

# Radiosity

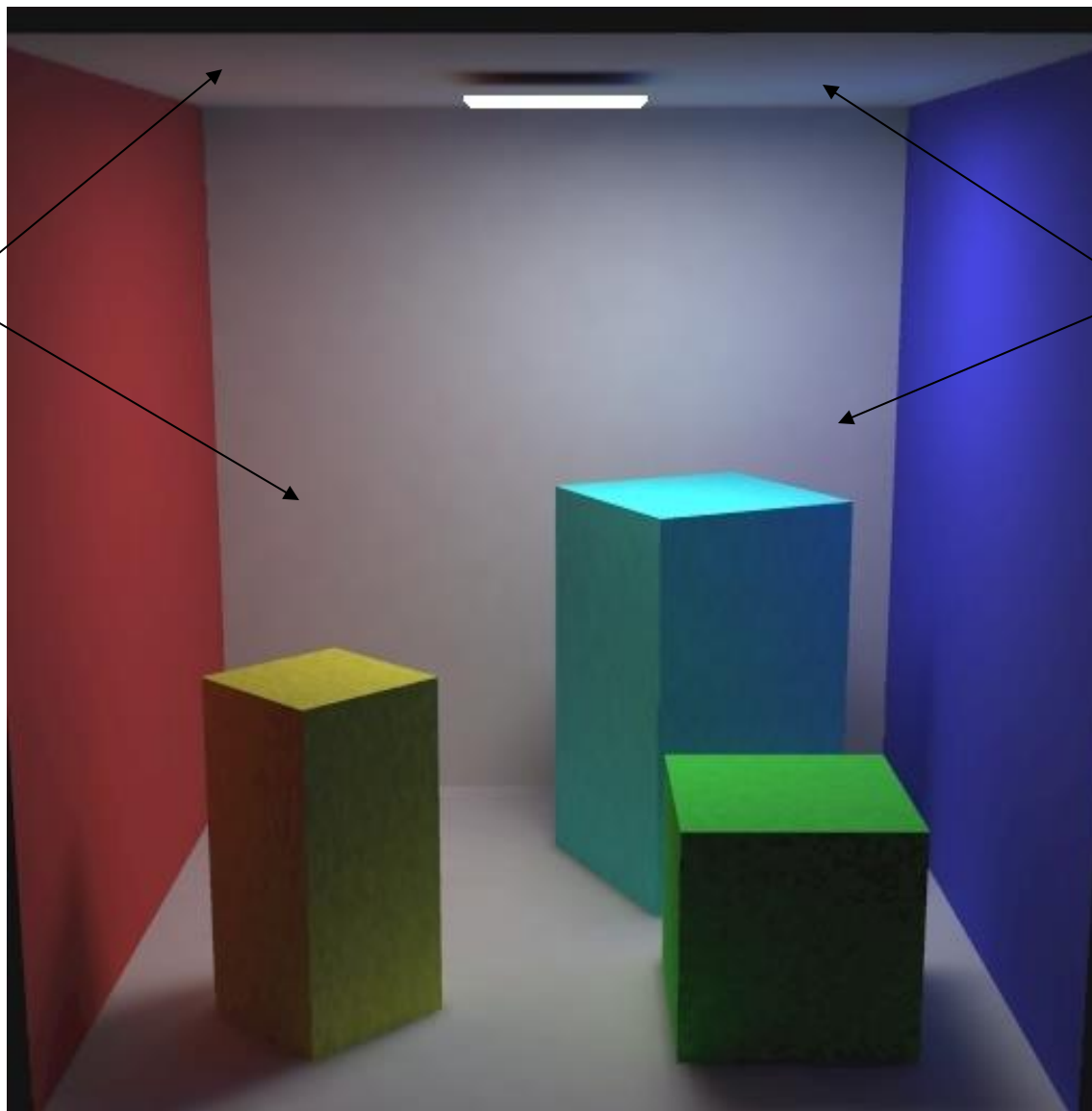
Thanks to Kavita Bala, Pat Hanrahan, Doug James, Ledah Casburn

# Global Illumination



**Modeling: Stephen Duck; Rendering: Henrik Wann Jensen**

# Cornell Box



red hue

blue hue

# Lighting Effects

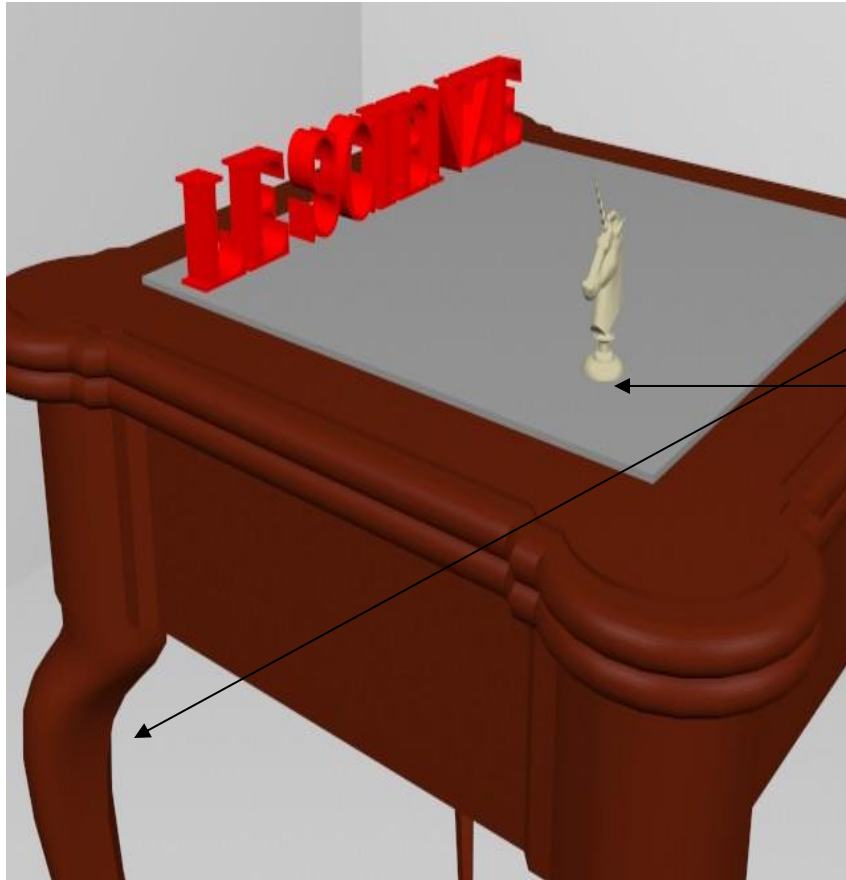


The ambient lighting in the upper-right image is approximated by a constant value. This is typical of most scanline algorithms. The middle and lower-left images were rendered with a ray tracing global illumination algorithm.



The middle image was rendered with no ambient light calculations. The lower-left image was rendered with several levels of diffuse re-reflection to give a better approximation of the ambient light in this scene.

# Phong Shading



Plastic looking scene

- no object interactions
- no shadows

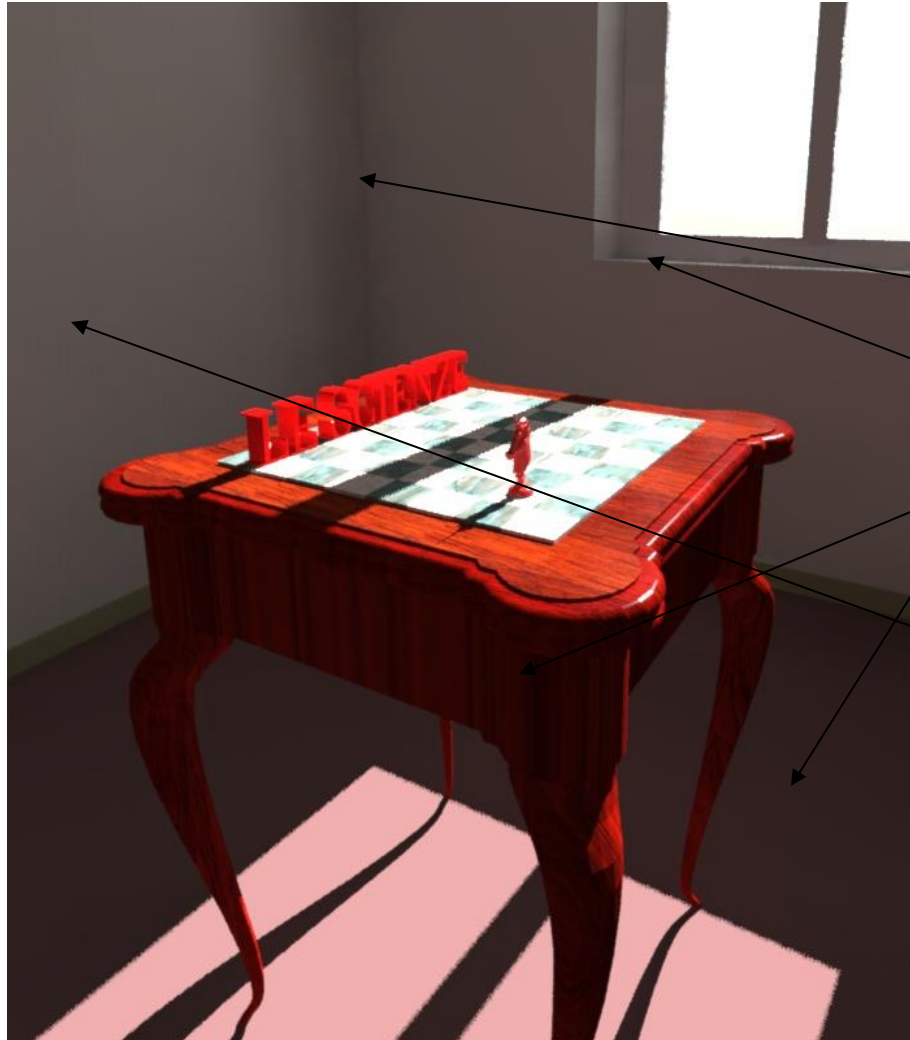
# Ray Tracing



Scene doesn't look realistic enough.

- where is the corner of room?
- is window flush with wall?
- is the carpet and wood supposed to be this dark?

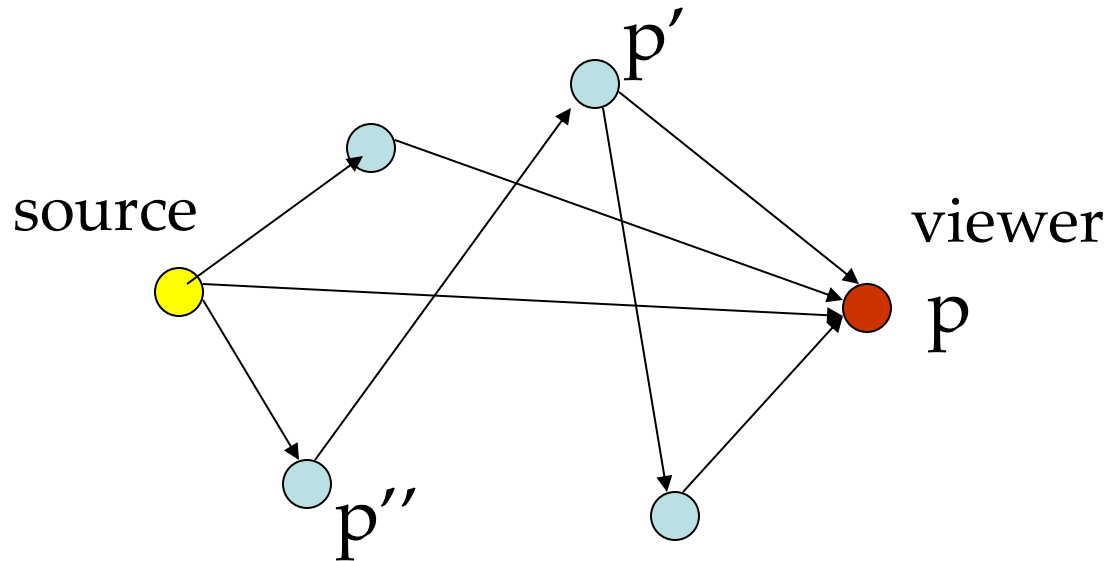
# Radiosity – today's topic



Indirect lighting affects realism.

- room has a corner
- window has depth
- carpet and wood on table is lighter
- walls look more pink

# The Rendering Equation – Graph Style



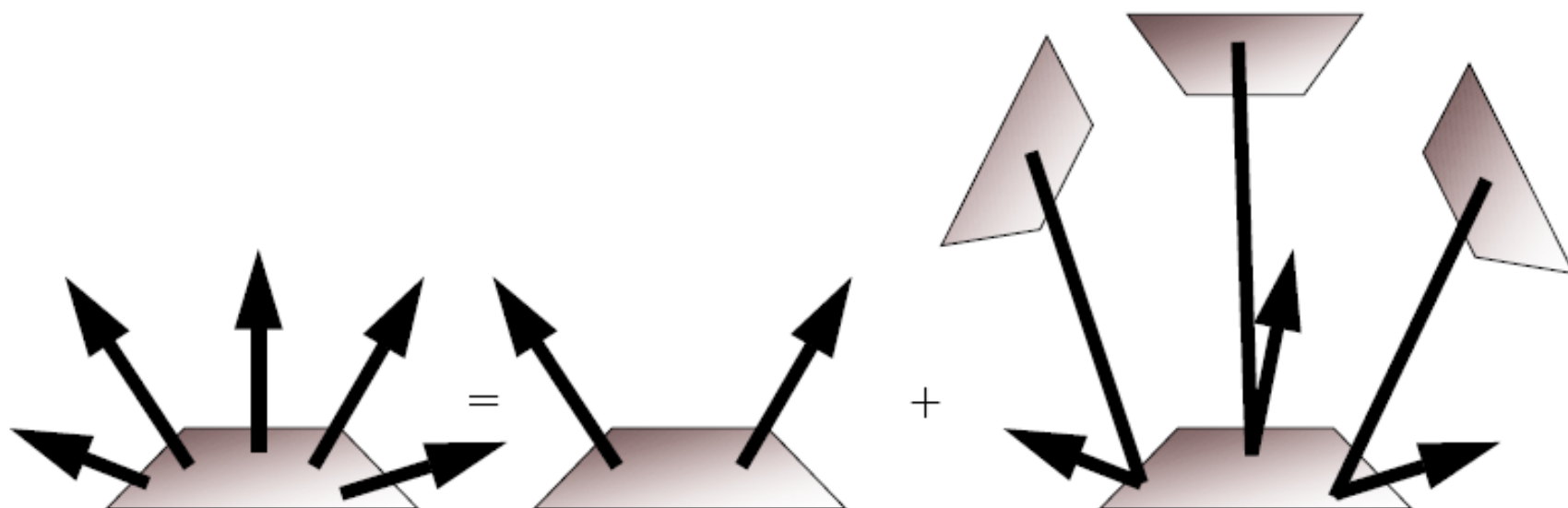
$$i(\mathbf{p}, \mathbf{p}') = v(\mathbf{p}, \mathbf{p}')(\epsilon(\mathbf{p}, \mathbf{p}') + \int \rho(\mathbf{p}, \mathbf{p}', \mathbf{p}'')i(\mathbf{p}', \mathbf{p}'')d\mathbf{p}'')$$

Visibility  
(shadows)      Emission  
(light source)

Reflectance from  
Surfaces

# Conservation of Energy

---



Emitted power = self-emitted power + received & reflected power

# Diffuse Interreflections - Radiosity

- Consider lambertian surfaces and sources.
- Radiance independent of viewing direction.
- Consider total power leaving per unit area of a surface.
- Can simulate soft shadows and color bleeding from diffuse surfaces.
- Used abundantly in heat transfer literature

# Irradiance, Radiosity

- Irradiance  $E$  is the power **received** per unit surface area
  - Units:  $\text{W}/\text{m}^2$
- Radiosity
  - Power per unit area **leaving** the surface

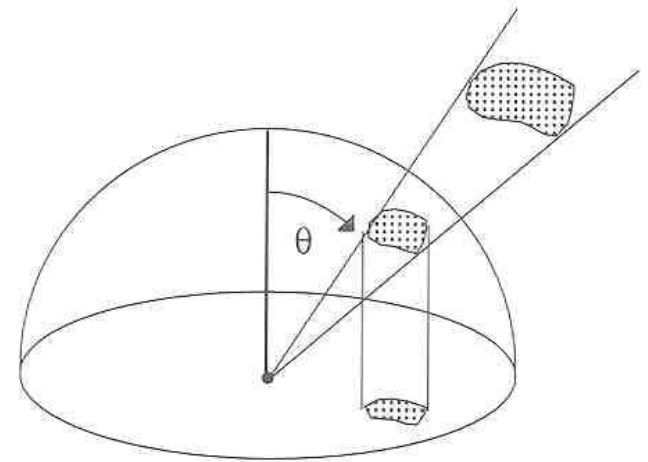
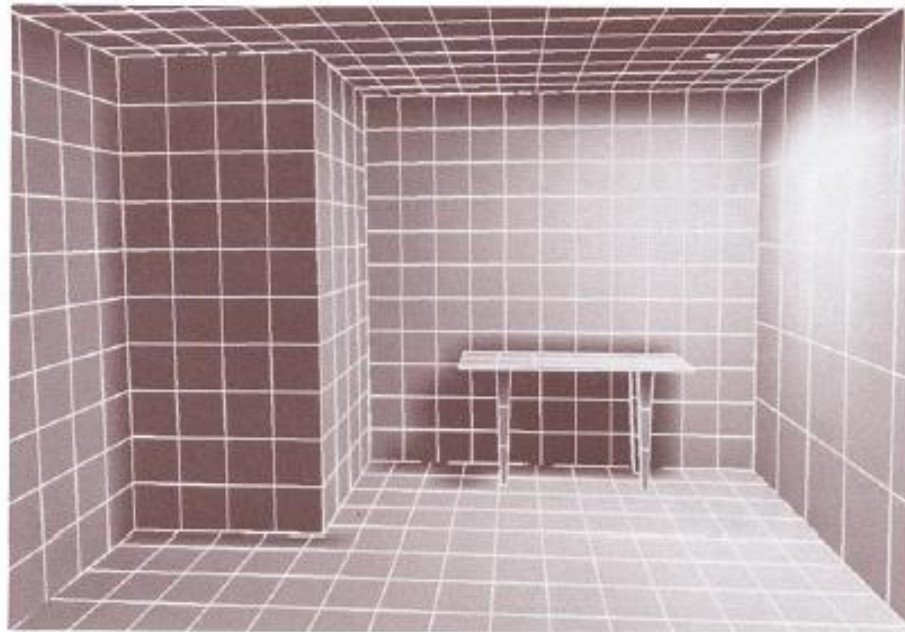
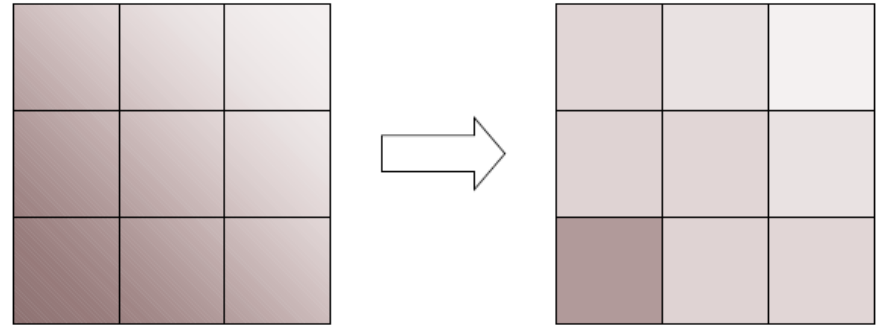


Figure 2.8: Projection of differential area.

# Planar piecewise constancy assumption

- Subdivide scene into small “uniform” polygons



**Table in room sequence from Cohen and Wallace**

# Power Equation

- Power from each polygon:

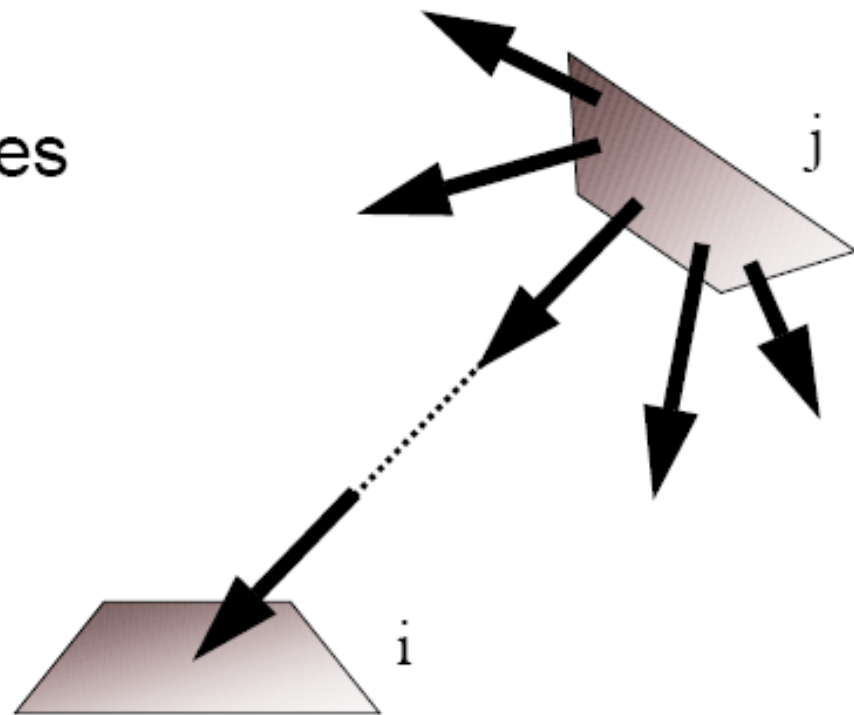
$$\forall i : \Phi_i = \Phi_{ei} + \rho_i \sum_{j=1}^N \Phi_j F(i \rightarrow j)$$

- Linear System of Equations:

- $\Phi_i$  : power of patch  $i$  (unknown)
- $\Phi_{e,i}$  : emission of patch  $i$  (known)
- $\rho_i$  : reflectivity of patch  $i$  (known)
- $F(j \rightarrow i)$ : form-factor (coefficients of matrix)

# Form Factor

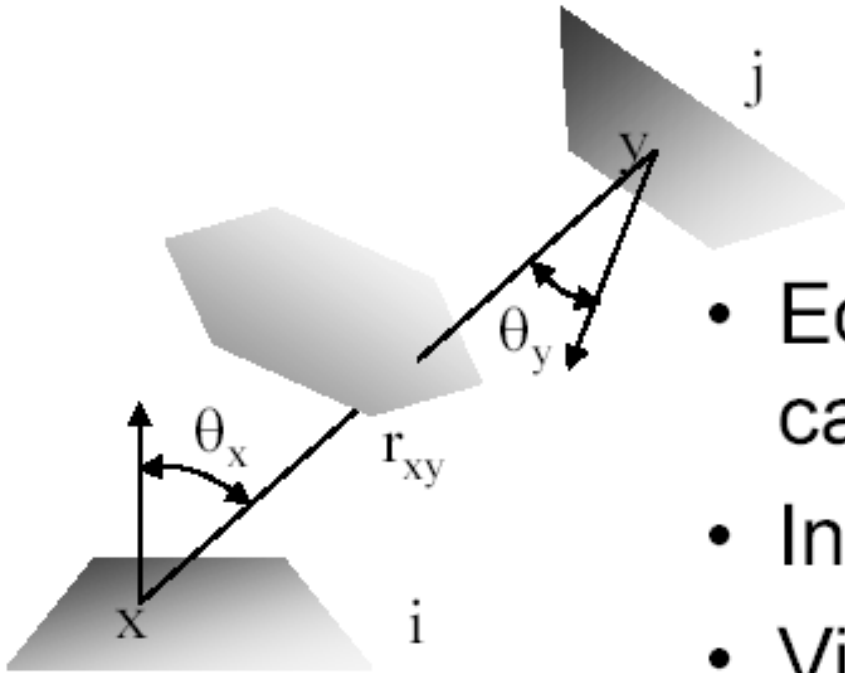
- $F_{j \rightarrow i}$  = the fraction of power emitted by  $j$ , which is received by  $i$
- Area
  - if  $i$  is smaller, it receives less power
- Orientation
  - if  $i$  faces  $j$ , it receives more power
- Distance
  - if  $i$  is further away, it receives less power



# Form Factor

---

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_y dA_x$$



- Equations for special cases (polygons)
- In general hard problem
- Visibility makes it harder

# Form Factors Invariant

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_y dA_x$$

$$F(i \rightarrow j) = \frac{1}{A_i} \int_{A_j} \int_{A_i} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_x dA_y$$

$$F(i \rightarrow j)A_i = F(j \rightarrow i)A_j$$

# Form Factor Computation

$$F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) dA_y dA_y$$

- Schroeder and Hanrahan derived an analytic expression for polygonal surfaces.
- In general, computing double integral is hard.
- Use Monte Carlo Integration.

# Being Smart about Form Factors

Form factors depend only on scene geometry. If geometry is constant, they only need to be calculated once.

Solution of the radiosity system is independent of viewing conditions, so if only the viewer position changes, it only needs to be solved once—can walk around the scene in real-time after it's initially generated

# Being Smart about Form Factors

Form factors are complicated. Full numeric approximation of these is expensive—many special cases may be solved analytically.

Because we assume that radiosity is constant across a patch, two patches are typically assumed to be fully inter-visible or not at all inter-visible. That means that patches have to be small enough to resolve shadows and other complexities

# How to perform visibility testing?

Two basic methods, both of which have aliasing problems:

- Raycasting (typically slow)

- Hemicube method (z-buffer exploit)

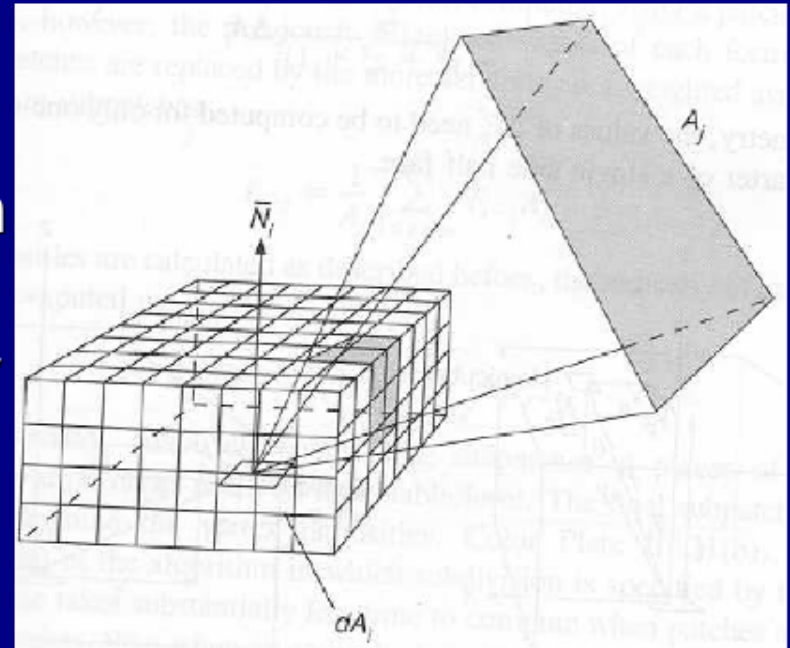
Anti-aliasing may be performed in both cases

# Hemicube Visibility Testing

Render the entire scene from the perspective of the center of the current patch

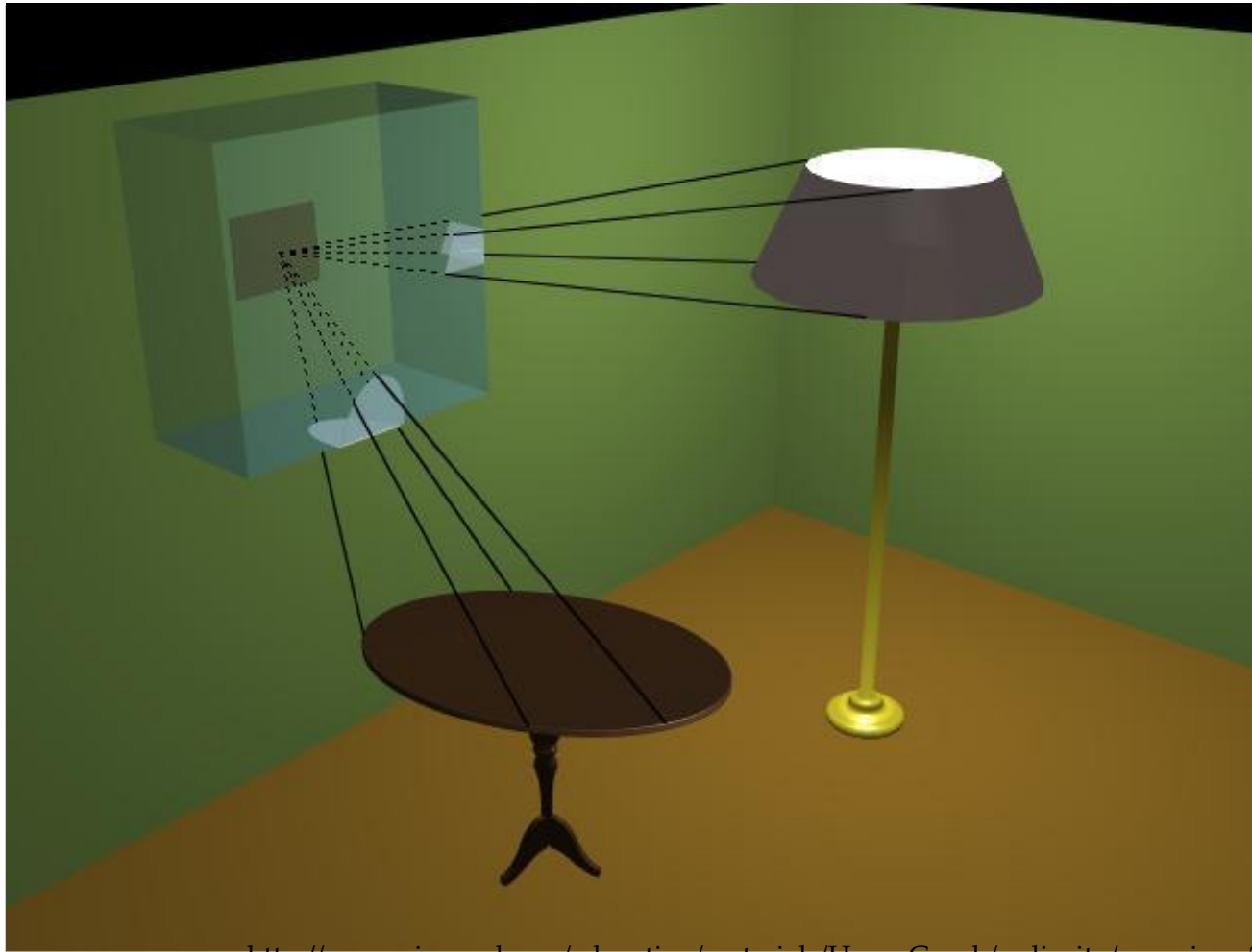
Rather than color, store patch identifiers, using the z-buffer to determine visibility

Takes advantage of graphics hardware



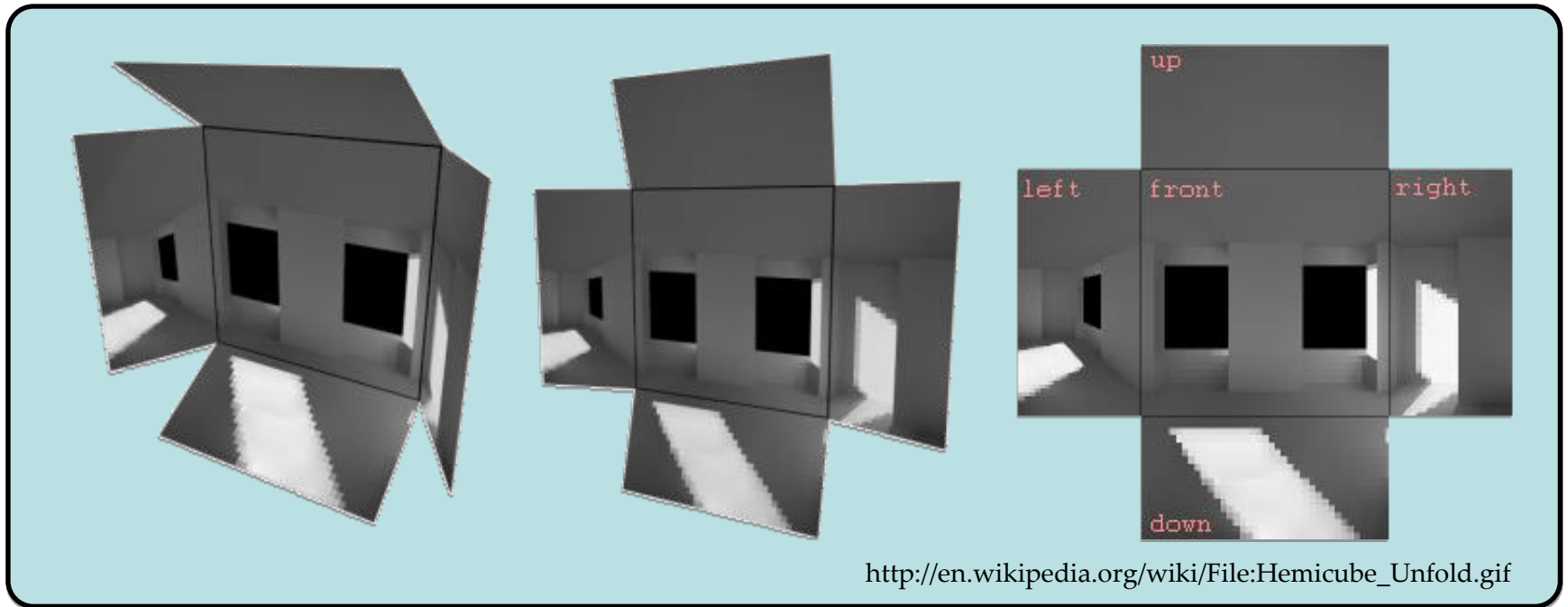
R. Ramamoorthi

# Hemicube in Action



[http://www.siggraph.org/education/materials/HyperGraph/radiosity/overview\\_2.htm](http://www.siggraph.org/education/materials/HyperGraph/radiosity/overview_2.htm)

# Hemicube in Action



# Power $\rightarrow$ Radiosity

---

$$\Phi_i = \Phi_{e,i} + \rho_i \sum_{j=1}^N \Phi_j F(j \rightarrow i)$$



Divide by  $A_i$

$$\frac{\Phi_i}{A_i} = \frac{\Phi_{e,i}}{A_i} + \rho_i \sum_{j=1}^N \frac{\Phi_j F(j \rightarrow i)}{A_i}$$
$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N \frac{\Phi_j \frac{F(i \rightarrow j) A_i}{A_j}}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N \frac{\Phi_j F(i \rightarrow j)}{A_j}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N B_j F(i \rightarrow j)$$

# Linear System of Radiosity Equations

$$\forall \text{patches } i: \quad B_i = B_{ei} + \rho_i \sum_j F_{i \rightarrow j} B_j$$

$$\begin{bmatrix}
 1 - \rho_1 F_{1 \rightarrow 1} & -\rho_1 F_{1 \rightarrow 2} & \cdots & -\rho_1 F_{1 \rightarrow n} \\
 -\rho_2 F_{2 \rightarrow 1} & 1 - \rho_2 F_{2 \rightarrow 2} & \cdots & -\rho_2 F_{2 \rightarrow n} \\
 \cdots & \cdots & \cdots & \cdots \\
 -\rho_n F_{n \rightarrow 1} & -\rho_n F_{n \rightarrow 2} & \cdots & 1 - \rho_n F_{n \rightarrow n}
 \end{bmatrix}
 \begin{bmatrix}
 B_1 \\
 B_2 \\
 \cdots \\
 B_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 B_{e1} \\
 B_{e2} \\
 \cdots \\
 B_{en}
 \end{bmatrix}$$

Known
Unknown
Known

- Matrix Inversion to Solve for Radiosities.

# Iterative approaches

---

- Jacobi iteration
- Start with initial guess for energy distribution (light sources)
- Update radiosity/power of all patches based on the previous guess

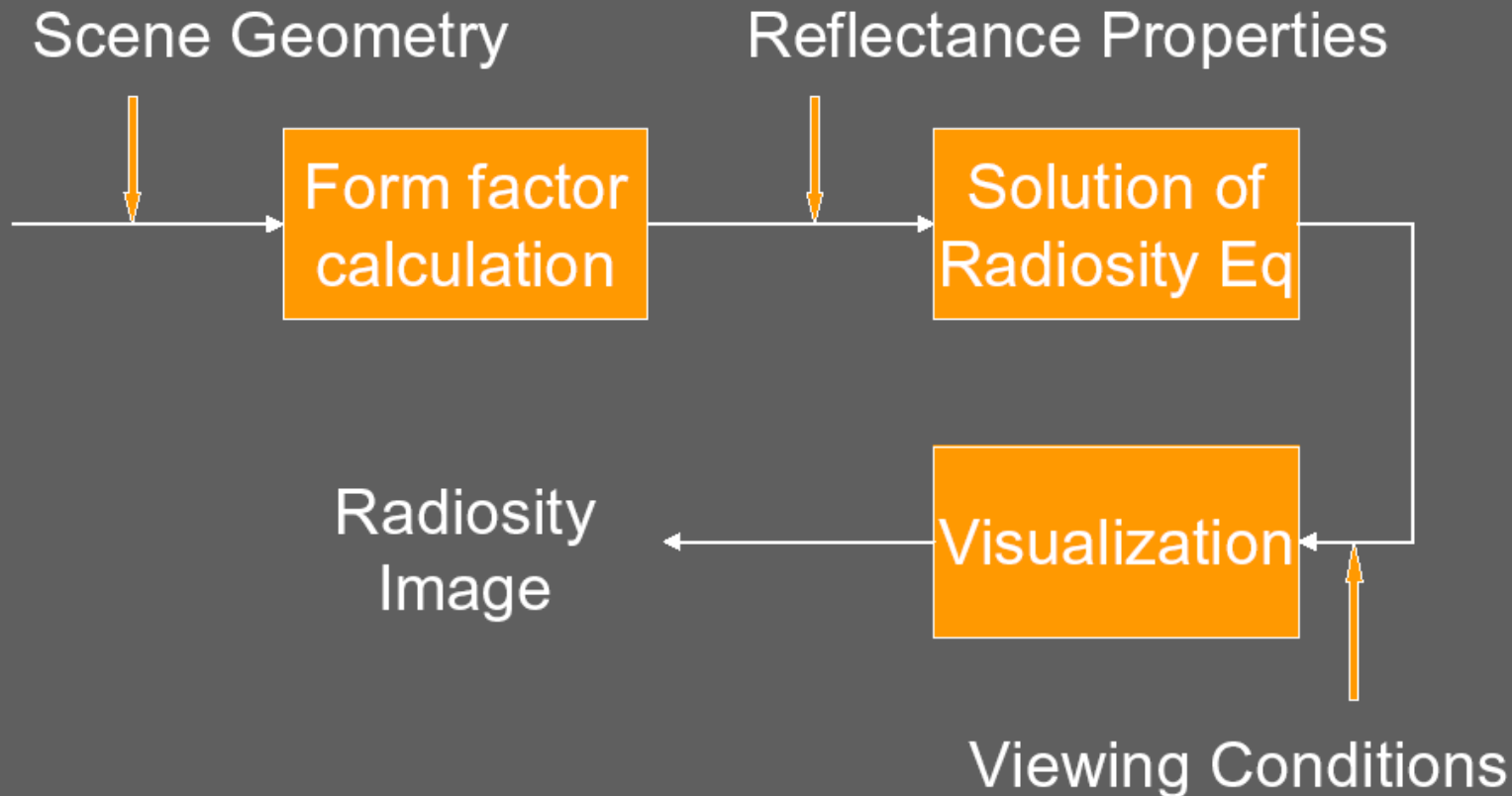
$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N B_j F(i \rightarrow j)$$

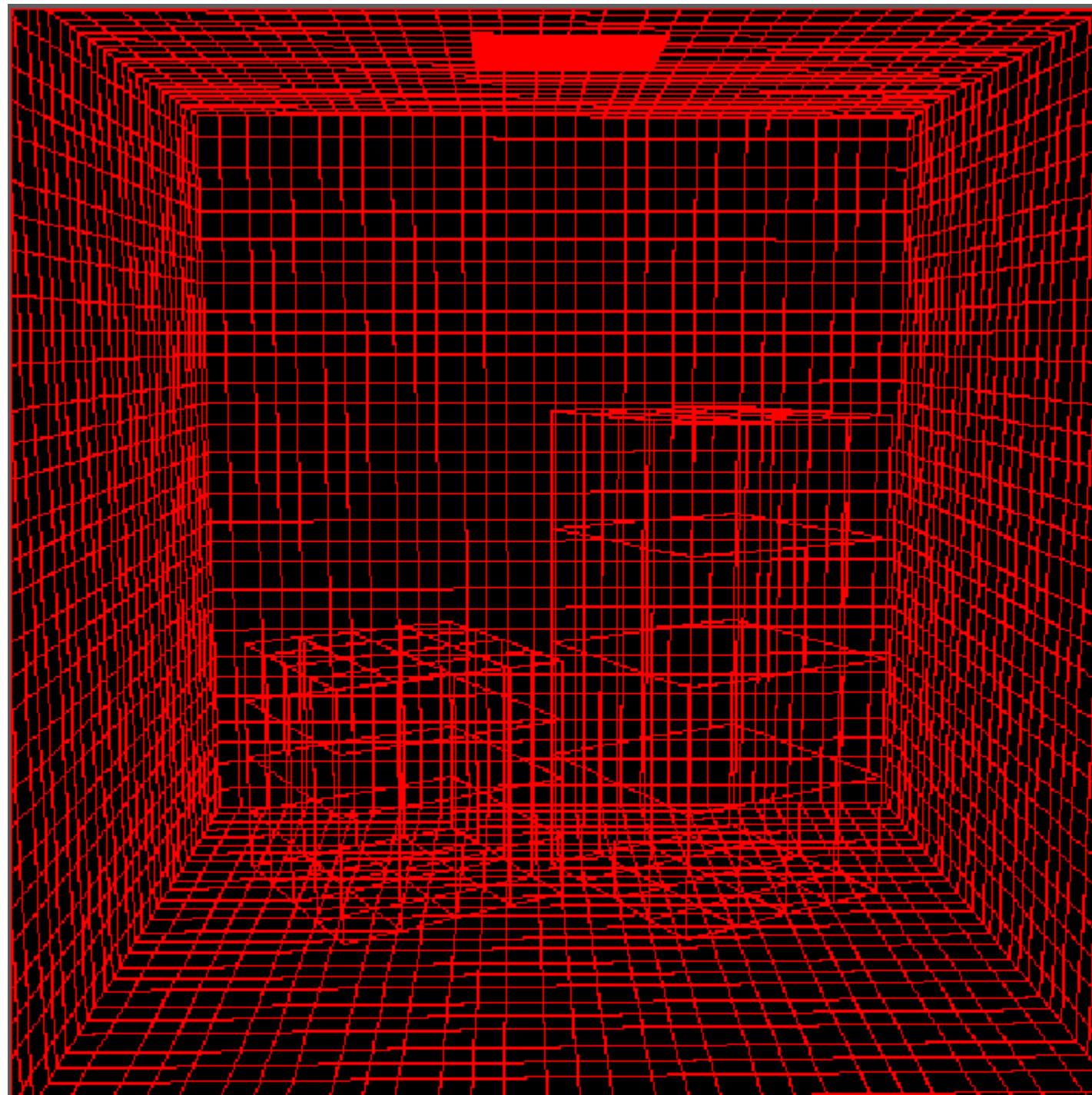
new value

old values

- Repeat until converged

# Radiosity "Pipeline"

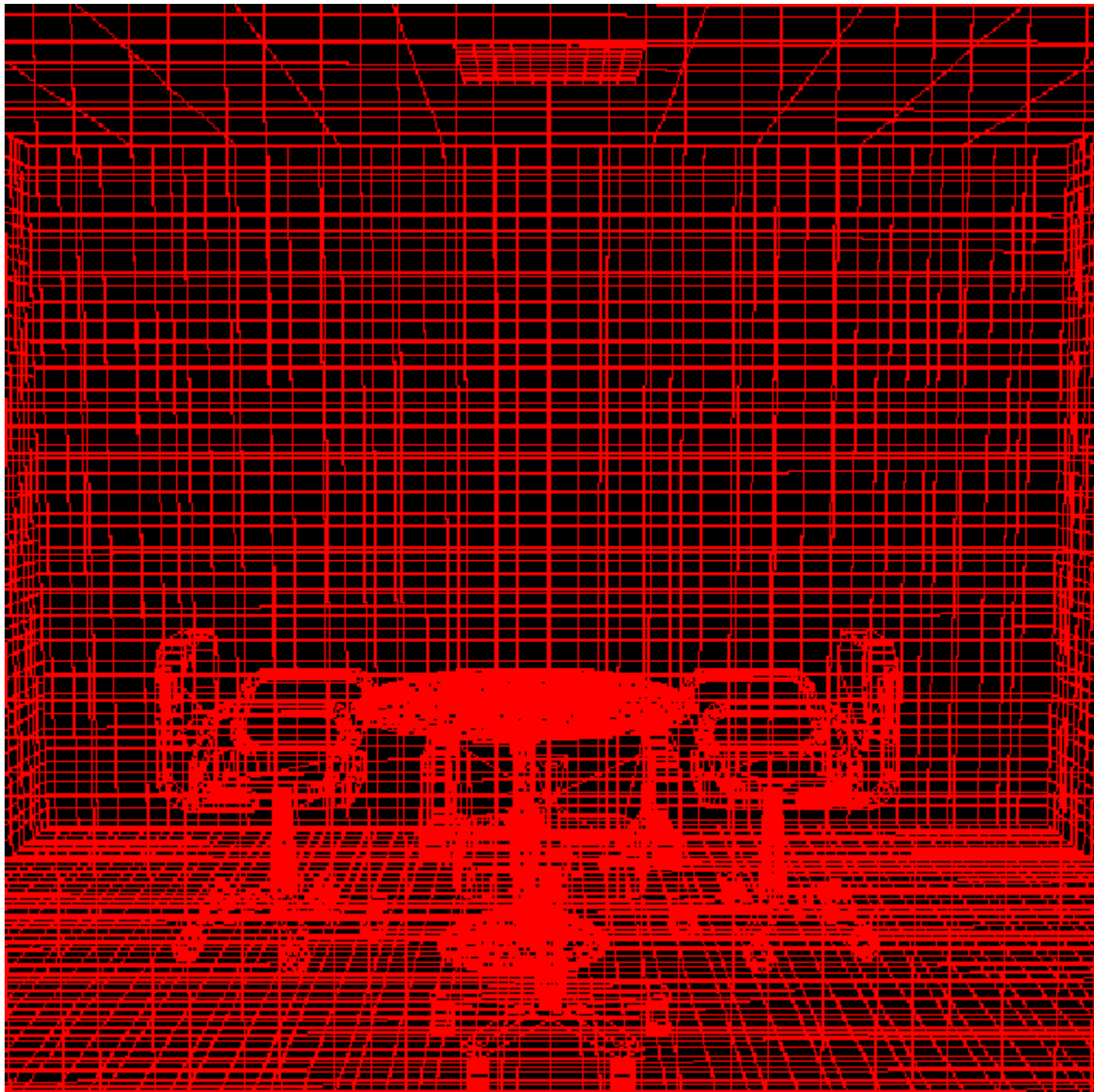




Wireframe



- Classical Approach
- No Interpolation



Wireframe



- Classical Approach

- Low Res

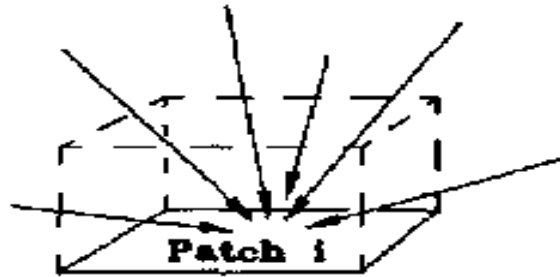


- Classical Approach
- High Res
- More accurate

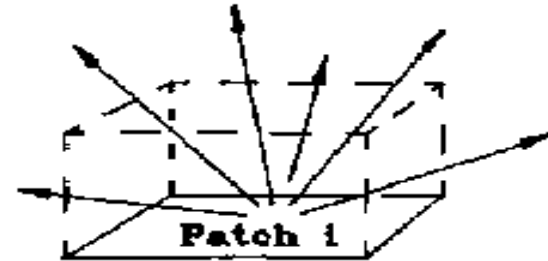


- Classical Approach
- High Res
- Interpolated

# Progressive Solution



**GATHERING**



**SHOOTING**

vs.

$$\begin{bmatrix} \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{x} \end{bmatrix} + \begin{bmatrix} \text{XXXXXXXXXX} \end{bmatrix} \begin{bmatrix} \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \end{bmatrix}$$

$$B_1 = E_1 + \sum_{j=1}^N (\rho_j R_{1j}) B_j$$

$$\begin{bmatrix} \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \end{bmatrix} = \begin{bmatrix} \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \end{bmatrix} + \begin{bmatrix} \mathbf{x} \end{bmatrix} \begin{bmatrix} \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \\ \text{XXXXXXXXXX} \end{bmatrix}$$

For all j:

$$B_j = B_j + B_1 (\rho_j E_{j1})$$

where:  $E_{j1} = R_{j1} A_1 / A_j$

**Figure 1: Gathering vs. Shooting**



## **PROGRESSIVE SOLUTION**

**The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.**

# Sample Scenes



# Sample Scenes



**From Cohen, Chen, Wallace and Greenberg 1988**

# Sample Scenes



# Sample Scenes



# Sample Scenes



# Radiosity

# Summary

---

**Classic radiosity = finite element method**

## **Assumptions**

- **Diffuse reflectance**
- **Usually polygonal surfaces**

## **Advantages**

- **Soft shadows and indirect lighting**
- **View independent solution**
- **Precompute for a set of light sources**
- **Useful for walkthroughs**

## Review: Local vs. Global Illumination

- Global illumination: **Ray tracing**
  - **Realistic specular** reflection/transmission
  - Simplified diffuse reflection\*
- Global illumination: **Radiosity**
  - **Realistic diffuse** reflection
  - Diffuse-only: No specular interaction\*



indirect

direct

both

# Radiosity Examples



<http://www.autodesk.com/us/lightscape/examples/html/index.htm>

# Raytracing Examples



<http://www.povray.org/>

# Raytracing Examples



# Radiosity Examples



<http://www.autodesk.com/us/lightscape/examples/html/index.htm>

# Image vs. Object Space

- Image space: **Ray tracing**
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)
- Object space: **Radiosity**
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL

# Two Pass Solution

- First Pass: Diffuse Interreflections

View independent, global diffuse illumination computed with radiosity.

- Second Pass: Specular Interreflections

View dependent, global specular illumination computed with ray-tracing.

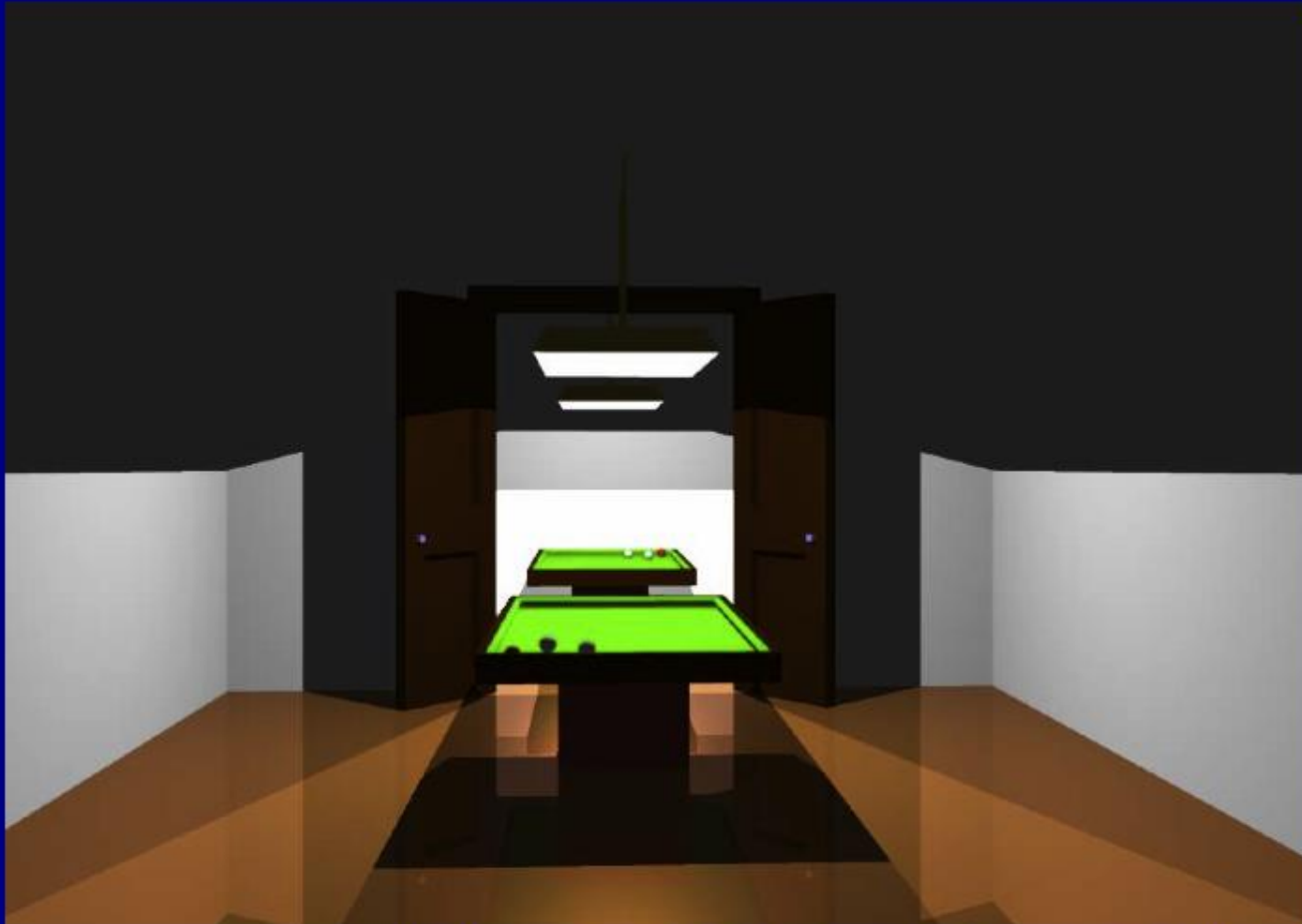
- Combine strengths of radiosity and ray-tracing.

# First Pass Result

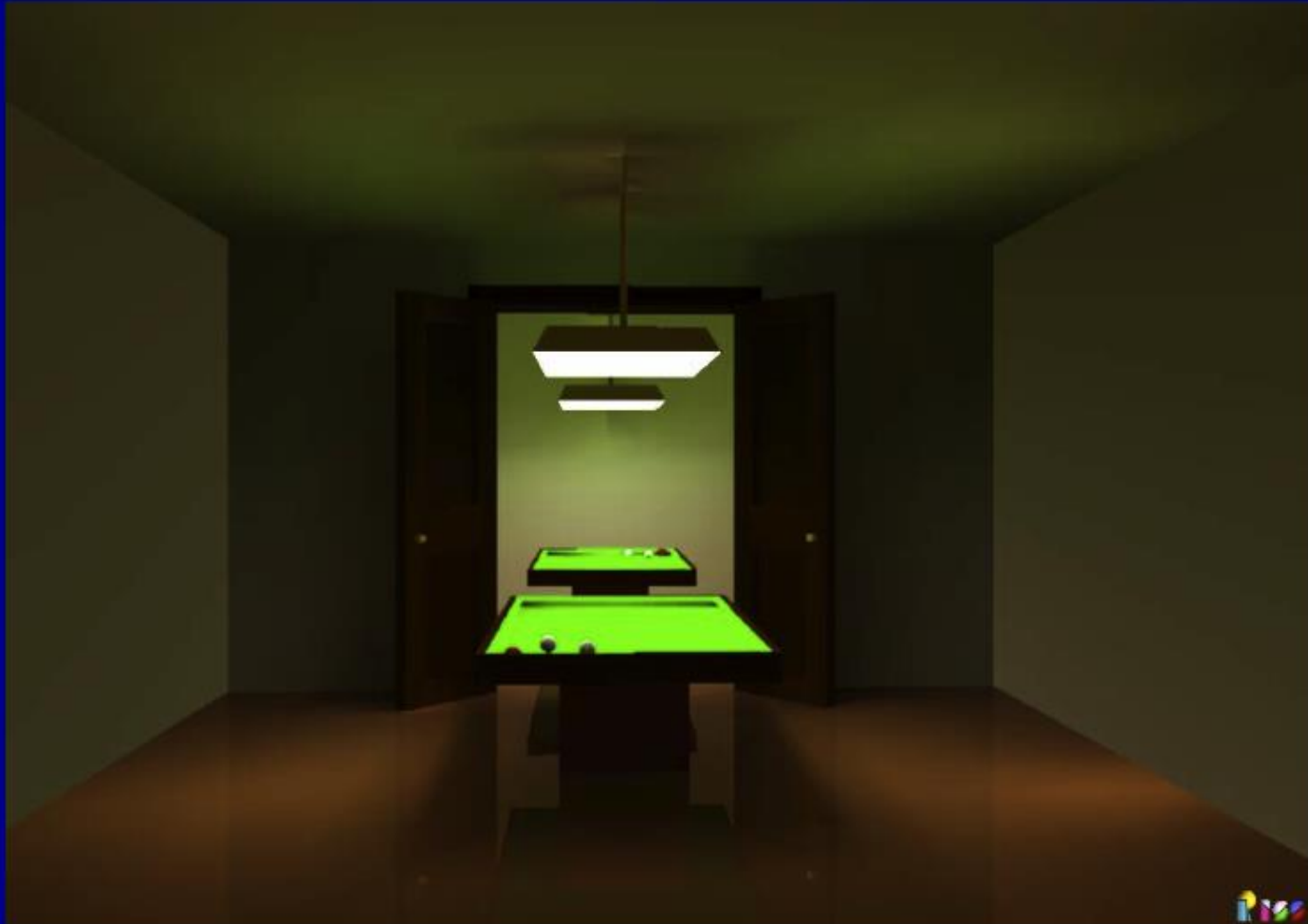


# Second Pass Result

(radiosity info. not yet used, just raytracing)



# Combined (Final) Result



# Practice Problems

List as many pros and cons as you can think of for pure ray tracing vs. radiosity approaches.

Give one scheme for combining ray tracing and radiosity to get some of the best of both worlds.

# Practice Problems

What is the property of radiosity that makes it view independent?

Explain in detail the shooting and gathering schemes for computing radiosity. Which do you think would be more efficient and why?

What lighting effects are easier to capture using a radiosity approach? Which are easier to capture using ray tracing?