; f(v) = v^{1/2.5} = v^{0.4}$$

# Neighborhood Operations



# Convolution



$$F = \begin{bmatrix} 0.2 & 0.1 & -1.0 \\ 0.3 & 0.0 & 0.9 \\ 0.1 & 0.3 & -1.0 \end{bmatrix}$$

$$I' = F * I$$

# Properties of Convolution

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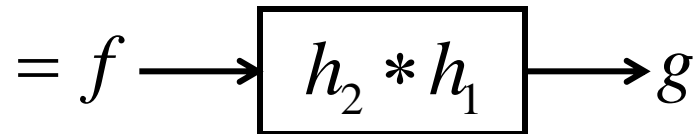
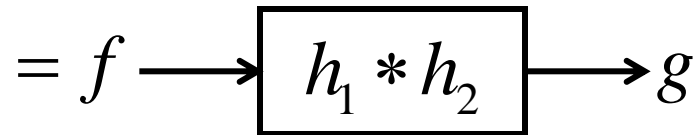
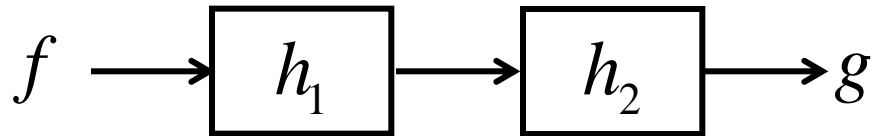
- Commutative

$$a * b = b * a$$

- Associative

$$(a * b) * c = a * (b * c)$$

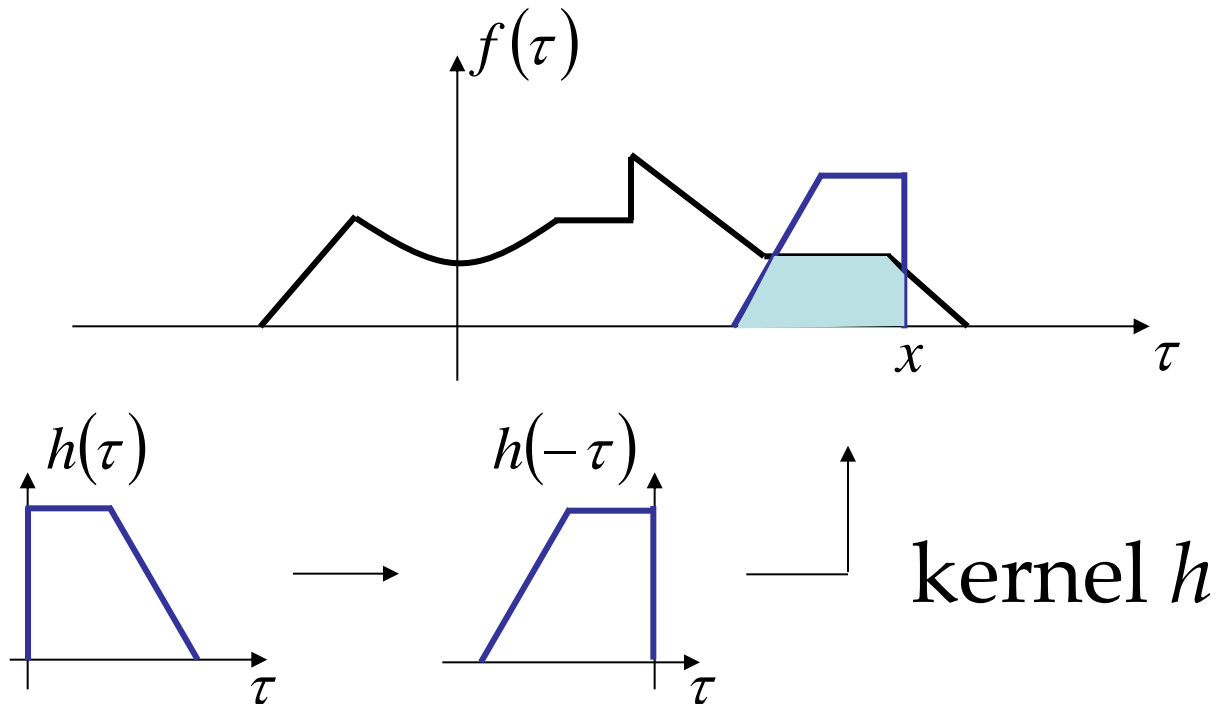
- Cascade system



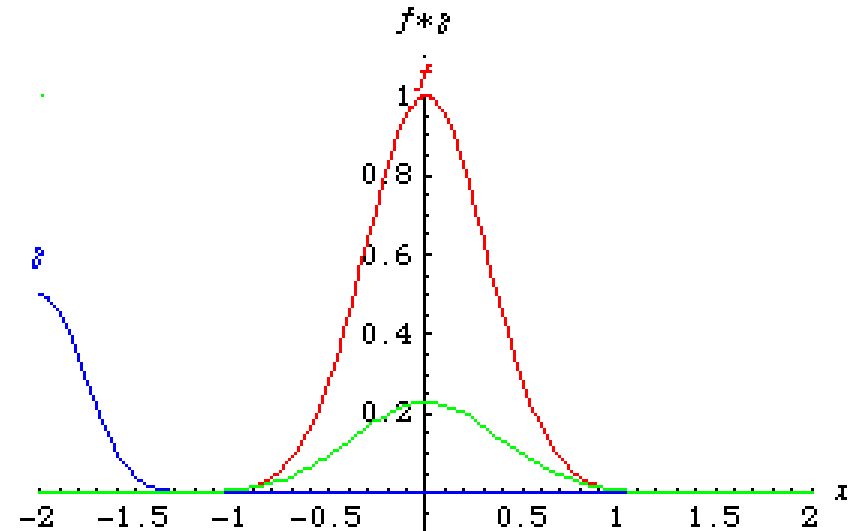
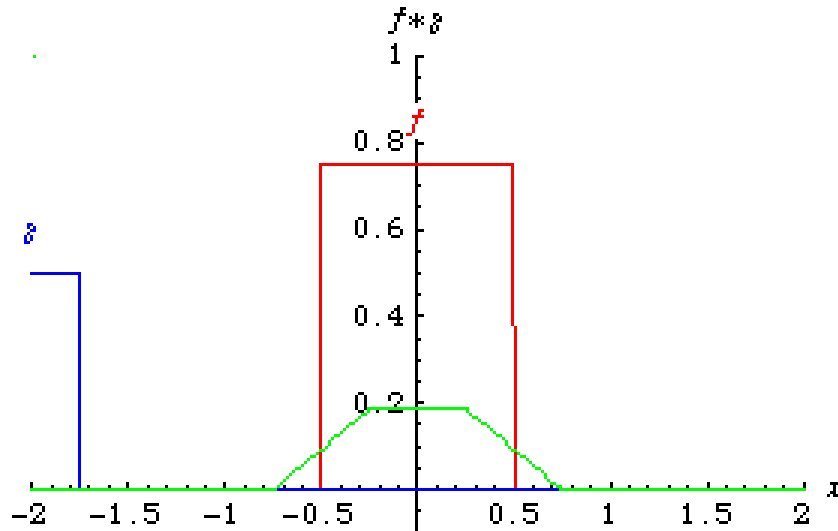
# Convolution

Convolution is linear and shift invariant

$$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)d\tau \quad g = f * h$$



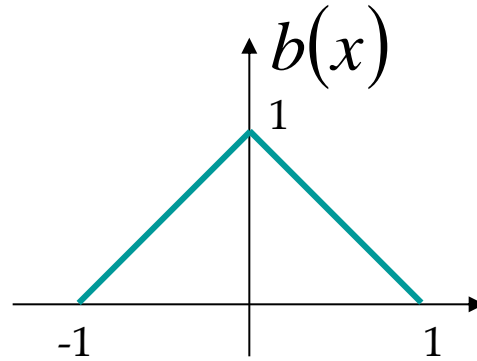
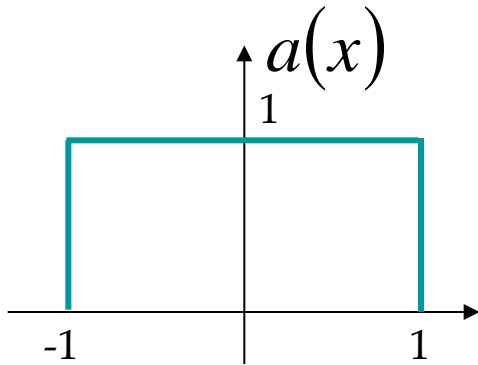
# Convolution - Example



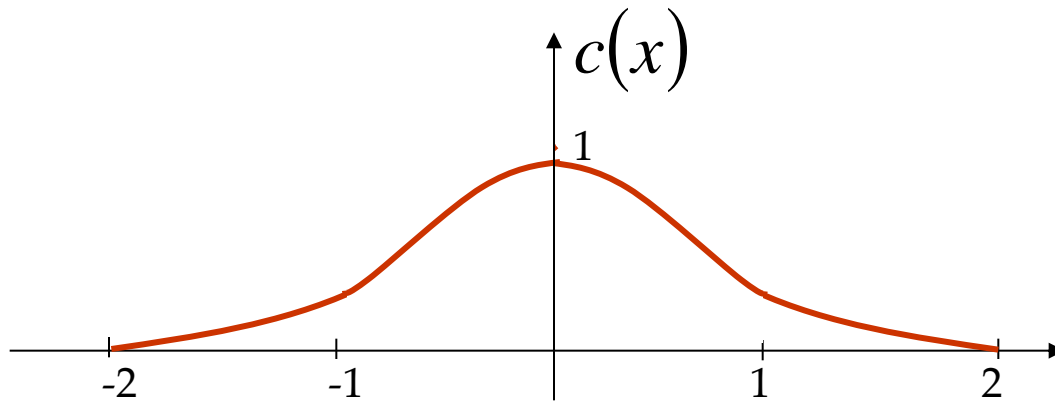
—  $f$   
—  $g$   
—  $f * g$

# Convolution - Example

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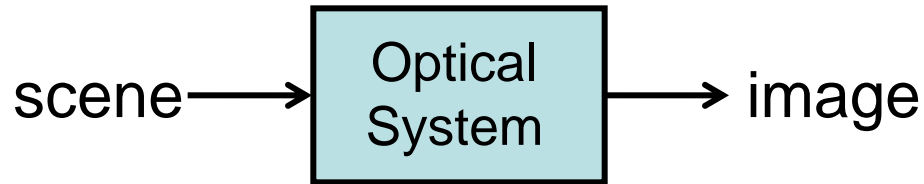


$$\downarrow c = a * b$$

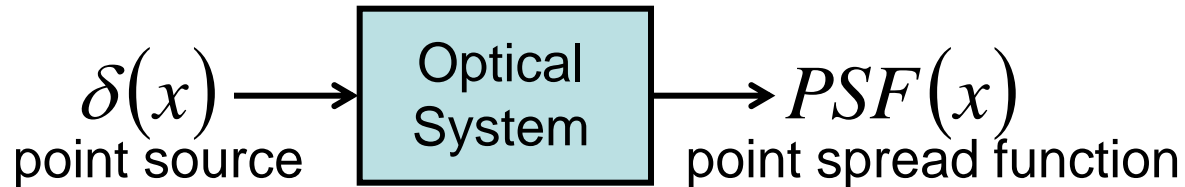


# Point Spread Function

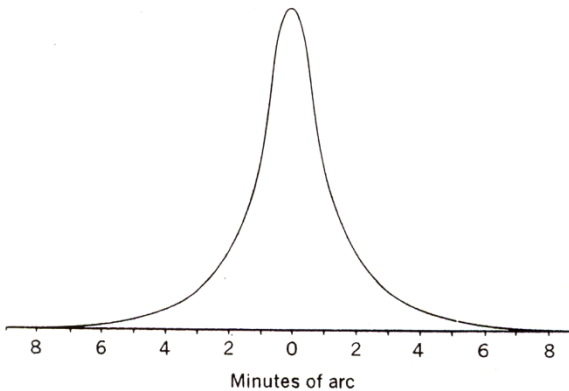
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- Ideally, the optical system should be a Dirac delta function.
- However, optical systems are never ideal.



- Point spread function of Human Eyes



# Point Spread Function

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normal vision



myopia



hyperopia

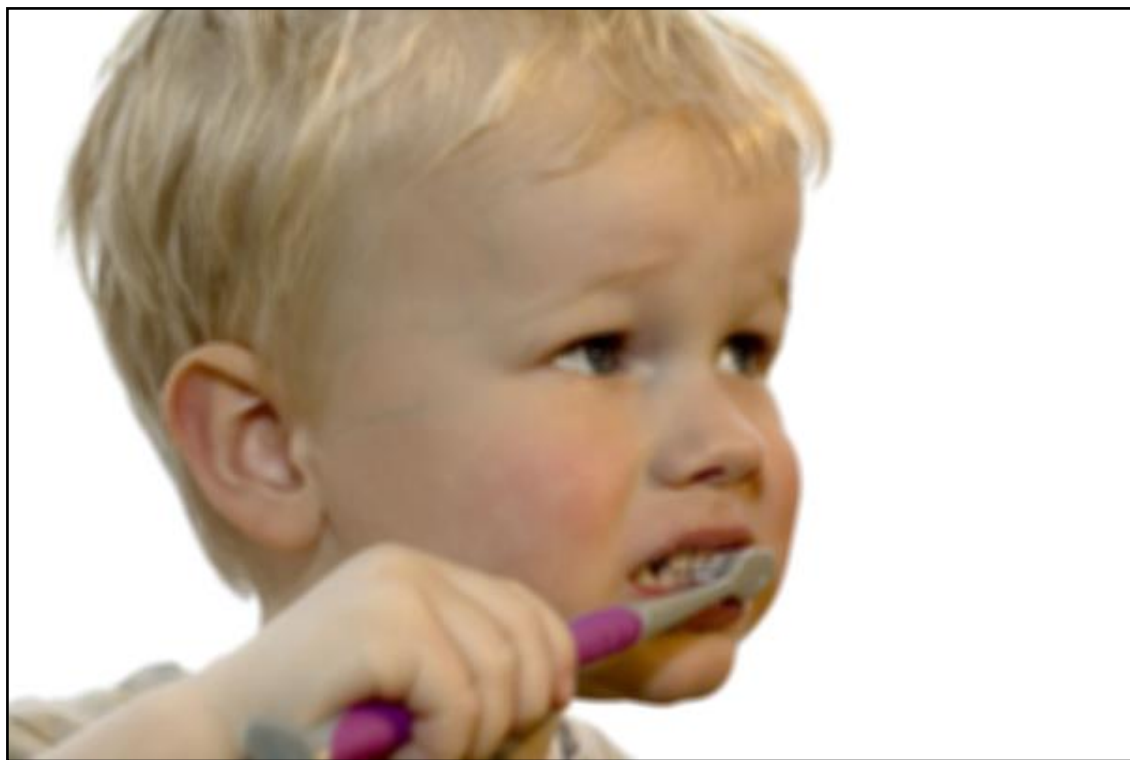


astigmatism

# Original Image



# Blurred Image



# Gaussian Smoothing

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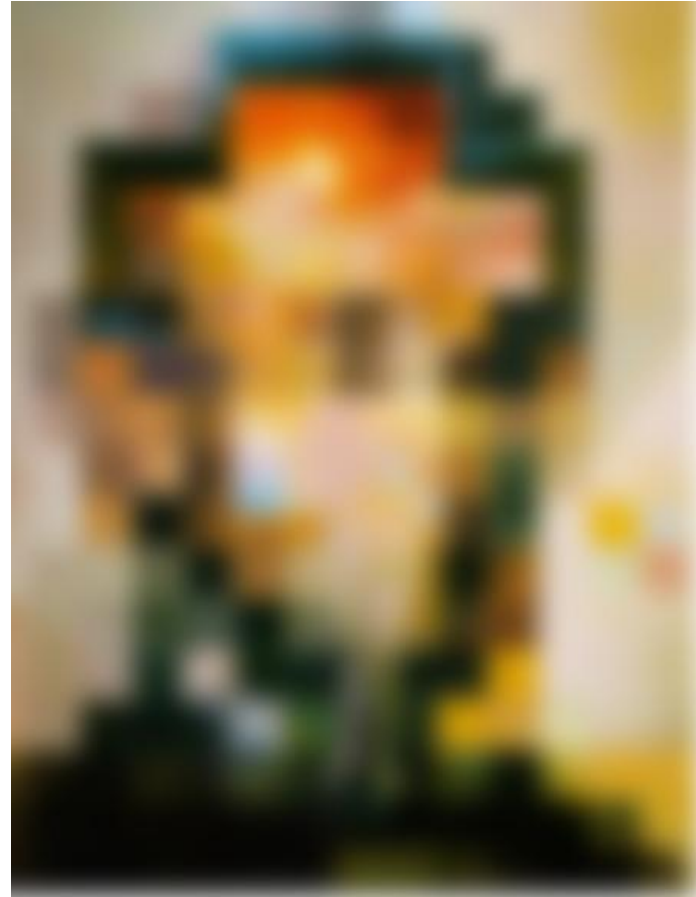
by Charles Allen Gillbert



by Harmon & Julesz

# Gaussian Smoothing

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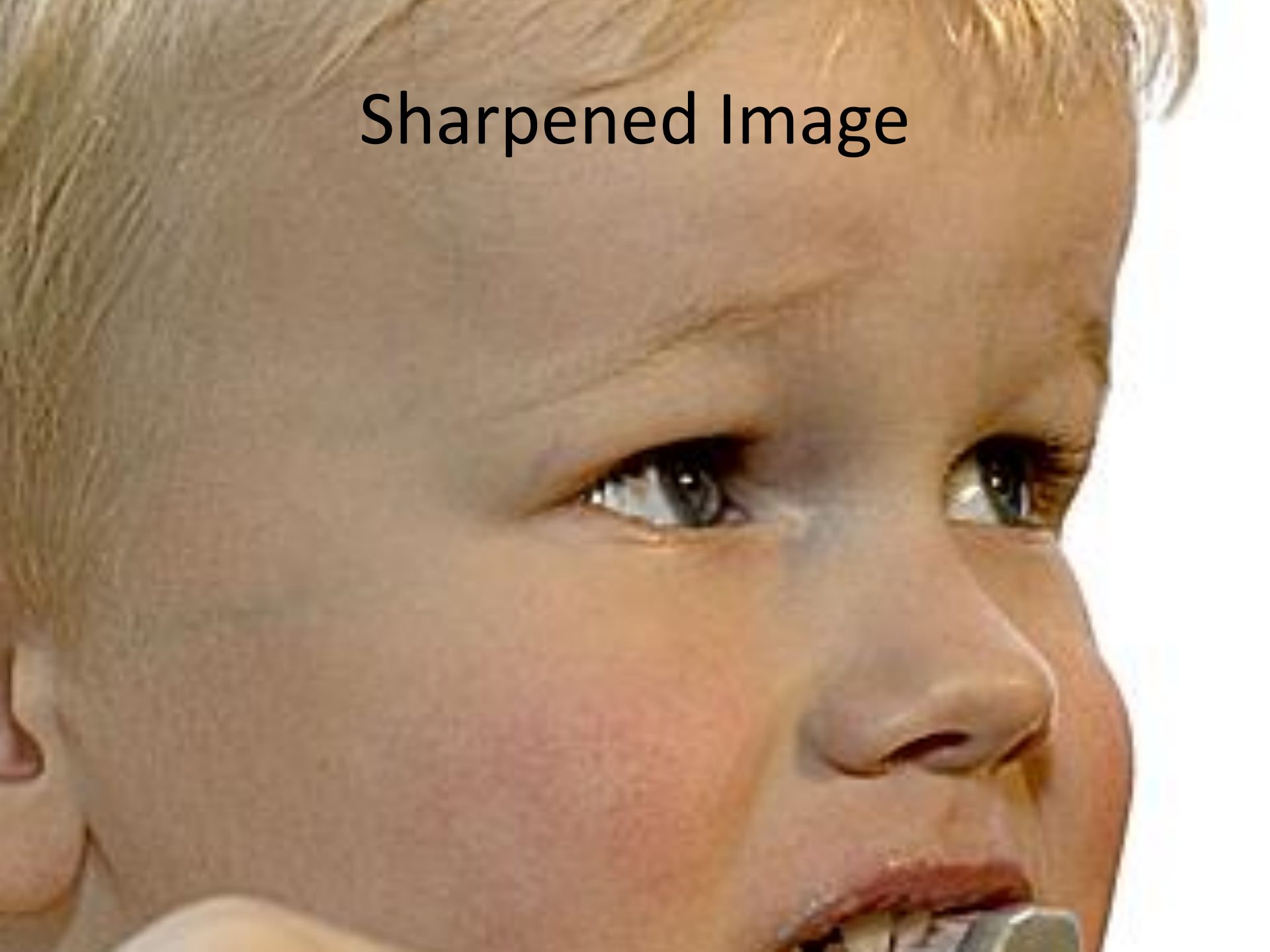
# Original Image



# Sharpened Image



Sharpened Image





# Noise



# Blurred Noise

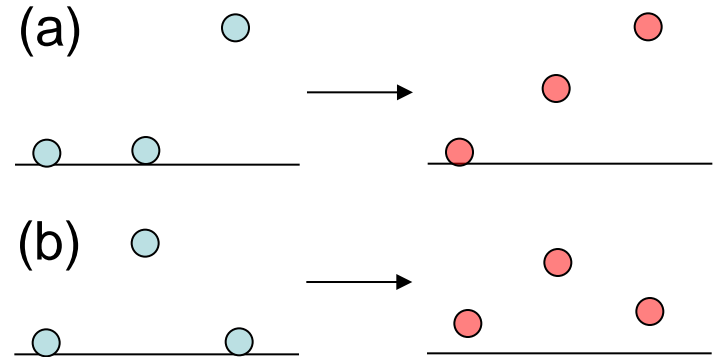


# Median Filter

- Smoothing is averaging

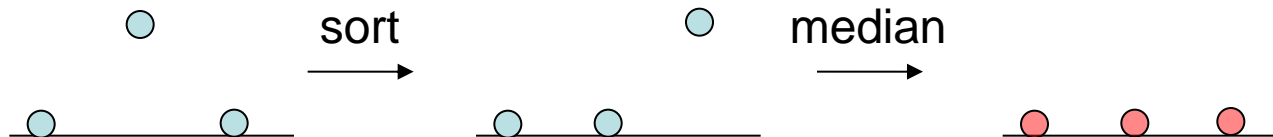
(a) Blurs edges

(b) Sensitive to outliers



- Median filtering

- Sort  $N^2 - 1$  values around the pixel
- Select middle value (median)



- Non-linear (Cannot be implemented with convolution)

# Median Filter



Can this be described as a convolution?

# Original Image



# Example: Noise Reduction



Image with noise



Median filter (5x5)

# Salt and pepper noise

Gaussian

Median

3x3



5x5



7x7



# Gaussian noise

Gaussian

Median



# Example: Noise Reduction



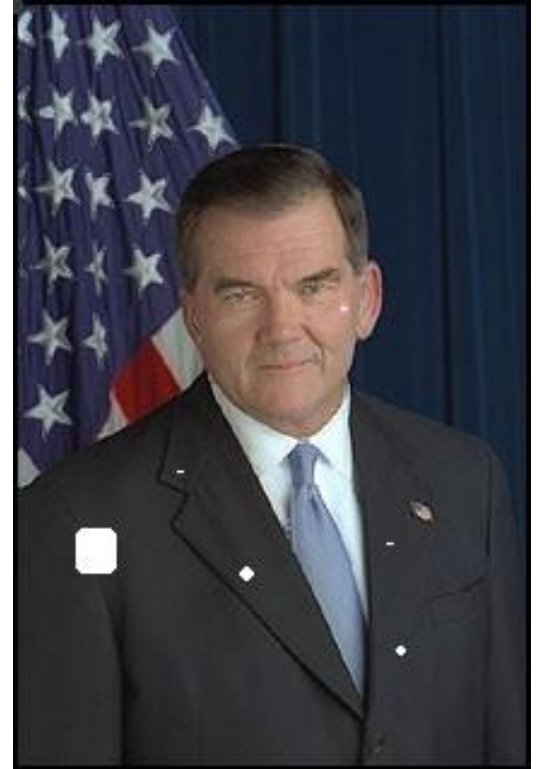
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Original image



Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Image with noise



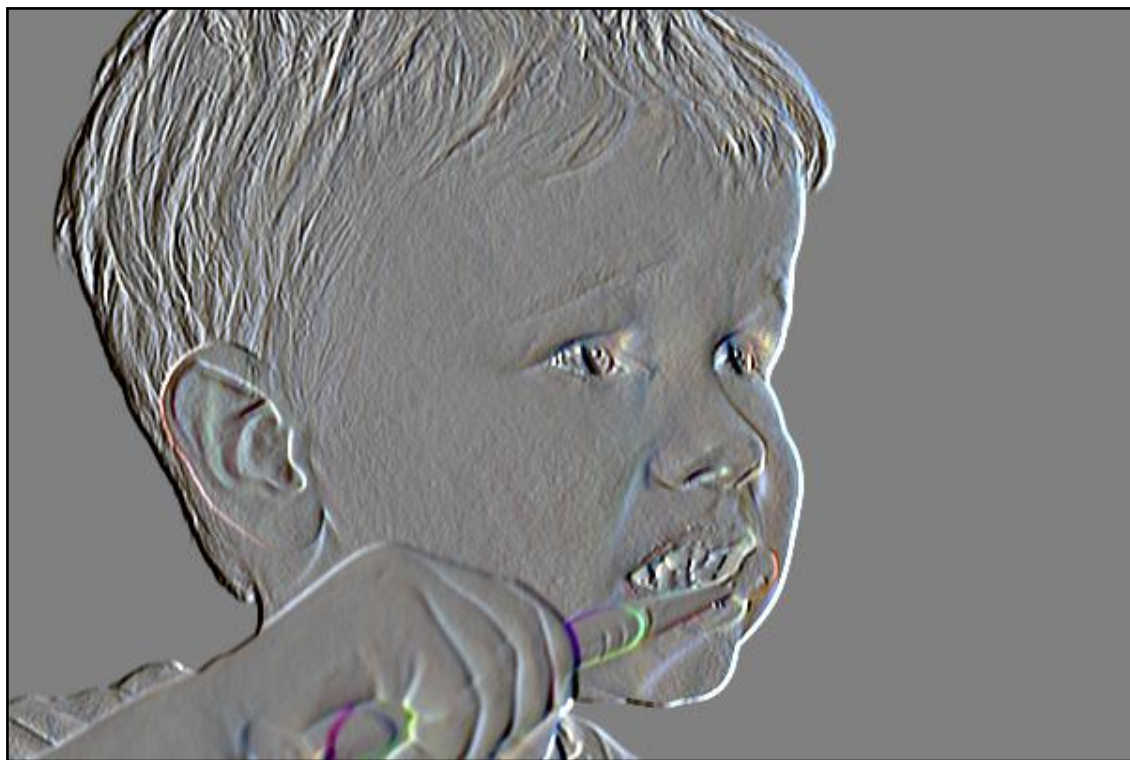
Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.

Median filter (5x5)

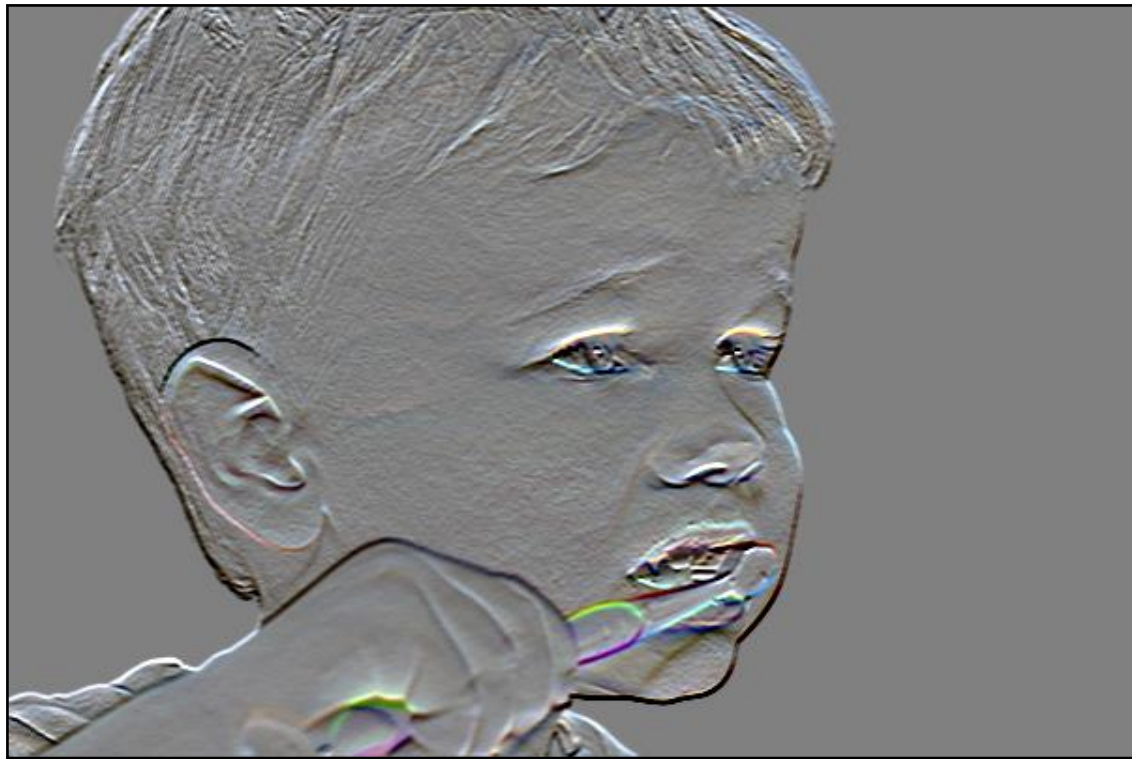
# Original Image



# X-Edge Detection



# Y-Edge Detection



# General Edge Detection



Can this be described as a convolution?

# Image Processing

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- Some operations preserve the range but change the domain of  $f$ :

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can this perform?

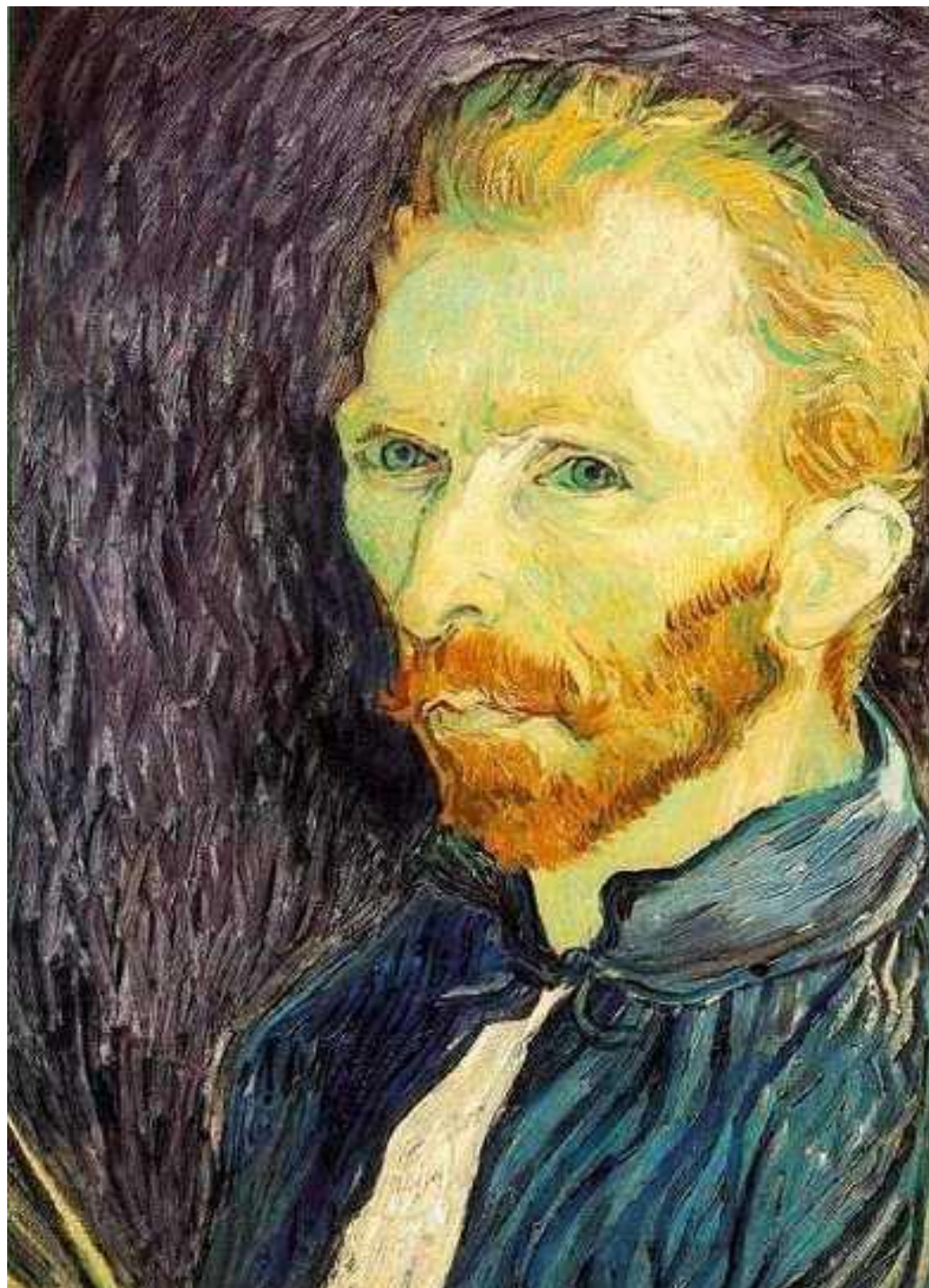
- Still other operations operate on both the domain and the range of  $f$ .

# Image Scaling

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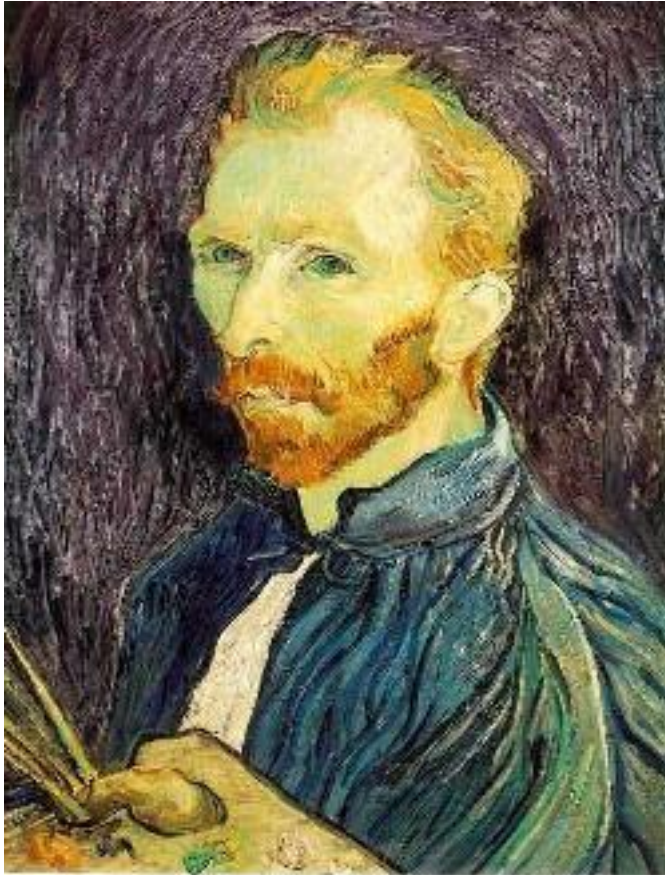
This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?



# Image Sub-Sampling

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1/4

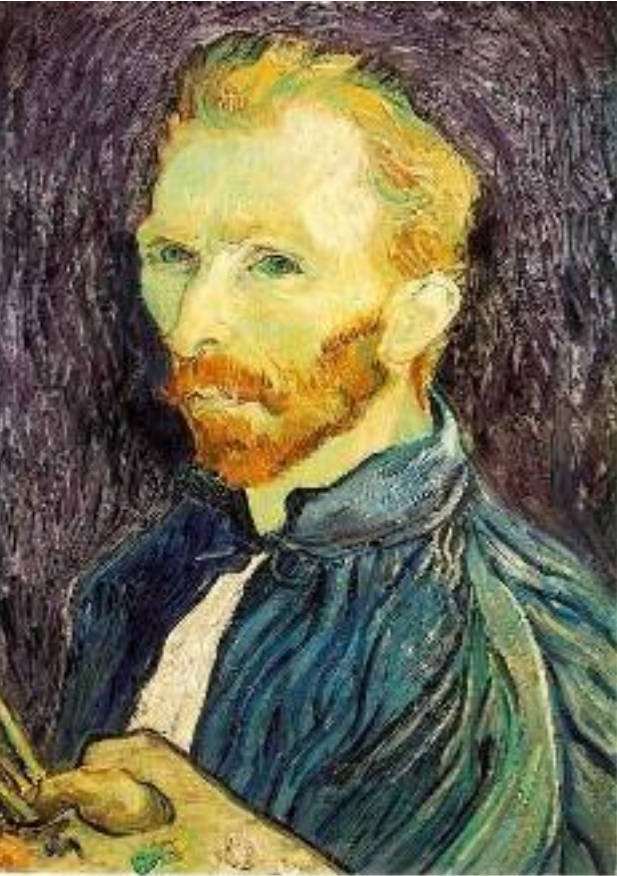


1/8

Throw away every other row and column to create a  $1/2$  size image  
- called *image sub-sampling*

# Image Sub-Sampling

---



$1/2$



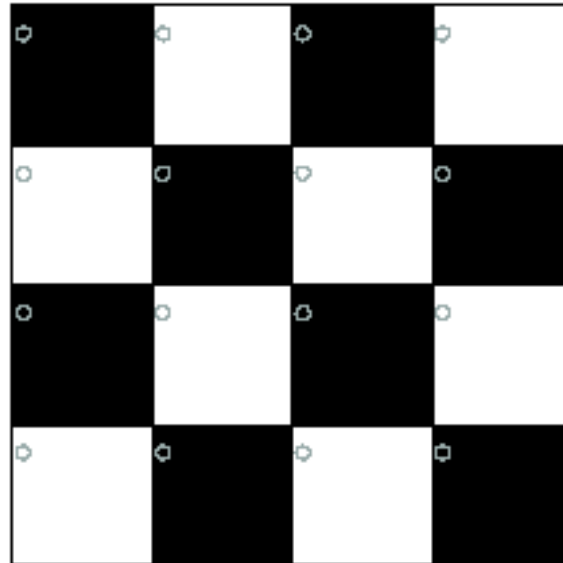
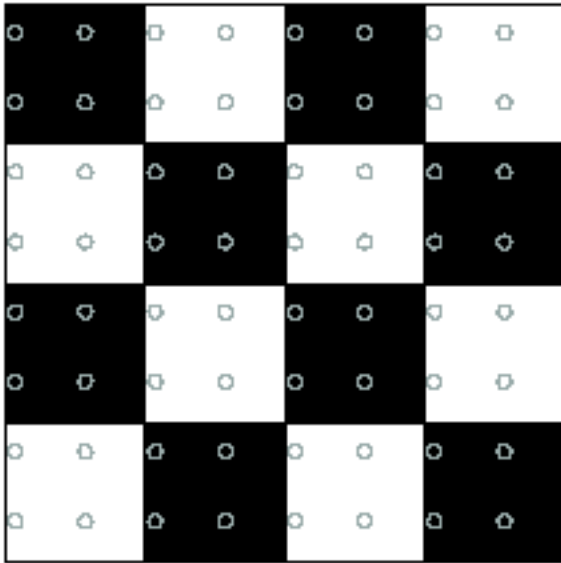
$1/4$  (2x zoom)



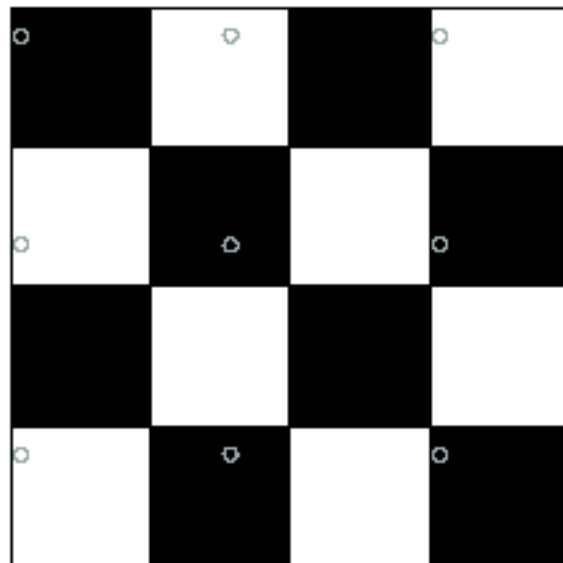
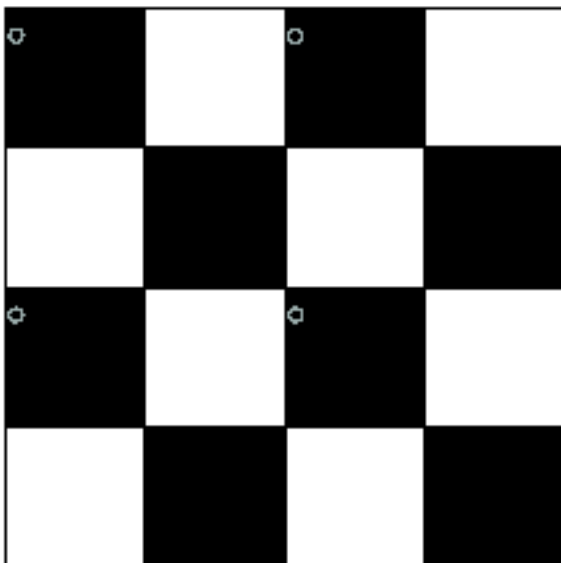
$1/8$  (4x zoom)

# Good and Bad Sampling

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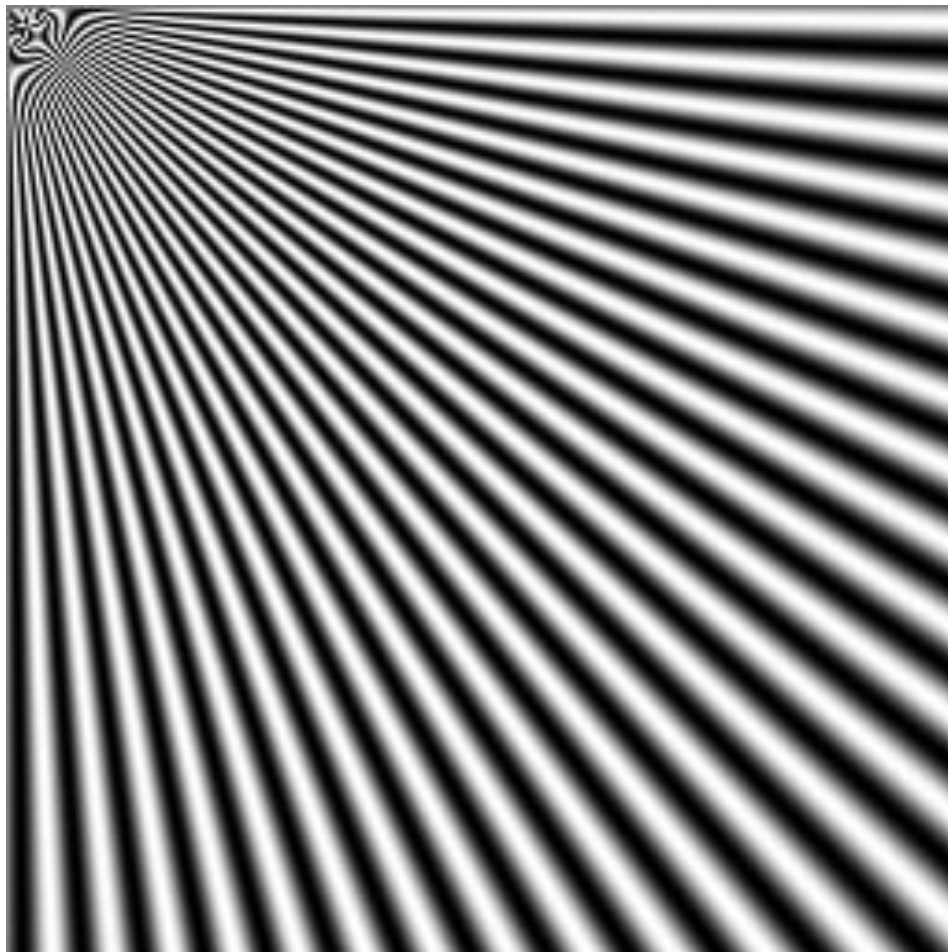
Good sampling:  
• Sample often or,  
• Sample wisely



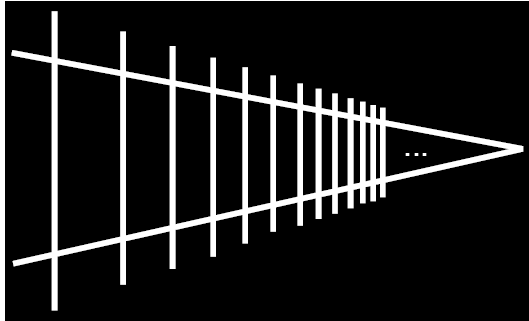
Bad sampling:  
• see aliasing in action!

# Aliasing

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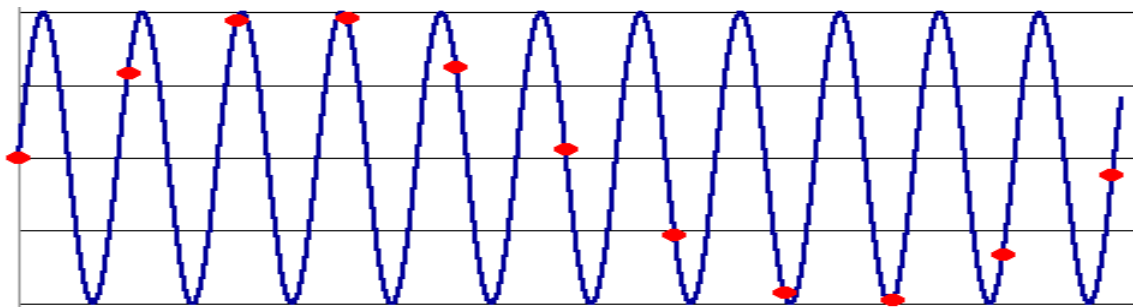


# Alias: n., an assumed name



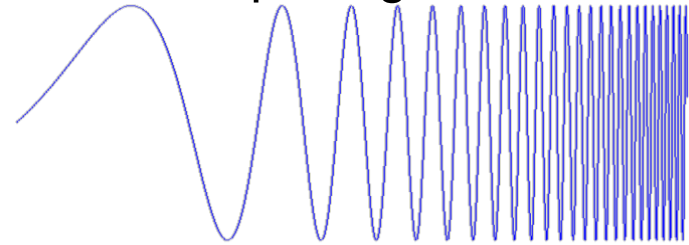
Picket fence receding into the distance will produce aliasing...

WHY?

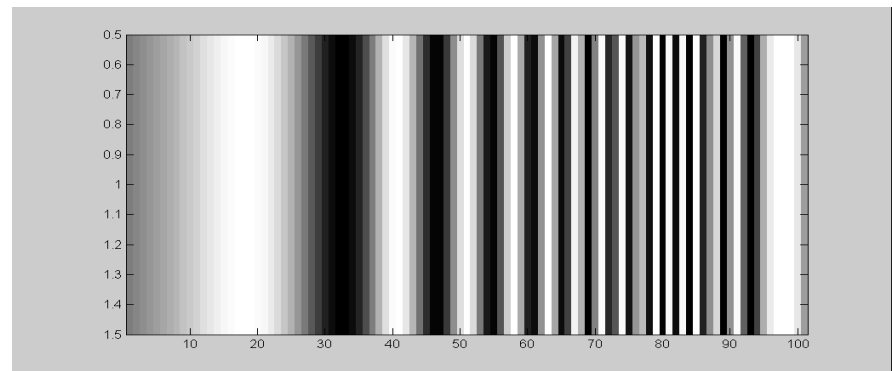


Not enough samples

Input signal:



Matlab output:



```
x = 0:.05:5; imagesc(sin((2.^x).*x))
```

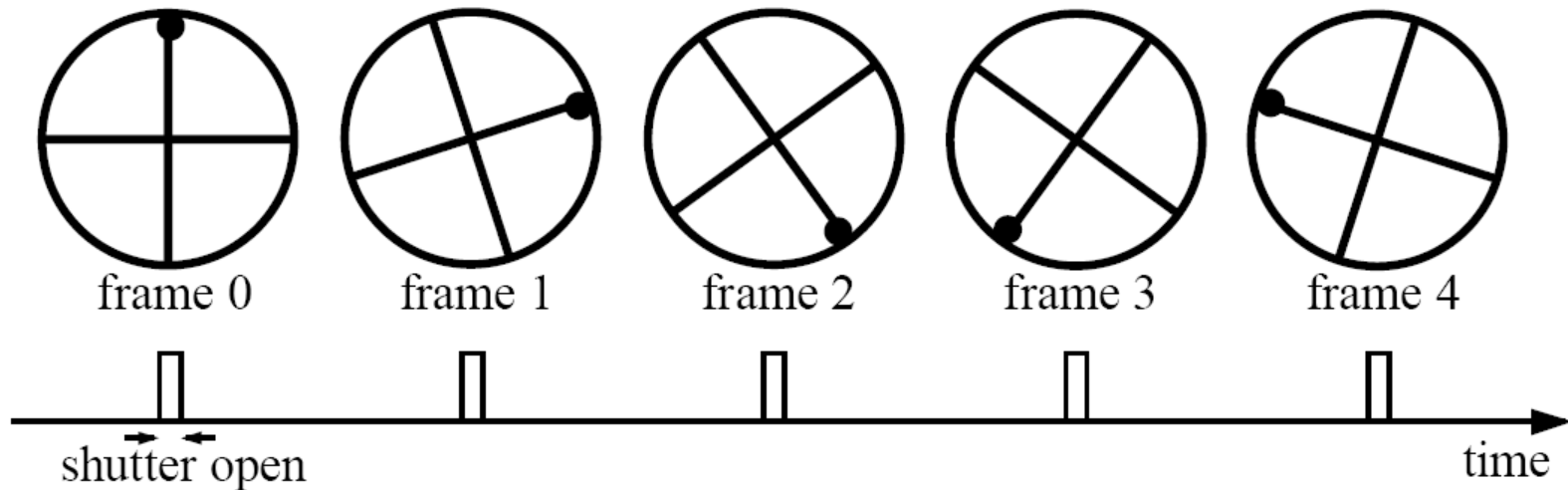
Alias!

# Really bad in video

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

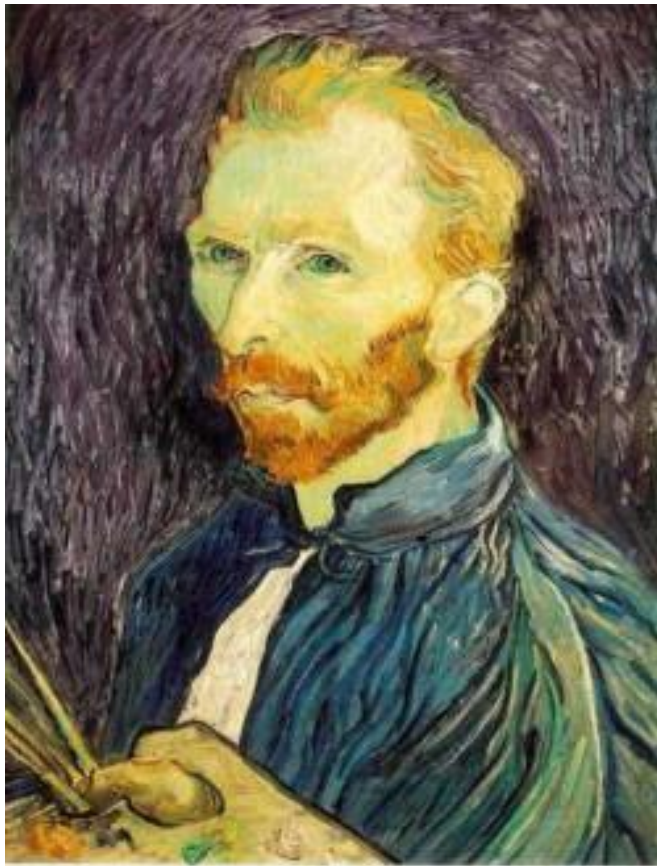
If camera shutter is only open for a fraction of a frame time (frame time =  $1/30$  sec. for video,  $1/24$  sec. for film):



Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)

# Sub-Sampling with Gaussian Pre-Filtering

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Gaussian 1/2



G 1/4



G 1/8

- Solution: filter the image, *then* subsample
  - Filter size should double for each  $\frac{1}{2}$  size reduction. Why?

# Sub-Sampling with Gaussian Pre-Filtering

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Gaussian  $1/2$



G  $1/4$



G  $1/8$

# Compare with...

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1/2



1/4 (2x zoom)



1/8 (4x zoom)



Canon D60 (w/ anti-alias filter)



Sigma SD9 (w/o anti-alias filter)



512

256

128

64

32

16

8



Figure from David Forsyth

# Original Image



# Warped Image

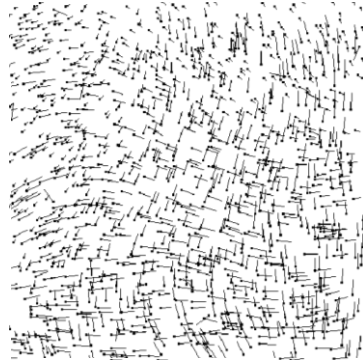


# Warped Image



orig

+



vector field

=



warped

how?

# Advection (just like a fluid)

